Path Integrals for the Universe

Neil Turok

interference

basic to quantum physics

universal

all known physics

$$\Psi = \int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D}\psi - \lambda H \overline{\psi}\psi + |DH|^2 - V(H)\right)}$$

dark energy

$$\psi = (u_L, d_L, u_R, d_R, u_L, d_L, u_R, d_R, u_L, d_L, u_R, d_R, e_L, \nu_L, e_R, \nu_R) \times 3$$

dark matter
Boyle, Finn, NT
2018

a l l k n o w n p h y s i c s

$$\Psi = \int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \bar{\psi} i \not{D} \psi - \lambda H \bar{\psi}\psi + |DH|^2 - V(H)\right)}$$

Schrödinger, Feynman, Einstein, Maxwell-Yang-Mills, Kobayashi-Maskawa, Lagrange, Euler, Planck, Newton, Dirac, Yukawa, Higgs

dark energy

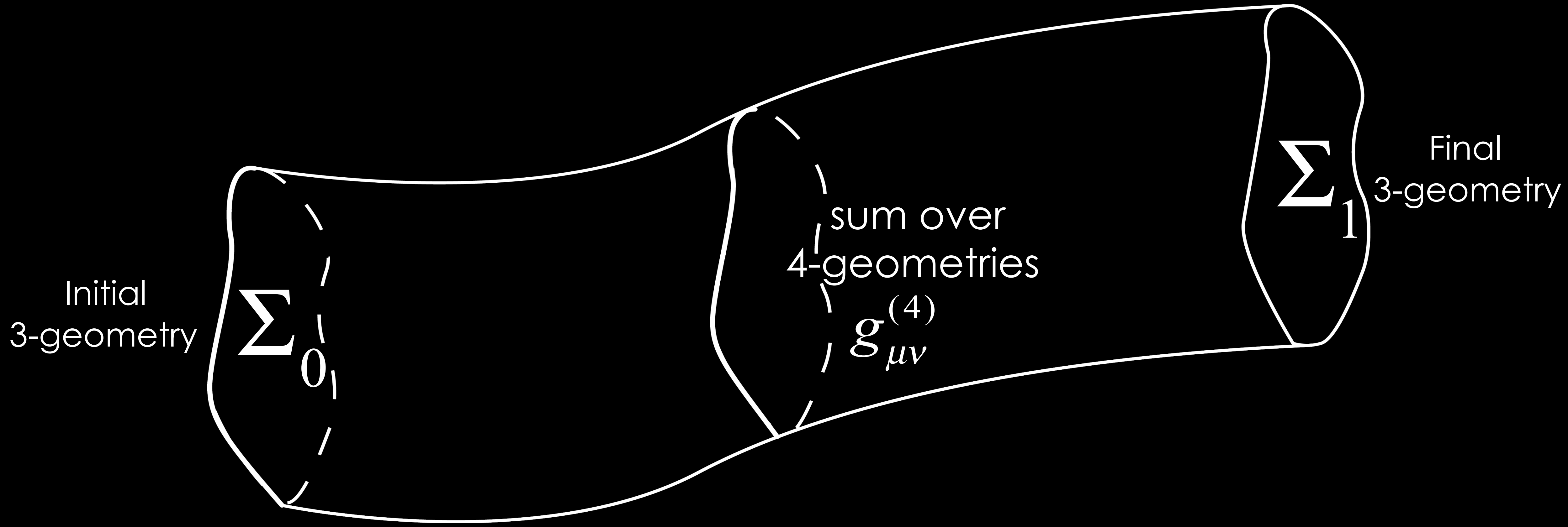
$$\psi = (u_L, d_L, u_R, d_R, u_L, d_L, u_R, d_R, u_L, d_L, u_R, d_R, e_L, \nu_L, e_R, \nu_R) \times 3$$

But “the Lorentzian path integral is ill defined”

