Positive moments for scattering amplitudes

from

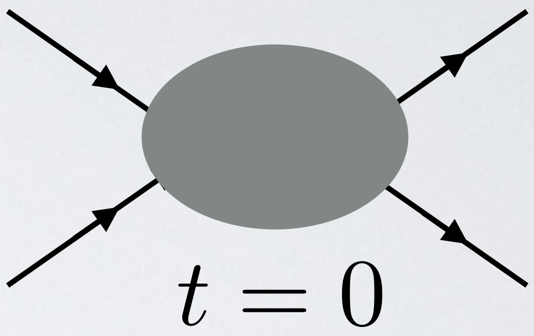
Bellazzini, Elias Miro, Rattazzi, Riembau, Riva
2011.00037 [hep-th]

(see also Tolley, Wang, Zhou, 2011.02400 [hep-th], and probably others)

Quentin Bonnefoy, 12/11/20, Journal club of the DESY theory group

Positivity bounds : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]



$\mathcal{A}(s)$

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Ingredients :

- **analyticity**
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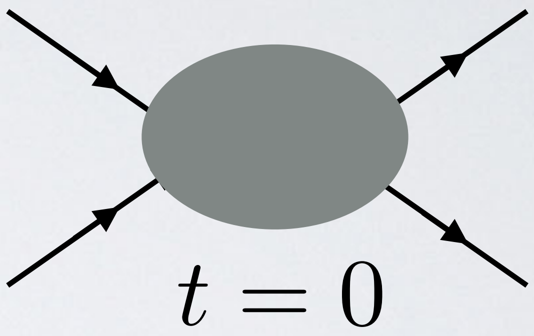

