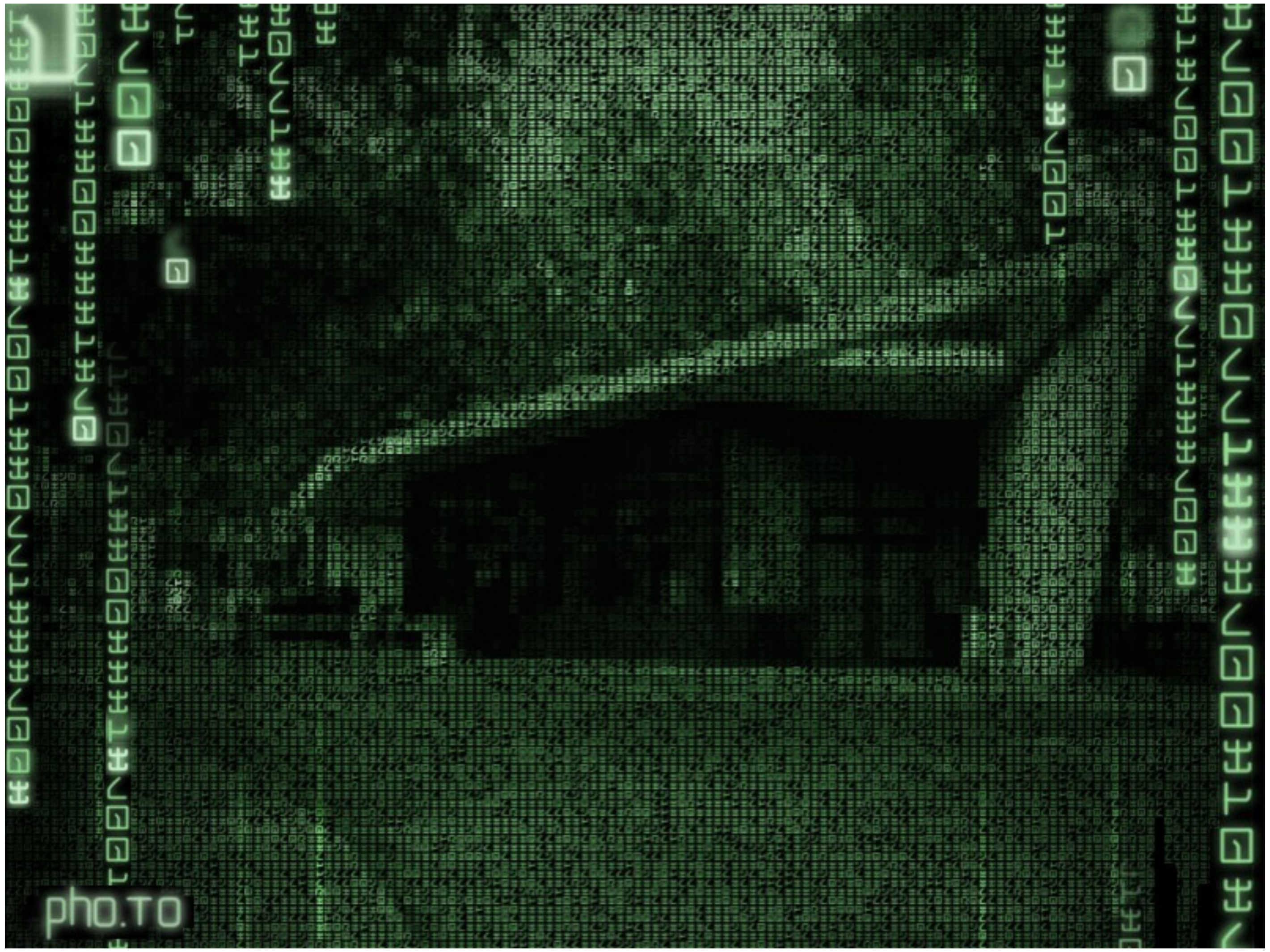
Are we living in the Matrix?

