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Two-loop QED corrections to μe scattering

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In collaboration with:
Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Tramontano

A loop summit 2021
July 27th, 2021
Cadenabbia, Italy

MAX-PLANCK-INSTITUT FÜR PHYSIK

Motivations

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

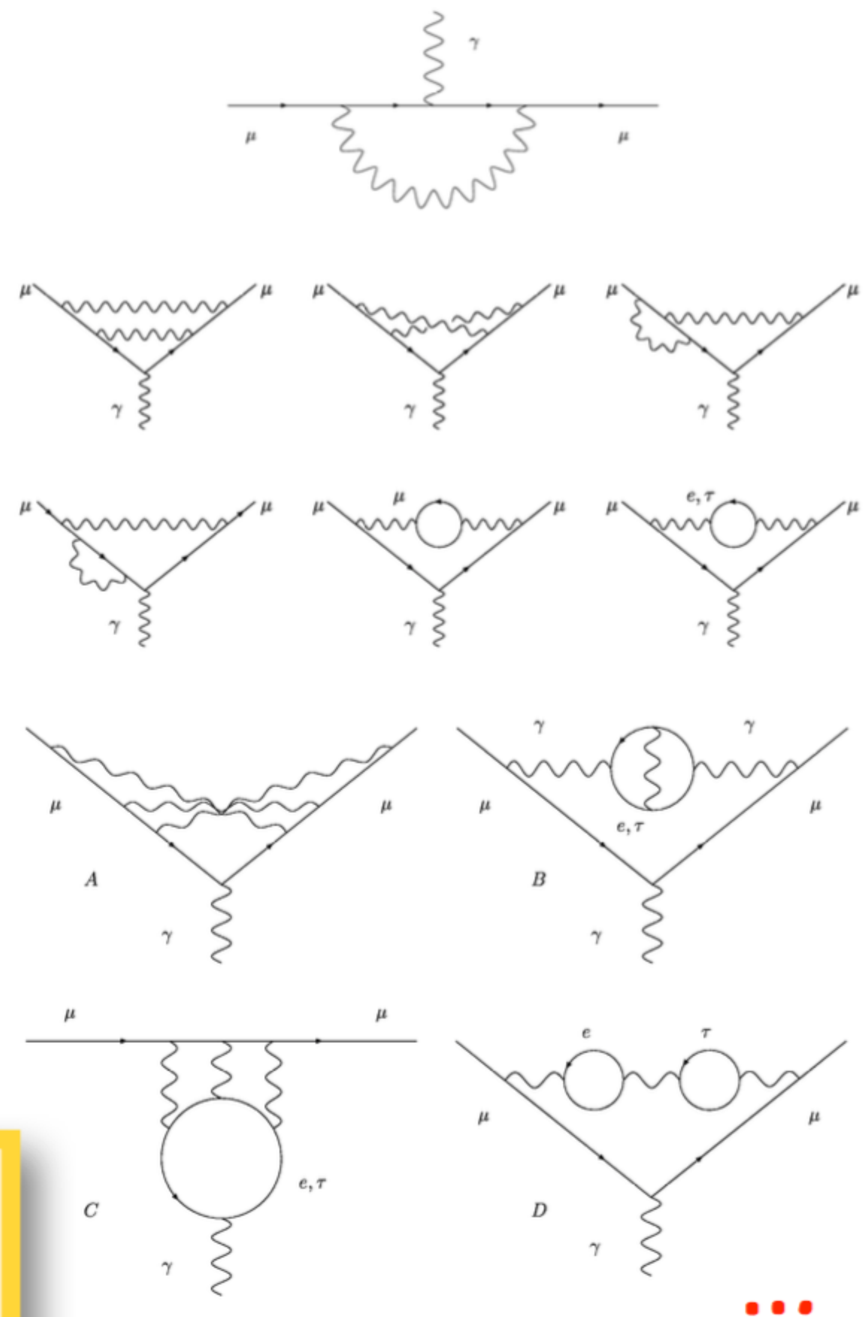
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8780\,(60)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
S. Laporta, PLB 2017 (mass independent term). COMPLETED^2!