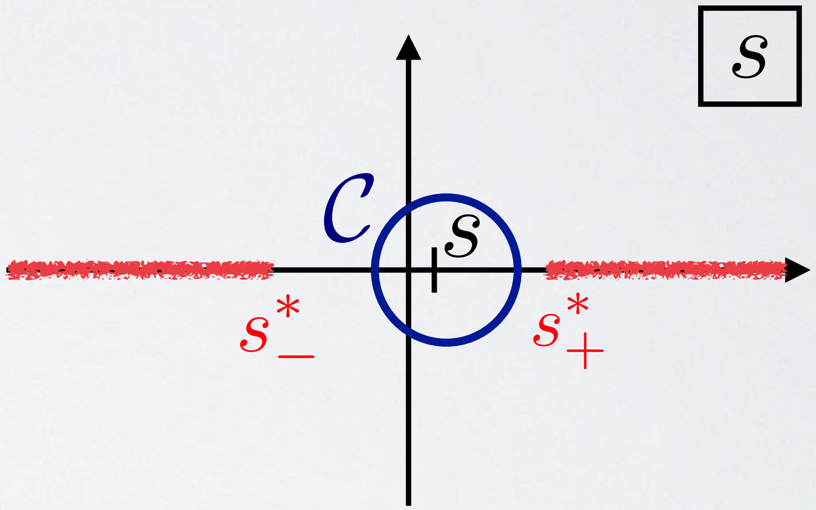

$$\mathcal{A}(s) = \int_{\mathcal{C}} \frac{\mathcal{A}(s')}{2\pi i(s' - s)}$$

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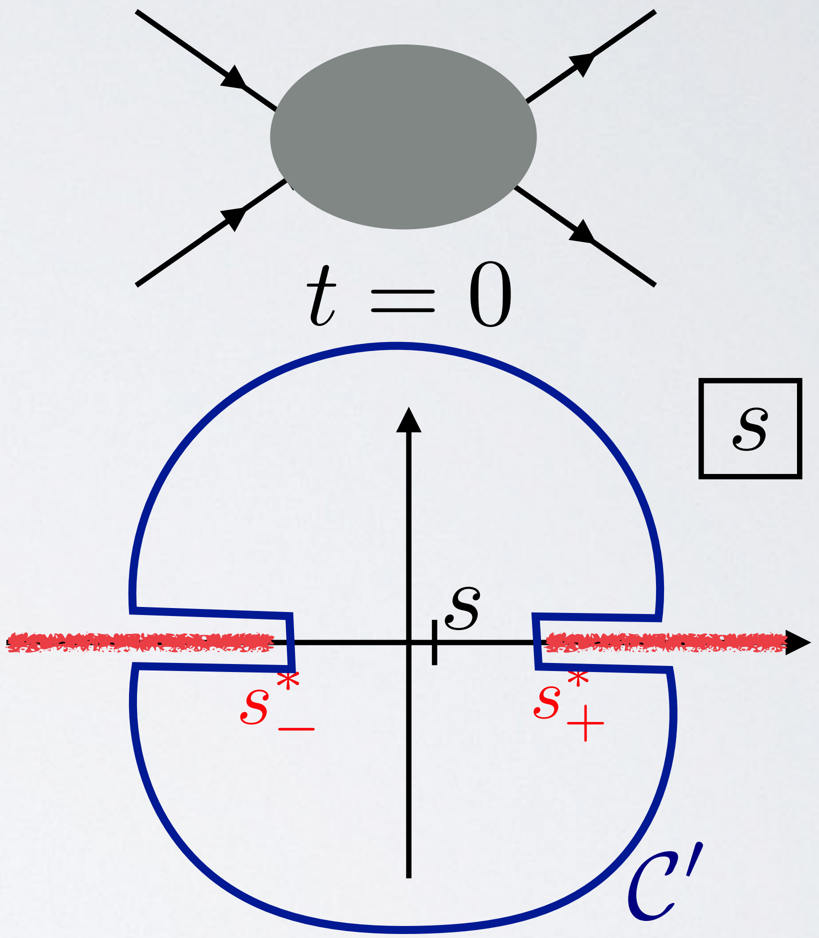

$$\mathcal{A}(s) = \int_{\mathcal{C}'} \frac{\mathcal{A}(s')}{2\pi i(s'-s)}$$

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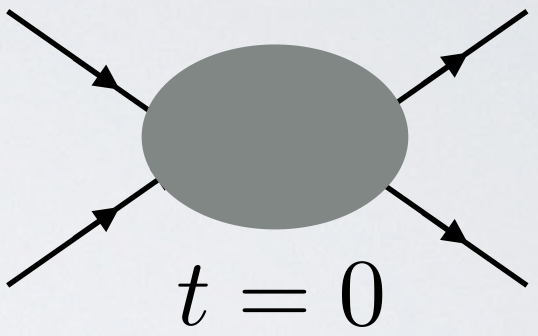

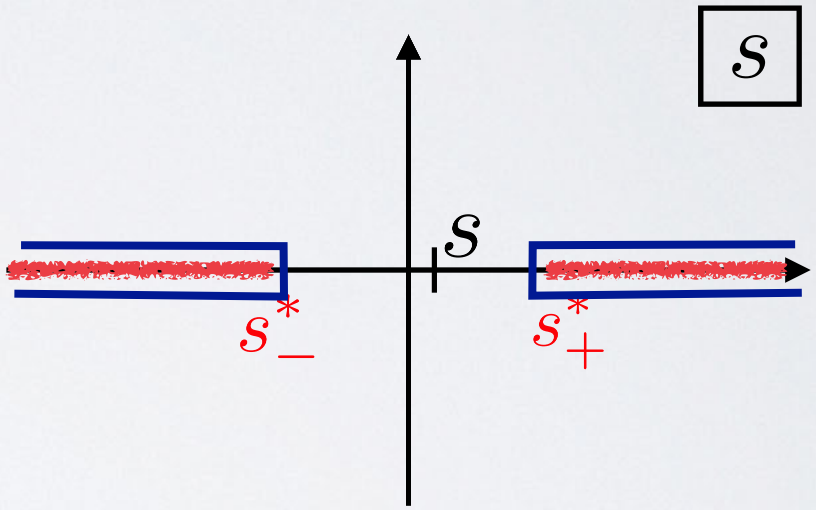

$$\mathcal{A}(s) = \int_{-\infty}^{s_-^*} \frac{\text{Im}\mathcal{A}(s')}{2\pi i(s'-s)} + \int_{s_+^*}^{+\infty} \frac{\text{Im}\mathcal{A}(s')}{2\pi i(s'-s)}$$

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Ingredients :