e.g. Mukhanov +Winitzki 2005

Wheeler, Feynman,
De Witt, Teitelboim …

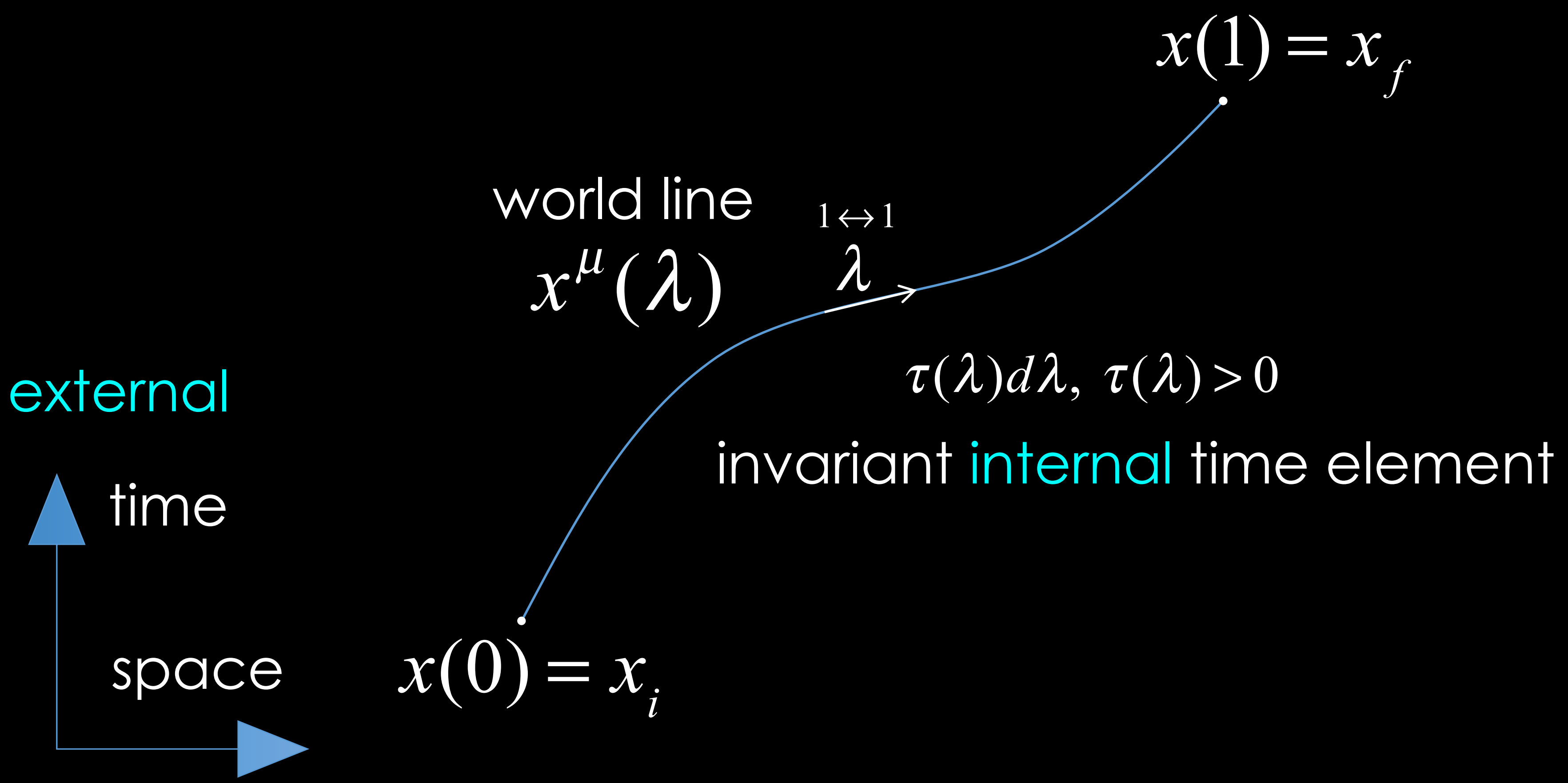
quantum geometrodynamics



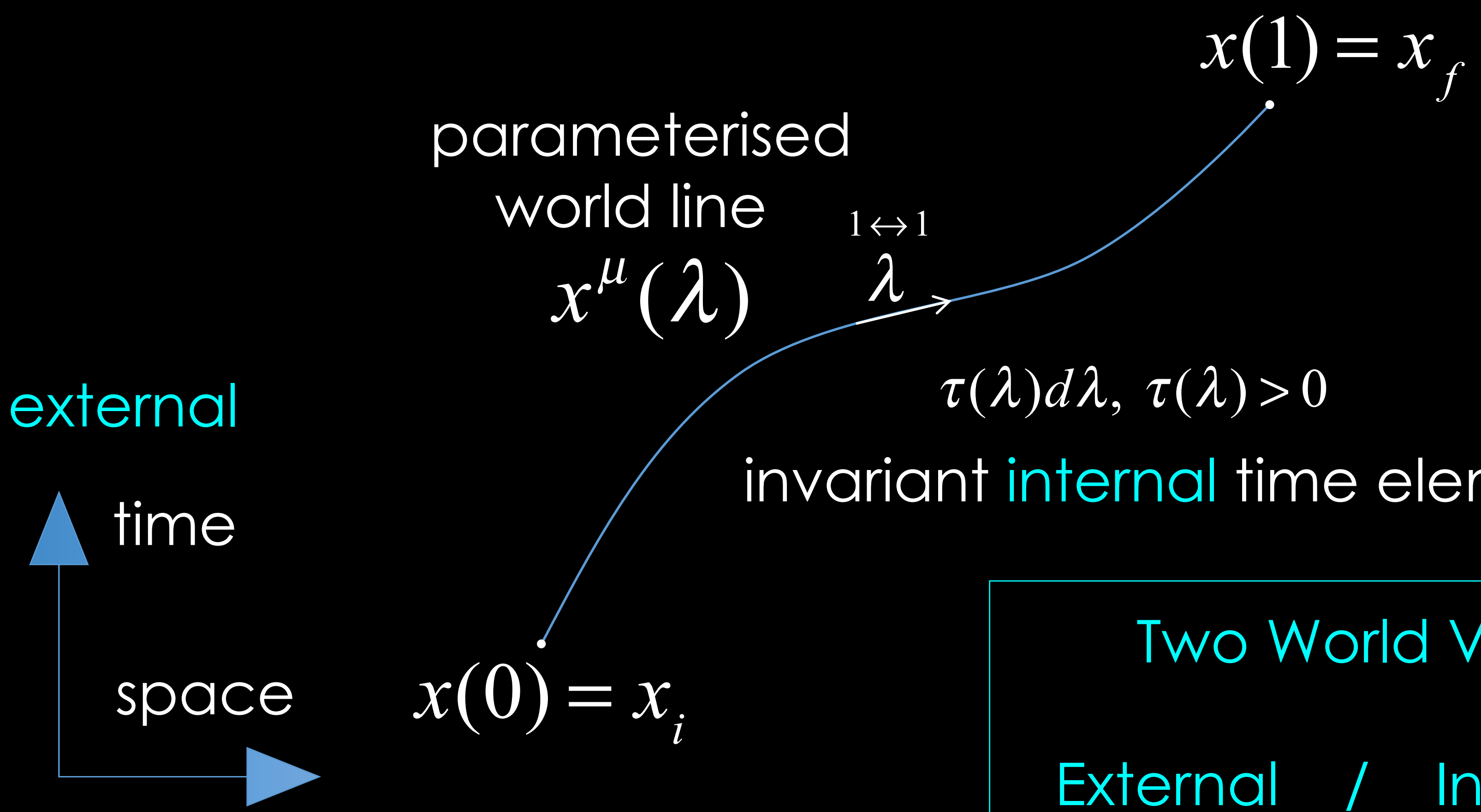
can we make progress with a single-universe picture?

(Feynman/Schwinger/Tomonaga)

Quantized relativistic particle



Quantized relativistic particle



Two World Views

External / Internal

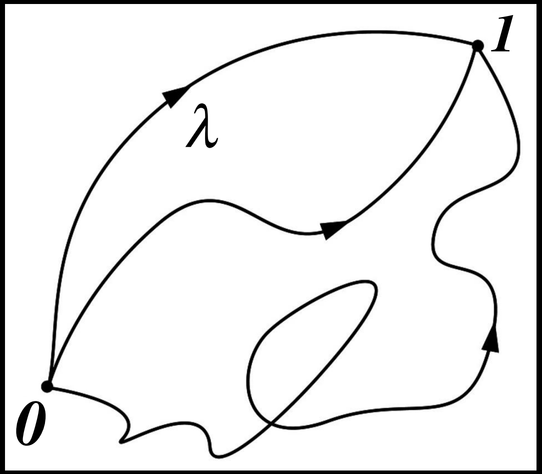
Particle Physics / Cosmology

amplitude for a particle initially at $x_0^\mu \equiv (t_0, \vec{x}_0)$ to be found at x_1^μ

Internal time unobserved so integrate over all allowed (+ve) values

propagator: $K(x_1, x_0) = \int_{0^+}^{\infty} d\tau \int_{x_0}^{x_1} Dx\, e^{i\frac{S}{\hbar}}$

action: $S = \frac{m}{2}\int_0^1 d\lambda \left(\frac{\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}{\tau} - \tau\right)$



As we shall see, answer unique, equivalent to $i\epsilon$

FT wrt space $\widetilde{K(t_1, t_0, \vec{k})} = e^{i\frac{\pi}{4}} \int_0^\infty d\tau \sqrt{\frac{\mu}{2\pi\tau}}\, e^{-i\frac{\mu}{2}\left(\frac{(t_1 - t_0)^2}{\tau} + \frac{\omega_k^2}{\mu^2}\tau\right)} = \frac{\mu}{\omega_k} e^{-i\omega_k |t_1 - t_0|}$

cf QFT propagator

Particles propagate forward in time, antiparticles backward

dimensionless: $\delta(t_1 - t_0) \int d\tau \quad c = 1; \vec{p} = \hbar\vec{k}; E = \hbar\omega; m = \hbar\mu; q = \hbar\, e$ etc

Spacetime amplitude approach to relativistic QM

$\psi(x) = \psi(t, \vec{x})$ amplitude for particle to be at spacetime point $t, \vec{x}$

Inner product:

$$(\psi, \chi) = \int d^4x \; \psi^*(x)\chi(x) \Longrightarrow$$ positive probabilities (cf KG norm)

Regularize covariantly: use μ' on left and μ on right in inner product

On-shell states: $\psi^+_{\vec{k}} = ce^{-i(\omega_k t - \vec{k}\cdot\vec{x})}$; $\omega_k \equiv \sqrt{k^2 + \mu^2}$

$$(\psi^+_{\vec{k}'}, \psi^+_{\vec{k}}) = \lim_{\mu' \to \mu} |c|^2 \, \delta(\sqrt{k^2 + \mu'^2} - \sqrt{k^2 + \mu^2})\,|_{\text{coefft of } \delta(\mu' - \mu)} = |c|^2 2\,\omega_k$$

Derivations simplified, *e.g.*, energy-time uncertainty relation, semi-classical and non-relativistic limits etc.

Highly oscillatory integrals

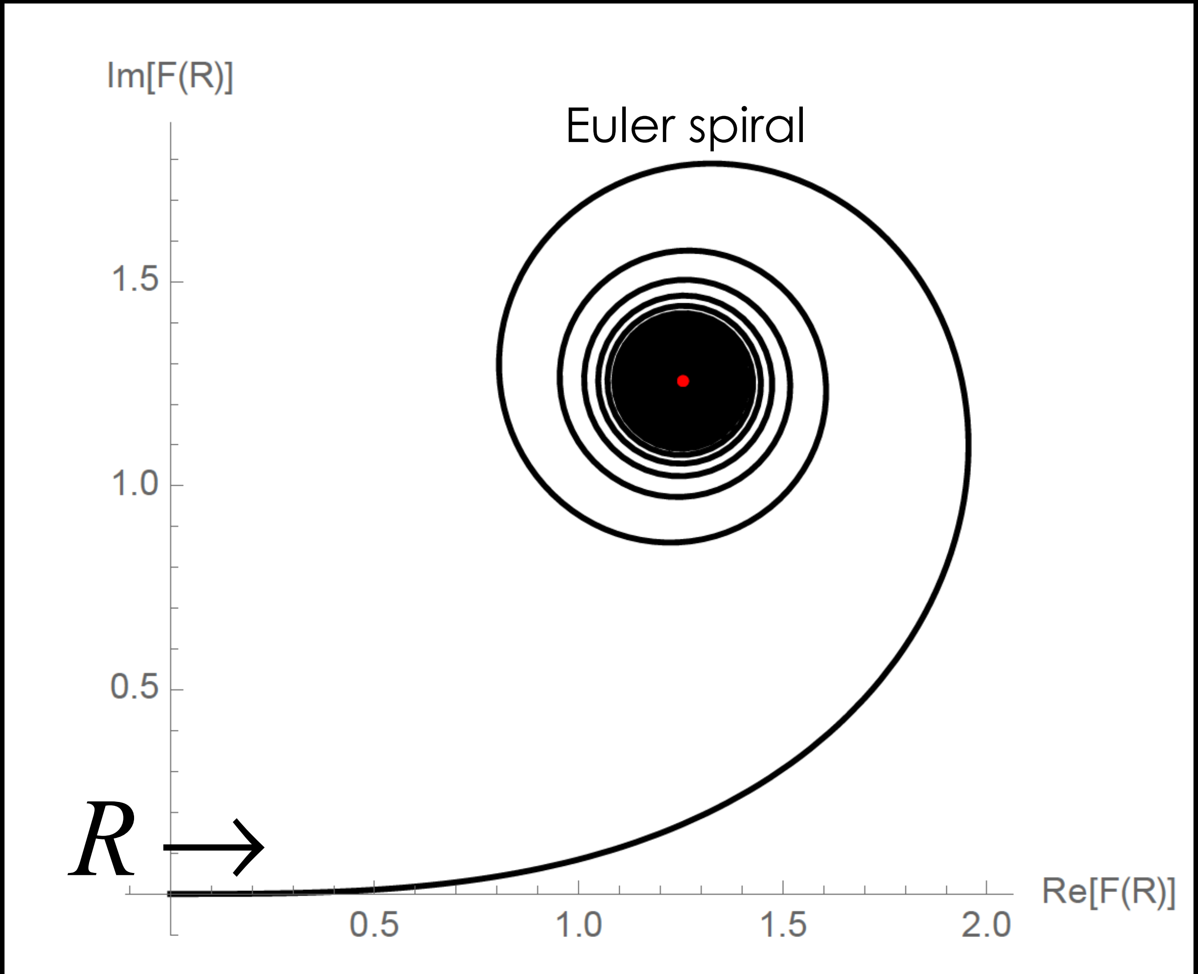
e.g., Fresnel integral

$$F(R) = \int_{-R}^{+R} e^{ix^2}\,dx$$

$$I = \lim_{R\to\infty} F(R) = e^{i\frac{\pi}{4}}\sqrt{\pi}$$

Conditionally, not absolutely convergent

Higher dimensional case?



