Multiple parton interactions in QCD

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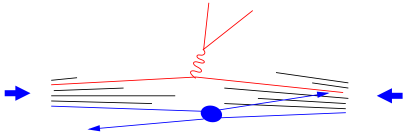
Multi-parton interactions

- generically take place in hadron-hadron collisions
  at high c.m. energy several interactions can be hard
- effects average out in sufficiently inclusive quantities
  but do affect final state properties



- estimated to be important for many processes at LHC
  see e.g. Procs. of the Workshop on HERA and the LHC, 2005 and 2008
- many studies:
  theory: Treleani et al; Artru, Mekhfi; Frankfurt, Strikman, Weiss; ...
  experiment: Tevatron, ...
  Monte Carlo generators: Sjostrand et al; Gieseke et al

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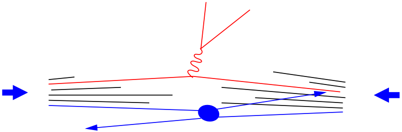
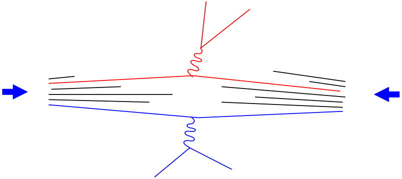


- so far no systematic derivation in QCD
- aim of this talk: discuss some steps in this direction

work in progress with A. Schäfer

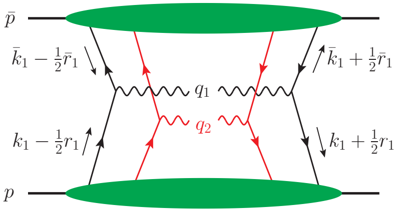
Theoretical framework

- require all interactions to have hard scale
  - predictive power from factorization and pert. theory
  - not applicable to purely soft phenomena (underlying event etc.)
- consider double Drell-Yan process



  - other color singlet particles $(W, Z, H, \dots)$ in full analogy
  - jet production highly relevant in practice
    but gluon exchange with spectator partons $\gg$ complicated
- keep transverse momentum of photons differential
  - since are interested in final-state details
  - need k_T dependent parton distributions

Basic structure: momentum



- colliding right- and left-moving partons: $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$

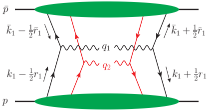
  $k_1^+ \pm \frac{1}{2} r_1^+ \approx q_1^+ \quad \Rightarrow r_1^+ \approx 0$

  $\Rightarrow$ longitudinal mom. fraction fixed by final state: $x_1 = \frac{k_1^+}{p^+} \approx \frac{q_1^+}{p^+}$

  same argument: $\bar{r}_1^- \approx 0$ and $\bar{k}_1^- \approx q_1^-$

- $\boldsymbol{k}_1 + \bar{\boldsymbol{k}}_1 \pm \frac{1}{2}(\boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1) = \boldsymbol{q}_1 \quad \Rightarrow \quad \boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$

  transverse parton momenta not the same
  on left and right of final-state cut



Basic structure: cross section

- cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \Bigg[\prod_{i=1}^{2} \hat{\sigma}_i \big(q_i^2 = x_i \bar{x}_i s \big) \Bigg]$$

$$\times \Bigg[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \Bigg] \int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_i)\, \bar{F}(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y}_i)$$

where $\boldsymbol{y}_1$ is Fourier conjugate to $\boldsymbol{r}_1$ and $\bar{\boldsymbol{r}}_1$

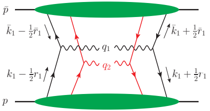
- have parton-level cross sections $\hat{\sigma}_i$ and multi-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^-\, d^2\boldsymbol{z}_2}{(2\pi)^3}\, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^-\, d^2\boldsymbol{z}_1}{(2\pi)^3}\, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times\, 2p^+ \int dy_1^-\, \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\, \bar{q}\big(y_1 - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y_1 + \tfrac{1}{2}z_1\big)|p\rangle_{\,z_i^+ = y_1^+ = 0}$$

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation involved

Basic structure: cross section



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- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\rightarrow$ collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$$