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- **polynomial boundedness** (Froissart bound)
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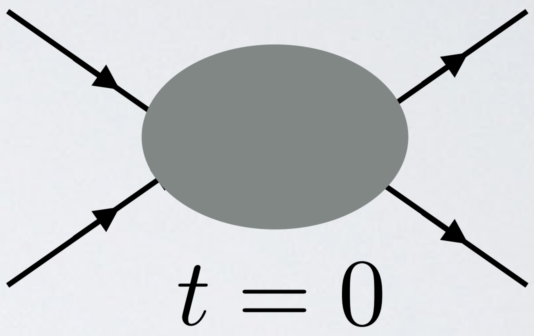

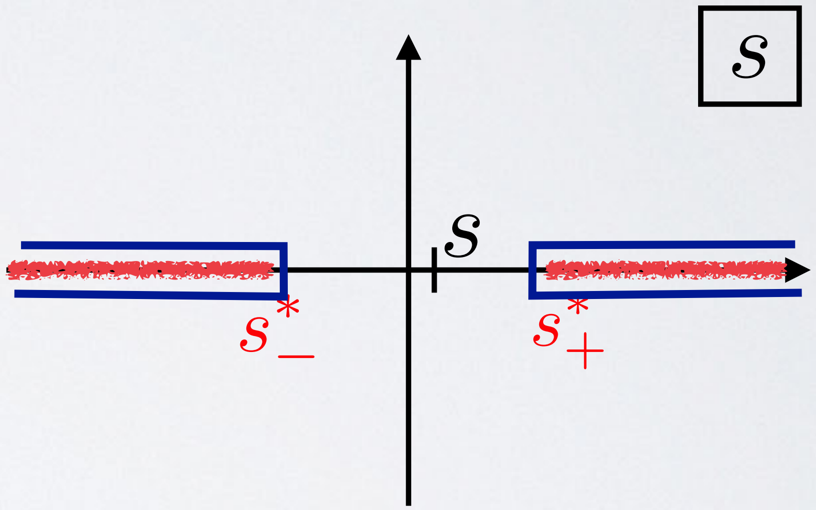

$$\widetilde{\mathcal{A}}(s) = \int_{-\infty}^{s_-^*} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s'-s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s'-s)^3}$$

Positivity bounds : sign constraints on amplitudes

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Ingredients :

- **analyticity**
- **polynomial boundedness** (Froissart bound)
- **crossing symmetry**
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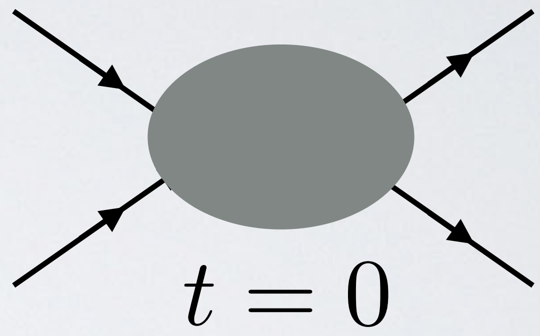

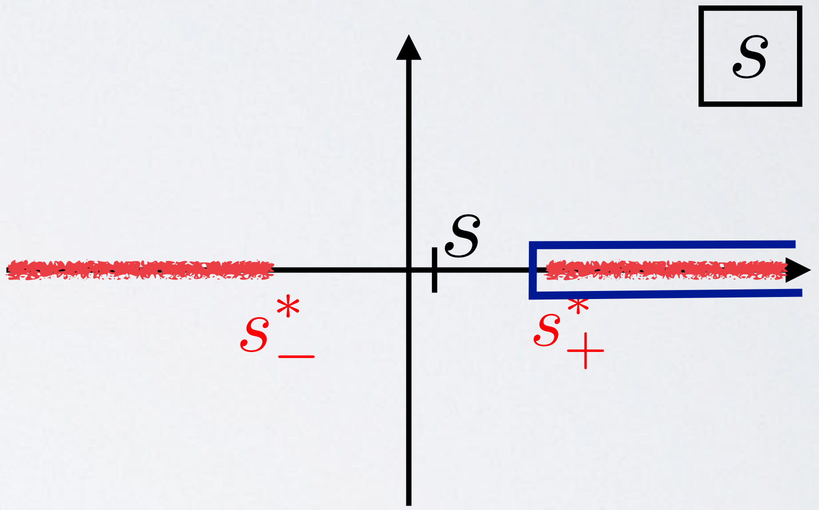

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2 - s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - 4m^2 + s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - s)^3}$$

