Systematic Uncertainties

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Definition

A definition: Systematics are whatever you still have to do after you have your initial result (but your time is already running out...)

A real definition:
Measurement uncertainty due to uncertainties on external input or due to uncertainties not due to the statistics of your data.

Remarks:
- The term systematic „uncertainty“ is preferred, as your measurement hopefully does not contain errors....
- Often no clear recipies how to determine the systematic uncertainty.
  -> Needs experience from own analyses and closely following (or reading about the details of) other analyses
- Sometimes the value assigned is based on an „educated guess“
  -> Needs „gut feelings“ based on experience

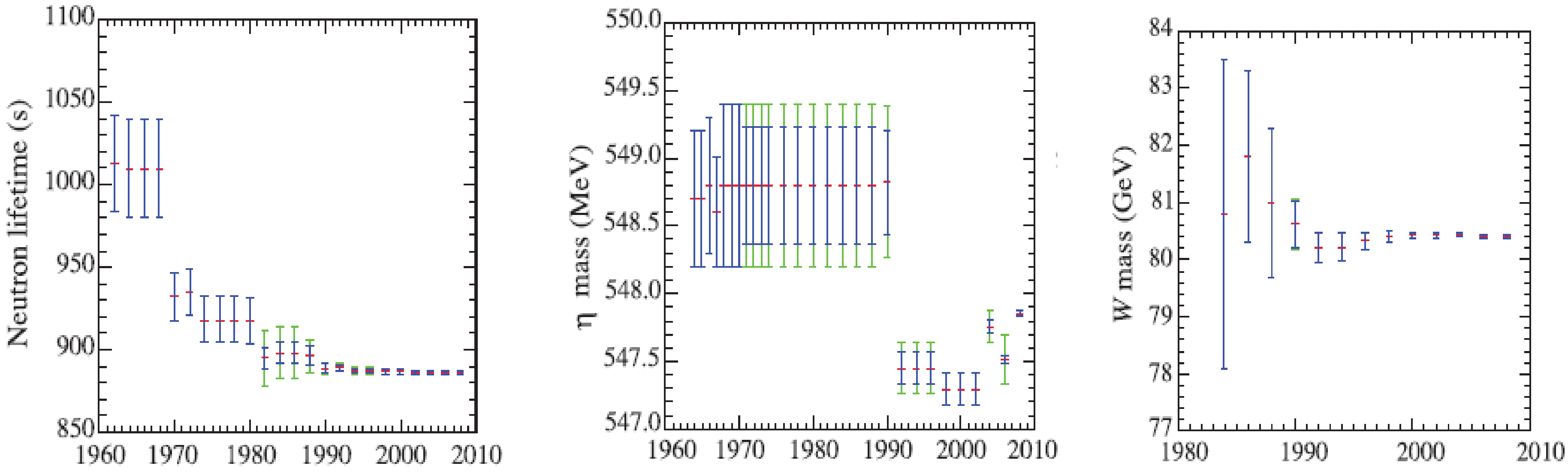
This lecture cannot provide experience,
but hopefully some ideas, strategies...

Examples of systematic uncertainties

- Background
- Acceptance
- Efficiencies
- Detector resolution
- Detector calibration (energy scales)
- MC simulation
- Theoretical models/input
- External experimental parameters (branching ratio,...)
- „External“ parameters (lumi, ...)
- Varying exp. conditions (temperature, air pressure, ...)
- You (the biased experimentalist)
- ....many more....
- And finally....the unknowns

Variation of particle properties with time

PDG:



Some of the (later) results biased by earlier results and thus „similar“?

PDG: „Older data are discarded in favor of newer data when it is felt that the newer data have smaller systematic errors, or have more checks on systematic errors, or have made corrections unknown at the time of the older experiments, or simply have much smaller errors.“

Outline

- Definition (done)
- The (sometimes) fine line between statistical and systematic uncertainties
- Some examples:
  - Avoiding systematic uncertainties
  - Detecting systematic uncertainties
  - Assigning systematic uncertainties

Statistical or systematic uncertainty

Example:
Your W-> l ν analysis: $$\sigma = \frac{N - N_{BG}}{Acc \cdot efficiency \cdot L}$$

he efficiency: Statistical or systematic uncertainty?

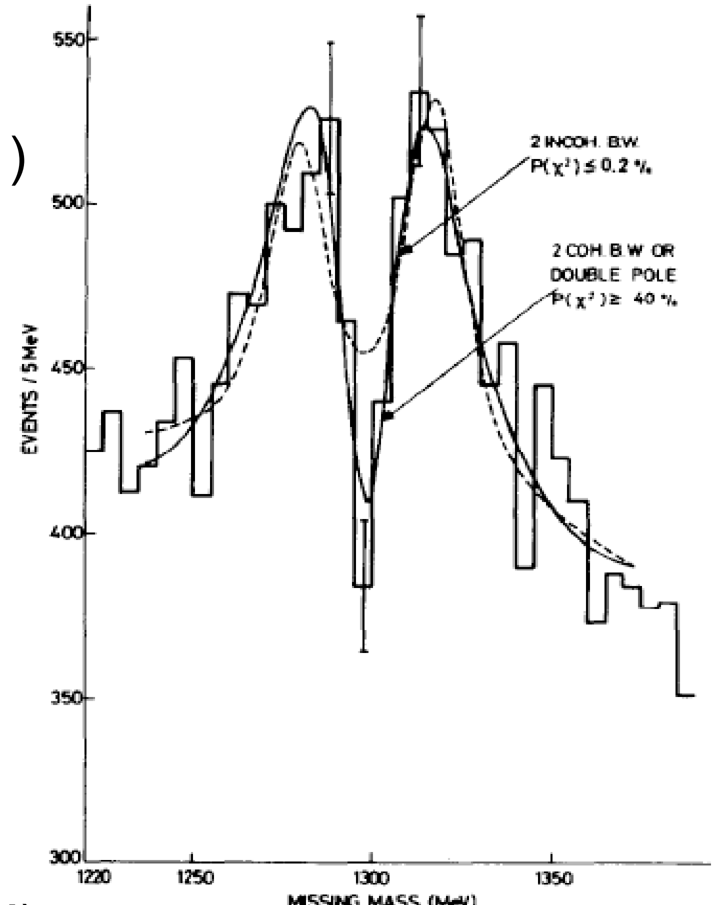
1) In the beginning, you might have to get the efficiency from MC
   -> systematic uncertainty
2) More data arrives: Your friendly colleague gives you a first lepton efficiency based on data from his Z studies
   (cross section of W order of magnitude bigger than cross section of Z)
   -> not truely „external“ parameter (correlated) -> assign as statistical uncertainty
3) Some decent data set available:
   The efficiency from the Z studies by now has a small statistical uncertainty:
   stat. << sys. unc. inherent in your colleagues method or/and
   stat. << sys. unc. arising from the fact that his efficiency maybe does not neccessarily apply exactly to your case
   -> systematic uncertainty
4) Somewhere in between 2) and 3) you have to consider a systematic and a statistical component from your efficiency to your overall uncertainty

Avoiding systematic uncertainties

- Biased experimentalist
  - Don‘t tune your cuts by looking at your signal region
  - Tune cuts in background region, on different channel, on MC, ...
  - „Blind analysis“: Part of data is covered (or modified) until all the analysis is fixed
- Acceptance, MC, Background,...
  - Is your cut really needed or does it have large overlap with other cuts? Fewer cuts are usually better....
  - Don‘t use cuts that are not well modeled in MC (if relying on MC), usually better to live with more but well known background (e.g., acceptance from MC for cross section measurement)
- The unknowns
  - Find the unknowns by talking to (more experienced) colleagues

Example: Biased experimentalist

- Cern 1967: Report of narrow dip (6 standard deviations) in the A2 resonance
- Next: Other experiments also report dip ($< 3\sigma$) (suspicion: some that were also looking but did not see anything did not report on it?)
- Later: dip disappears with more data



What has happened:

- A dip in an early run (statistical fluctuation) was noticed and suspected to be real
- Data was looked at as they came in...and was checked for problems much more carefully/strict when no dip showed up (if you look long enough you will (always) find a problem, especially in early LHC running!)

Initial statistical fluctuation became a significant false discovery!

Outlier/data rejection: The textbook

- Chauvenet‘s criterion: Reject data point if:

  probability x N (number of data points) < 0.5
- Example: 8 values taken with one being 2σ (5% probability) away from the mean

  -> 0.05 x 8 = 0.4 -> reject

  In other words: up to 10 events -> reject outside 2 sigma
- Only works for gaussian distribution. One often has tails...
- Only good for the case of exactly one outlier...
- Probablility < 0.5 x 1/N ....why 0.5?
- Having a prescription does not mean that one can blindly follow it ....
- No generally applicable/valid prescription for data rejection.
- This textbook example is not commonly used

Outlier/Data rejection: The reality

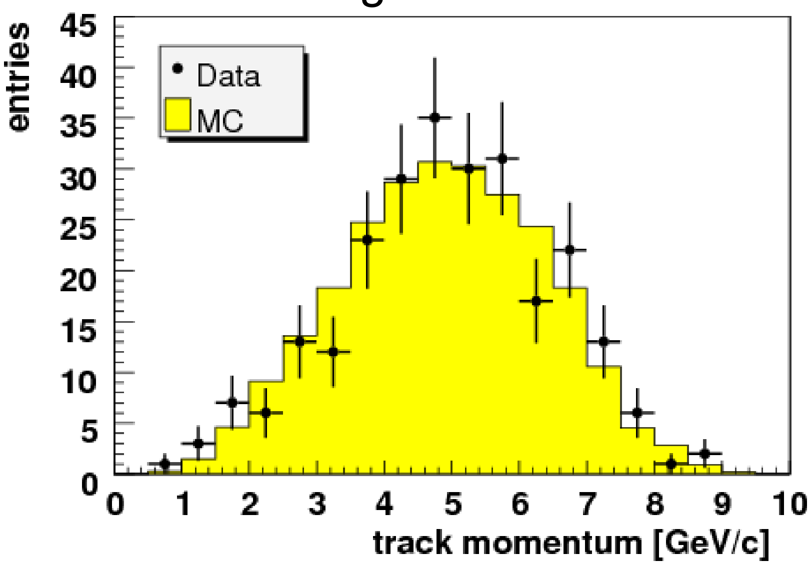
- Quality of early LHC data will be questionable and be taken under rapidly changing conditions

  -> Will have to reject data, but be careful
- Try to understand why the data was an outlier
- Have external reasons for cutting data
- Pay attenion: Dou you only start searching for problems because you have a result you did not expect? -> self-biased experimentalist
- Dont let your result „make“ the (cut) selection -> very much self-biased experimentalist

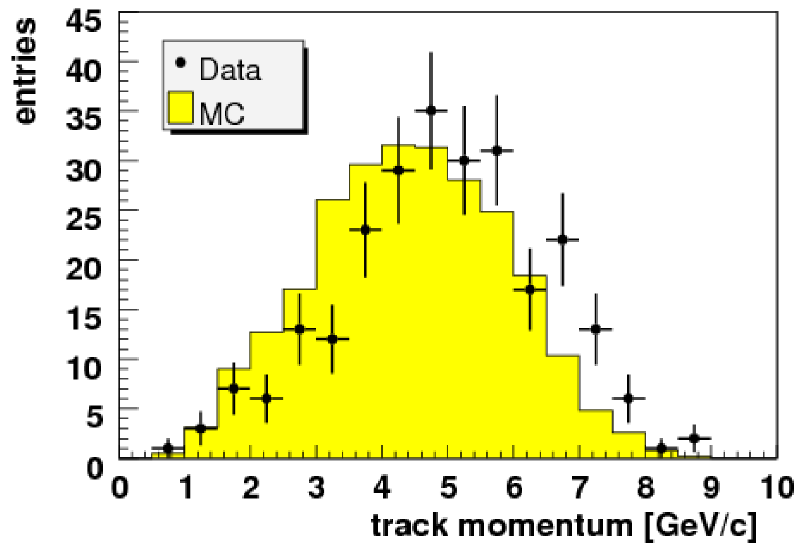
Detecting systematic uncertainties

Example: Data-MC comparison -> look at all possible variables

good



bad



Most problems can be seen by eye

Note: MC (no stat. unc. shown) should always have negligible statistical uncertainty compared to the one from data.