David Tong



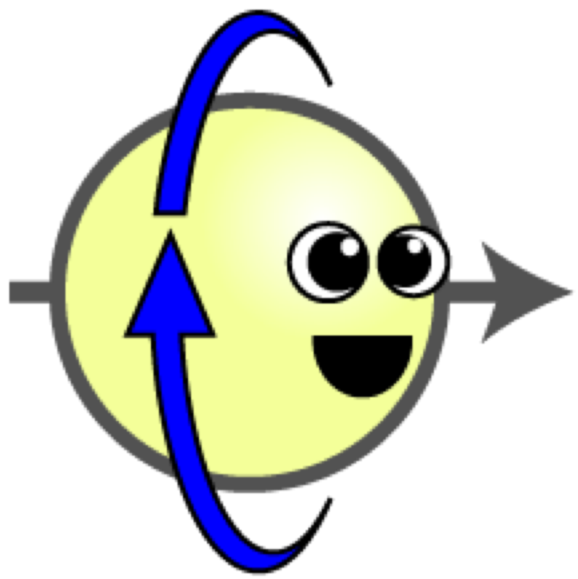
THE ROYAL SOCIETY

SIMONS FOUNDATION

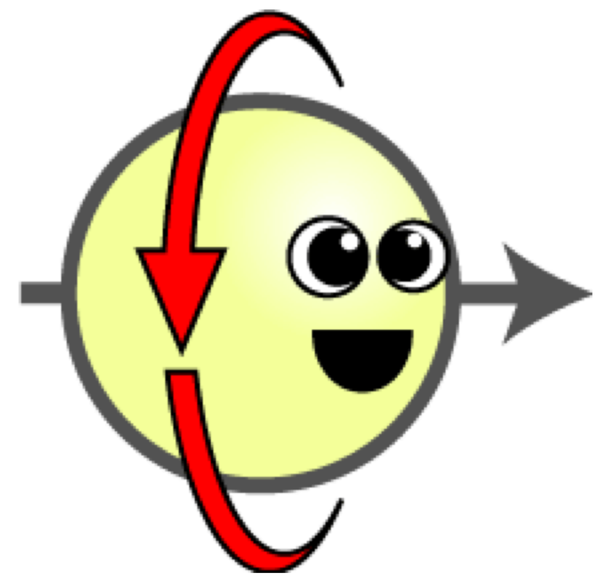
Science & Technology Facilities Council

Parity Violation

Chiral Fermions



left-handed fermion

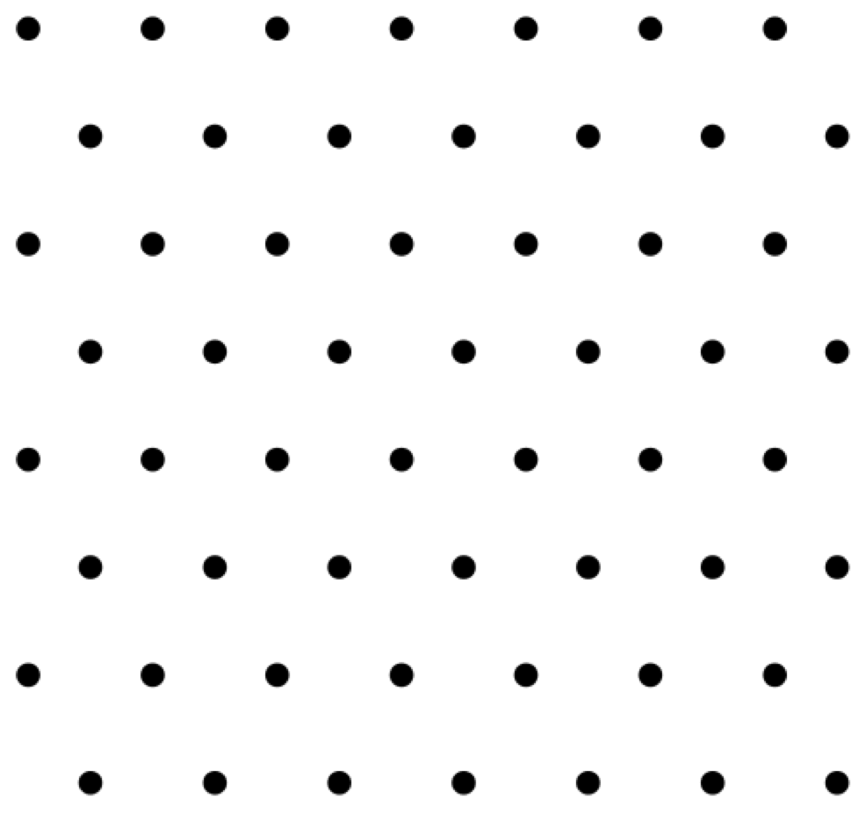


right-handed fermion

Parity violation ⇒ these experience different forces

Trouble Simulating Chiral Fermions

Replace continuous objects with discrete approximations

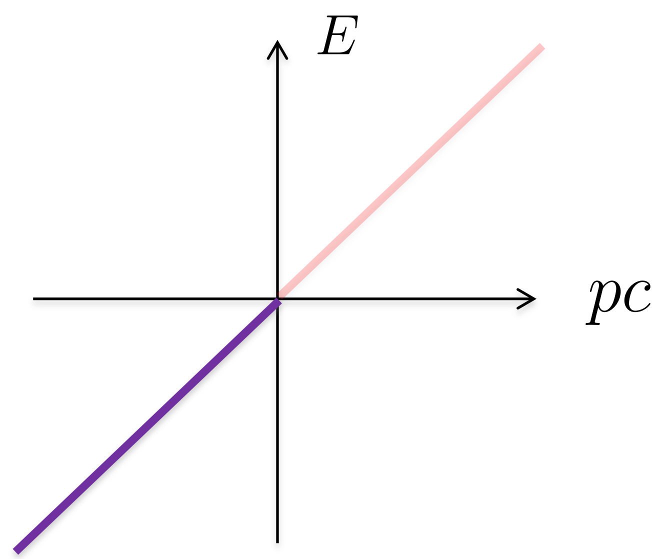


