Hard job of starting the Universe

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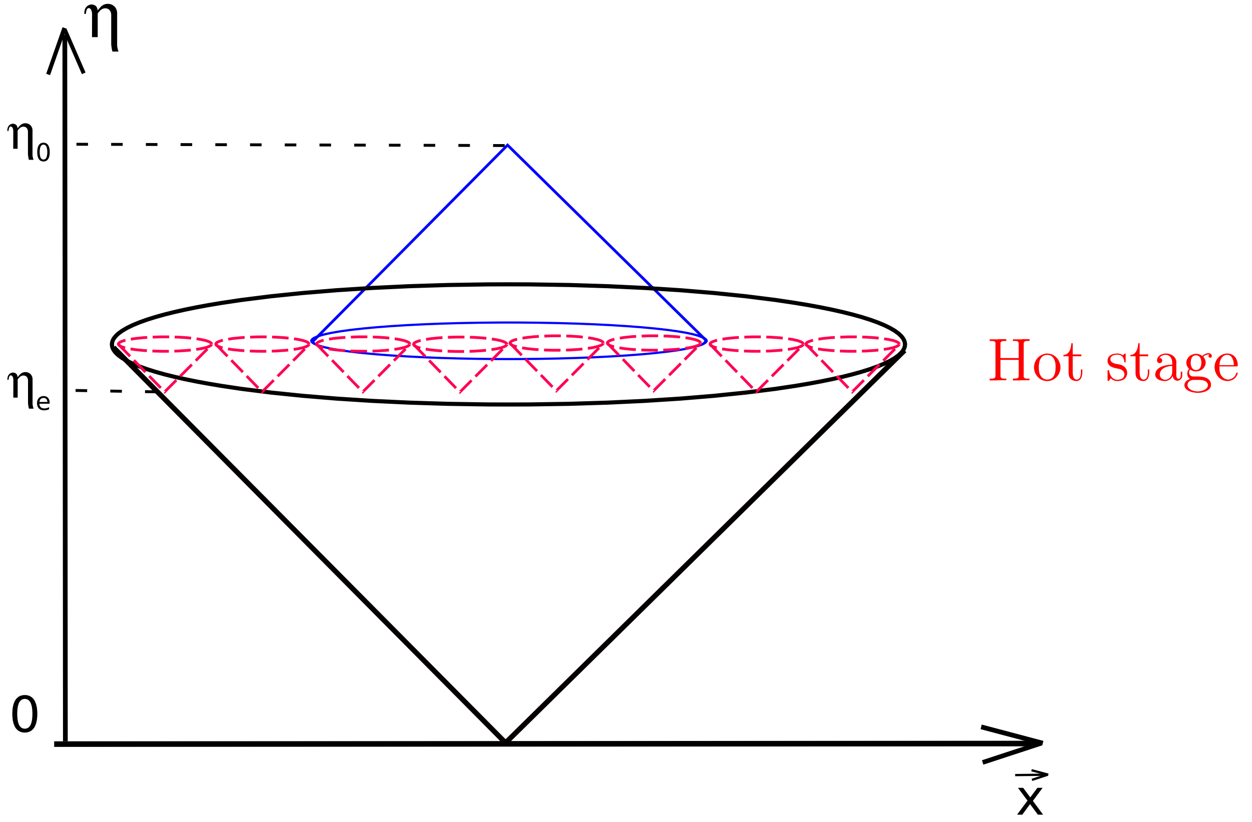
Experiment (CMB): density perturbations whose wavelengths were larger than Hubble size all the way to recombination

$\Longrightarrow$ There was epoch preceding hot stage

One of the sides of horizon problem.



Causal structure, in proper time η, required by observations.

Dominating hypothesis: inflation

Even within inflationary paradigm: What was before?

[Maybe purely academic question]

- Classical GR: Penrose theorem (under certian assumptions!)
  $\Longrightarrow$ singularity in the past
- "Ethernal inflation" is past geodesically incomplete

  Borde, Guth, Vilenkin' 03
- Many other arguments for non-inflationary initial epoch

  e.g., recent: Di Tucci, Feldbridge, Lehners, Turok' 19

Moreover, inflation is still not a theorem

Are there other options?