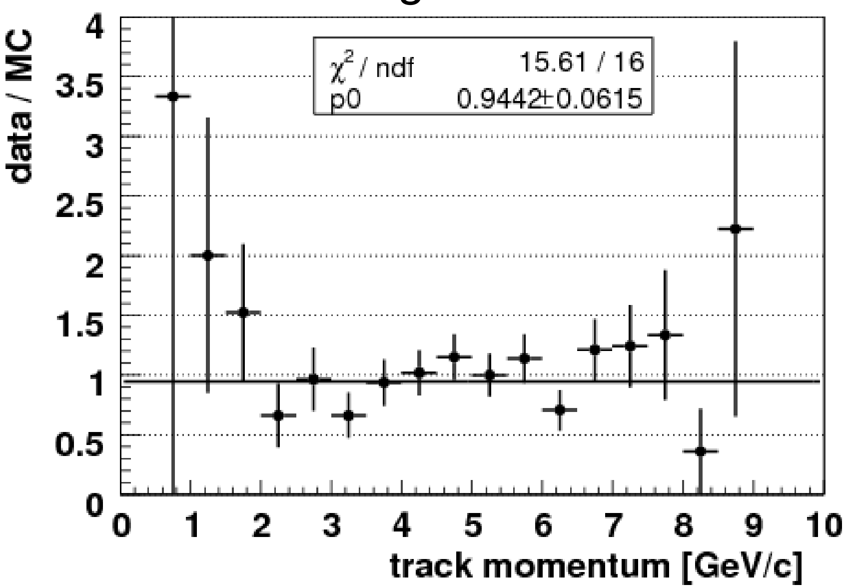
-> An uncertainty should never arise from limited MC statistics.

-> Generate at least 10 times more MC data than you have real data.

.....likely difficult at LHC......

Divide data by MC

good



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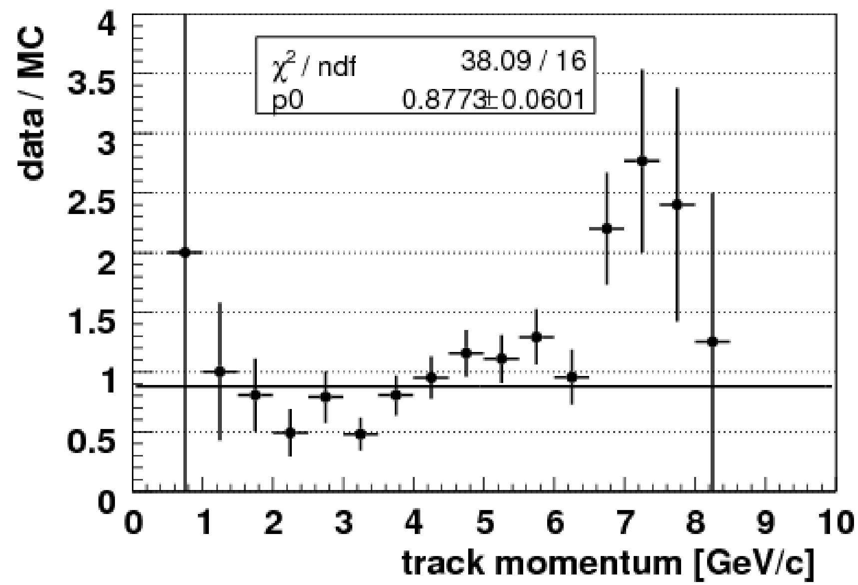


- Deviations better visible when plotting data/MC
- Significance of disagreement:
  -> Fit a constant line, check χ^2 /dof

Stability of result

- Result stable over time?

  - compare results for different time periods, e.g., before and after shutdown, or change of beam conditions, or change of detector setup, day and night (temperature), nice weather versus bad weather (air pressure), ...
- Results stable in different detector areas (if symmetric) ?

  - upper half versus lower half?

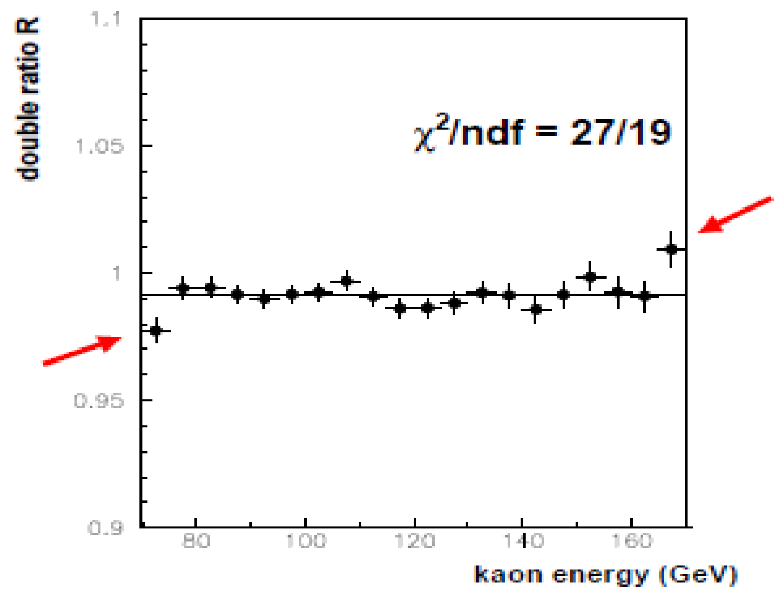
  - forward versus backward (if no physics reason)?
- Result stable using different methods?

  - when you have two methods that should give the same result you should do them both
- Result stable as function of analysis variables? ->

Example: CP violation @NA48

Double ratio of decay widths:

$$R = \frac{\Gamma(K_L^0 \quad \pi^0\pi^0)}{\Gamma(K_S^0 \quad \pi^0\pi^0)} / \frac{\Gamma\,(K_L^0 \quad \pi^+\pi^-)}{\Gamma\,(K_S^0 \quad \pi^+\pi^-)}$$



<u>Analysis in bins of kaon energy:</u>

-> Disagreement at the edges.
No reason for this behavior found.

How bad is it?

⋗ χ^2 / DOF = 27/19 ....and how bad is that?

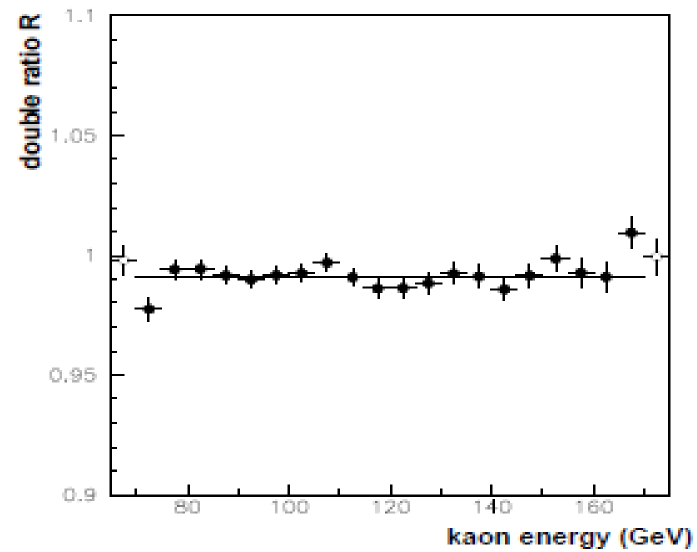
Rough estimate: $\sigma_{\chi^2 distribution} \quad \sqrt{2 \quad n_{dof}} \quad 6.2$ -> 1.3 σ effect

Better estimate: Probability (27,19) = 10,5 % [Root: TMath::Prob(27,19)]

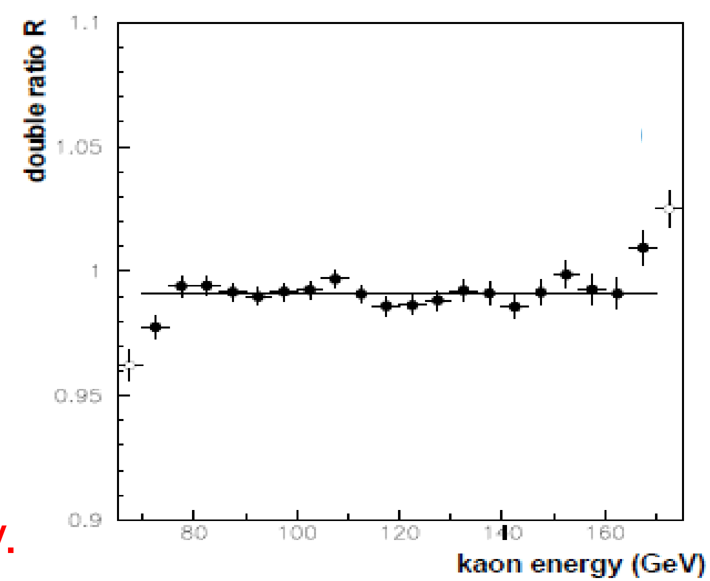
Not really unlikely to be statistical fluctuation if it weren‘t the outermost guys....

How to check...?

<u>How can one check?</u>
-> Enlarge test region if possible...
-> Additional bins okay
-> no systematic uncertainty assigned



<u>Hypothetical question:</u>
If it had looked like that ->
...Now you have to understand the effect



<u>Then:</u>
Did you understand it -> Can you correct for it?
If not, do one of the following:

- Discard outer bins if independent information justifies this.
- Last resort: Determine systematic uncertainty.

HowTo assign systematic uncertainties

Simplest case:

Uncertainty (standard deviation) on parameter x (branching ratio, ...) is known.