$+ \; 750.80\,(89)\,(\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015 & 2017



Adding up, I get:

$a_\mu^{QED} = 116584718.932\,(20)(23) \times 10^{-11}$
from coeffs, mainly from 4-loop unc ⮠ ⮡ from α (Cs)
with $\alpha = 1/137.035999046(27)$ [0.2ppb] 2018

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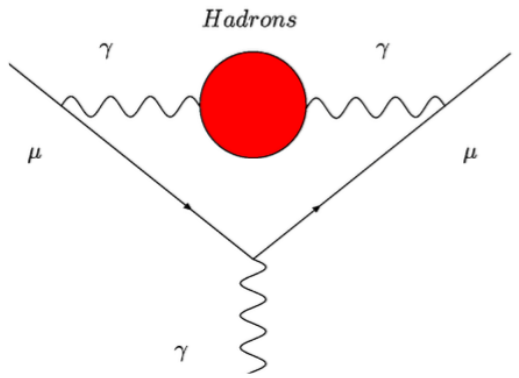
Motivations

New space-like proposal for HLO

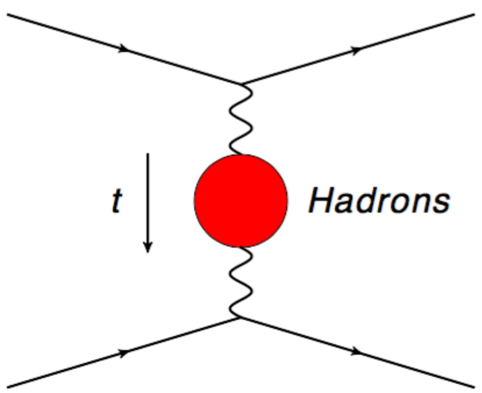
- At present, the leading hadronic contribution a_μ^{HLO} is computed via the time-like formula:



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- Alternatively, exchanging the x and s integrations in a_μ^{HLO}



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

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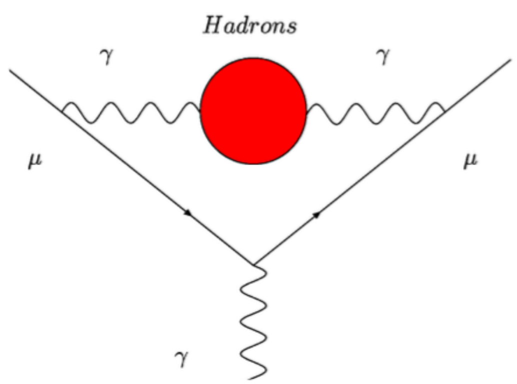

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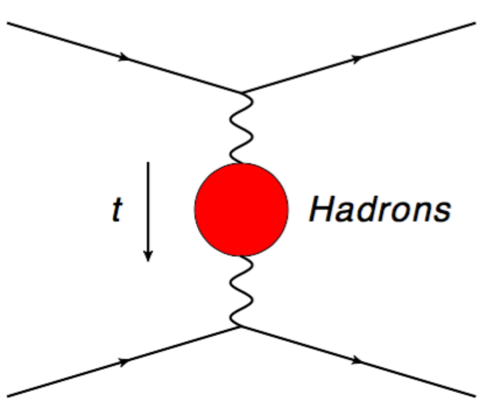

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Muon-electron scattering: The MUonE Project

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987

Outline

- Motivation
- μe-scattering at two loops
  — integr*and*/integr*al* reduction
  — UV renormalisation
- IR pole predictions
- Results & checks
- Outlook

Motivations

Results @NLO

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

Anatomy

- Born matrix element
  tree-level & n-pt process

- Real contribution
  Tree-level (n+1)-particles

- Virtual Contribution
  one-loop n-particles

- No assumption made in the newest result
- QED & EW effects
- Full lepton mass dependence
- Fully differential fixed order MC @ NLO

[Carloni Calame, Alacevich, Chiesa, Montagna, Nicrosini, Piccinini (2018)]

$$\hat{\sigma}_{NLO} \sim \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^R_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\hat{\sigma}^V_{NLO}$$ + MC integration

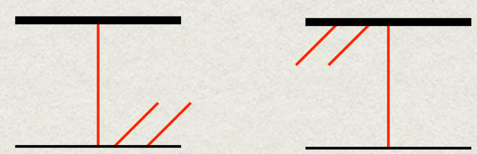
Motivations

what about @NNLO

Anatomy

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

- Real-Real contribution
  Tree-level (n+2)-particles



- Real-Virtual Contribution
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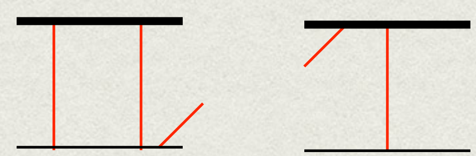