Need long (in conformal time) classical epoch to generate perturbations.

(Our patch of) the Universe may well have started
from "classical singularity"
(something outside classical field theory)

However, it is legitimate to ask whether one can avoid singularity in classical field theory or one inevitably has to deal with quantum gravity. Focus of this talk.

Off hand possibilities (either before or instead of inflation)

- Bouncing Universe: Start from contracting stage
($H < 0$) $\Longrightarrow$ bounce $\Longrightarrow$ expansion ($H > 0$)

- Genesis

Creminelli, Nicolis, Trincherini' 2010

Start from Minkowski, empty space ($H = 0$), then energy density builds up, Universe starts to expand ($H > 0$), expansion accelerates.

Both require $\dot{H} > 0$.

Obstacle in classical GR (hard job...): Null Energy Condition (NEC) obeyed by conventional matter:

$$T_{\mu\nu} n^\mu n^\nu > 0$$

for any null vector n^μ, such that $n_\mu n^\mu = 0$.

- Quite robust
- Penrose theorem

Penrose' 1965

In cosmology: if the NEC holds, and spatial curvature is negligible, there is initial singularity

No bounce, no Genesis.

NB: One way to get around: exploit spatial curvature to make bounce

Starobinsky' 1978

Problematic because of Belinsky–Lifshits–Khalatnikov phenomenon.

- A combination of Einstein equations (spatially flat):

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density; $T_{ij} = \delta_{ij} p$ = effective pressure.

- The Null Energy Condition:

  $T_{\mu\nu} n^\mu n^\nu > 0$, $n^\mu = (1,1,0,0) \Longrightarrow \rho + p > 0 \Longrightarrow dH/dt < 0$,

  Hubble parameter was greater early on. No bounce

  Penrose: there was a singularity in the past, $H = \infty$.

- Another side of the NEC:
  Covariant energy-momentum conservation:

$$\frac{d\rho}{dt} = -3H(\rho + p)$$

NEC: energy density decreases during expansion,
except for $p = -\rho$, cosmological constant. No Genesis

NEC is not violated in conventional field theories

with Lagrangians involving first derivatives only.

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

Prototype example: scalar field theory with field π and

$$L = F(X, \pi)$$

with $X = (\partial\pi)^2 \equiv g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X}\partial_\mu\pi\partial_\nu\pi - g_{\mu\nu}F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X}X - F \ , \qquad T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X}X = 2\frac{\partial F}{\partial X}\dot{\pi}^2$$

NEC-violation: $\partial F/\partial X_c < 0$. But perturbations $\pi = \pi_c(t) + \delta\pi(\vec{x},t)$

$$L_{\delta\pi} = \mathscr{F}\ \partial_t\delta\pi \cdot \partial_t\delta\pi - \frac{\partial F}{\partial X_c}\ \partial_i\delta\pi \cdot \partial_i\delta\pi + \dots$$

- $\mathscr{F} < 0$: both terms have wrong sign. Hyperbolic equation of motion, but negative energies $\Longleftrightarrow$ ghosts: $E = -\sqrt{p^2 + m^2}$
  Catastrophic vacuum instability

- $\mathscr{F} > 0$: only gradient term has wrong sign. Elliptic equation of motion $\Longrightarrow$ gradient instability

$$E^2 = -(p^2 + m^2) \quad \Longrightarrow \quad \delta\pi \propto \mathrm{e}^{|E|t}$$

Also catastrophic

NB. Loophole: $\partial F/\partial X_c = 0$. Higher derivative terms (understood in effective field theory sense) become important and help.

Ghost condensate

Arkani-Hamed et. al.' 2003

Twist: scalar-tensor theories with second derivatives in the Lagrangian.

Danger: higher order equations of motion $\Longrightarrow$ extra degrees of freedom = Ostrogradsky ghosts (hard job...)

Not necessarily!