Replace the continuum of space(time) with a discrete lattice

Chiral Fermions on the Lattice

Consider a particle in d=1+1 dimensions.

$$E = +pc$$

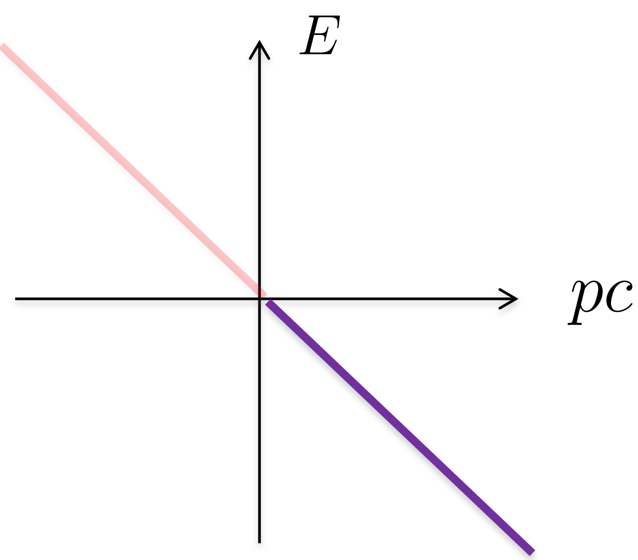


This is right-moving fermion $v = \frac{dE}{dp}$

Chiral Fermions on the Lattice

Consider a particle in d=1+1 dimensions.

$$E = -pc$$



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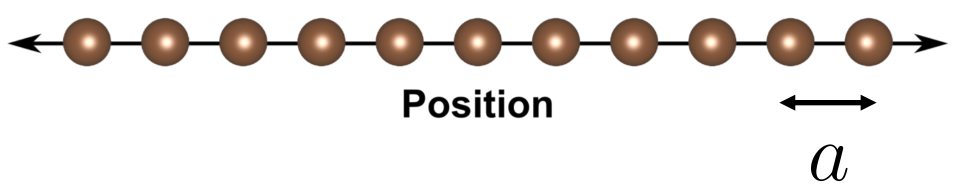
Chiral Fermions on the Lattice