-> Vary x by σ_x -> result varies by σ_{result}

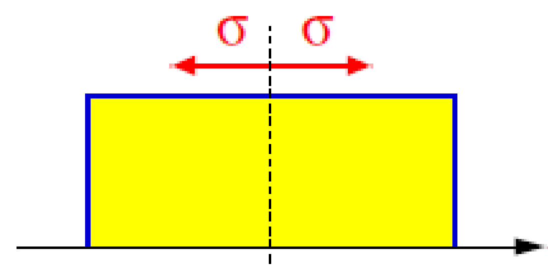
Still easy:

Possible range for input parameter x (min. x and max. x) is known.

-> Assume uniform probability over full range (if reasonable).

-> $\sigma_x = \frac{1}{\sqrt{12}}(x_{\max} - x_{\min}) - 0.3(x_{\max} \quad x_{\min})$

(„Gain“ of 60% compared to naive $\sigma_x = 0.5(x_{\max} - x_{\min})$)



Example: You measure an asymmetry A = (B-C) / (B+C). The asymmetry is due to the asymmetry from your signal and your background process:

$$A_{meas} = f_{sig} A_{Sig} + f_{BG} A_{BG}$$

In case you have no idea about the background asymmetry, it still is bound to [-1,1]. → $\sigma_{A_{BG}} = \frac{2}{\sqrt{12}}$

Cut variations:

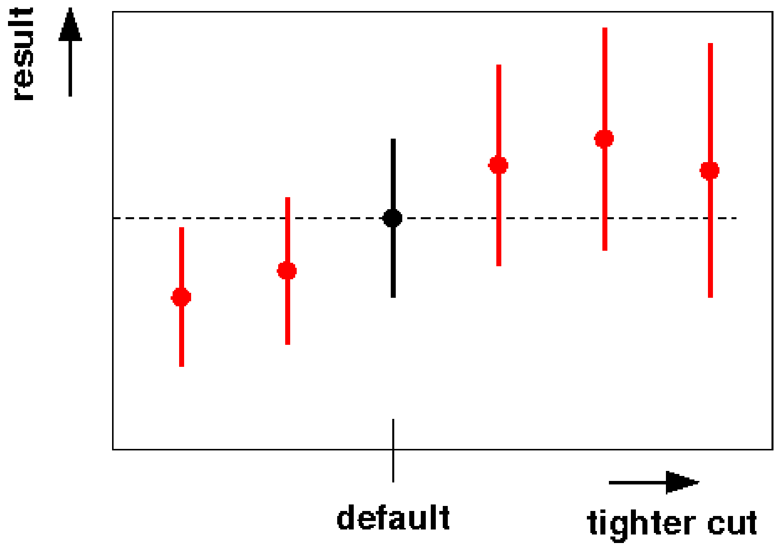
Cut variations commonly used to check stability of result.

But: Difficult to learn something from the result!

Usually not a good way to determine systematic uncertainty

Two possibilities:

1. Result is stable -> good, done
2. Result is not stable
   - -> will not tell you why
   - -> cannot just assign sys. unc.
   - -> look at underlying distributions



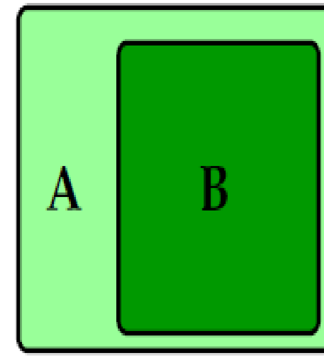
In most cases systematic uncertainty can only be assigend if reason for variation is „understood“.
(but if reason understood there might be better ways than assigning sys. unc.)

But first, how to work with cut variations ->

Correlated Data sets

Cut variations usually lead to fully correlated data sets:

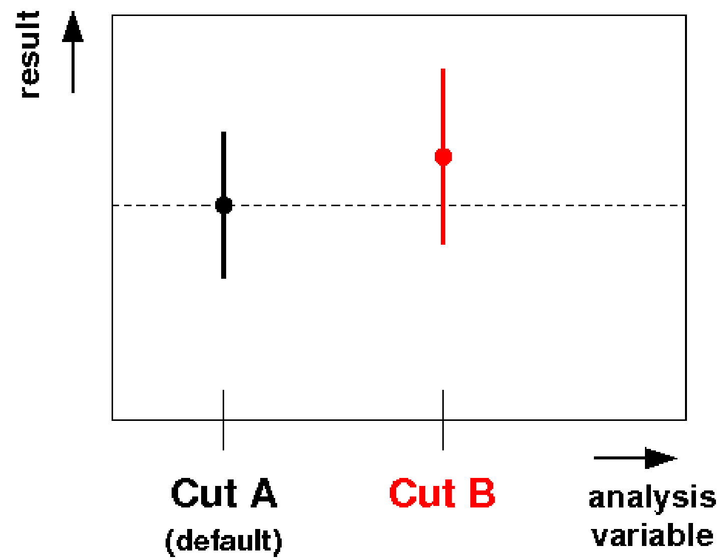
Default cut: sample A with result $x_A \quad \sigma_A$
Tighter cut: sample B, fully contained in A , result $x_B \quad \sigma_B$
-> correlated errors, i.e., stat. unc. not meaningful
-> Significance of difference?



➔ Consider sample C = A w/o B
Use standard prescription for averaging results (weighted average):

$$\bar{x} = x_A = \sum_i \frac{x_i}{\sigma_i^2} \Big/ \sum_i \frac{1}{\sigma_i^2} = \frac{x_B/\sigma_B^2 + x_C/\sigma_C^2}{1/\sigma_B^2 + 1/\sigma_C^2}$$

$$\sigma_{\bar{X}}^2 = \sigma_A^2 = 1 \Big/ \sum_i \frac{1}{\sigma_i^2} = \frac{1}{1/\sigma_B^2 + 1/\sigma_C^2}$$

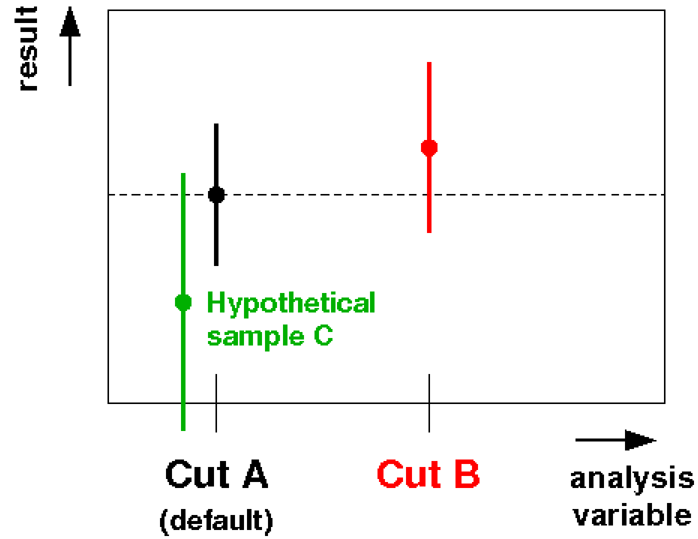


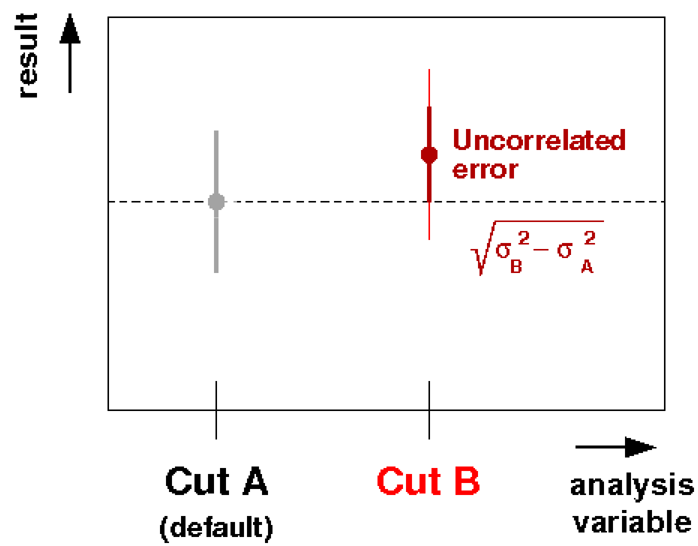
Uncorrelated error

$$\sigma_A^2 = \frac{1}{1/\sigma_B^2 + 1/\sigma_C^2} \quad \rightarrow \quad \frac{1}{\sigma_C^2} = \frac{1}{\sigma_A^2} - \frac{1}{\sigma_B^2}$$

Stat. unc. meaningful

$$x_A = \frac{x_B/\sigma_B^2 + x_C/\sigma_C^2}{1/\sigma_B^2 + 1/\sigma_C^2} \quad \rightarrow \quad x_C = \frac{x_A/\sigma_A^2 - x_B/\sigma_B^2}{1/\sigma_A^2 - 1/\sigma_B^2}$$



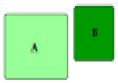
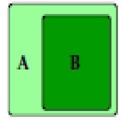
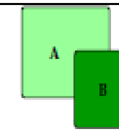


Significance of the difference:

$$\frac{x_C - x_B}{\sqrt{\sigma_B^2 + \sigma_C^2}} = \ldots = \frac{x_A - x_B}{\sqrt{\sigma_B^2 - \sigma_A^2}}$$

$$\sigma_{uncorrelated}^2 = |\sigma_B^2 - \sigma_A^2|$$

A useful table

 Independent		 Completely Correlated		 Partially Correlated	
f	σ_f	f	σ_f	f	σ_f
$E_A - E_B$	$\sqrt{\sigma_A^2 + \sigma_B^2}$	$E_A - E_B$	$\sqrt{\lvert\sigma_A^2 - \sigma_B^2\rvert}$	$E_A - E_B$	$\sqrt{\sigma_A^2 + \sigma_B^2 - 2\frac{\sigma_A^2\sigma_B^2}{\sigma_{A\cap B}^2}}$
$\frac{E_A - E_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}$	1	$\frac{E_A - E_B}{\sqrt{\lvert\sigma_A^2 - \sigma_B^2\rvert}}$	1	$\frac{E_A - E_B}{\sqrt{\sigma_A^2 + \sigma_B^2 - 2\frac{\sigma_A^2\sigma_B^2}{\sigma_{A\cap B}^2}}}$	1
$\frac{E_A}{E_B}$	$\frac{E_A}{E_B}\sqrt{\frac{\sigma_A^2}{{E_A}^2} + \frac{\sigma_B^2}{{E_B}^2}}$	$\frac{E_A}{E_B}$	$\frac{E_A}{E_B}\sqrt{\frac{\sigma_A^2}{{E_A}^2} + \frac{\sigma_B^2}{{E_B}^2} - \frac{2}{E_A E_B}\sigma_A^2}$	$\frac{E_A}{E_B}$	$\frac{E_A}{E_B}\sqrt{\frac{\sigma_A^2}{{E_A}^2} + \frac{\sigma_B^2}{{E_B}^2} - \frac{2}{E_A E_B}\frac{\sigma_A^2\sigma_B^2}{\sigma_{A\cap B}^2}}$

Lara De Nardo, HERMES internal note

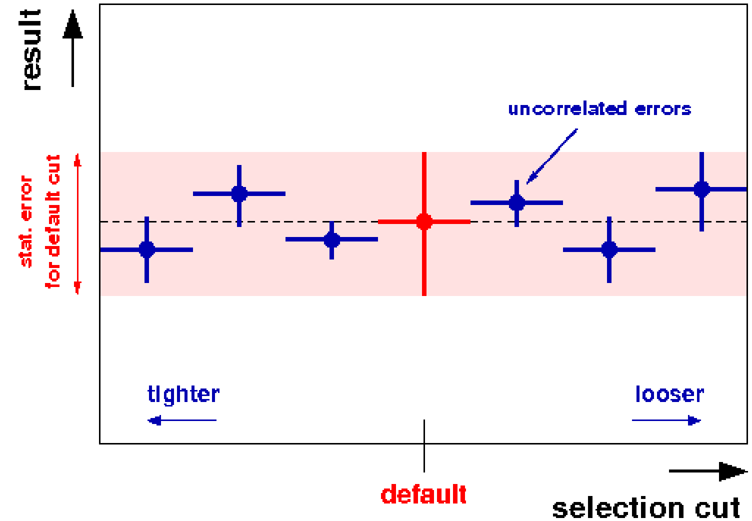
Now look at the cut variations again

Using the uncorrelated errors we can now judge on the significance of the difference.