2d: square cutoff $$\lim_{R\to\infty}\int_{-R}^{R}dx\int_{-R}^{R}dy e^{i(x^2+y^2)} = \lim_{R\to\infty}F(R)^2 = i\,\pi$$

2d: round cutoff $$\lim_{R\to\infty}2\pi\int_0^R rdr\; e^{ir^2} = \lim_{R\to\infty}\frac{\pi}{i}(e^{iR^2}-1)=?$$

d dims: sharp cutoff $$\int_0^R r^{d-1}dr\; e^{ir^m}\sim\frac{e^{\frac{i\pi d}{2m}}\Gamma(\frac{d}{m})}{m}-\frac{i}{m}e^{iR^m}R^{d-m}+..$$

integer $d, m, d > m$

d dims: smooth cutoff (allows cancellations, more physical) $$\int_0^\infty r^{d-1}dr\; e^{ir^m}e^{-\left(\frac{r}{R}\right)^m}\sim\frac{e^{\frac{i\pi d}{2m}}\Gamma\left(\frac{d}{m}\right)}{m}(1-\frac{i\,d}{m\,R^m}+..)$$

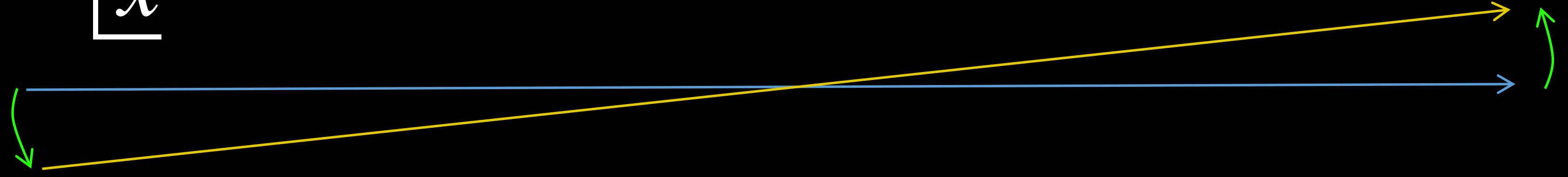
(see also F.N.H. Robinson," Macroscopic Electromagnetism")

Q: can one obtain the result for an infinite, smooth cutoff
without introducing a cutoff at all?

A: Yes: complex analysis (Cauchy theorem)

x

e.g. e^{ix^4} deform at large x



higher dimensions:
Picard-Lefschetz

general method for performing highly oscillatory integrals exactly
via steepest descent in arbitrary finite dimension

we flow the contour onto a series of relevant “Lefschetz thimbles”

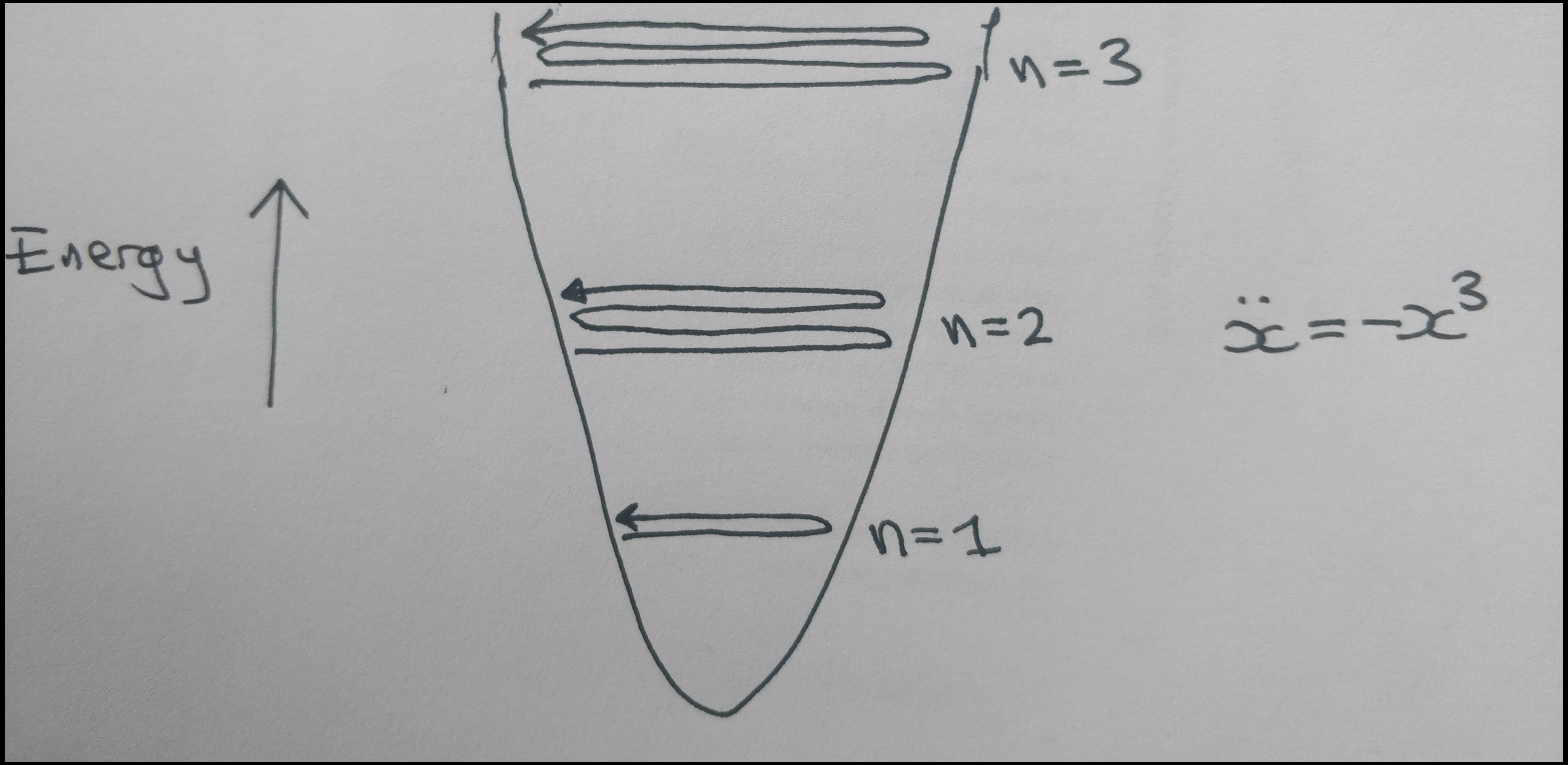
new approach, provides a definition of Lorentzian path integral for gravity