But a simple fact from quantum mechanics

Space is discrete



$\Longrightarrow$

Momentum is periodic

$$p \in \left[-\frac{\hbar\pi}{a}, \frac{\hbar\pi}{a} \right)$$

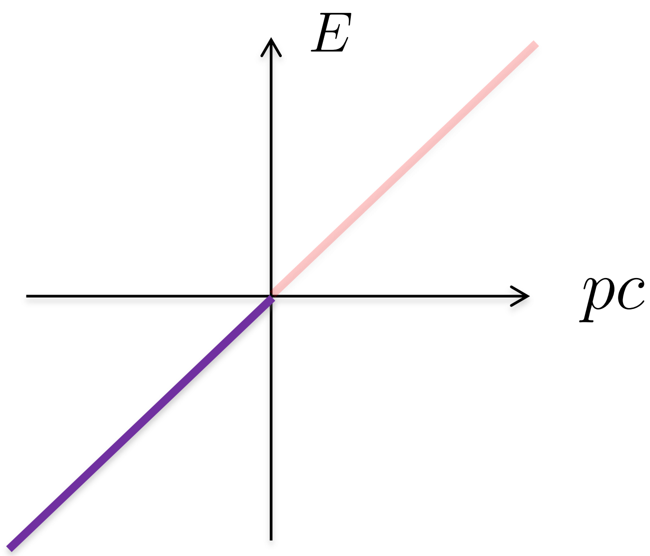
This is the *Brillouin zone*

A Right-Moving Fermion on the Lattice

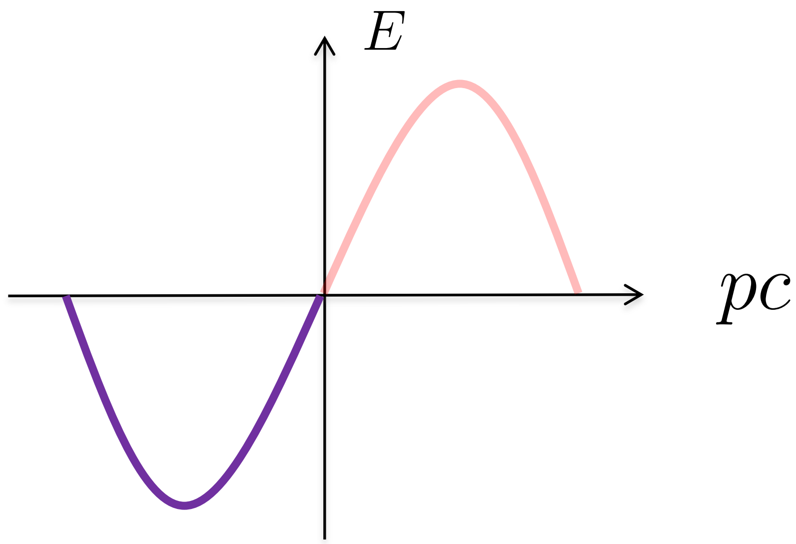
Start with a right-moving fermion.

On a lattice, energy must be a continuous, periodic function of momentum.

<u>Continuum</u> $E = +pc$

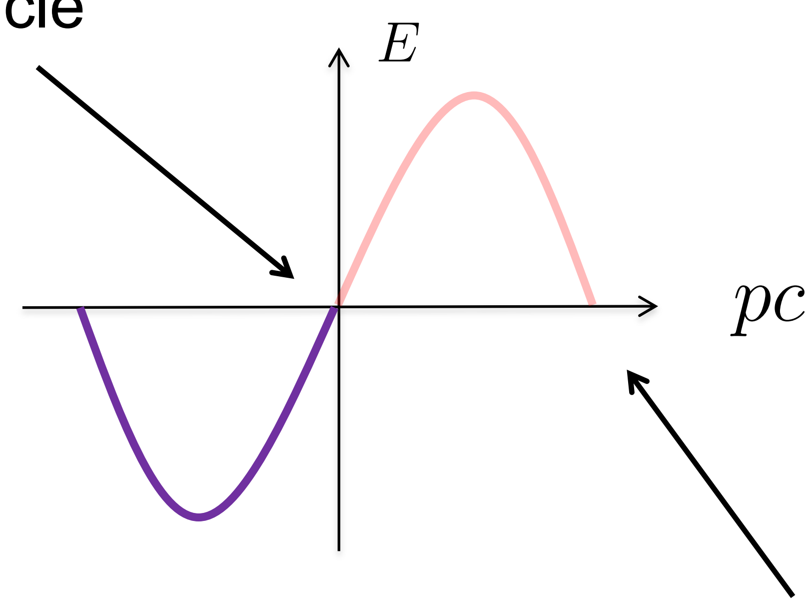


<u>Lattice</u> $E \sim \sin\left(\frac{pa}{2\hbar}\right)$



The Nielsen-Ninomiya Theorem

Start from right-moving particle



The lattice generates a new, low-energy state.
This is a left-moving particle.

- Can’t put a single right-moving fermion on a lattice.
- Moreover, left- and right-moving fermions experience same forces.

Nielsen and Ninomiya ‘81

Quantum Anomalies

Most chiral theories do not make sense

Bell and Jackiw ’69; Adler 69;
Bouchiat, Illiopoulos and Meyer ‘72; Georgi and Glashow ‘72; Gross and Jackiw ‘72

Quantum Anomalies

Consistency conditions need to be obeyed. Either...

- Left-handed and right-handed fermions feel the same force

Or

- More delicate balancing act between left-handed and right-handed

Only this second option violates parity

An Example: The Standard Model

strong → weak → hypercharge

$$G = SU(3) \times SU(2) \times U(1)$$

Take one generation of quarks and leptons to have usual properties under strong and weak force but arbitrary *integer* charges under hypercharge

After a change of variables, consistency conditions (ignoring gravity) require

$$X^3 + Y^3 = Z^3$$

with X, Y and Z integers. Unique solution, e.g $1^3 + 0^3 = 1^3$ gives observed charges.