Significance of difference at most 1σ

-> usually no sys. unc. should be applied

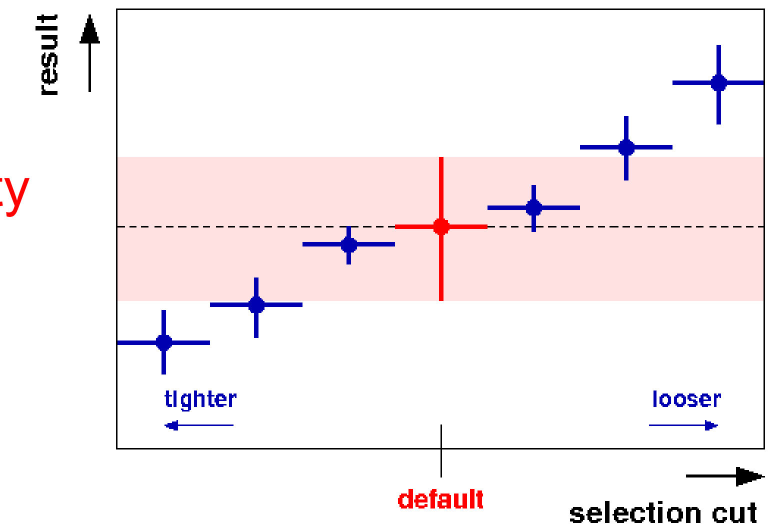
Don‘t be conservative (in order to hide possibly undetected other issues?)



Impossible to assign systematic uncertainty

-> Effect has to be understood

-> check underlying distributions



More possible scenarios

Variation should be understood:

-> check underlying distributions

-> what happens for tighter/looser cuts?

....try hard......

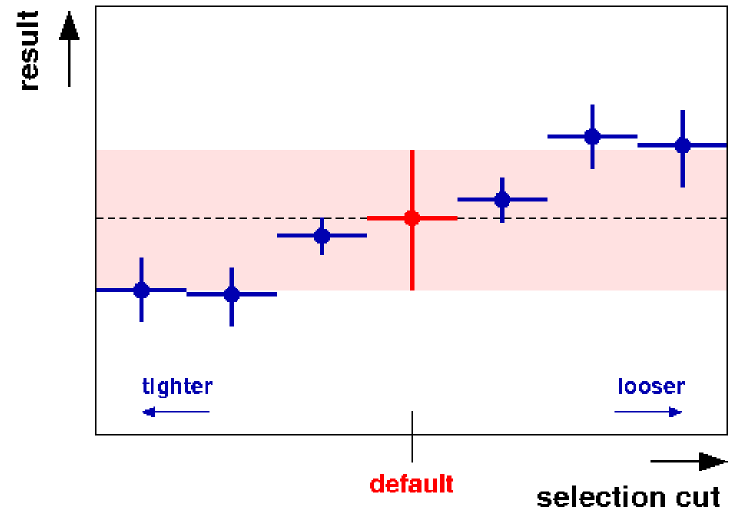
....harder........

...If clueless at the end of the day

-> variation ~ systematic uncertainty

Not nice: sys. unc. ~ stat. unc.

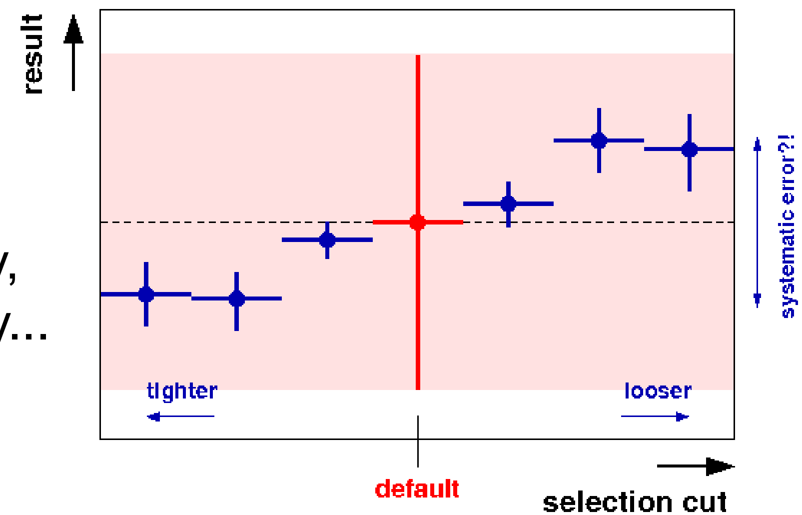
-> large contribution to overall uncertainty



Same as above applies, but:

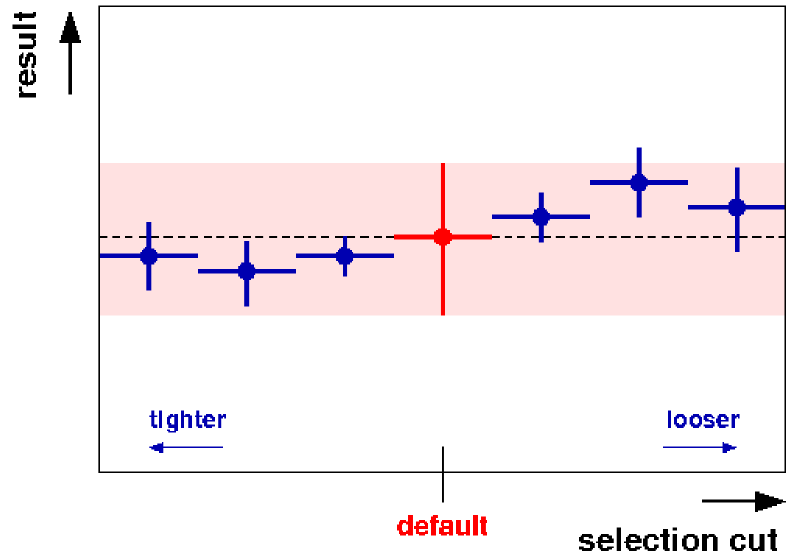
systematic uncertainty << statistical uncertainty,

-> don‘t try too hard if you have bigger fish to fry...



Cut Variations: Examples

Tricky: can be statistical fluctuation or systematic effect
-> look at underlying distributions
-> check with even looser/tighter cuts
-> ....
-> If you find nothing (else) suspicious,
be bold -> no systematic uncertainty.
In case of doubt -> variation ~ sys. unc.



Summary:

- Cut variations are usually only useful to check the stability of your result
- When using cut variations, pay attention to correlated data sets and calculate significance of difference
- If your result is not stable, find the reason...and don‘t just assign the difference as systematics!

Small statistics

Data-MC comparison with small statistics:

- Systematic differences can be hidden by stat. uncertainties
- Multidimensional (as function of correlated variables at the same time) comparison not possible, also no fine binning
- Statistical fluctuation can fake systematic effects

Can you enlarge the data sample?

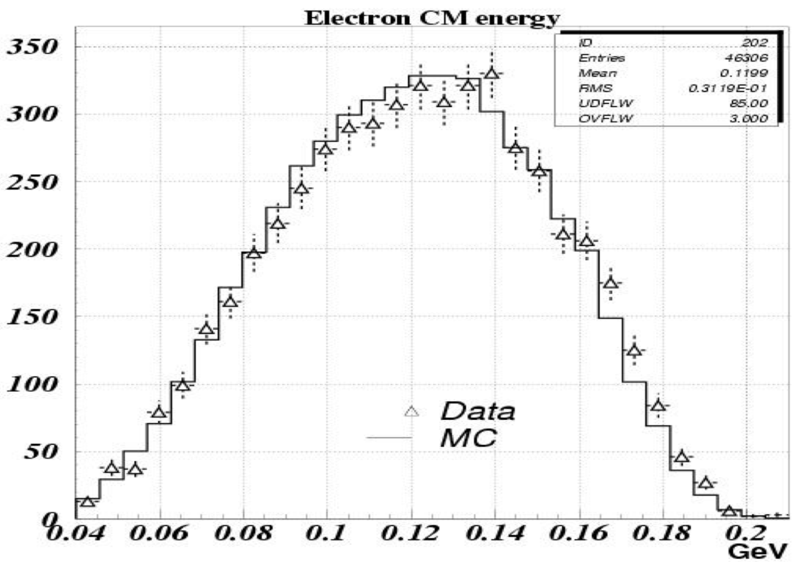
- Release cuts (-> enlarge background)
- Look at different (control) channel -> next example

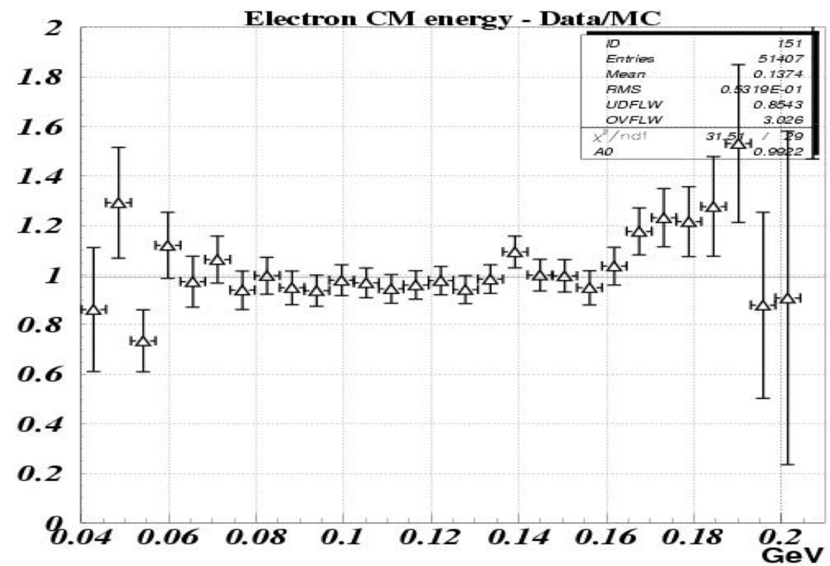
In general, be careful:

- Is the additional data representative (different kinematic region, channel, ...) ?
- Extrapolating to your area of interest might involve additional uncertainties, especially if your signal sits in a tail.....