We used this to disprove the Hartle-Hawking and Vilenkin proposals

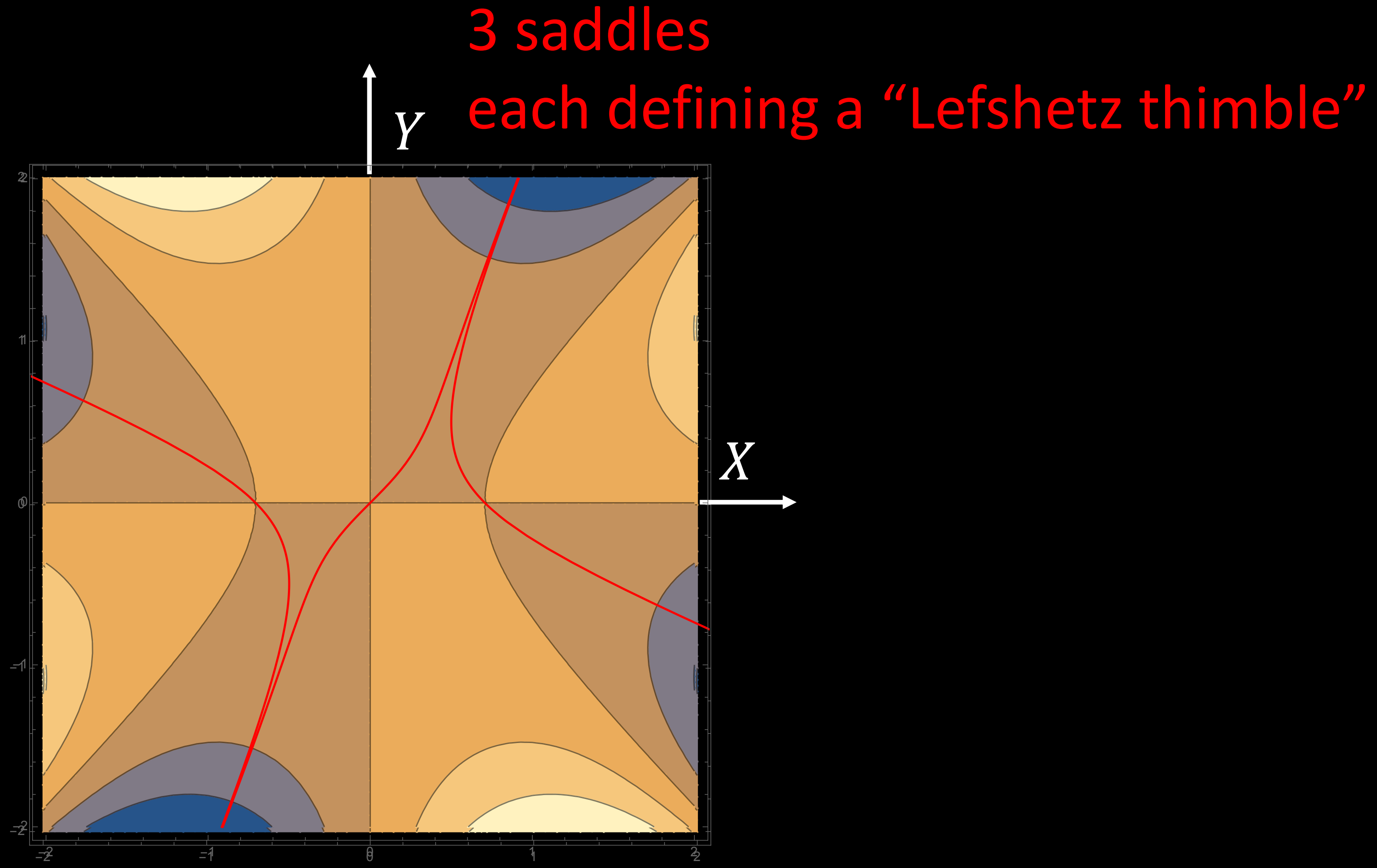
Feldbrugge
Lehners
NT

Defining the Lorentzian path integral: e.g., anharmonic oscillator

$$S = \int \left(\tfrac{1}{2}\dot{x}^2 - \tfrac{1}{4}x^4\right)dt$$

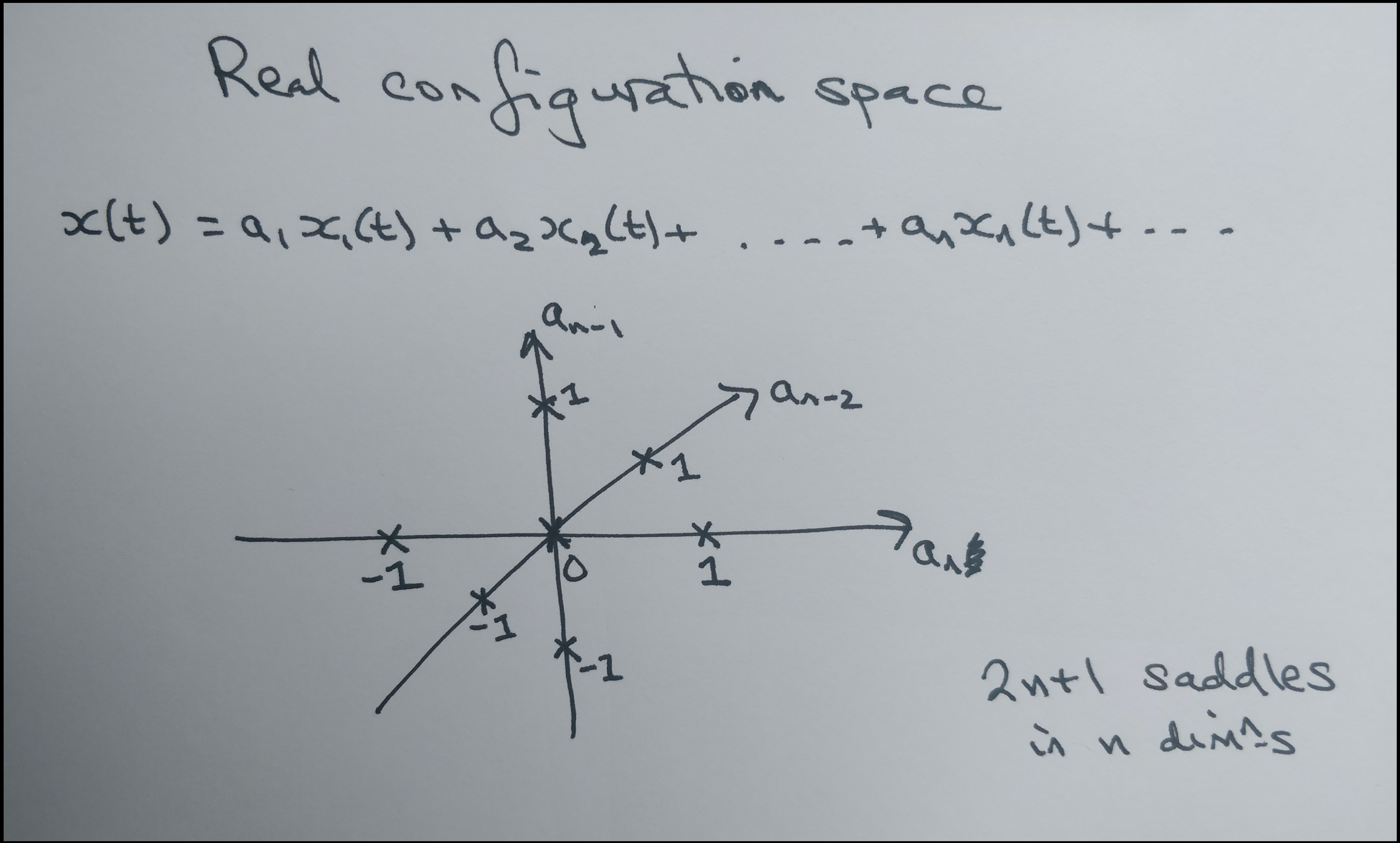


Toy version:



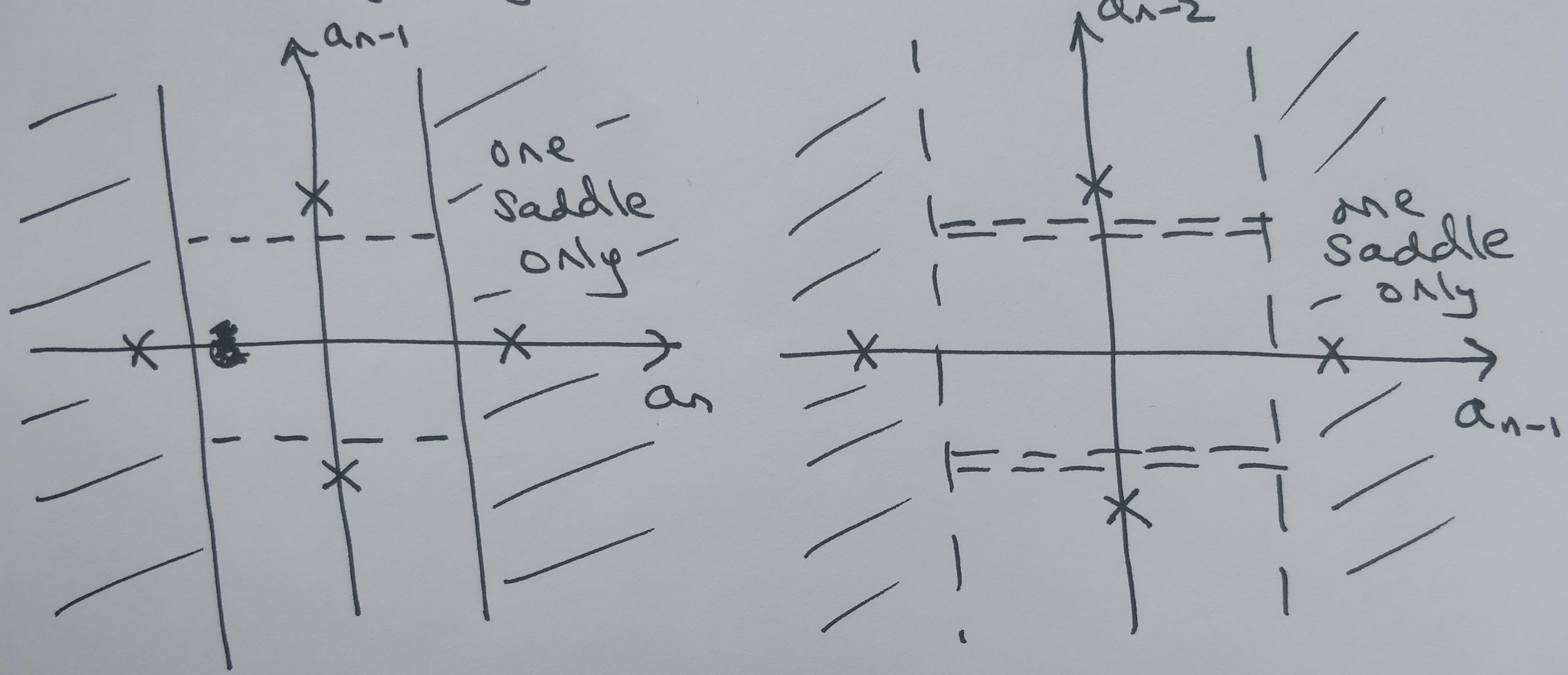
$$Re[i(x^2\text{-}x^4)];\ \ x = X + iY$$

For simplicity, consider the case $x(0) = x(1) = 0$; there are an infinite number of classical solutions



infinite set of thimbles: sum over a finite number provides an intrinsic regularization