Lohitsiri and Tong ‘19

Quantum Anomalies

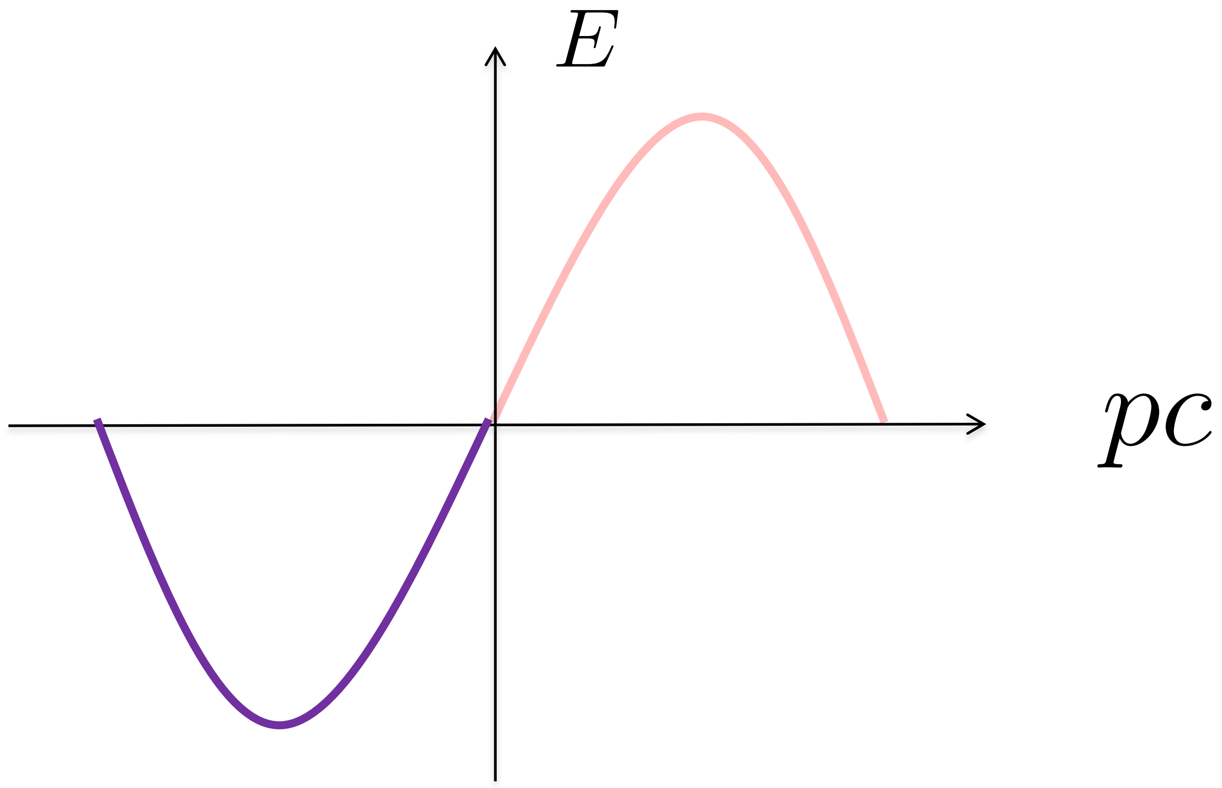
Most chiral theories do not make sense. We must have either

The lattice chooses this option

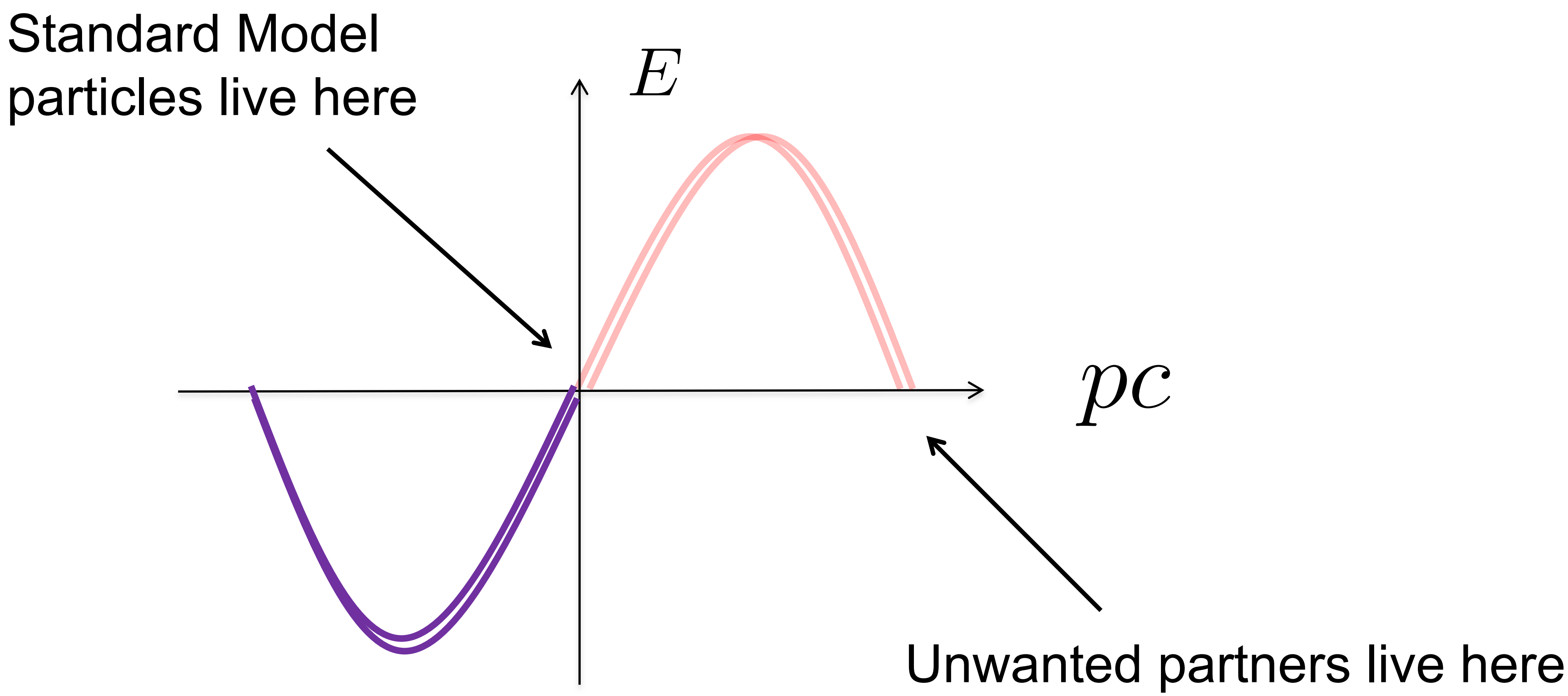
- Left-handed and right-handed fermions feel the same force
- More delicate balancing act between left-handed and right-handed