Data-MC comparison

Example: Rare decay: $K^+ \quad \pi^0 e^+ \nu \gamma$



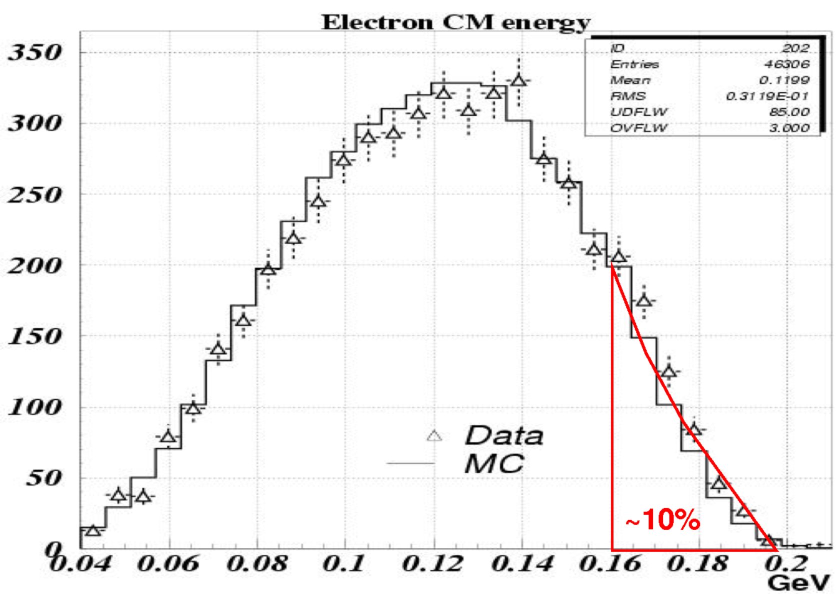


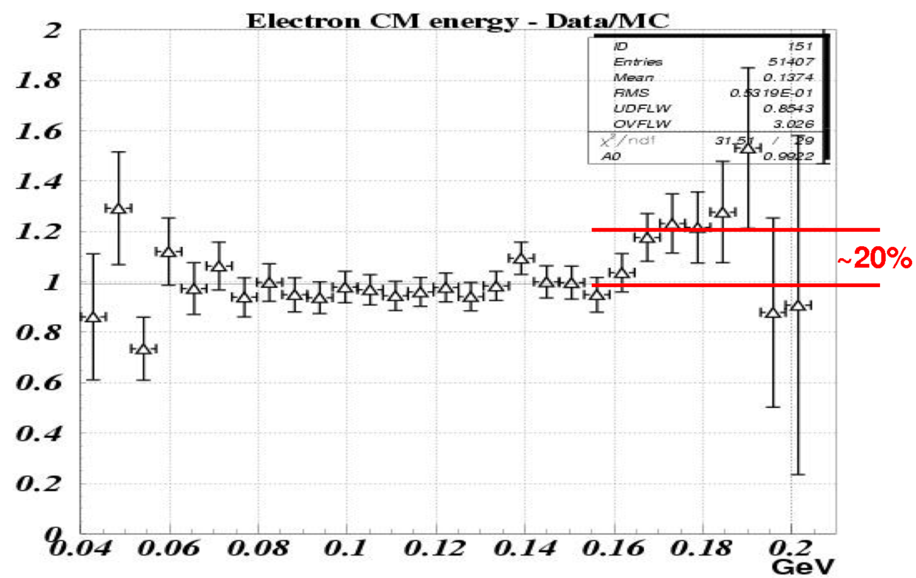
χ^2 / DOF = 31.5/29 is okay, but obvious disagreement beyond 0.16 GeV

Unable to find source for this ->

Estimate systematic uncertainty

....estimate systematic uncertainty:





Estimate systematic uncertainty:

- 10 % of all data above 0.16 GeV
- 20 % more data than MC above 0.16 GeV

-> Systematic uncertainty of +/- 2% on decay rate

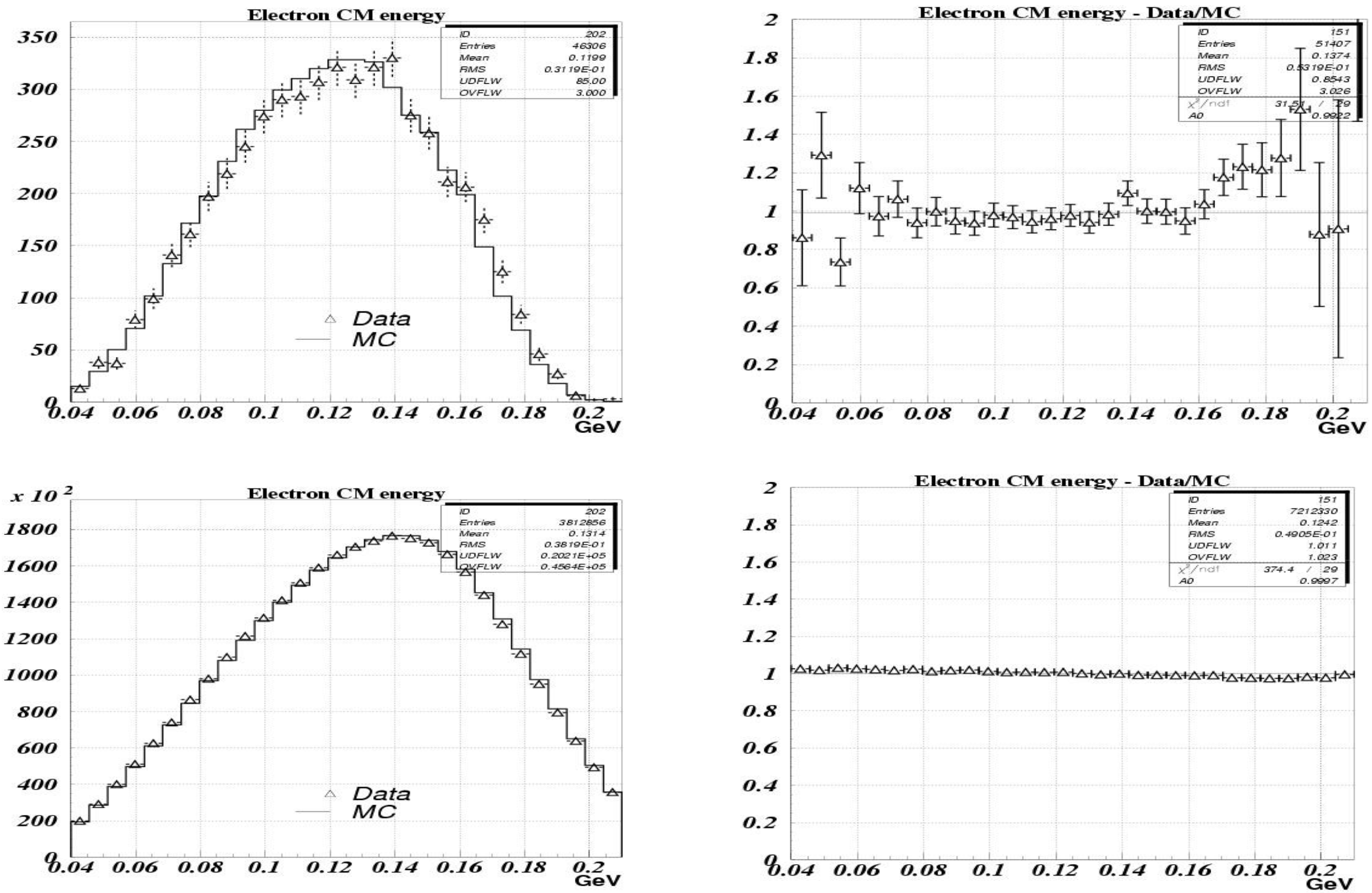
-> Largest single uncertainty in analysis

-> Try to do better...

Look at control channel

$K^+ \quad \pi^0 e^+ \nu\gamma$

$K^+ \quad \pi^0 e^+ \nu$



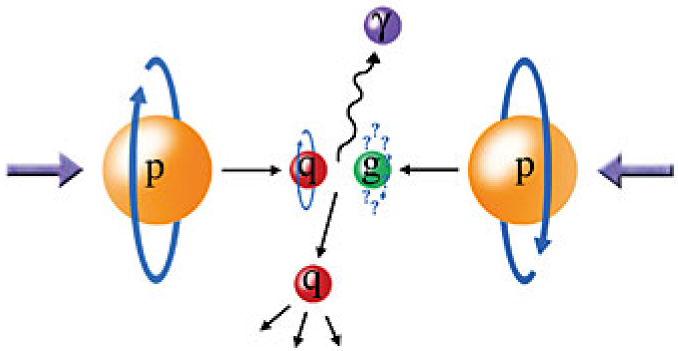
-> No discrepancy in more abundant control channel

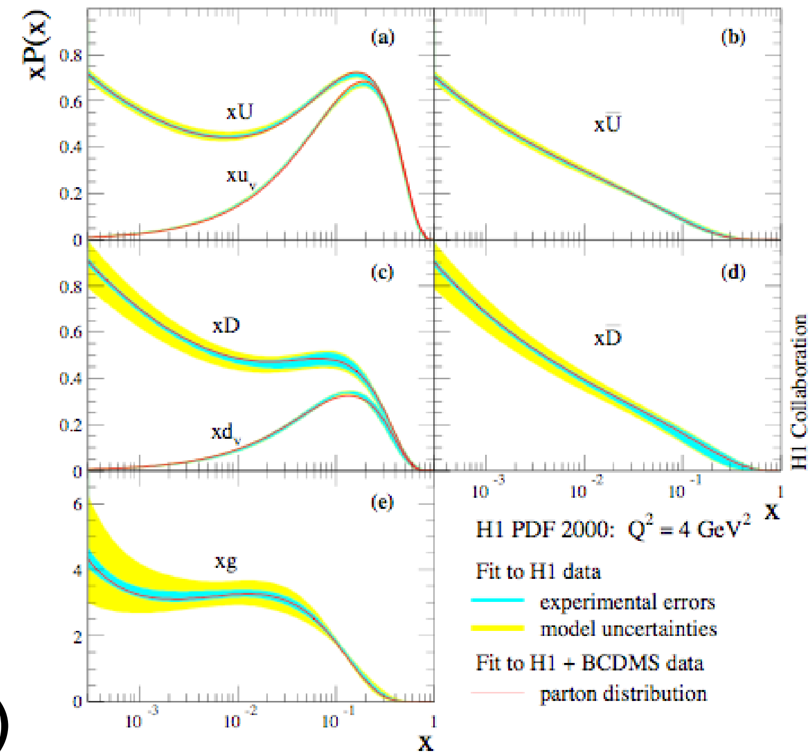
-> No estimation of systematics this way, but does not seem to be a detector problem!