- ❑ `GoSam`
  Improvements for QED/EW
  Chiesa, Ossola and Tramontano

- Virtual-Virtual Contribution
  two-loop n-particles

  + 1L$\otimes$1L

- ❑ Preliminary calculation of 3-pt form factors
  [Bernreuther, Bonciani, Ferroglia, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi (2003-2005)]

[Carloni Calame, Chiesa, Hasan, Montagna, Nicrosini, Piccinini (2020)]

$$\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions & MC integrations

Motivations

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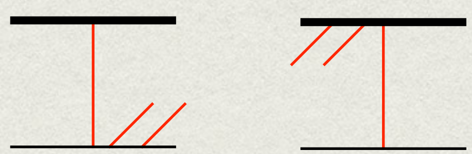
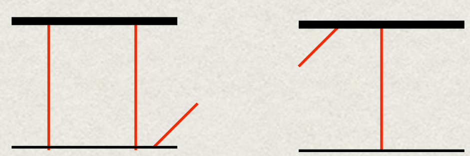
☆ **This talk**

- Real-Real contribution
  Tree-level (n+2)-particles



☑ Complete UV renormalised 4-pt amplitude

- Real-Virtual Contribution
  one-loop (n+1)-particles



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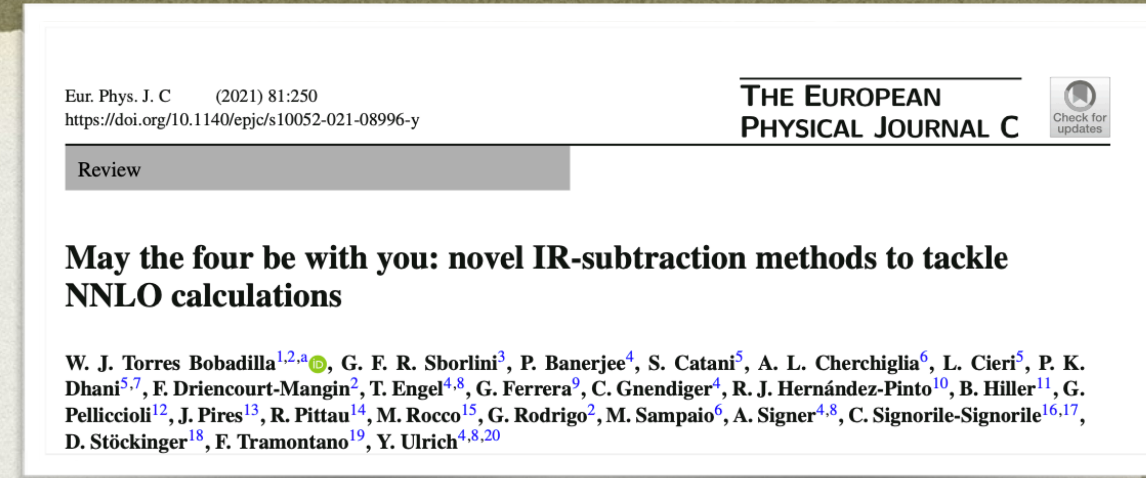
Eur. Phys. J. C (2021) 81:250
https://doi.org/10.1140/epjc/s10052-021-08996-y
THE EUROPEAN PHYSICAL JOURNAL C
Check for updates
Review

May the four be with you: novel IR-subtraction methods to tackle NNLO calculations

W. J. Torres Bobadilla[1,2,a], G. F. R. Sborlini[3], P. Banerjee[4], S. Catani[5], A. L. Cherchiglia[6], L. Cieri[5], P. K. Dhani[5,7], F. Driencourt-Mangin[2], T. Engel[4,8], G. Ferrera[9], C. Gnendiger[4], R. J. Hernández-Pinto[10], B. Hiller[11], G. Pelliccioli[12], J. Pires[13], R. Pittau[14], M. Rocco[15], G. Rodrigo[2], M. Sampaio[6], A. Signer[4,8], C. Signorile-Signorile[16,17], D. Stöckinger[18], F. Tramontano[19], Y. Ulrich[4,8,20]

what about @NNLO

Anatomy

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

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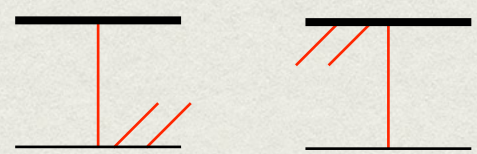
☑ Complete UV renormalised 4-pt amplitude