Positivity bounds : sign constraints on amplitudes

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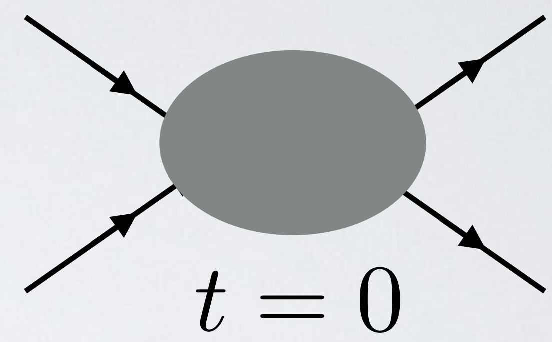
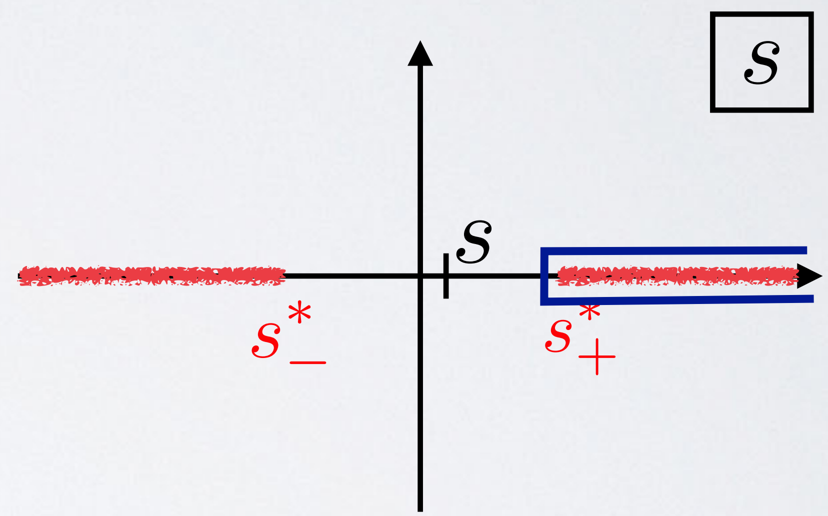


- **analyticity**
- **polynomial boundedness** (Froissart bound)
- **crossing symmetry**
- **unitarity**



> 0

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2-s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-4m^2+s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

Positivity bounds : sign constraints on amplitudes

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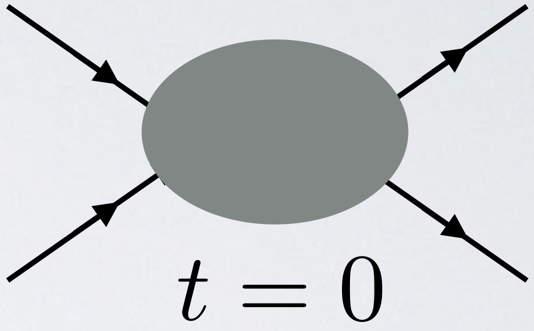

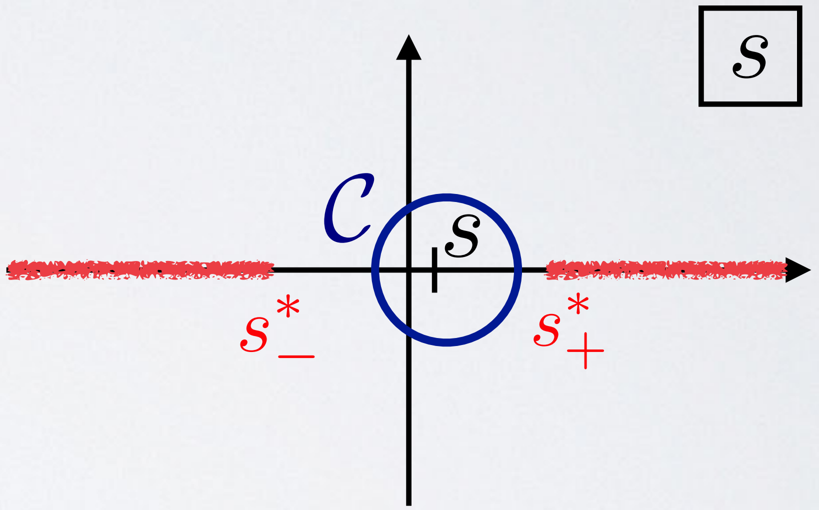

$$\widetilde{\mathcal{A}}(s) = \int_{\mathcal{C}} \frac{\mathcal{A}(s')}{2\pi i (s' - s)^3} > 0$$