PDFs: They even come with a recipie!

PDFs (Parton Distribution Functions):
QCD-Fits using a certain paramaterization and various boundary conditions and assumptions

PDF is universal!
➔ Calculate the pp cross section





$$\sigma_{pp} \qquad {}_{f} \quad pdf_1(x_B^1) \qquad pdf_2(x_B^2) \qquad \hat{\sigma}_{part}$$

Unsicherheiten in den PDFs ➔
Unsicherheiten bei Vorhersagen (z.B. LHC)

U=u+c≈ u, D=d+s≈ d

Fits to a single data set can „easily“ take into account the stat. and sys. uncertainties of that measurement...

Data in global fits

MSTW, arXiv:0901.0002

Most fits are „global“,
i.e., they fit „all“ the available data ➔

Data sets are from colliders and fixed target, from ep, pp, eA, ν A, ..., i.e., their probed x-range and their sensitivity to a certain parton is very different. Their systematic uncertainties are also not necessarily derived in a consistent way......

➔Until recently, only the result (central value) of the fit was available and fed into your favourite MC generator.....

Data set	LO	NLO	NNLO
BCDMS μp F_2 [32]	165 / 153	182 / 163	170 / 163
BCDMS μd F_2 [102]	162 / 142	190 / 151	188 / 151
NMC μp F_2 [33]	137 / 115	121 / 123	115 / 123
NMC μd F_2 [33]	120 / 115	102 / 123	93 / 123
NMC $\mu n/\mu p$ [103]	131 / 137	130 / 148	135 / 148
E665 μp F_2 [104]	59 / 53	57 / 53	63 / 53
E665 μd F_2 [104]	49 / 53	53 / 53	63 / 53
SLAC ep F_2 [105, 106]	24 / 18	30 / 37	31 / 37
SLAC ed F_2 [105, 106]	12 / 18	30 / 38	26 / 38
NMC/BCDMS/SLAC F_L [32–34]	28 / 24	38 / 31	32 / 31
E866/NuSea pp DY [107]	239 / 184	228 / 184	237 / 184
E866/NuSea pd/pp DY [108]	14 / 15	14 / 15	14 / 15
NuTeV νN F_2 [37]	49 / 49	49 / 53	46 / 53
CHORUS νN F_2 [38]	21 / 37	26 / 42	29 / 42
NuTeV νN xF_3 [37]	62 / 45	40 / 45	34 / 45
CHORUS νN xF_3 [38]	44 / 33	31 / 33	26 / 33
CCFR $\nu N \to \mu\mu X$ [39]	63 / 86	66 / 86	69 / 86
NuTeV $\nu N \to \mu\mu X$ [39]	44 / 40	39 / 40	45 / 40
H1 MB 99 e^+p NC [31]	9 / 8	9 / 8	7 / 8
H1 MB 97 e^+p NC [109]	46 / 64	42 / 64	51 / 64
H1 low Q^2 96–97 e^+p NC [109]	54 / 80	44 / 80	45 / 80
H1 high Q^2 98–99 e^-p NC [110]	134 / 126	122 / 126	124 / 126
H1 high Q^2 99–00 e^+p NC [35]	153 / 147	131 / 147	133 / 147
ZEUS SVX 95 e^+p NC [111]	35 / 30	35 / 30	35 / 30
ZEUS 96–97 e^+p NC [112]	118 / 144	86 / 144	86 / 144
ZEUS 98–99 e^-p NC [113]	61 / 92	54 / 92	54 / 92
ZEUS 99–00 e^+p NC [114]	75 / 90	63 / 90	65 / 90
H1 99–00 e^+p CC [35]	28 / 28	29 / 28	29 / 28
ZEUS 99–00 e^+p CC [36]	36 / 30	38 / 30	37 / 30
H1/ZEUS ep $F_2^{\rm charm}$ [41–47]	110 / 83	107 / 83	95 / 83
H1 99–00 e^+p incl. jets [59]	109 / 24	19 / 24	—
ZEUS 96–97 e^+p incl. jets [57]	88 / 30	30 / 30	—
ZEUS 98–00 $e^{\pm}p$ incl. jets [58]	102 / 30	17 / 30	—
DØ II $p\bar{p}$ incl. jets [56]	193 / 110	114 / 110	123 / 110
CDF II $p\bar{p}$ incl. jets [54]	143 / 76	56 / 76	54 / 76
CDF II $W \to \ell\nu$ asym. [48]	50 / 22	29 / 22	30 / 22
DØ II $W \to \ell\nu$ asym. [49]	23 / 10	25 / 10	25 / 10
DØ II Z rap. [53]	25 / 28	19 / 28	17 / 28
CDF II Z rap. [52]	52 / 29	49 / 29	50 / 29
All data sets	**3066 / 2598**	**2543 / 2699**	**2480 / 2615**

The parametrization

$$
\begin{aligned}
xu_v(x, Q_0^2) &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\,\sqrt{x}+\gamma_u\, x),\\
xd_v(x, Q_0^2) &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\,\sqrt{x}+\gamma_d\, x),\\
xS(x, Q_0^2) &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\,\sqrt{x}+\gamma_S\, x),\\
x\Delta(x, Q_0^2) &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2),\\
xg(x, Q_0^2) &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\,\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}},\\
x(s+\bar{s})(x, Q_0^2) &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\,\sqrt{x}+\gamma_S\, x),\\
x(s-\bar{s})(x, Q_0^2) &= A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0),
\end{aligned}
$$

All input parameters are allowed to vary.
Unfortunately highly correlated and even partially redundant („full“ compensation possible).

➔ MSTW uses subset of 20 (sufficiently independent) parameters, others fixed at best values.

The PDF error sets

Parameter	LO	NLO	NNLO
$\alpha_S(Q_0^2)$	0.68183	0.49128	0.45077
$\alpha_S(M_Z^2)$	0.13939	0.12018	0.11707
A_u	1.4335	0.25871	0.22250
η_1	$0.45232^{+0.022}_{-0.018}$	$0.29065^{+0.019}_{-0.013}$	$0.27871^{+0.018}_{-0.014}$
η_2	$3.0409^{+0.079}_{-0.067}$	$3.2432^{+0.062}_{-0.039}$	$3.3627^{+0.061}_{-0.044}$
ϵ_u	$-2.3737^{+0.54}_{-0.48}$	$4.0603^{+1.6}_{-2.3}$	$4.4343^{+2.4}_{-2.7}$
γ_u	8.9924	30.687	38.599
A_d	5.0903	12.288	17.938
η_3	$0.71978^{+0.057}_{-0.082}$	$0.96809^{+0.11}_{-0.11}$	$1.0839^{+0.12}_{-0.11}$
$\eta_4-\eta_2$	$2.0835^{+0.32}_{-0.45}$	$2.7003^{+0.50}_{-0.52}$	$2.7865^{+0.50}_{-0.44}$
ϵ_d	$-4.3654^{+0.28}_{-0.22}$	$-3.8911^{+0.31}_{-0.29}$	$-3.6387^{+0.27}_{-0.28}$
γ_d	7.4730	6.0542	5.2577
A_S	$0.59964^{+0.036}_{-0.030}$	$0.31620^{+0.030}_{-0.021}$	$0.64942^{+0.047}_{-0.041}$
δ_S	−0.16276	−0.21515	−0.11912
η_S	$8.8801^{+0.33}_{-0.33}$	$9.2726^{+0.23}_{-0.33}$	$9.4189^{+0.25}_{-0.33}$
ϵ_S	$-2.9012^{+0.33}_{-0.37}$	$-2.6022^{+0.71}_{-0.96}$	$-2.6287^{+0.49}_{-0.51}$
γ_S	16.865	30.785	18.065
$\int_0^1 dx\, \Delta(x,Q_0^2)$	$0.091031^{+0.012}_{-0.009}$	$0.087673^{+0.013}_{-0.011}$	$0.078167^{+0.012}_{-0.0091}$
A_Δ	8.9413	8.1084	16.244
η_Δ	$1.8760^{+0.24}_{-0.30}$	$1.8691^{+0.23}_{-0.32}$	$2.0741^{+0.18}_{-0.35}$
γ_Δ	$8.4703^{+2.0}_{-0.3}$	$13.609^{+1.1}_{-0.6}$	$6.7640^{+0.77}_{-0.41}$
δ_Δ	−36.507	−59.289	−36.090
A_g	0.0012216	1.0805	3.4055
δ_g	$-0.83657^{+0.15}_{-0.14}$	$-0.42848^{+0.066}_{-0.057}$	$-0.12178^{+0.23}_{-0.16}$
η_g	$2.3882^{+0.51}_{-0.50}$	$3.0225^{+0.43}_{-0.36}$	$2.9278^{+0.68}_{-0.41}$
ϵ_g	-38.997^{+36}_{-35}	−2.2922	−2.3210
γ_g	1445.5^{+880}_{-750}	3.4894	1.9233
$A_{g'}$	—	−1.1168	−1.6189
$\delta_{g'}$	—	$-0.42776^{+0.053}_{-0.047}$	$-0.23999^{+0.14}_{-0.10}$
$\eta_{g'}$	—	$32.869^{+6.5}_{-5.9}$	$24.792^{+6.5}_{-5.2}$
A_+	$0.10302^{+0.029}_{-0.017}$	$0.047915^{+0.0096}_{-0.0076}$	$0.10455^{+0.019}_{-0.016}$
η_+	$13.242^{+2.9}_{-1.4}$	$9.7466^{+1.0}_{-0.8}$	$9.8689^{+1.0}_{-0.6}$
A_-	$-0.011523^{+0.009}_{-0.018}$	$-0.011629^{+0.009}_{-0.023}$	$-0.0093692^{+0.006}_{-0.024}$
η_-	10.285^{+16}_{-6}	11.261^{+22}_{-6}	9.5783^{+26}_{-5}
x_0	0.017414	0.016050	0.018556
r_1	−0.39484	−0.57631	−0.80834
r_2	−1.0719	0.81878	1.2669
r_3	−0.28973	−0.083208	0.15098

Combinations of 20 parameters are expressed in eigenvectors and eigenvalues of covariance matrix

➔ Eigenvectors are orthogonal

➔ Pairs („up-down variations“) of eigenvector PDF sets span the hypersphere with a radius T corresponding to the allowed tolerance for required confidence interval,

e.g., for a 68% confidence level □ $T=\sqrt{\Delta\chi^2}=1$

The recipie:

The (asymmetric) uncertainty on a quantity (e.g., cross section) is derived by separately adding all (20 in case of MSTW) „up“ and all „down“ fluctuations on that quantity in quadrature (orthogonal eigenvectors).
If a pair of eigenvector PDF sets causes the quantitiy to fluctuate in one direction add once the maximum and once zero ➔ see example