Defining the path integral



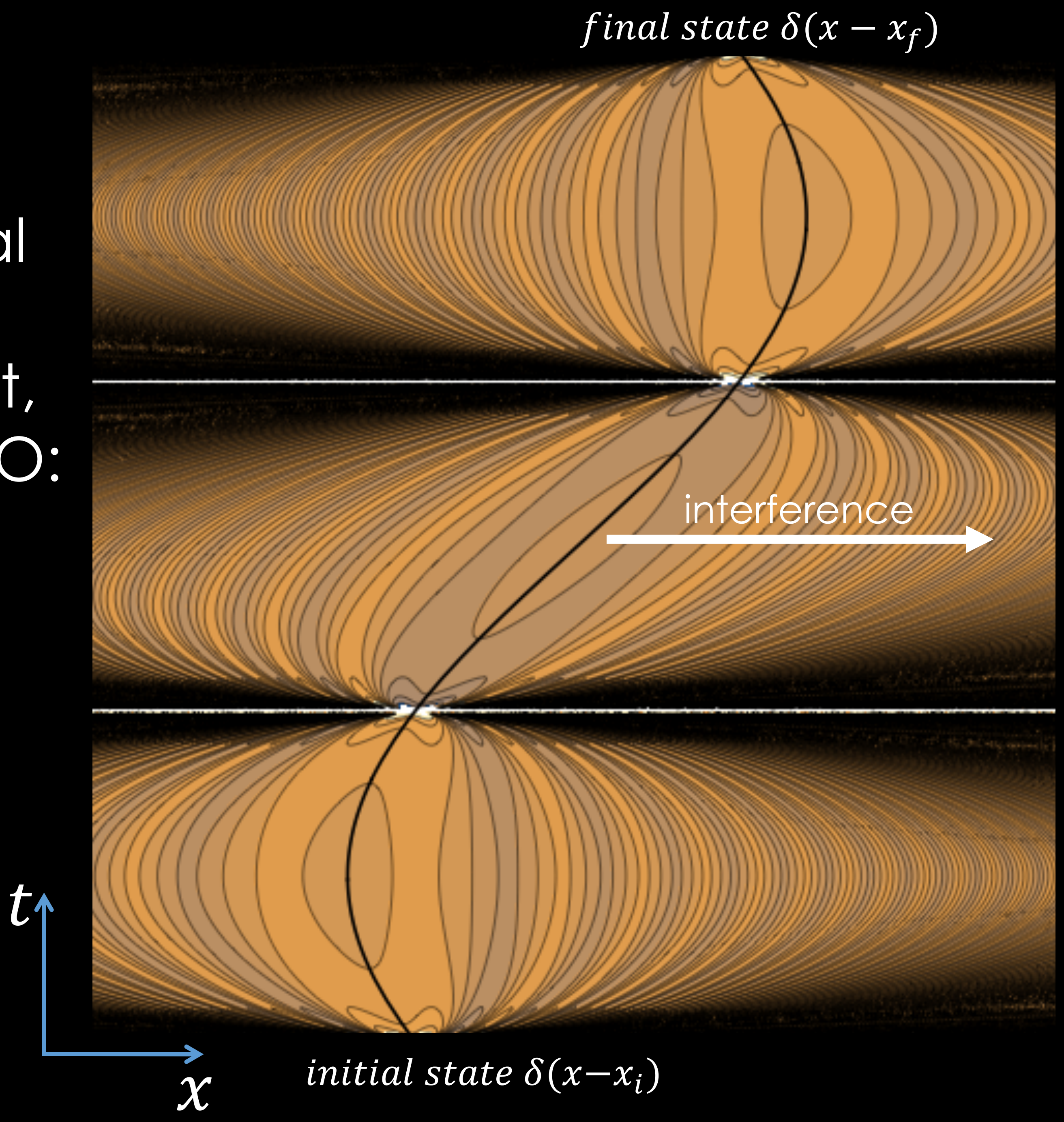
- each subintegral deformed to be absolutely convergent

emergence of spacetime:
look “inside” the path integral

theory of weak measurement,
e.g., SHO:

“weak density”
$\mathrm{Re}[\langle f|\delta(\hat{x}(t) - x)|i\rangle]$

governs the response
of a “weak measuring
device” (von Neumann)



relativistic pair creation in an electric field ("Schwinger process")

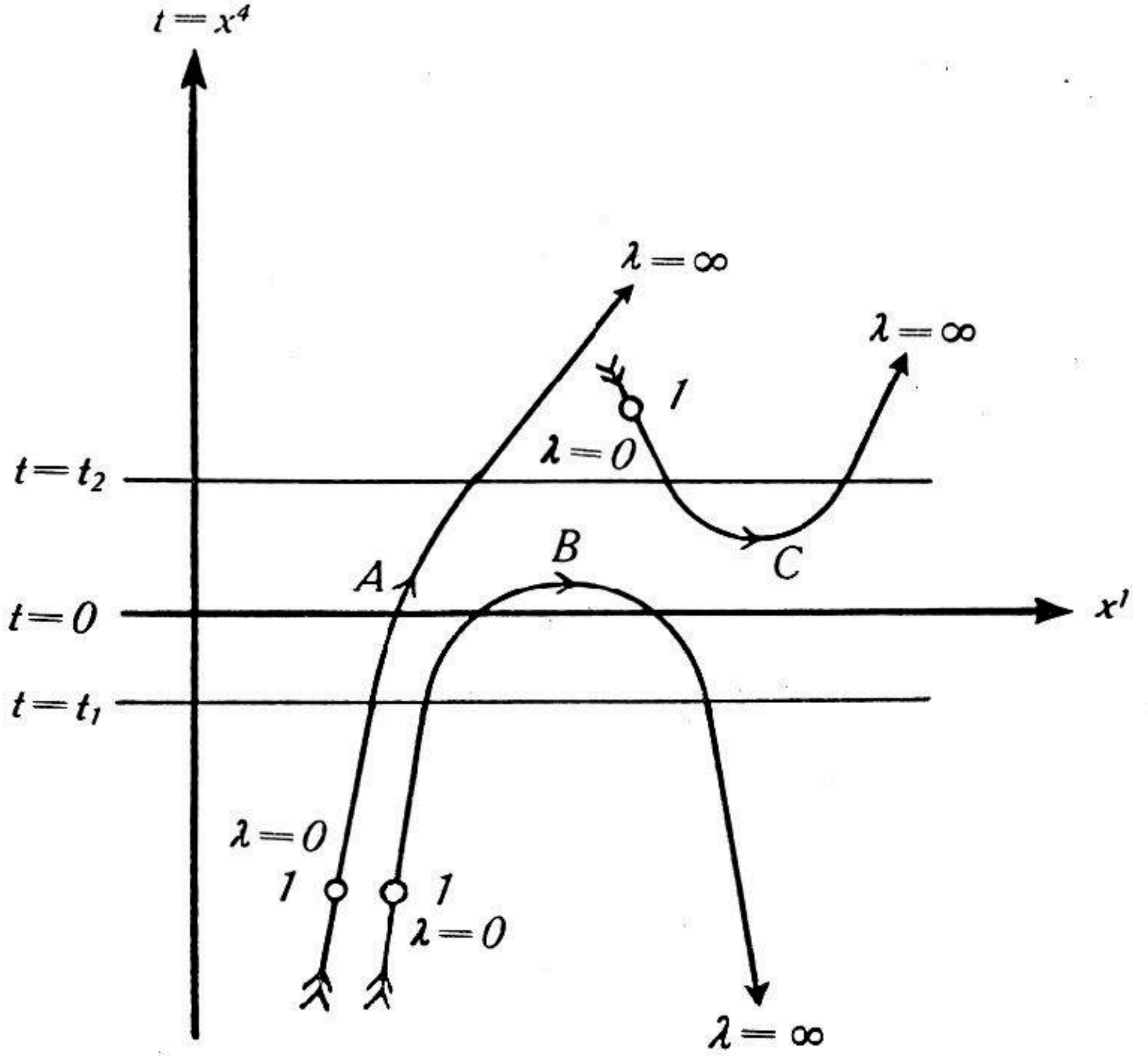
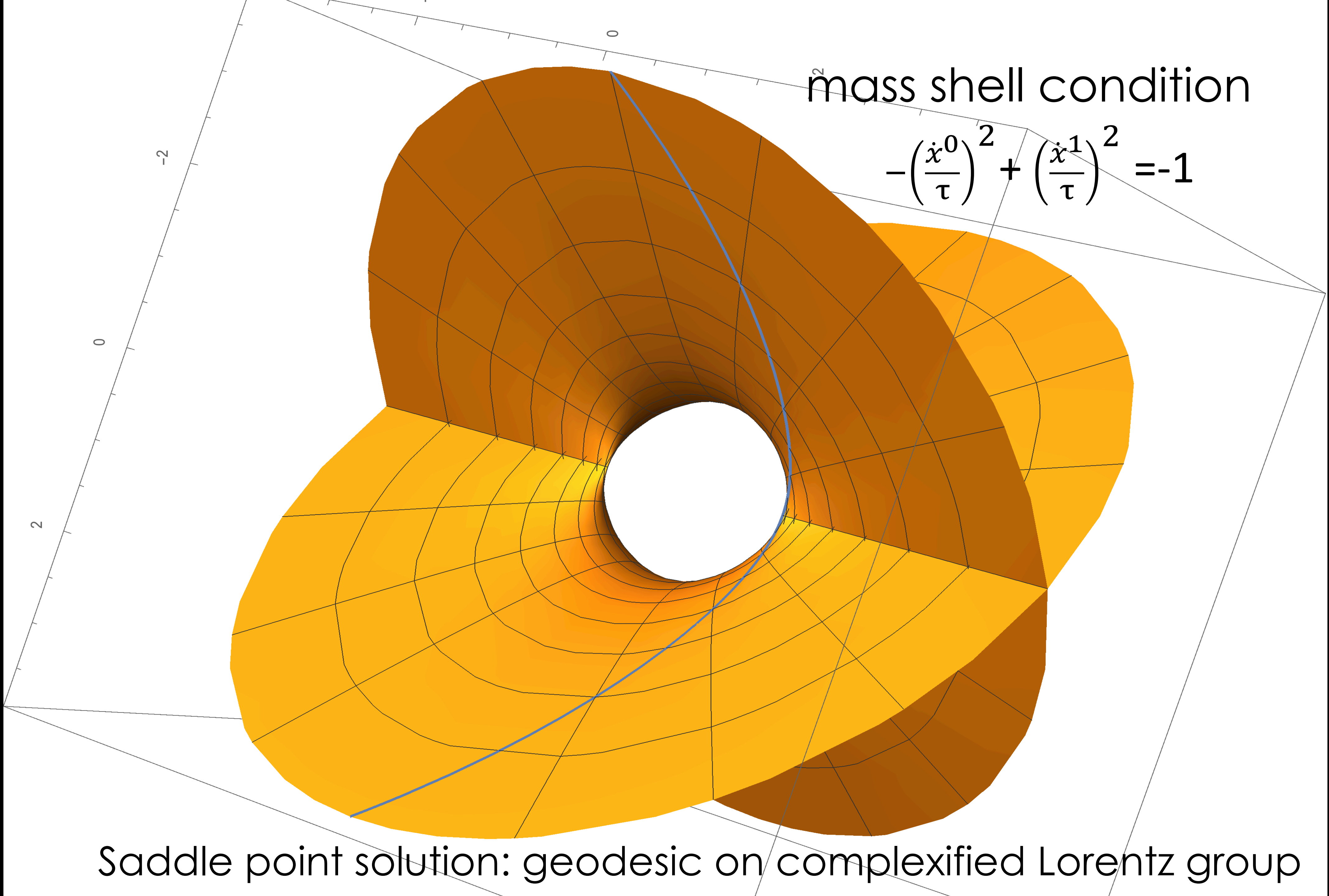