At small s : computed in the EFT,
bound on EFT coefficients

[see papers of the authors, Remmen/Rodd, de Rham/Tolley/Zhou, Zhang/Wang...]

How to go further (in simple scalar models)**?**

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2 - s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s' - 4m^2 + s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s' - s)^3} > 0$$

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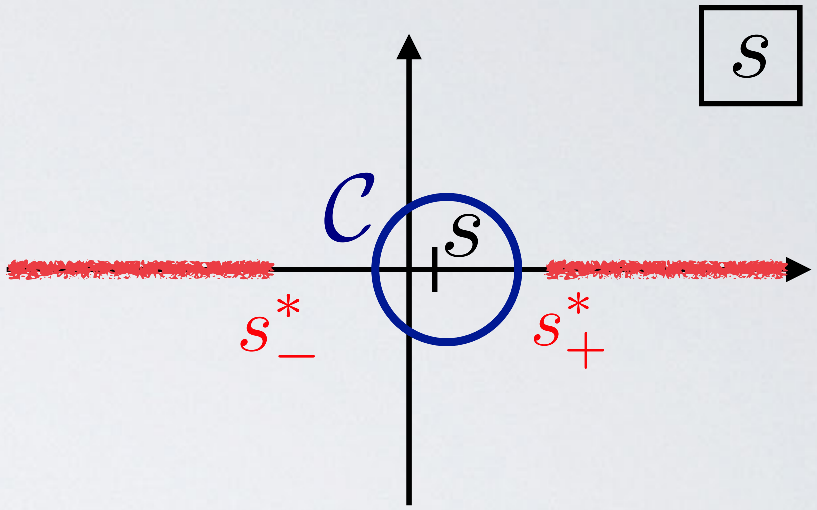
Integrands are positive :

$$\int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3} f(s') > 0$$

New quantities : **arcs**



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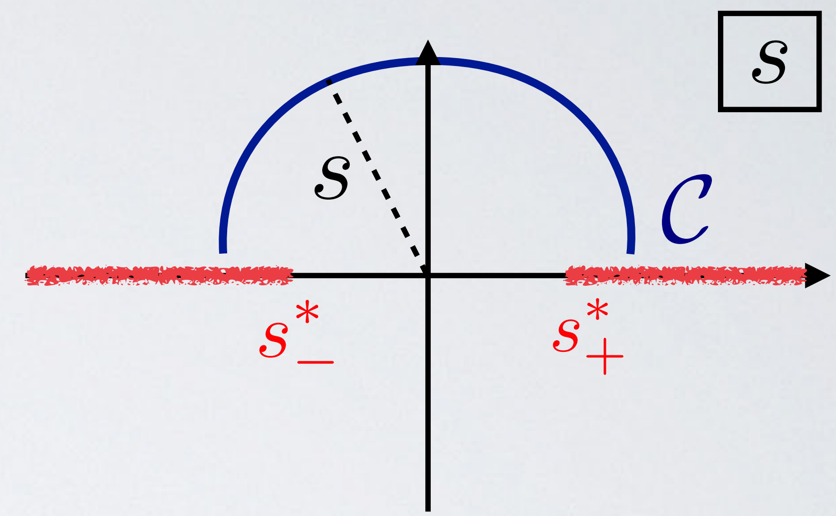
$$a_n(s) \equiv \frac{1}{\pi i} \int_{\mathcal{C}(s)} ds' \, \frac{\mathcal{A}(s')}{s'^{2n+3}}$$