- Example # 1: Horndeski Horndeski' 1974

  aka Euler hierarchies, aka generalized Galileons, aka KGB, aka generalized Fab Four

  - Second derivatives in Lagrangian, second order field equations
  - Simplest case: Creminelli, Nicolis, Trincherini' 10, Deffayet, Pujolas, Sawicki, Vikman' 10, Kobayashi, Yamaguchi, Yokoyama' 10

$$L = -\frac{1}{16\pi G}R + F(\pi, X) - K(\pi, X)\Box\pi$$

    where again $X = (\partial\pi)^2$.
  - Explicit examples of stable NEC-violation.

Simple example: scale-invariant model, $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln \lambda$

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \,, \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial\pi)^2$$

Homogeneous solution in Minkowski space (attractor)

$$\mathrm{e}^{\pi_c} = \frac{1}{\sqrt{Y_*}\,|t|} \,, \qquad t < 0$$

- $Y \equiv \mathrm{e}^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* = \mathrm{const}$, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F_Y - 2Y_* K + 2Y_*^2 K_Y = 0$$

$$F_Y = dF/dY \,.$$

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c}\mathscr{F}(\partial_t\delta\pi)^2 - \mathscr{G}(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

Absence of ghosts: $\mathscr{F} = Z_Y \equiv dZ/dY > 0$ at $Y = Y_*$, no problem.

- NEC-violation and absence of gradient instabilities:

$$\rho + p = \mathrm{e}^{4\pi_c}(F_Y - 2K + Y_* K_Y) \cdot 2Y_* \quad < 0$$

$$\mathscr{G} = \mathrm{e}^{2\pi_c}(F_Y - 2K + 4Y_* K_Y) \quad > 0$$

Easy to arrange.

NB: $\rho = 0$, $p < 0$ $\qquad$ $p \to 0$ as $t \to -\infty$

When coupled to gravity $\Longrightarrow$ early stage of Genesis.

Creminelli, Nicolis, Trincherini' 10,

General Horndeski theory

Require: both "Einstein" equations and π-field equation second order

Four arbitrary functions

Horndeski' 1974; Deffayet, Esposito-Farese, Vikman' 09

$$L = F(\pi, X) - K(\pi, X)\Box\pi$$
$$+ G_4(\pi, X)R + G_{4,X}\cdot\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$
$$+ G_5\cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X}\cdot\left[(\Box\pi)^3 - 3\Box\pi\cdot(\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]$$

- Modified gravity (scalar-tensor).

  NB: NEC is no longer relevant. Think in terms of Null Convergence Condition (NCC)

  Tipler' 1978

$$R_{\mu\nu}n^\mu n^\nu > 0\,.$$

- NCC can be violated without instabilities.
- NB: always in Jordan frame.

However, bounce and Genesis problematic. Hard job...

Libanov, Mironov, V.R.' 16;
Kobayashi' 16; Ijjas, Steinhardt' 16

Choose unitary gauge $\delta\pi = 0$.

$$ds^2 = N^2 dt^2 - a^2 \mathrm{e}^{2\zeta}(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}k_{kj})(N^i dt + dx^i)(N^j dt + dx^j)$$

Dynamical variables in scalar sector: transverse traceless h_{ij} and ζ.

Consider Genesis for definiteness (same argument for bounce):

$$a(t) \to \text{const as } t \to -\infty$$

$$a(t) \to \infty \text{ as } t \to +\infty$$

Upon solving for constraints, find quadratic Lagrangians for perturbations

$$L_S = \mathscr{G}_{\mathscr{S}} \dot{\zeta}^2 - a^{-2} \mathscr{F}_{\mathscr{S}} (\partial_i \zeta)^2 \, , \quad L_T = \mathscr{G}_{\mathscr{T}} \dot{h_{ij}}^2 - a^{-2} \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2$$

NB: $\mathscr{G}_{\mathscr{T}}$, $\mathscr{F}_{\mathscr{T}}$ = effective M_{Pl}^2. Stable background $\Longleftrightarrow$
$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} > 0$.