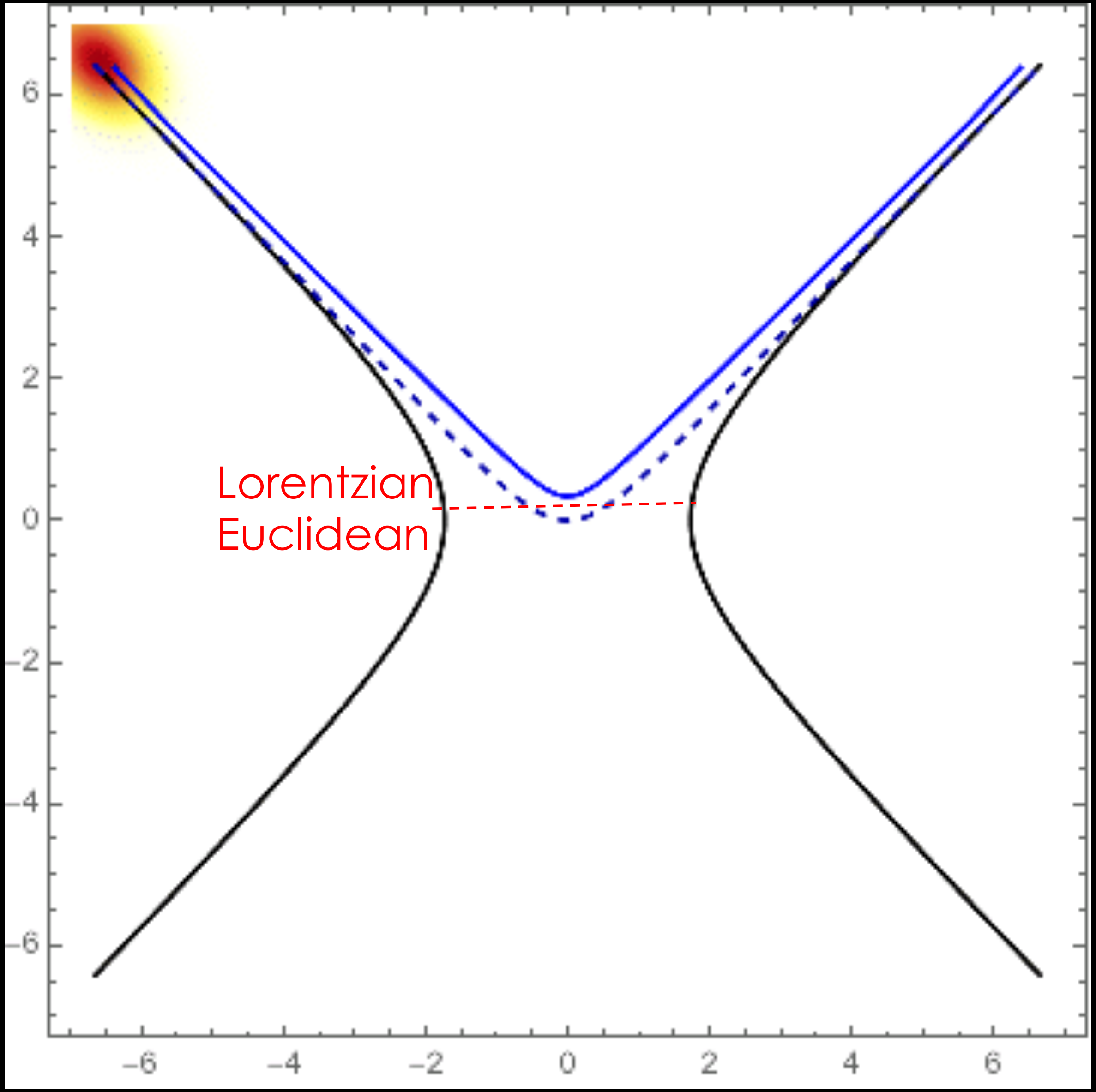
Fig. 1.

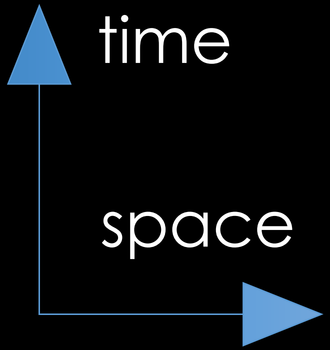
Ernst Stueckelberg 1941


mass shell condition
$-\left(\frac{\dot{x}^0}{\tau}\right)^2 + \left(\frac{\dot{x}^1}{\tau}\right)^2 = -1$
Saddle point solution: geodesic on complexified Lorentz group

Feldbrugge
Fertig
Sberna
NT, to appear

Study the *real, internal dynamics in spacetime* using "weak values"



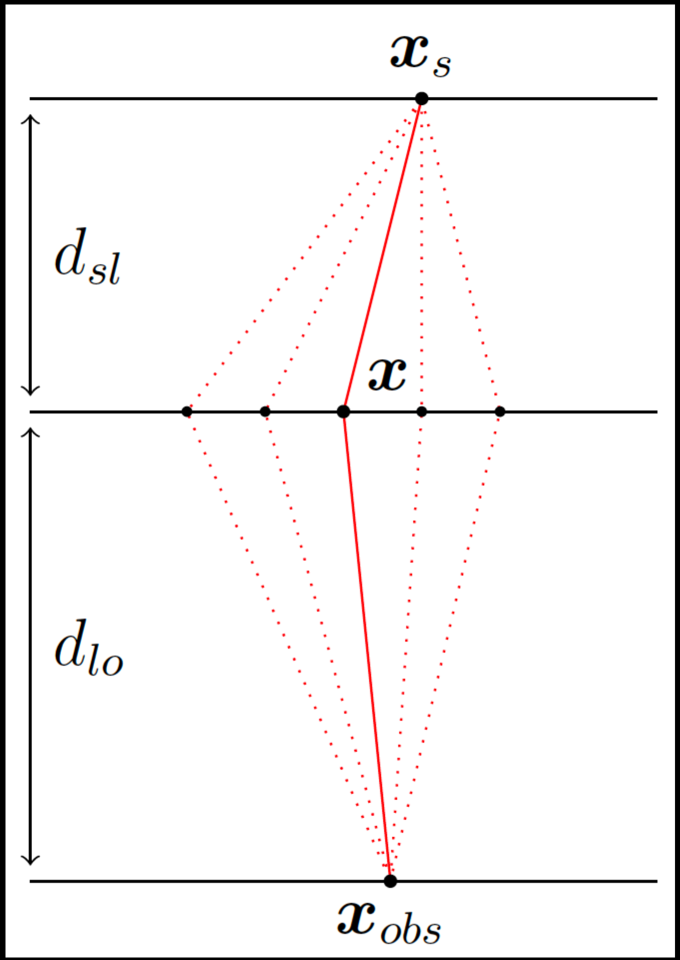


application to radio astronomy

+J. Feldbrugge, U-L. Pen

(1909.04632; 2008.01154)