Basic structure: distributions

- k_T dependent distribution

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^-\, d^2\boldsymbol{z}_2}{(2\pi)^3}\, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^-\, d^2\boldsymbol{z}_1}{(2\pi)^3}\, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
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- collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+}$$
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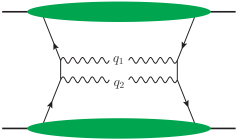
still $\boldsymbol{y}_1 \neq \boldsymbol{0}$ $\Rightarrow$ fields at different transverse positions
$\Rightarrow$ not a twist-four operator
but product of two twist-two operators

Power behavior: single versus double hard scattering

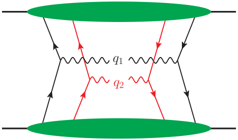
- from scattering formulae readily find

$$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\,d^2\boldsymbol{q}_1\;dx_2\,d\bar{x}_2\,d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2} \qquad Q^2 \sim q_i^2,\; \Lambda^2 \sim \mathrm{GeV}^2$$

for both



and



$\Rightarrow$ double scattering not power suppressed

Power behavior: single versus double hard scattering

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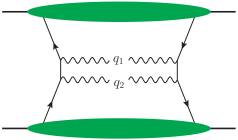
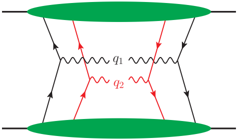
for both  and 

$\Rightarrow$ double scattering not power suppressed

- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

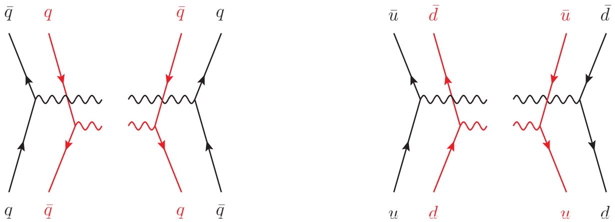
single: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim 1$$ since $\int d^2(\boldsymbol{q}_1+\boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1-\boldsymbol{q}_2) \sim Q^2$

double: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space in transv. momenta

Interference effects

- so far: distributions with operators $\bar{q}_2\, q_2\; \bar{q}_1\, q_1 \;\rightsquigarrow$ double parton densities
- but also have interference contributions (no probability interpretation)

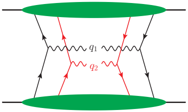


operator: $\bar{q}_2\, \bar{q}_2\; q_1\, q_1$

operator: $\bar{u}_2\, d_2\; \bar{d}_1\, u_1$

- must be included in cross section formula
  but is discarded in existing estimates

Spin structure



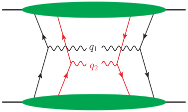
$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
$$\times\, 2p^+ \int dy_1^- \; \langle p| \bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\; \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1\, q(y_1 + \tfrac{1}{2}z_1)|p\rangle$$

- at leading twist for quarks: $\Gamma_i = \frac{1}{2}\gamma^+,\ \frac{1}{2}\gamma^+\gamma_5,\ \frac{1}{2}\sigma^{+\alpha}$
- spin correlations even in unpolarized target, e.g.

  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^{\uparrow} q_2^{\uparrow} + q_1^{\downarrow} q_2^{\downarrow} - q_1^{\uparrow} q_2^{\downarrow} - q_1^{\downarrow} q_2^{\uparrow}$$

  note: not suppressed by hard scattering in double Drell-Yan
- transverse spin correlations from $\Gamma_1 = \Gamma_2 = \frac{1}{2}\sigma^{+\alpha}$
  $\rightsquigarrow$ correlated decay planes of the two γ^*
- to parameterize $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$ for unpol. target: 30 scalar functions
  still 8 functions to parameterize $F(x_i, \boldsymbol{y}_1)$
- expect spin effects to decrease for small x_1, x_2
  but will see an exception later

Color structure



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- operators $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ can couple to color singlet or octet:

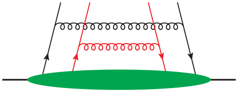
$$F_1 \to (\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1)$$
$$F_8 \to (\bar{q}_2 t^a q_2)\,(\bar{q}_1 t^a q_1) = \tfrac{1}{2}(\bar{q}_2 \mathbb{1} q_1)\,(\bar{q}_1 \mathbb{1} q_2) - \tfrac{1}{2N_c}\,(\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1)$$