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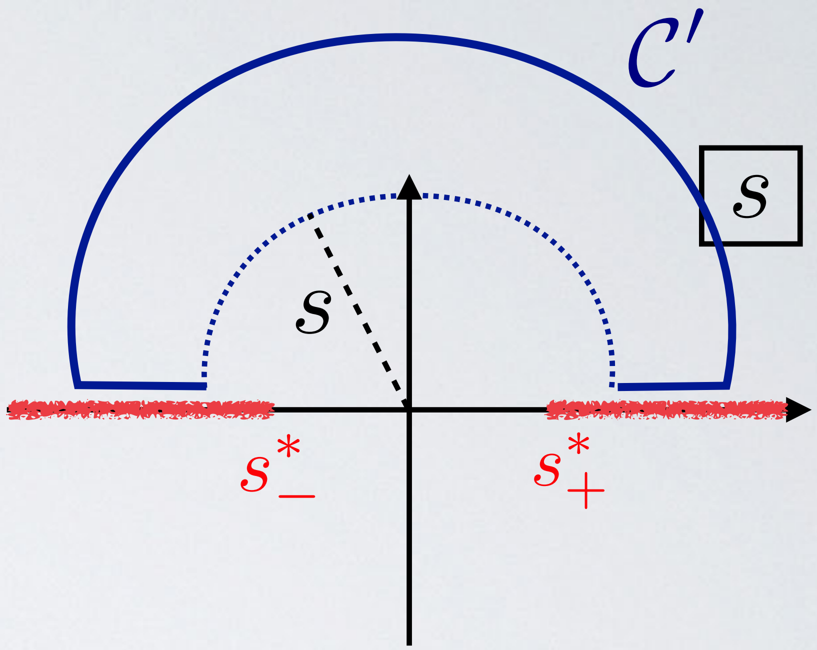

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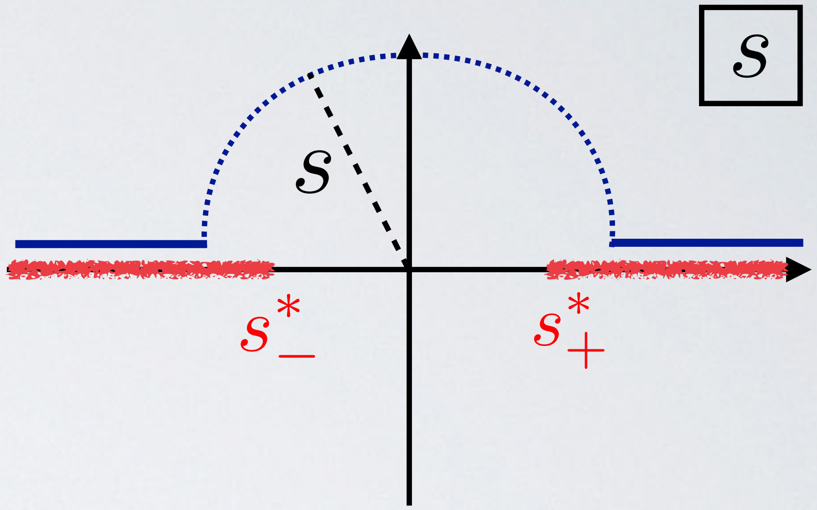

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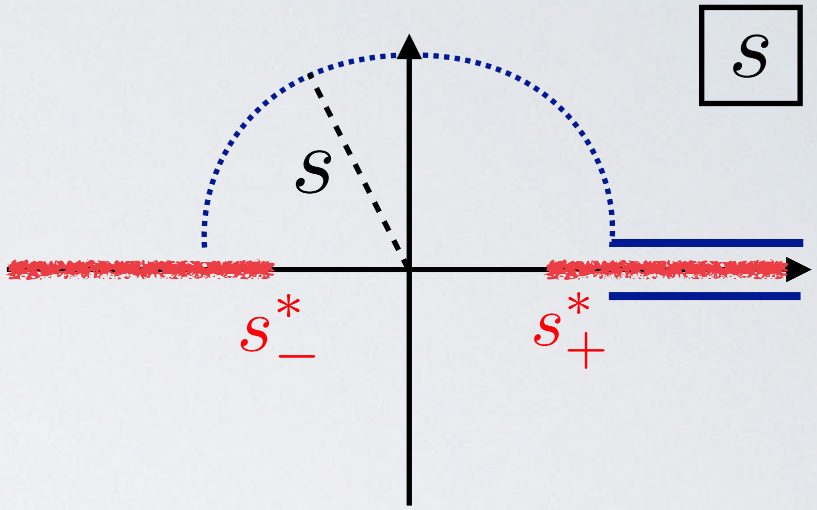