Key relation (by explicit calculation in general Horndeski)

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}{}^2(t)}{\Theta(t)}$$

where $\Theta(t) = -2HG_4 + \dot{\pi}XK_X + \dots$, a complicated expression.

Main property: ξ never crosses zero.

$$\xi(t_f) - \xi(t_i) = \int_{t_i}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

Impossible for $\mathscr{F}_{\mathscr{S}} > 0$, $\mathscr{F}_{\mathscr{T}} > 0$, and

$$\int_{-\infty}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty\,, \quad \int_{t_i}^{+\infty} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty$$

even if Θ crosses zero (i.e., $\xi = \infty$) at some moment of time.

Genesis, bounce $\Longleftrightarrow$ $\mathscr{F}_{\mathscr{S}} < 0$ and/or $\mathscr{F}_{\mathscr{T}} < 0$ at some time $\Longleftrightarrow$ gradient instability or ghost

Ways out (?), still in Horndeski.

- $a(t) \to 0$ as $t \to -\infty$, so that

$$\int_{-\infty}^{t} a(t)\, dt < \infty$$

Say $a = t^{-2}$. Hubble and its derivatives vanish as $t \to -\infty$.

Problem: past geodesic incompleteness.

- $\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$, so that

Wetterich' 2015; Kobayashi '2016; Ijjas, Steinhardt '2016

$$\int_{-\infty}^{t_f} dt\, \left(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}\right) < \infty$$

No-go theorem does not work.

But gravity tricky as $t \to -\infty$: effective Planck mass vanishes.

Strong coupling?

- $\mathscr{G}_\mathscr{T}, \mathscr{F}_\mathscr{T}, \mathscr{G}_\mathscr{S}, \mathscr{F}_\mathscr{S} \to 0$ as $t \to -\infty$.

  Can one trust classical field theory treatment of cosmological evolution?

Study couplings of h_{ij} and ζ, find strong coupling energy scales as $t \to \infty$, compare with classical energy scale $E_{class} = H, \dot{H}/H$.

Concrete example of Genesis: classical energy scale $E_{class} \propto |t|^{-1}$, while strong coupling scales $E_{strong} \propto |t|^{-\alpha}$ with $\alpha < 1$ in part of parameter space.

Ageeva, Evseev, Melichev, V.R.' 18, 20;
Ageeva, V.R., Petrov' 20

Classical treatment of evolution legitimate.

Overall picture: Universe starts at very low quantum gravity scale $E_{strong} \propto |t|^{-\alpha}$ but expands so slowly that $E_{class} \ll E_{strong}$. Standard Model scales are above E_{strong}. Gravity is the strongest force.

- Common wisdom: geodesic incompleteness of tensor/scalar modes

$$\int_{-\infty}^{t} dt \; a(t) \; \mathscr{F}_{\mathscr{T},\mathscr{S}} < \infty$$

  But: geodesic incompleteness is a confusing issue for massless particles

Horndeski is not the most general scalar-tensor theory with tensor + one scalar modes $\Longrightarrow$ No Ostrogradsky ghost

- Variation of action may give higher order field equations, but they may combine in such a way that the resulting equations are second order.

  Degenerate Higher-Order Scalar Tensor theories, DHOST

  Langlois, Noui' 16; Crisostomi, Koyama, Tasinato' 16

- Relatively simple subclass: "beyond Horndeski" theories

  Zumalacárregui, Gacia-Bellido' 2014; Gleyzes, Langlois, Piazza, Vernizzi' 2014

  Example of additional (to Horndeski) term

$$F_4(\pi,X)\varepsilon^{\mu\nu\lambda\rho}\varepsilon^{\mu'\nu'\lambda'}{}_{\rho}\ \pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\lambda\lambda'}$$

- Way to understand (sometimes): disformal transformation

$$g_{\mu\nu} \to \Omega(\pi,X)g_{\mu\nu} + \Lambda(\pi,X)\partial_\mu\pi\partial_\nu\pi$$

  Horndeski $\to$ beyond Horndeski

  NB: This is formal trick: Ω, Λ may be singular

No-go theorem no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

One again has

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

but now

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}(\mathscr{G}_{\mathscr{T}} + 2F_4X^2)}{\boldsymbol{\Theta}(t)}$$

can cross zero.