Nature chose this option

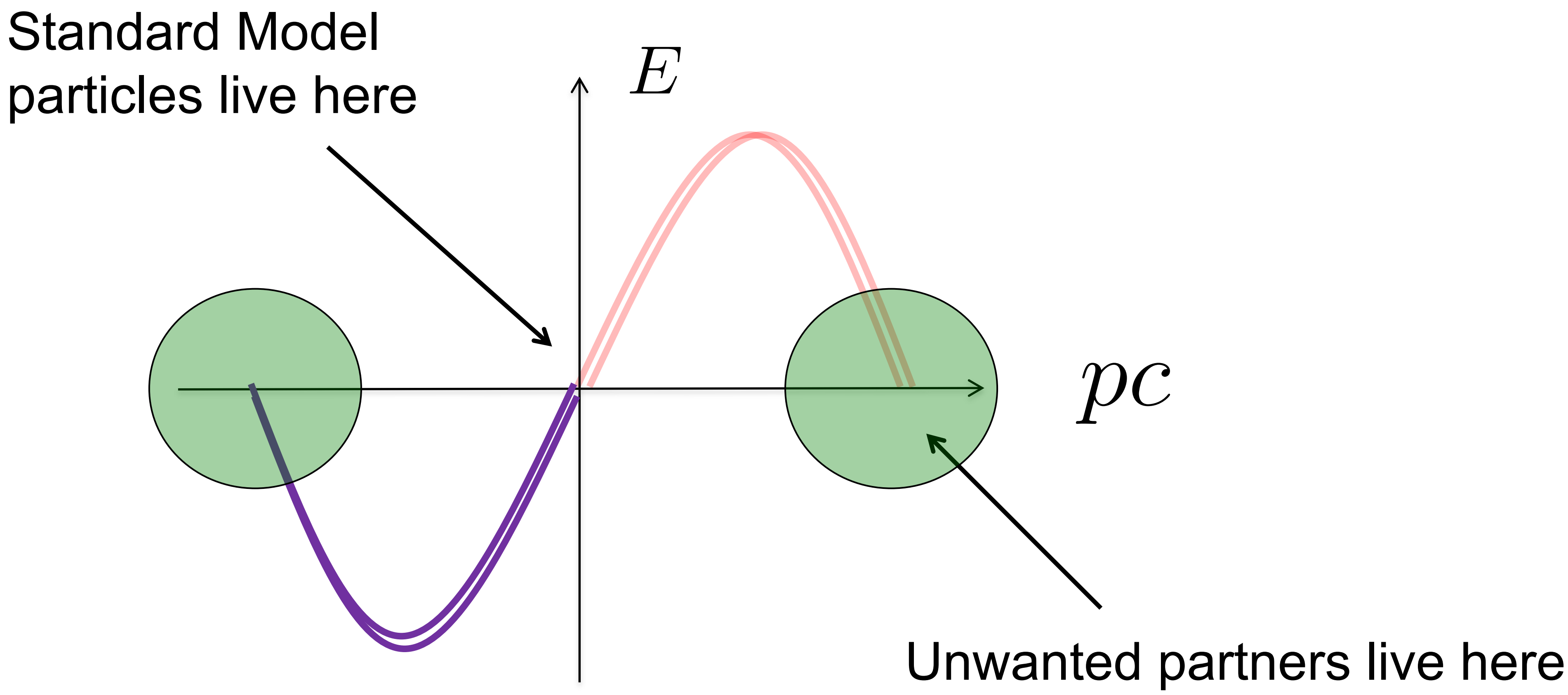
Evading the Nielsen-Ninomiya Theorem



Evading the Nielsen-Ninomiya Theorem



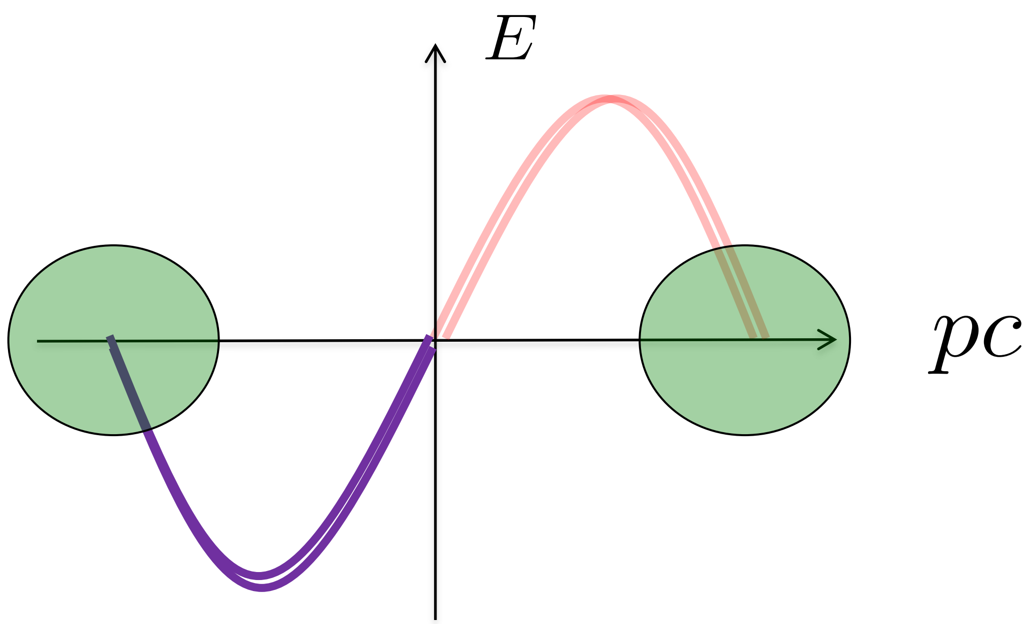
Evading the Nielsen-Ninomiya Theorem



Turn on some strong interactions that give the unwanted partners a large mass

Difficulty: We want to give mass to *chiral* fermions without breaking any symmetry!

Giving Chiral Fermions a Mass: A First Attempt



Eichten and Preskill '86
+ many others since

An old idea: use four-fermion couplings to give a mass to the partners

$$\mathcal{L} = \psi\psi\psi\psi$$

Irrelevant, so could never work on the continuum. But it *might* work on the lattice.

Gapping Chiral Fermions

Goal: Find a way to give a mass to chiral fermion without breaking symmetries

e.g. give a mass to one generation of the Standard Model
without breaking electroweak symmetry

Razamat and Tong '20

How to Gap Chiral Fermions

The Rules of the Game

- Start from free massless fermions realising a non-anomalous chiral symmetry G

Add extra degrees of freedom and flow to the IR. The goal is to gap everything while preserving G.

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- Scalars.
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- Fermions.
  - If these are charged under *G,* they must come in left/right conjugate pairs.
- Gauge Fields.
  - These gauge a different symmetry *H* providing
    - $[H,G] = 0$
    - There are no mixed anomalies with G.
    - There are scalars that allow a phase in which *H* is Higgsed.