[2011.00037]

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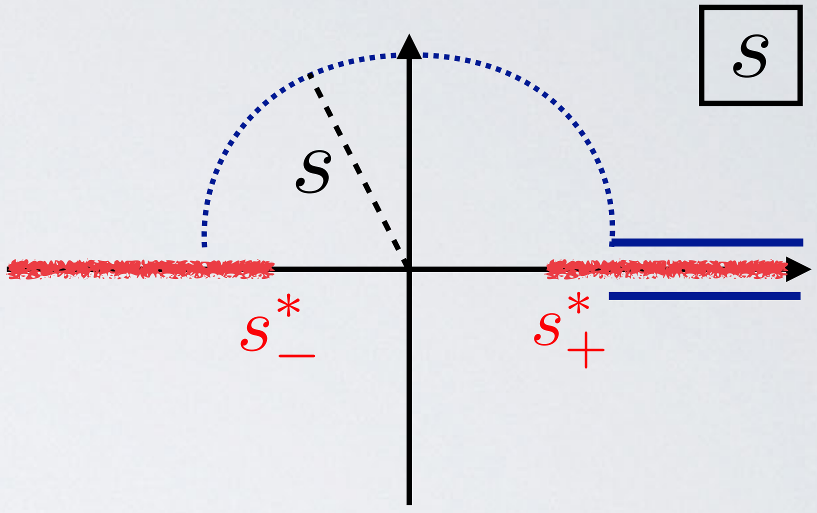

Positive, and **computable at small s in the EFT**, for the same reasons at before

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

More bounds :

$$\int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^{2n+3}} f(s') > 0$$

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$$d\mu(x) \equiv \frac{dx}{\pi}\mathrm{Im}\mathcal{A}\left(\frac{s}{\sqrt{x}}\right)$$

$$\int_0^1 d\mu(x) x^n (1-x^k) > 0$$

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yields an inequality on the arcs/moments :

$$(-1)^k \Delta^k a_n > 0 \quad \text{with} \quad \Delta a_n \equiv s^2 a_{n-1} - a_n$$

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Largest set of inequalities on the moments

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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What about the **projection on low-energy arcs?**

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[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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$$(H_N^l)_{ij} = a_{i+j+l}$$
$$i, j \leq \lfloor (N-l)/2 \rfloor$$

$$H_N^0 \succ 0 \, , \, H_N^1 \succ 0$$
$$H_{N-1}^0 - s^2 H_N^1 \succ 0 \, , \, H_{N-1}^1 - s^2 H_N^2 \succ 0$$

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Non-linear, s-dependent bounds (more and more stringent when s grows)

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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At tree-level

$$a_n(s) \equiv \frac{1}{\pi i} \int_{\mathcal{C}(s)} ds' \, \frac{\mathcal{A}(s')}{s'^{2n+3}}$$

$$\Longrightarrow \quad a_n = c_{2n+2}$$

$$\mathcal{A}(s) = c_0 + c_2 s^2 + c_4 s^4 ...$$

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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s-independent bounds

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$$\mathcal{A}(s) = c_0 + c_2 s^2 + c_4 s^4 \ldots$$

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s-dependent bounds :

upper bounds on the cutoff Ex : $s^2_{\max} < \frac{c_4}{c_6}$

bounds on « soft » theories Ex : $c_2 > s^2 c_4$

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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Tree-level UV completion

$$\mathrm{Im}\mathcal{A}(s) = \frac{\pi}{2}\sum_{i=1}^{P} g_k^2 M_k^2 \delta(s - M_k^2) \implies$$

$d\mu(x)$ is a sum of deltas at $x = \dfrac{s}{M_k^2}$

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With enough arcs, one **reconstructs the full UV theory :**

- number of particles from $\det(H_{2P}^0) = 0$
- spectrum from $\det(H_{2P-2}^0 - s^2 H_{2P-1}^1) = 0$
- couplings from $a_n = \sum \dfrac{g_k^2}{M_k^{4n+4}}$