- Real-Real contribution
  Tree-level (n+2)-particles



- Real-Virtual Contribution
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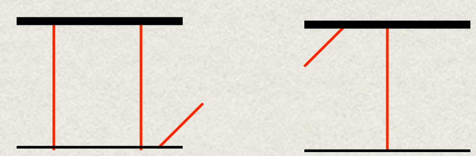

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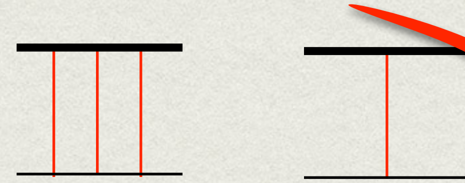

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+ Subtractions & MC integrations

$e^+e^- \to \mu^+\mu^-$

- emu scattering —> di-muon production
- Close connection to $q\bar{q} \to t\bar{t}$
  (completely known numerically in literature)

[Bonciani, Ferroglia, Gehrmann, Studerus (2009)]
[Barnreuther, Czakon, Fiedler (2013)]

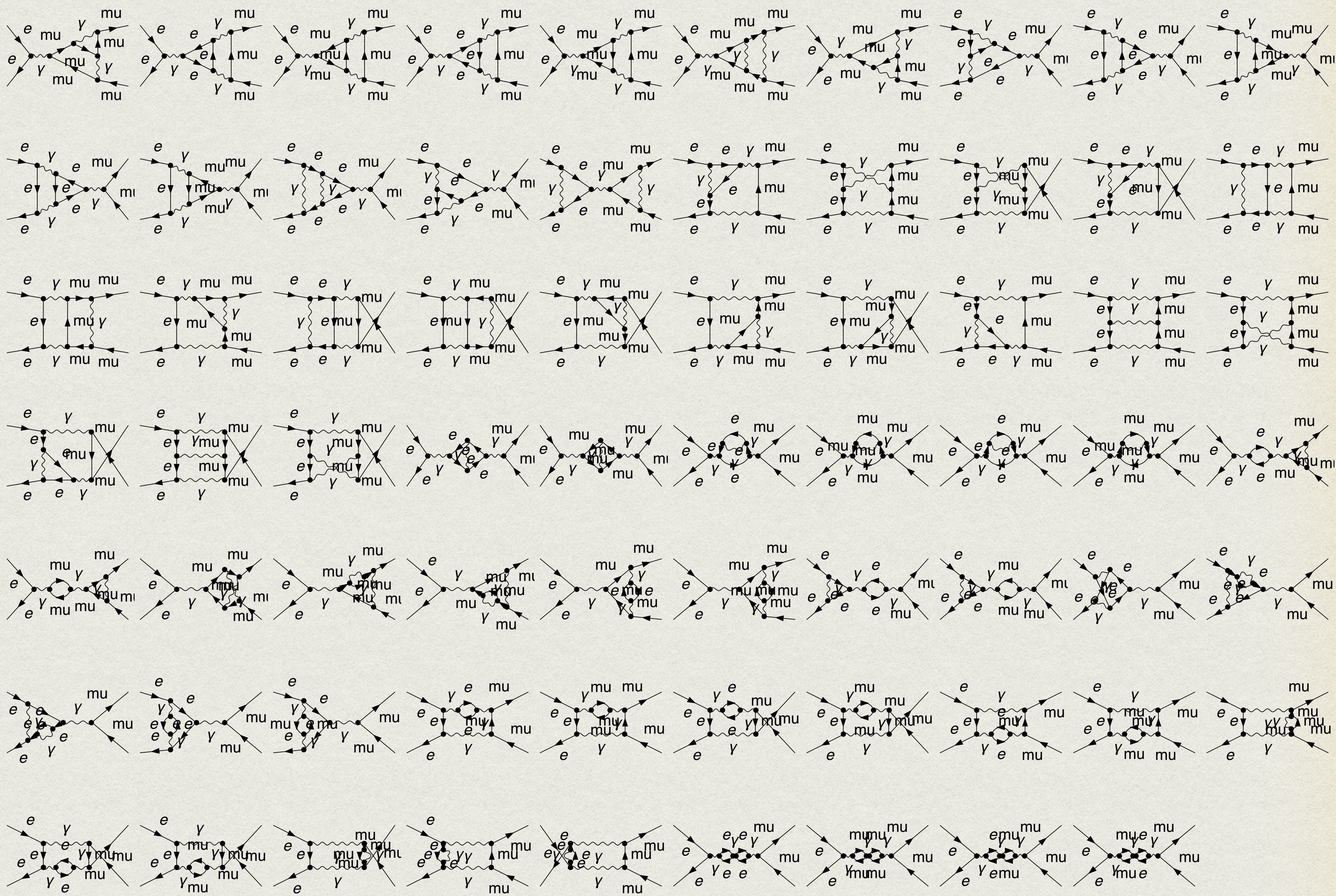
- Checks from QCD to QED

$e^+e^- \to \mu^+\mu^-$ @ *two loops*

69 diagrams
@ 2-loop

Automatically
organised in
groups

Integrand
reductions by
means of **AIDA**



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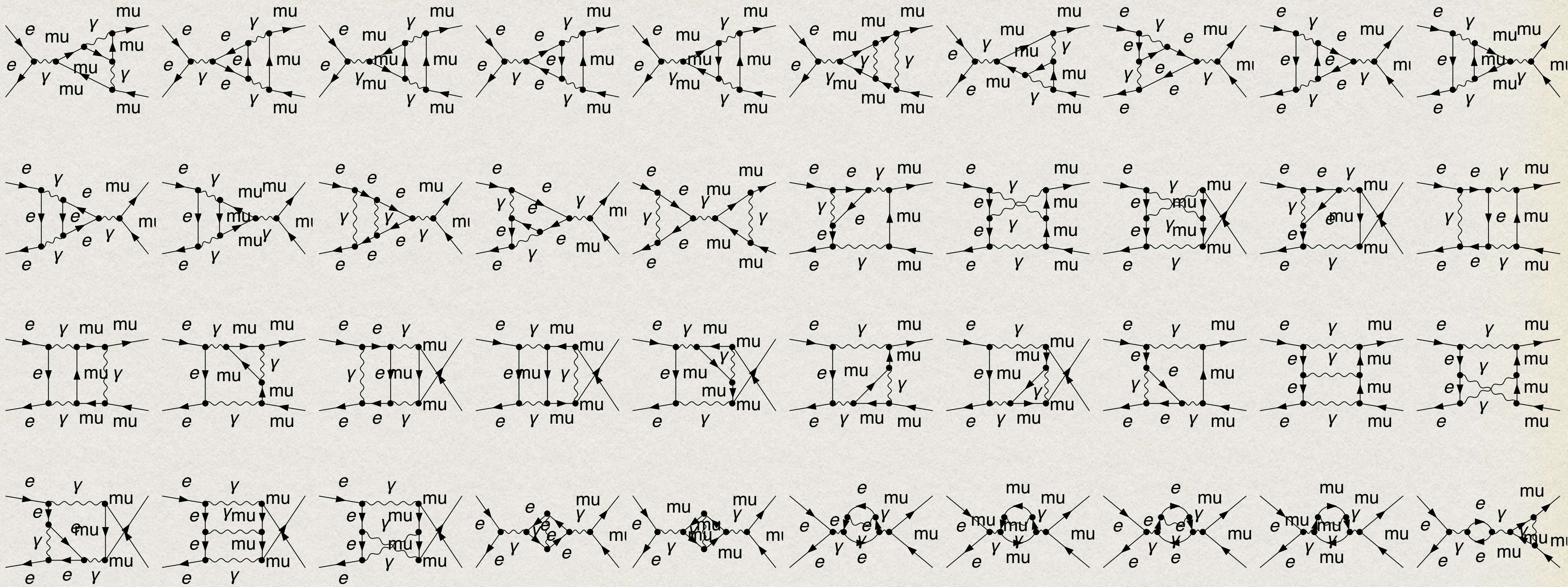


Automatically organised in groups

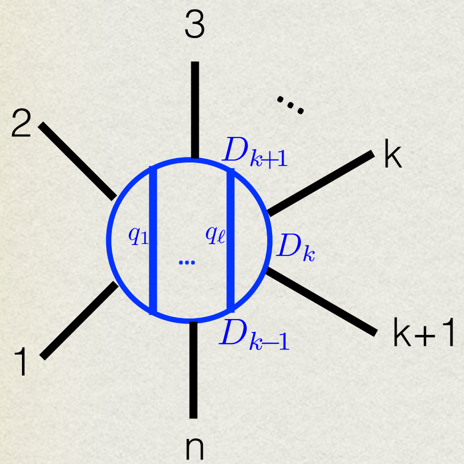
$$\mathcal{M}^{(2)} = A^{(2)} + \boldsymbol{n_l}\, B_l^{(2)} + \boldsymbol{n_h}\, C_h^{(2)} + \boldsymbol{n_l^2}\, D_l^{(2)} + \boldsymbol{n_h n_l}\, E_{hl}^{(2)} + \boldsymbol{n_h^2}\, F_h^{(2)}$$