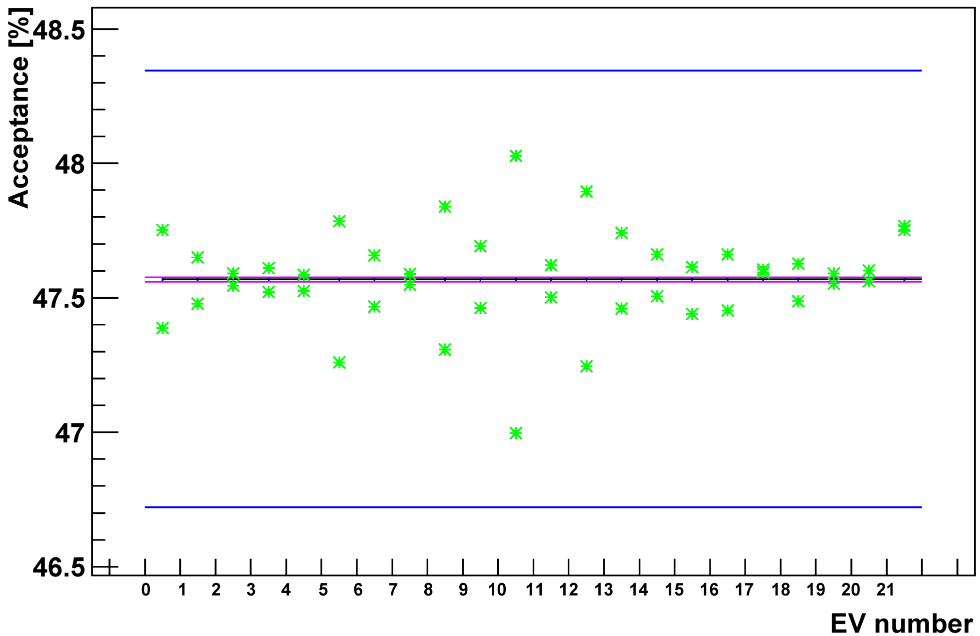
Example: Acceptance

- Variation of PDF results in variation in derived acceptance
  - Study impact of PDF variation around best fit value
- MSTW2008/CTEQ66 provide set of 40/44 variations of mean PDF (error sets)
- Calculate 40/44 acceptances using error sets
- Add up deviations (up and down separately) from mean acceptance in quadrature to get (asymmetric) systematic uncertainty
- Technically done via event reweighting ("LHAPDF"):
  - weight (w) for each event with respect to central value (CV) PDF

$$w = \frac{PDF_{ES}(x_1, Q^2, id1) \cdot PDF_{ES}(x_2, Q^2, id2)}{PDF_{CV}(x_1, Q^2, id1) \cdot PDF_{CV}(x_2, Q^2, id2)}$$

- id1,id2: quark flavours, x1,x2: Bjorken-x, Q^2: scale

Result and warnings



CTEQ NLO error set fed in PYTHIA:
Acceptance
(for Z->ee in ATLAS with some cuts on the eta and pT of the electrons)
is 47.6 + 0.8 – 0.9 %

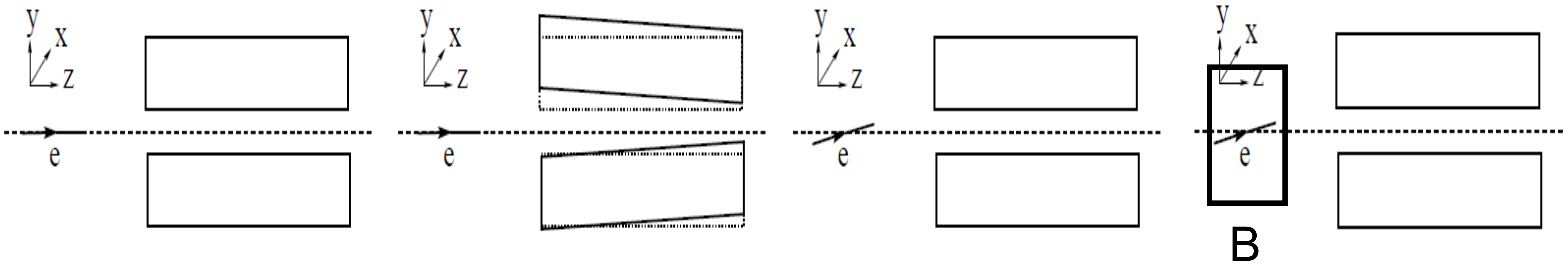
Warning, these uncertainties usually do not take into account:

- Form of input Parametrization
- Higher order QCD
- Higher order EW
- Nuclear corrections for neutrino data
- Choice of data sets
- ...

Minimize uncertainties „All in one“

If systematic uncertainties are not correlated you can (usually) add them in quadrature. If they are/might be correlated you have to add them linearly ➔ can get large, while in fact they might partially cancel.

➔Try to address uncertainties that might be correlated „All in one“ shot.

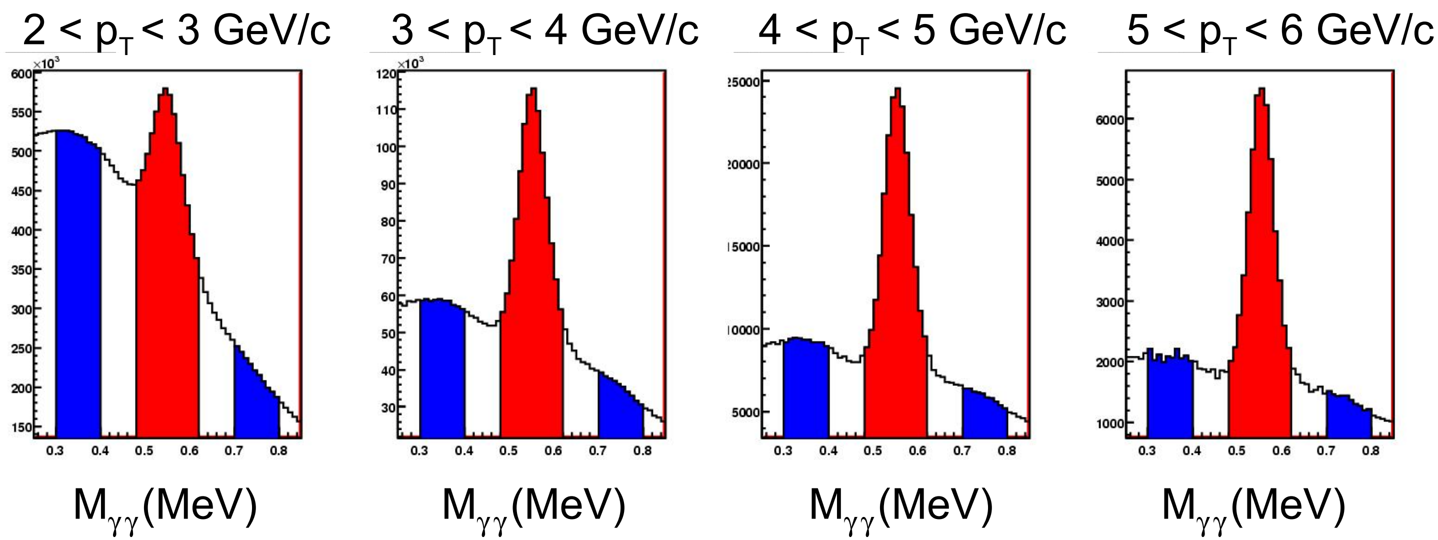


Example misalignment:
- misaligned (forward) spectrometer
- misaligned beam
- effect of transverse magnetic field (holding field for transverse target) on incoming and scattered electron

If possible: Have all effects modeled in the same MC and vary them all at the same time. (Indeed, some cancellations were found (HERMES@DESY)

Peak extraction (PHENIX@BNL)

$\eta \to \gamma\gamma$

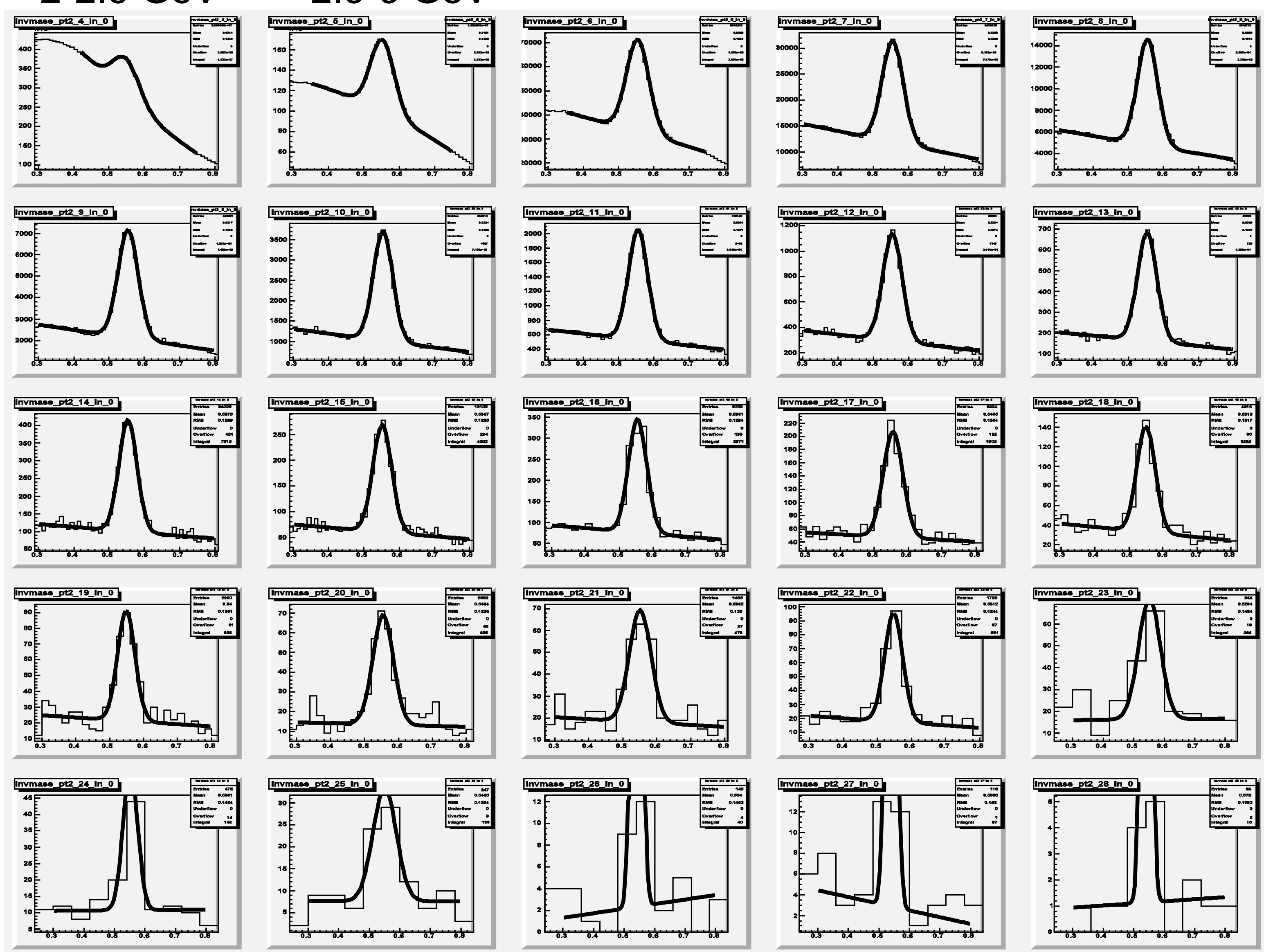


Signal/Background extraction:

- Fit to Signal+Background
- Sideband
- Same charge backgound (not in this case)
- Mixed background (usually for large combinatorial background, e.g., high multiplicity in heavy ion collisions
- Have an excellent MC descripton? Get it from MC directly....