- in double Drell-Yan have equal weight: $F_1 \bar{F}_1 + F_8 \bar{F}_8$
- gauge invariance $\rightsquigarrow$ Wilson lines between field pairs in color singlet
  after $\int d^2\boldsymbol{k}_i$ still complicated Wilson line structure for $(\bar{q}_2 \mathbb{1} q_1)\,(\bar{q}_1 \mathbb{1} q_2)$

spin and color correlations discussed by M. Mekhfi, PRD32 (1985)
but apparently not followed up in literature

High q_T

- consider region $\Lambda \ll q_T \ll Q$, where $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n = hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- case (a): $|\boldsymbol{r}_1| \sim \Lambda$, i.e. $|\boldsymbol{y}| \sim 1/\Lambda$ of hadronic size
  $\rightsquigarrow$ independent hard scatters for pair 1 and 2



- color factors: C_F^2 for singlet F_1
  $\left(\frac{1}{2N_c}\right)^2$ for octet F_8
- singlet also favored in gluon sector

power behavior: $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \sim \frac{\Lambda^2}{q_T^4}$ and $s \frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{\Lambda^2}{Q^2 q_T^4}$

.

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  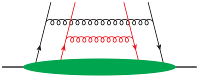

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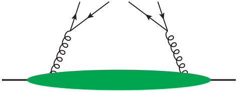
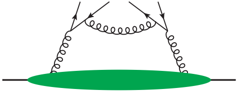
- case (b): $|\boldsymbol{r}_1| \sim |\boldsymbol{q}_i|$, i.e. $|\boldsymbol{y}| \ll 1/\Lambda$ small

  $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \sim \frac{1}{q_T^2}$ and $s \frac{d\sigma}{d^2\boldsymbol{q}_1 \, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2 q_T^2}$

  equal contributions to $\Gamma_1 = \Gamma_2 = \frac{1}{2}\gamma^+$ and $\Gamma_1 = \Gamma_2 = \frac{1}{2}\gamma^+\gamma_5$
- case (a) is power suppressed by $\frac{\Lambda^2}{q_T^2}$, but has small-x enhancement

Evolution of collinear distributions

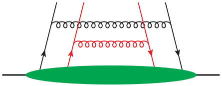
consider only color singlet combination F_1, situation for F_8 more complicated

$$F(x_i, \boldsymbol{y}_1) = \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+}$$
$$\times\, 2p^+ \int dy_1^- \,\langle p|\bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\, \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1\, q(y_1 + \tfrac{1}{2}z_1)|p\rangle \Big|_{z_i^+ = y_1^+ = 0,\; \boldsymbol{z}_i = \boldsymbol{0}}$$

- $F(x_i, \boldsymbol{y}_1)$ for $\boldsymbol{y}_1 \neq \boldsymbol{0}$:

  separate DGLAP evolution for pair 1 and 2

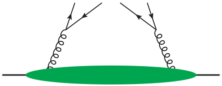
  $$\frac{d}{d\log\mu} F(x_i, \boldsymbol{y}_1) = P \otimes_{x_1} F + P \otimes_{x_2} F$$



- $\int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{y}_1)$:

  extra term from $2 \to 4$ parton transition

  recent study by Gaunt and Stirling



- $\rightsquigarrow$ consistency problem for frequently made ansatz

  $$F(x_i, \boldsymbol{y}_1; \mu) = G(\boldsymbol{y}_1) \left[\int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{y}_1) \right]_\mu$$

  if on r.h.s. include $2 \to 4$ term in evolution

Sudakov factors

- cross section differential in $\boldsymbol{q}_i$ contains Sudakov logarithms
- can adapt Collins-Soper-Sterman formalism

  (originally developed for single Drell-Yan and similar proc's)

  $\rightsquigarrow$ include and resum Sudakov logs in k_T dependent parton distr's

results of analysis (simplified):

- for $q_T \sim \Lambda$
  - Sudakov factors mix singlet and octet distr's F_1 and F_8
  - generically Sudakov factors of same size for singlet and octet
- for $q_T \gg \Lambda$ and $\boldsymbol{y}_1 \sim 1/\Lambda$
  - singlet F_1 decouples from octet F_8
  - octet distr'n has extra suppression by factor $\sim \exp\Big[\text{const.}\, \alpha_s \log \frac{|\boldsymbol{z}_i|}{|\boldsymbol{y}_1|}\Big]$ with $|\boldsymbol{z}_i|/|\boldsymbol{y}_1| \sim \Lambda/q_T$