Integrand reductions by means of **AIDA**

$$\mathcal{M}^{(2)} = \frac{\mathcal{M}^{(2)}_{-4}}{\epsilon^4} + \ldots + \frac{\mathcal{M}^{(2)}_{-1}}{\epsilon} + \mathcal{M}^{(2)}_0 + \mathcal{O}(\epsilon)$$

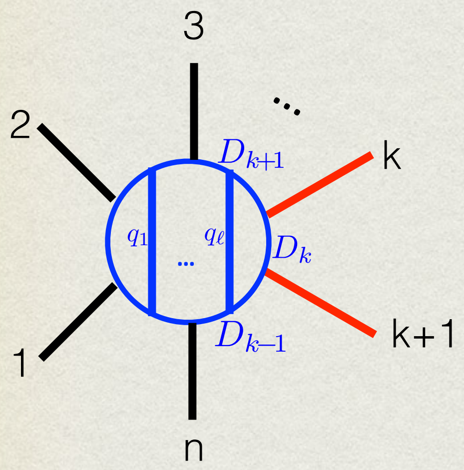
Integrand decomposition method

[Ossola, Papadopoulos, Pittau (2006)]
[Ellis, Giele, Kunszt, Melnikov (2007)]
[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]



$$= \frac{\mathcal{N}_{i_1\cdots i_m}(q_i)}{D_1\cdots D_k\cdots D_m}$$

- ☑ Applicable to any theory
- ☑ Ideal for helicity amplitudes
- ☑ Work for any number of external legs
- ☑ Straightforwardly automated



$$= \sum_{k=1}^{m} \frac{\mathcal{N}_{i_1\cdots i_{k+1} i_{k-1}\cdots i_m}(q_i)\, D_k}{D_1\cdots D_k\cdots D_m} + \frac{\Delta_{i_1\cdots i_k\cdots i_m}(q_i)}{D_1\cdots D_k\cdots D_m}$$

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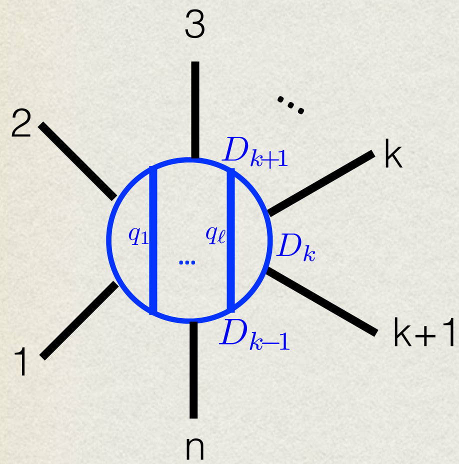

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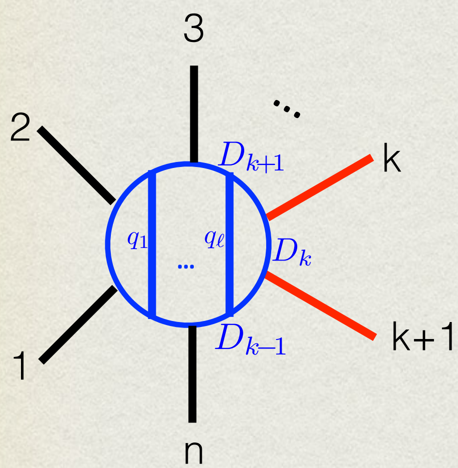

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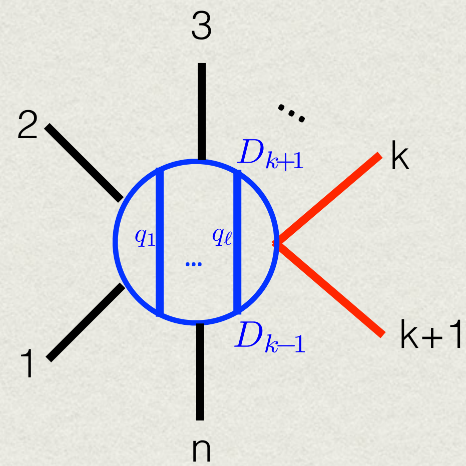

$$+ \frac{\Delta_{i_1 \cdots i_k \cdots i_m}(q_i)}{D_1 \cdots D_k \cdots D_m}$$

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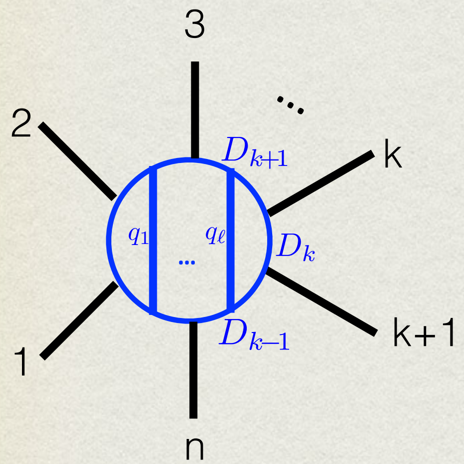