thin plasma lens



$$\int d\vec{x}_\perp e^{i\omega \int |d\vec{x}| \frac{n(\vec{x})}{c}}$$

Pythagoras

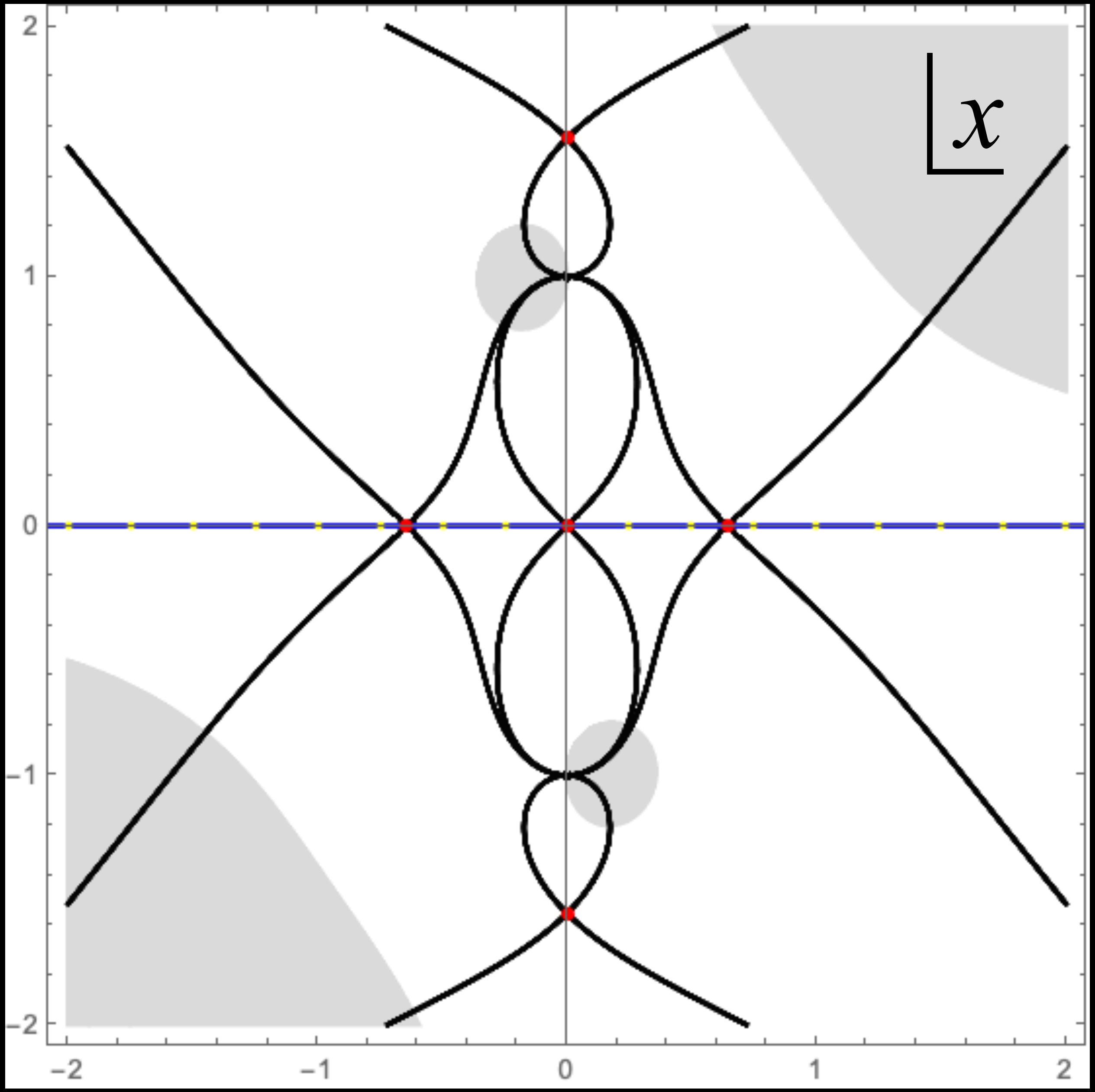
Refraction

$$\approx \int d\vec{x}_\perp e^{i\frac{\omega}{2c}\left[\frac{(\vec{x}_\perp - \vec{\mu})^2}{\bar{d}} - \int dz \frac{\omega_p^2(\vec{x}_\perp, z)}{\omega^2}\right]}$$

lensing strongest at low frequencies

$$\frac{1}{\bar{d}} \equiv \frac{1}{d_{sl}} + \frac{1}{d_{lo}}; \quad n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \; ; \quad \omega_p^2 = \frac{n_e(\vec{x})e^2}{\varepsilon_0 m_e}$$

Flowing
the contour
(case b)



Nakamura–Deguchi 1999

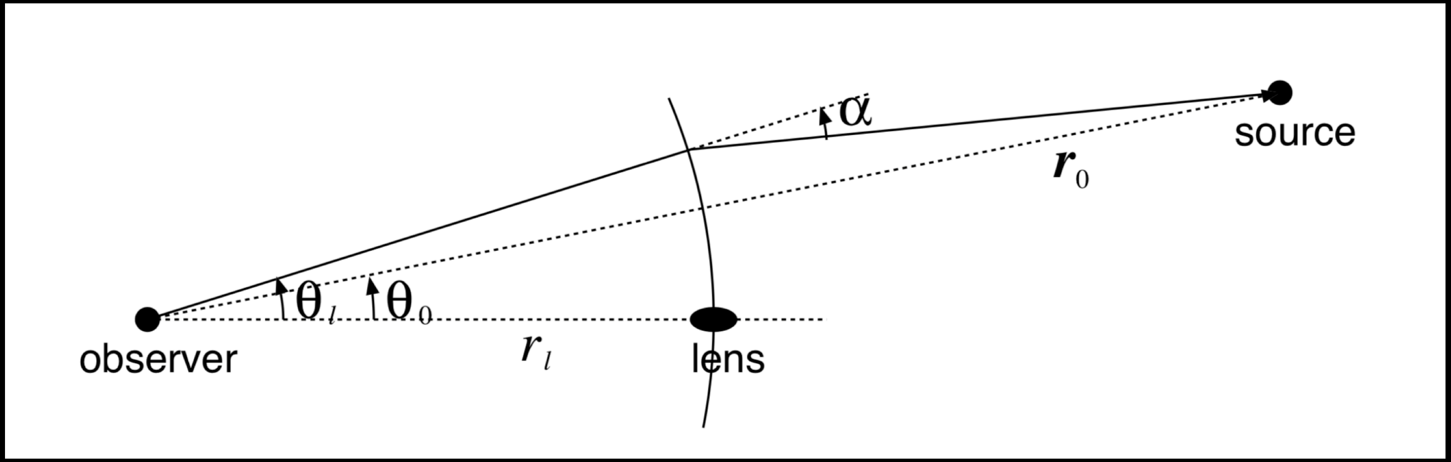
Gravitational Lensing

$$\Psi(\omega,\vec{\mu}) \sim \omega \int d^2\vec{x} e^{i\omega\left[\frac{1}{2}(\vec{x}-\vec{\mu})^2-\phi(\vec{x})\right]} \text{ where}$$

For a point mass in thin lens approx

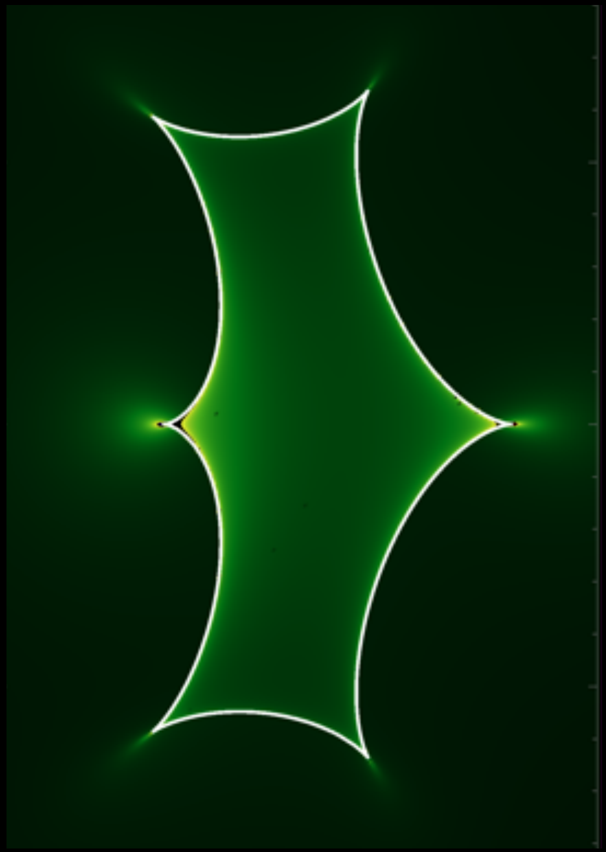
$\phi = \ln(x)$, ω is frequency in units of $r\theta_*^2$,

θ_* is Einstein angle, $\omega = 10^5 \frac{M}{M_\odot} \frac{\nu}{GHz}$

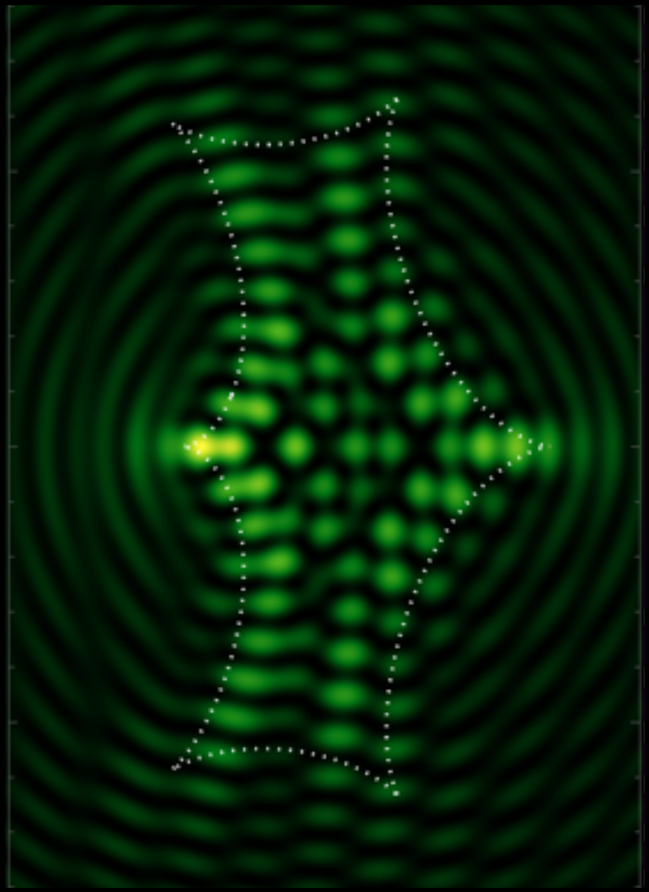


Wave optics effects in microlensing will contain vast information about e.g. lens masses