NB: $\boldsymbol{\Theta} = 0$ not a problem, gauge artifact

Ijjas' 17;

Mironov, V.R., Volkova' 18

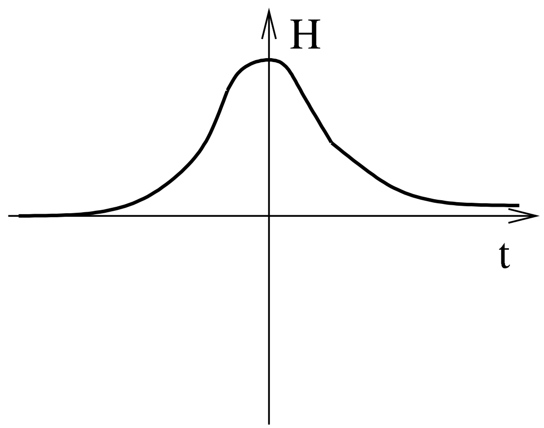
Intelligent design

Dubbed "Inverse method" by Ijjas, Steinhardt' 2016

- Choose background $\pi(t) = t$, no loss of generality

  Then $X = (\partial\pi)^2 = 1$. Field equations and stability conditions involve $f_0(t) = F(\pi(t))$, $f_1(t) = F_X(\pi(t))$, etc., all at $X = 1$.

- Choose your favorite $H(t)$ such that $H(t) \to \frac{1}{3t}$ as $|t| \to \infty$
  GR + Galileon = conventional massless scalar.



- Cook up Lagrangian functions in such a way that
  - Field equations are satisfied
  - Stability conditions are satisfied at all times

All this can be done both for Genesis and bounce

Mironov, V.R., Volkova' 18, 19

Moreover, one can design a model in such a way that

- Tensor perturbations are subluminal at all times (or luminal, if one wishes so)
- Scalar perturbations are subluminal at all times
- Bounce: GR is restored both as $t \to +\infty$ and as $t \to -\infty$.

However, there is still an issue to worry about: superluminality.

Theory with superluminal excitations cannot descend from healthy Lorentz-invariant UV-complete theory

Adams et. al.' 2006

Not an issue in DHOST theories per se.

But things change once one allows for extra field(s) ("matter")

– hard job...

DHOST with additional scalar field

Additional minimally coupled scalar: $L_\chi = (\partial\chi)^2$

New featue: DHOST pertutbations kinetically mix with $\delta\chi$ if $\dot\chi_c \neq 0$ in background: ion reads (modulo terms with less than two derivatives)

$$L^{(2)\,scalar}_{\pi+\chi} = G_{AB}\,\dot u^A \dot u^B - \frac{1}{a^2} F_{AB}\,\partial_i u^A \partial_i u^B$$

where $A, B = 1, 2$, $u^1 = \zeta$, $u^2 = \delta\chi$,

$$G_{AB} = \begin{pmatrix} \mathscr{G}_{\mathscr{S}} & \dot\chi_c g \\ \dot\chi_c g & 1 \end{pmatrix}, \qquad F_{AB} = \begin{pmatrix} \mathscr{F}_{\mathscr{S}} & \dot\chi_c f \\ \dot\chi_c f & 1 \end{pmatrix},$$

$g, f(\pi, X)$ = combinations of functions in DHOST Lagrangian.

One of the modes superluminal unless $g = f$

Imposing $g = f \Longrightarrow$ Very special DHOST theory (not beyond Horndeski). Work in progress.

To summarize

- Avoiding classical singularity (constructing bouncing and/or Genesis cosmology) does not appear impossible.
- This requires unusual fields with complicated Lagrangians involving second derivatives.
  - Absence of Ostrogradsky ghost, catastrophic instabilities and superluminality imposes strong (non-linear!) constraints on functions in Lagrangian.
- Is the price too high — job too hard?