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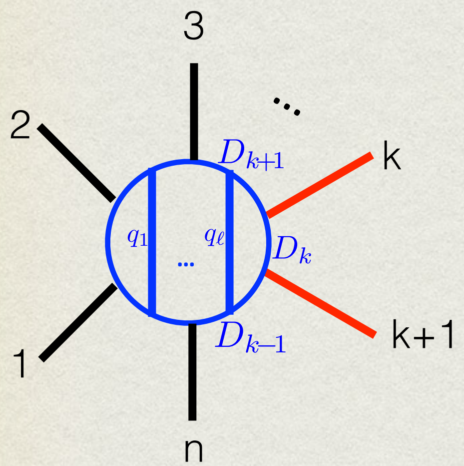

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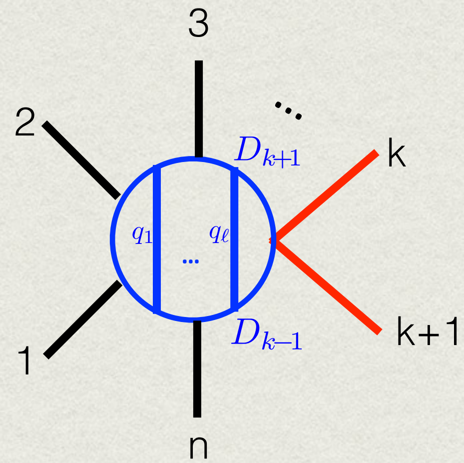

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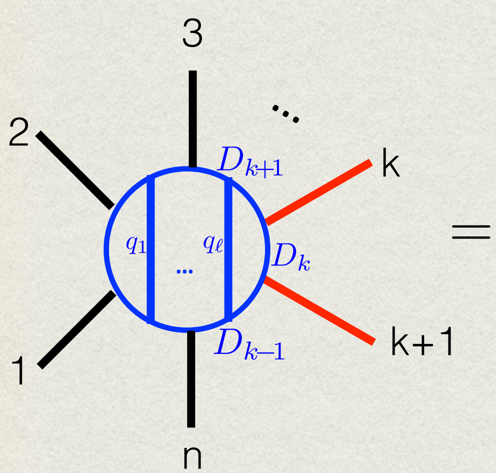
Amplitude decomposed into all possible multi-particle cuts



$$= \sum_{k=0}^{m} \sum_{\{1,\cdots,m\}} \frac{\Delta_{i_1\cdots i_k}(q_i)}{D_1\cdots D_k}$$

Numerator in terms of
Irreducible polynomials

Polynomial division module Groebner basis

[Mastrolia, Ossola (2011)]
[Zhang (2012-2016)]
[Badger, Frellesvig, Zhang (2012-2013)]
[Mastrolia, Mirabella, Ossola, Peraro (2012)]
...

Adaptive integrand decomposition (AID)

$$d = d_{\parallel} + d_{\perp}$$

$$d_{\parallel} = n - 1$$

$$d_{\perp} = (5-n) - 2\epsilon$$

- Splits d=4-2ε into parallel and orthogonal directions
- Nice properties for less than 5 external legs

[Collins (1984)]
[van Neerven and Vermaseren (1984)]
[Kreimer (1992)]

Loop momenta

$$\bar{l}_i^\alpha = \bar{l}_{\parallel i}^\alpha + \lambda_i^\alpha \longrightarrow \bar{l}_i^\alpha = \sum_{j=1}^{d_\parallel} x_{ji}\, e_j^\alpha\,, \qquad \lambda_i^\alpha = \sum_{j=d_\parallel+1}^{4} x_{ji}\, e_j^\alpha + \mu_i^\alpha\,, \qquad \lambda_{ij} = \sum_{l=d_\parallel+1}^{4} x_{li} x_{lj} + \mu_{ij}$$