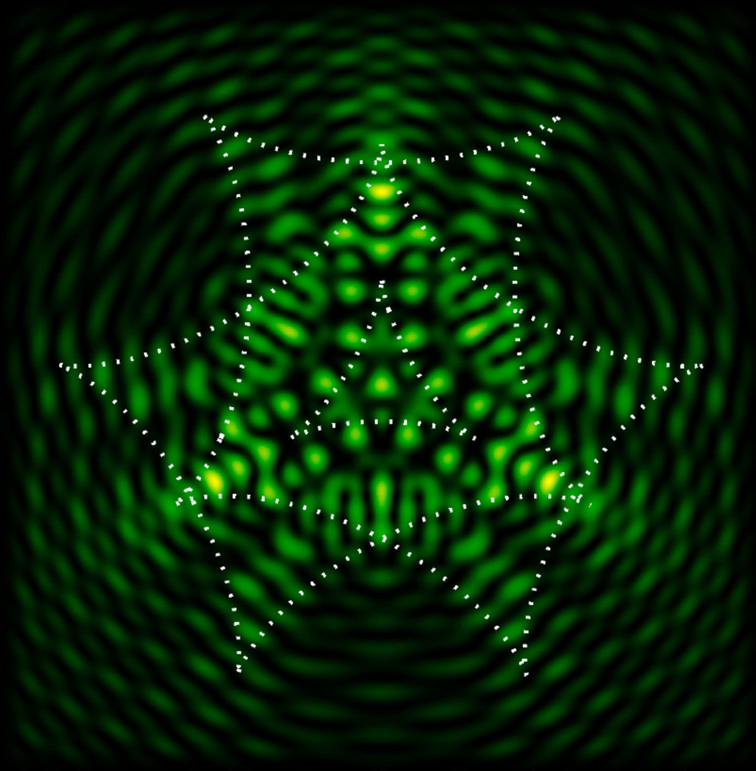
Geometric optics



Wave optics

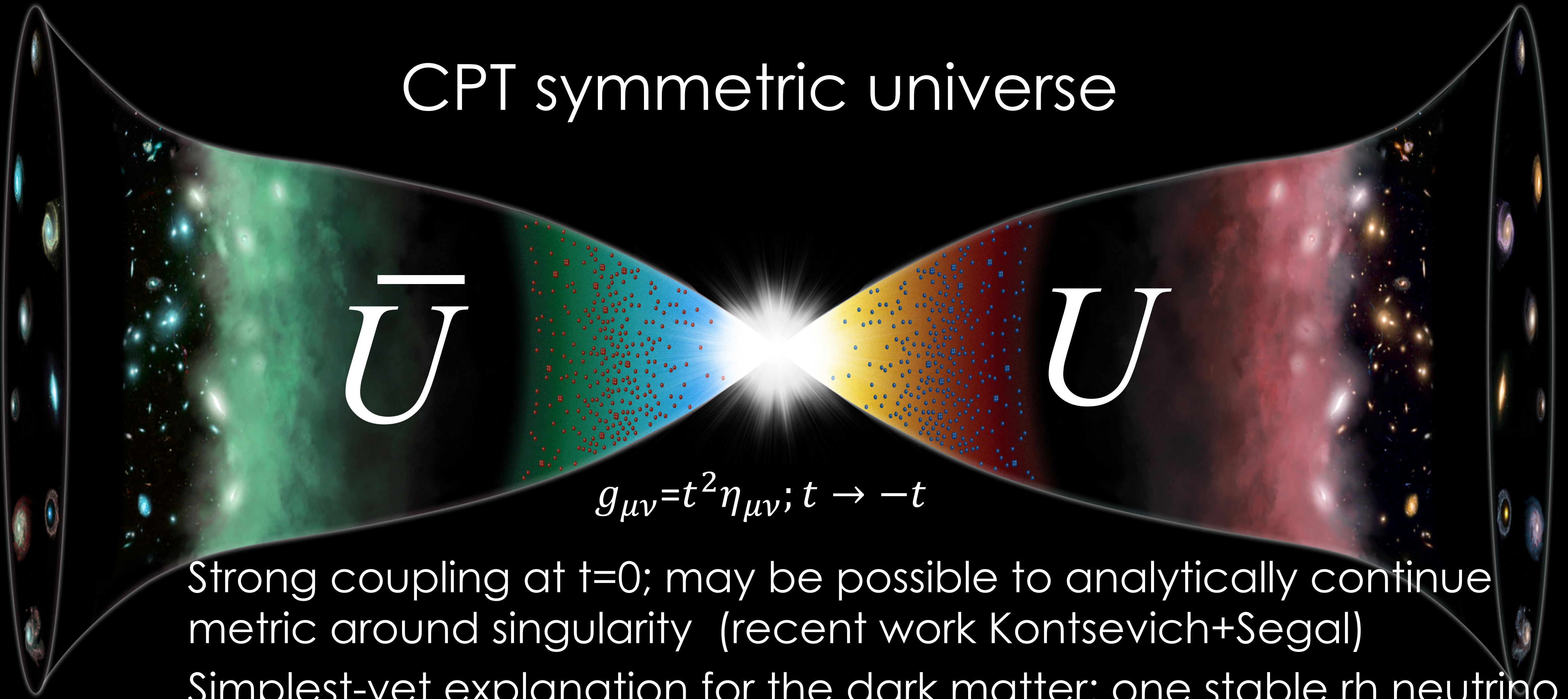


multiple redshifts (ie 3d lens)



w/ J. Feldbrugge (1909.04632; 2008.01154; 2010.03089)

CPT symmetric universe



$g_{\mu\nu}=t^2\eta_{\mu\nu}; t \to -t$

Strong coupling at t=0; may be possible to analytically continue metric around singularity (recent work Kontsevich+Segal)

Simplest-yet explanation for the dark matter: one stable rh neutrino stabilised by Z_2 symmetry $\nu_R^{(1)} \to -\nu_R^{(1)}$; $M_{\nu_R^{(1)}} = 4.8\times 10^8 GeV$

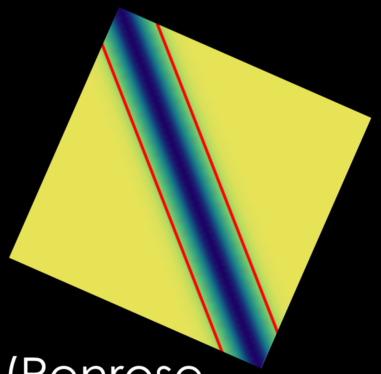
Prediction: lightest ν_L is massless

w/L. Boyle and K. Finn

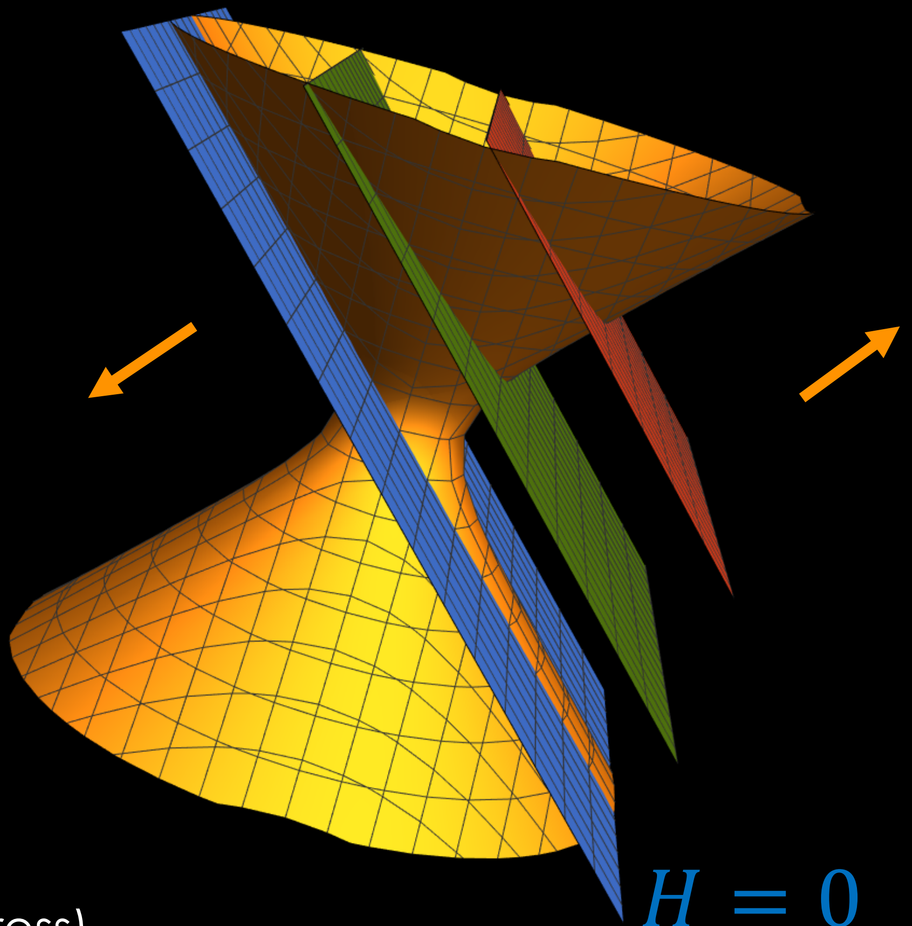
Phys.Rev.Lett. 121 (2018) no.25, 251301

nonsingular flat CPT-symmetric universe

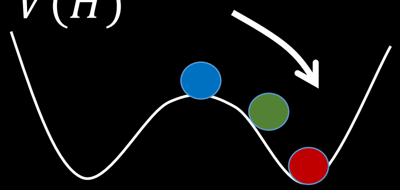
(cf null boundary condition
Aguirre+Gratton 2003)



(Penrose
Diagram)



Flat, radiation-dominated
universe



possibility of obtaining nearly
scale-invariant density variations
from the trace anomaly (in progress)

Next steps:

Prove existence of Lorentzian PI in quantum mechanics

interactions and classical fields in the world-line picture

gravity: cosmology and black holes

Maybe we already know (most of) the fundamental laws:
our job is to do the integrals…

Ваше здоровье, Валерий!