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

(left-handed)^c		right-handed		
leptons	quarks	electron	up quark	down quark
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$

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$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
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- Add three further pairs of fermions

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- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry

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- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry
- Supersymmetrize.
  - Add scalar superpartners for all fermions, and a *H* = SU(2) gaugino

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
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$(\mathbf{1}, \mathbf{2})_{-3}$	$(\bar{\mathbf{3}}, \mathbf{2})_{+1}$	$(\mathbf{1}, \mathbf{1})_{+6}$	$(\mathbf{3}, \mathbf{1})_{-4}$	$(\mathbf{3}, \mathbf{1})_{+2}$	$(\mathbf{1}, \mathbf{1})_{0}$
$(\mathbf{1}, \mathbf{2})_{-3}$				$(\mathbf{3}, \mathbf{1})_{+2}$	$(\mathbf{1}, \mathbf{1})_{0}$
$L' \longrightarrow (\mathbf{1}, \mathbf{2})_{+3}$				$D' \longrightarrow (\bar{\mathbf{3}}, \mathbf{1})_{-2}$	

- The H = SU(2) gauge theory is coupled to six doublets.
- This confines *without* breaking the global symmetry.

Seiberg '94

- The low-energy physics consists of 15 free mesons:

$$\epsilon_{ab} L^a L^b \qquad \epsilon_{ijk} D^i D^j \qquad L^a D^i \qquad L^a N \qquad D^i N$$

An Example: The Standard Model

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Razamat and Tong '20

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If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q^b_i + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}^i_b Q^b_i + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
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$(\mathbf{1}$					$\mathbf{1})_0$
$(\mathbf{1}$					$\mathbf{1})_0$

The theory is gapped while preserving G = SU(3) x SU(2) x U(1)!

$L' \longrightarrow (\mathbf{1}, \mathbf{2})_{+3}$ $\quad D' \longrightarrow (\bar{\mathbf{3}}, \mathbf{1})_{-2}$

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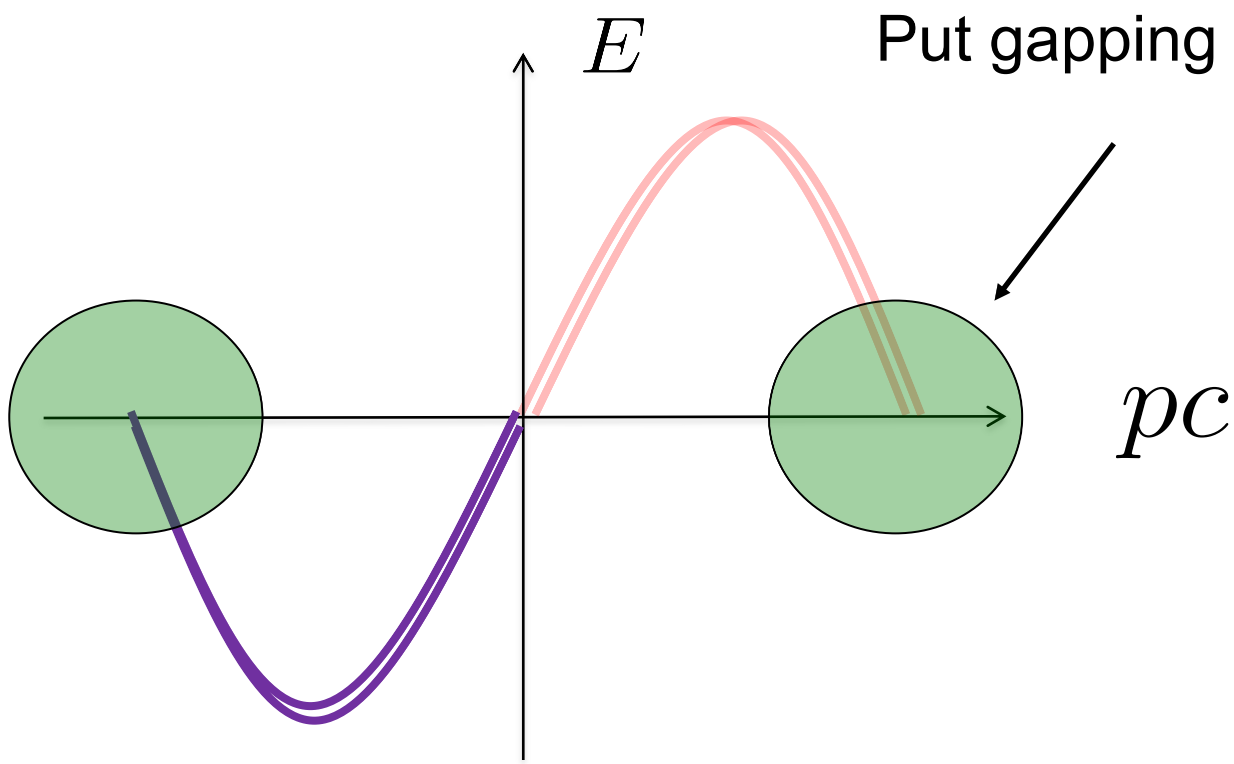
The Ultimate Goal

Implement this new way to give chiral fermions a mass on the lattice...



...and find a way to simulate the Standard Model on a computer

Thank you for your attention