Approximation by single-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
$$\times\, 2p^+ \int dy_1^- \, \langle p|\bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\, \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1\, q(y_1 + \tfrac{1}{2}z_1)|p\rangle$$

- between $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$ then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions

$$\langle p|\bar{q}_2\, q_2\;\bar{q}_1\, q_1|p\rangle \approx \sum_{p'} \langle p|\bar{q}_2\, q_2|p'\rangle\, \langle p'|\bar{q}_1\, q_1|p\rangle$$

- transverse momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ differ
  $\rightsquigarrow$ generalized parton distributions (GPDs)
- in physical terms: neglect correlations between parton 1 and 2

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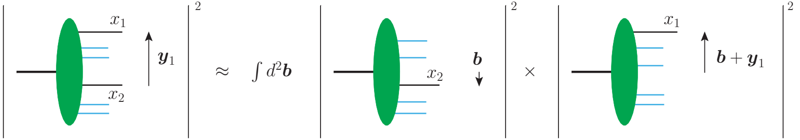
$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times \, 2p^+ \int dy_1^- \, \langle p| \bar{q}(-\tfrac{1}{2}z_2) \Gamma_2 \, q(\tfrac{1}{2}z_2) \, \bar{q}(y_1 - \tfrac{1}{2}z_1) \Gamma_1 \, q(y_1 + \tfrac{1}{2}z_1) |p\rangle$$

- between $\bar{q}_2 \, q_2$ and $\bar{q}_1 \, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$ then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions
- especially simple for collinear distributions:

$$F(x_i, \boldsymbol{y}_1) \; \approx \; \int d^2\boldsymbol{b} \; f(x_2, \boldsymbol{b}) \, f(x_1, \boldsymbol{b} + \boldsymbol{y}_1)$$

with $f(x, \boldsymbol{b}) =$ impact-parameter dependent distribution (M. Burkardt '02)

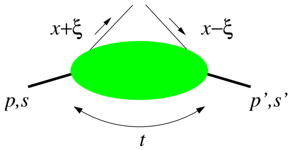


Interlude: some reminders about GPDs

- defined by matrix elements of
  - same quark-gluon operators as usual PDFs
  - between hadron states with different momentum (spin, flavor, ...)
- e.g. collinear distribution for unpolarized quarks:

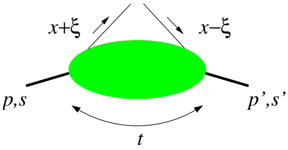
$$\int \frac{dz^-}{2\pi}\, e^{ixz^-(p+p')^+/2} \langle p', s' |\, \bar{q}(-\tfrac{1}{2}z)\, \tfrac{1}{2}\gamma^+ q(\tfrac{1}{2}z)\, | p, s\rangle \Big|_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$$

$$= H^q(x,\xi,t)\, \bar{u}(p',s')\gamma^+ u(p,s) + E^q(x,\xi,t)\, \bar{u}(p',s')\, \frac{i\sigma^{+\alpha}}{2m_p}(p'-p)_\alpha\, u(p,s)$$



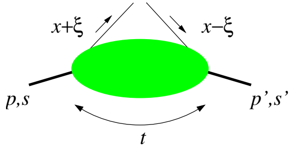
H: proton helicity conserving, E: proton helicity flip

GPDs: longitudinal momentum



- longitudinal momentum difference: ξ = skewness
- nonzero skewness $\rightsquigarrow$ modified evolution equations
  fully known at NLO, largely at NNLO
- no model independent way to get $H(x,\xi,0)$ from usual PDFs $H(x,0,0)$
- in physical processes:
  - ξ = fixed by external kinematics $\rightsquigarrow$ observable
  - x = loop variable $\rightsquigarrow$ appears under integral

GPDs: transverse momentum



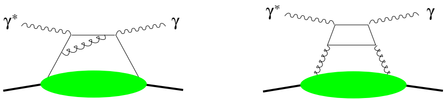
- at given ξ can trade Mandelstam t for transverse mom. transfer $\boldsymbol{\Delta}$

$$H(x,\xi,\boldsymbol{\Delta}) \xrightarrow{\text{Fourier trf.}} H(x,\xi,\boldsymbol{b})$$