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Loops in the EFT

$$a_n(s) \equiv \frac{1}{\pi i} \int_{\mathcal{C}(s)} ds' \, \frac{\mathcal{A}(s')}{s'^{2n+3}}$$

$$a_0 = c_2 + \frac{s^2}{2}\beta_4 + \frac{s^3}{3}\beta_5 + \frac{s^4}{4}\Big(\beta_6 + \frac{\beta_6'}{2}(4\log s - 1)\Big) + \cdots ,$$
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$$+ s^6\left[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)\right] + O(s^7)$$

$$(H_N^l)_{ij} = a_{i+j+l}$$
$$i, j \leq \lfloor (N-l)/2 \rfloor$$

$$H_N^0 \succ 0 \ , \ H_N^1 \succ 0$$
$$H_{N-1}^0 - s^2 H_N^1 \succ 0 \ , \ H_{N-1}^1 - s^2 H_N^2 \succ 0$$

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New s-dependence

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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New s-dependence

Running effects dominate the IR

$$a_0 \to c_2\ , \quad a_1 \to \beta_4 \log s\ , \quad a_{n\geq 2} \to -\frac{\beta_4}{(2n-2)s^{2n-2}}$$

and enter the bounds

$$c_4(s) - c_6(s)s^2 > -\frac{\beta_4}{2}$$

[2011.00037(+ 2011.02400+Arkani-Hamed/Huang (20XX?))]

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$$H^0_N \succ 0 \,,\; H^1_N \succ 0$$
$$H^0_{N-1} - s^2 H^1_N \succ 0 \,,\; H^1_{N-1} - s^2 H^2_N \succ 0$$

Loops in the EFT

$$a_n(s) \equiv \frac{1}{\pi i} \int_{\mathcal{C}(s)} ds' \frac{\mathcal{A}(s')}{s'^{2n+3}}$$

$$a_0 = c_2 + \frac{s^2}{2}\beta_4 + \frac{s^3}{3}\beta_5 + \frac{s^4}{4}\left(\beta_6 + \frac{\beta_6'}{2}(4\log s - 1)\right) + \cdots \,,$$
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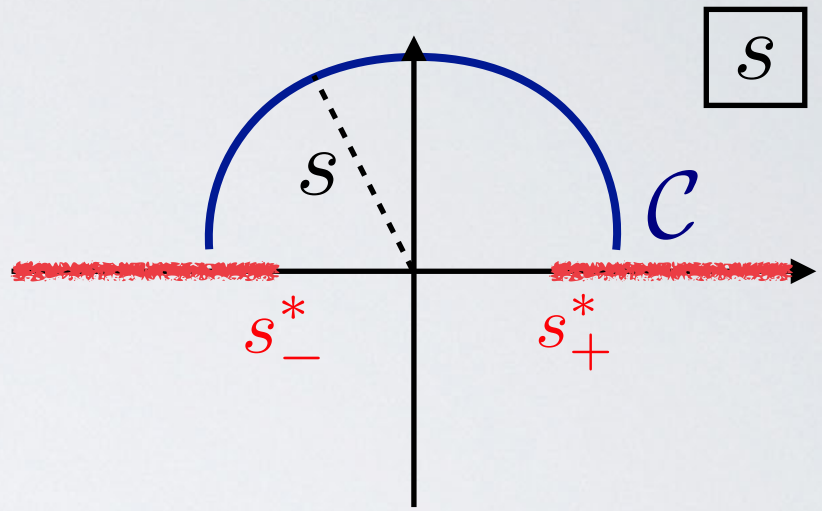
Strong corrections from soft theories ?
Still forbidden

TAKE AWAY

New quantities at finite s : **arcs**

$$a_n(s) \equiv \frac{1}{\pi i} \int_{\mathcal{C}(s)} ds' \, \frac{\mathcal{A}(s')}{s'^{2n+3}}$$



Most general bounds on arcs from the theory of moments :
s-dependence, non-linearity

At tree-level : bounds on the Wilson coefficients and on the EFT cutoff. UV info for weakly-coupled UV theories

At loop-level : quantum effects enter and sometimes dominate the bounds, the naive ones on Wilson coefficients can be violated. All bounds are s-dependent, they still rule out soft theories.