Other issues

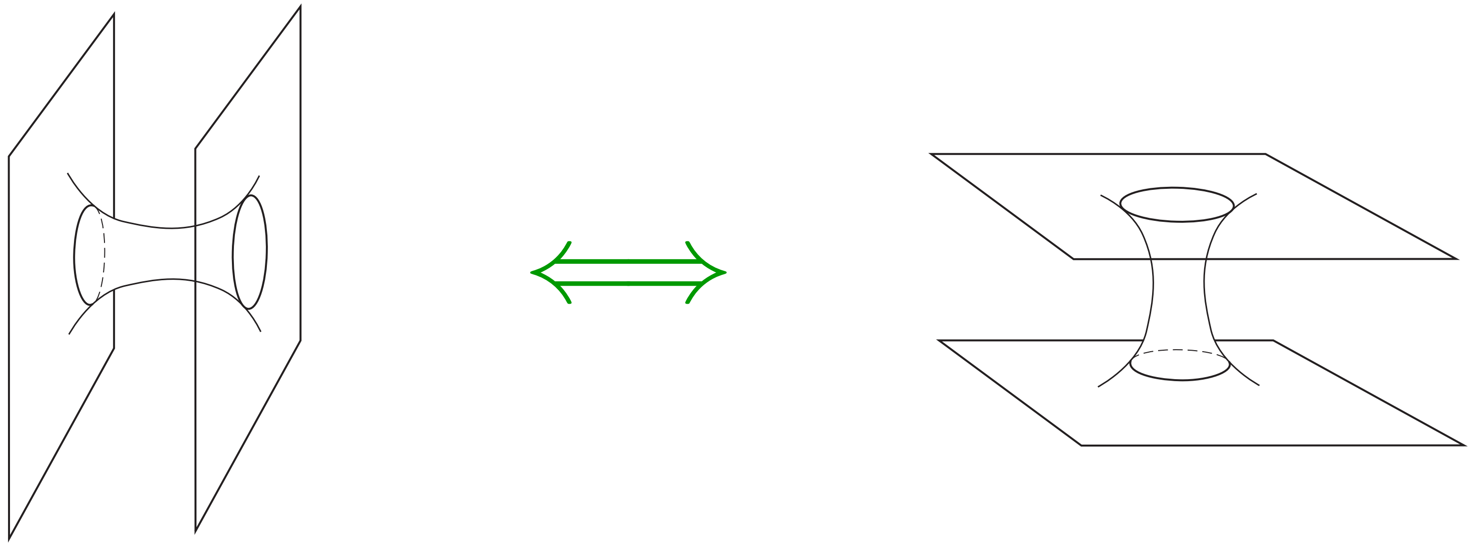
- Transition to hot epoch.
- Generation of density perturbations. Need a separate mechanism to generate nearly flat power spectrum.
- Tensor perturbations (gravity waves) with blue or peaked power spectrum (cf. NANOGrav)

Tahara, Kobayashi' 20

Instead of conclusion: where else DHOST may be instrumental?

- Lorentzian wormholes

Static wormhole $\Longleftrightarrow$ Bouncing Universe



No-go in NEC-preserving theories

No-go in Horndeski: no stable, static, spherically symmetric wormholes: always ghosts.

V.R.' 16; Evseev, Melichev' 18

Not obviously impossible in DHOST

Mironov, V.R., Volkova' 18; Francolini et. al.' 18

Studying stability HUGELY difficult.

- Creation of a universe in the laboratory

  - Question raised in mid-80's, right after invention of inflationary theory

    Berezin, Kuzmin, Tkachev' 1984; Guth, Farhi' 1986

    Idea: create, in a finite region of space, inflationary initial conditions $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

  - Do not need much energy: pour little more than Planckian energy into little more than Planckian volume.

    If NEC holds, no way: initial singularity

    Guth, Farhi' 1986; Berezin, Kuzmin, Tkachev' 1987

    How about DHOST theories?

Amazingly, many questions of pronciple still not answered.

Ahead: more to understand.