Many bins in pT with different shapes

2-2.5 GeV 2.5-3 GeV



One Method To Rule Them All?

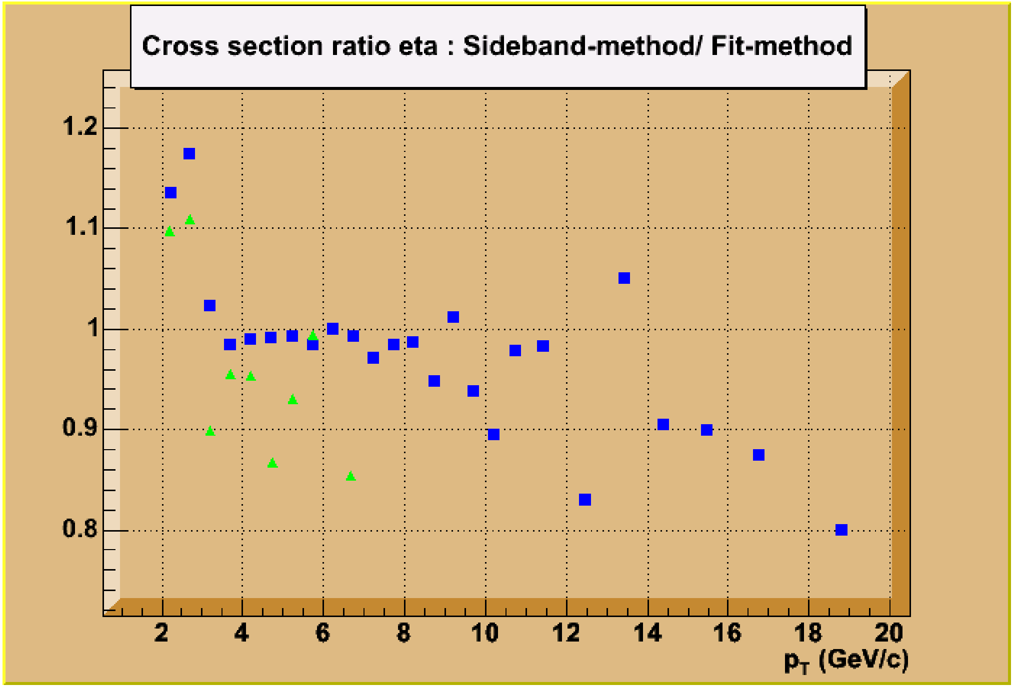
Not A Good Idea!

18-20 GeV

Peak extraction: Sideband/ Fit-Method

These are not automatically your systematic uncertainties:

- Sideband method needs somewhat linear background, not true in small pT bins
- The fits are maybe not good (enough) in large pT region



➔Use fit at small pt and sideband at large pt

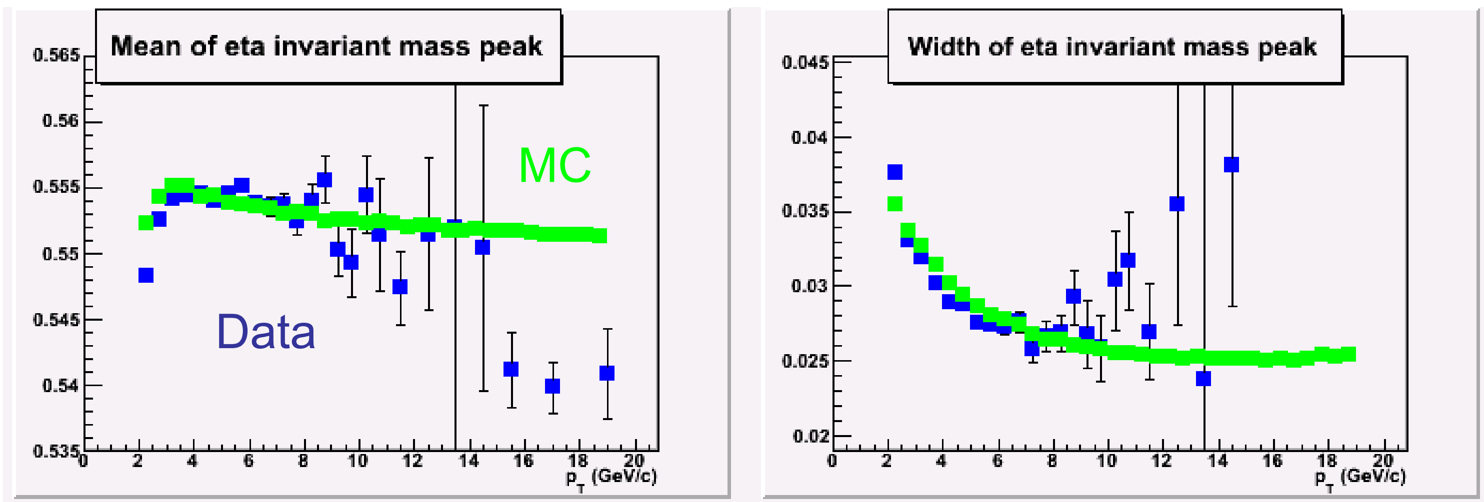
➔ Agreement in medium region, differences smaller 2%

Systematic uncertainty for Fit-Method:

- Use different fit functions (different function for signal and background)
- Use different fit ranges
- Check for differences when integrating over peak width of 2 or 3 sigma

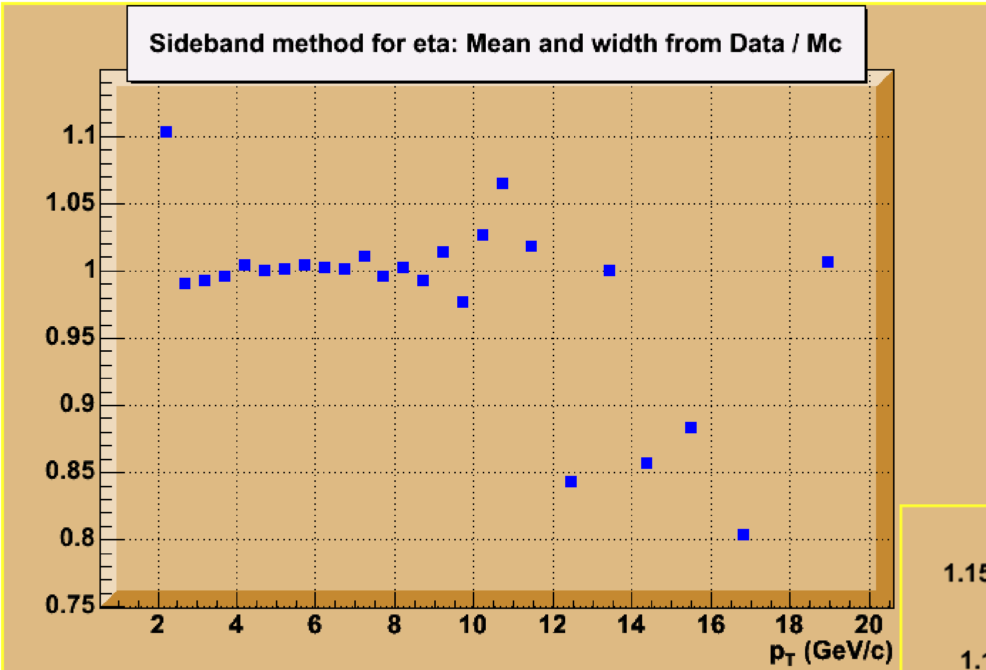
Peak extraction: Sidebands

Need to know width and mean of peak in order to know from where to where to count! Where do I get that from?
I have it from the fits, but fits aren‘t that good at large pt.



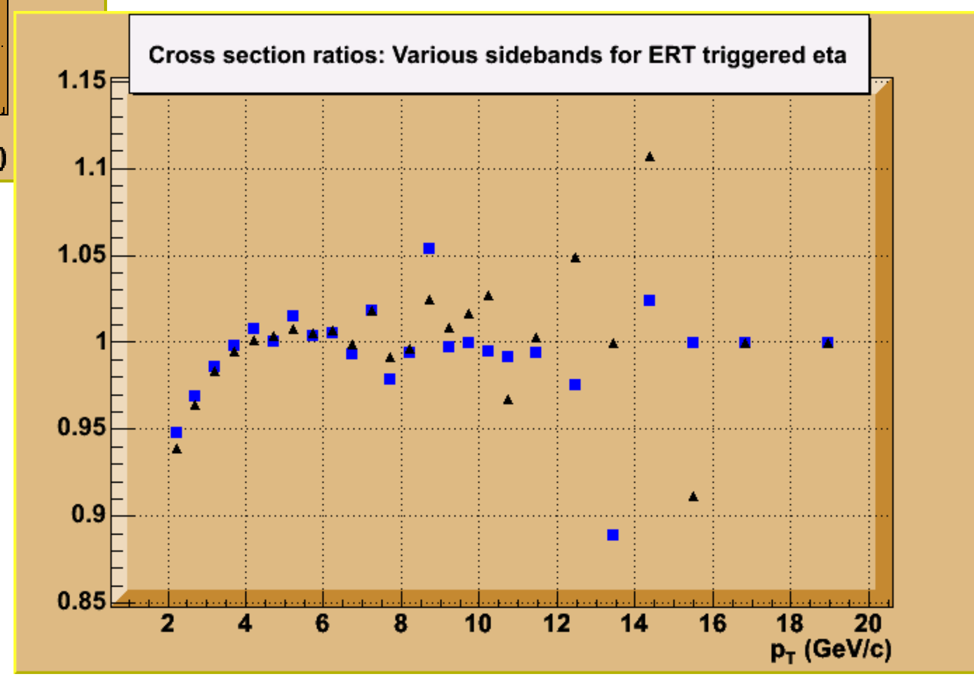
➔ Better take width from MC, way more stable (statistics).

Peak extraction: Sidebands



Mean and width from Data/MC:
Similar conclusions as before....

- Check different positions for sidebands
- Check different width of sidebands (larger sideband yields more statistics, but will extend to a region further away from peak)



Summary

- Plan ahead: Systematics need lots of time
- Think about all possible effects
- Check everything possible
- Try to understand what you see
- Free yourself from expectations
- Don‘t look look at the result while tuning cuts
- Talk to your colleagues
- Have good reasons for assigning sys. unc.
- Write all details into your analysis note