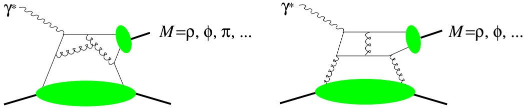
  $\boldsymbol{b}$ = impact parameter = distance of struck parton from proton center
- for $\xi = 0$ have density interpretation: $H(x,0,\boldsymbol{b}) = f(x,\boldsymbol{b})$
- important difference:
  - distance $\boldsymbol{y}_1$ in multi-parton distr's appears under integral
    $$d\sigma = \ldots \int d^2\boldsymbol{y}_1\, F(x_i,\boldsymbol{k}_i,\boldsymbol{y}_i)\, \bar{F}(\bar{x}_i,\bar{\boldsymbol{k}}_i,\boldsymbol{y}_i)$$
  - $\boldsymbol{\Delta}$ in GPDs experimentally observable

Processes involving GPDs: lepton-proton collisions

- deeply virtual Compton scattering
  best control over theory, full NLO, partial NNLO D. Müller et al.



- meson production: large Q^2 or heavy quarks $(J/\Psi, \Upsilon)$
  help for separating quark flavors and gluons
  radiative and power corrections important, even for $Q^2 \sim 10\,\mathrm{GeV}^2$



- data: HERA Collider (small x), HERMES and Jefferson Lab ($x \gtrsim 0.1$)
  hopefully soon: COMPASS ($x \sim 10^{-2}$), γp collisions at LHC

Multi-parton distributions and non-forward matrix elements

- slide 19 was oversimplified
  when insert proton states $|p\rangle\langle p|$ have two spin combinations:

$$\langle p_\uparrow|\bar{q}_2 q_2\ \bar{q}_1 q_1|p_\uparrow\rangle$$
$$\approx \sum_{p'} \Big[\langle p_\uparrow|\bar{q}_2 q_2|p'_\uparrow\rangle\, \langle p'_\uparrow|\bar{q}_1 q_1|p_\uparrow\rangle + \langle p_\uparrow|\bar{q}_2 q_2|p'_\downarrow\rangle\, \langle p'_\downarrow|\bar{q}_1 q_1|p_\uparrow\rangle \Big]$$

  includes proton helicity flip: $H(x_1,\ldots)H(x_2,\ldots) + E(x_1,\ldots)E(x_2,\ldots)$

- can also insert $\sum_X |X\rangle\langle X|$ in distributions describing interference, e.g.
  - fermion number: $\bar{q}_2\,\bar{q}_2\ q_1\, q_1 = \bar{q}_2\, q_1\ \bar{q}_2\, q_1$
  - color: $(\bar{q}_2 t^a q_2)\ (\bar{q}_1 t^a q_1) = \frac{1}{2}(\bar{q}_2 \mathbb{1} q_1)\ (\bar{q}_1 \mathbb{1} q_2) - \frac{1}{6}\,(\bar{q}_2 \mathbb{1} q_2)\ (\bar{q}_1 \mathbb{1} q_1)$

  where indices 1, 2 indicate momentum fraction x_1 or x_2

  get single-quark matrix elements of form $\langle p'|\bar{q}_2\, q_1|p\rangle$
  $\rightsquigarrow$ different longitudinal momenta of partons: nonzero skewness
  $\rightsquigarrow$ k_T dependent GPDs even for $\int d^2\boldsymbol{k}_1\ \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$

Summary

- multiple hard interactions not power suppressed
  for cross section differential in $|\boldsymbol{q}_i| \ll Q$
- nontrivial spin and color structure
  interference in fermion number and quark flavor
  size of these effects presently unknown
- some simplification for transv. mom. $|\boldsymbol{q}_i| \gg \Lambda$
  - collinear distributions as input
  - enhancement of color singlet combinations

  further studies needed
- need multi-parton distr's depending on transverse distance
  between partons

challenge: identify corrections to oversimplified approach
while retaining predictive power

Summary

under assumptions can relate multi-parton distr's to generalized parton distributions, which

- contain experimentally accessible information about transverse distribution of partons and its correlation with long. momenta
- offer a way to estimate size of interference-type multi-parton distr's, which are otherwise unknown
- allow study of general patterns such as correlations between x and b

GPDs may provide one piece of input for better understanding multiple interactions