- Numerator and denominators depend on different variables

[Mastrolia, Peraro, Primo (2016)]
[Mastrolia, Peraro, Primo, W.J.T. (2016)]

$$\int \prod_i d^{d_\parallel} \bar{l}_{\parallel i} \int \prod_{1\le i\le j\le \ell} d\lambda_{ij}\, G(\lambda_{ij})^{\frac{d_\perp - 1 - \ell}{2}} \int d\Theta_\perp \frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}\Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij})\cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

Straightforward integration of transverse components

Expand in Gegenbauer polynomials

$$\int d\,\Theta_\perp = \int_{-1}^{1} \prod_{i=1}^{4-d_\parallel} \prod_{j=1}^{\ell} d\cos\theta_{i+j-1\,j} (\sin\theta_{i+j-1\,j})^{d_\perp - i - j - 1}$$

$$\int_{-1}^{1} d\cos\theta (\sin\theta)^{2\alpha-1} C_n^{(\alpha)}(\cos\theta) C_m^{(\alpha)}(\cos\theta) = \delta_{mn} \frac{2^{1-2\alpha}\pi\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}$$

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and identification of spurious terms

Adaptive integrand decomposition (AID)

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Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in **3** steps

1) Divide and get $\Delta(\bar{l}_{\|i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of λ_{ij}

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

$$\frac{\mathcal{N}(\bar{l}_{\|i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

1)

$$\frac{\Delta(\bar{l}_{\|i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

2)

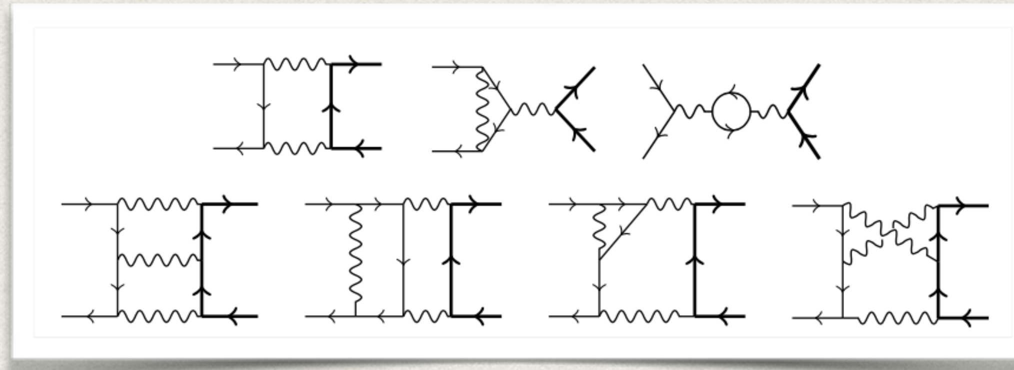
$$\frac{\Delta^{\text{int}}(\bar{l}_{\|i}, \lambda_{ij})}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

3)

$$\frac{\Delta'(\bar{l}_{\|i})}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

AIDA: a Mathematica implementation

[Mastrolia, Peraro, Primo, Ronca, W.J.T.]



Amplitude generation

`FeynArts-FeynCalc`
`Qgraph-FormCalc`

Algebraic decomposition ········ **Renormalisation**

Integrand decomposition
`AID`

IBP reduction
`Reduze, Kira, …`

$$\sum_k c_k\left(s,t,m^2,\epsilon\right) I_k^{(2)}\left(s,t,m^2,\epsilon\right)$$

$$I_k^{(2)} = \int \mathrm{d}^d q_1 \mathrm{d}^d q_2 \frac{1}{D_1^{\nu_1}\cdots D_{n_k}^{\nu_{n_k}}}$$

Loop integral evaluation - - - - - - **Analytic Continuation**

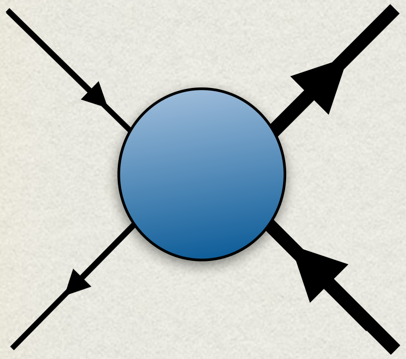
Numeric
`Fiesta, SecDec, …`

Analytic
Master Integrals
`Ginac, HandyG, …`

[Bonciani, Ferroglia, Gehrmann, von Manteuffel (2008-13)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]

Algebraic decomposition

$$e^+e^- \to \mu^+\mu^-$$



$$\mathcal{A}(\alpha) = 4\pi\alpha \left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \mathcal{A}^{(2)} + \mathcal{O}\left(\alpha^3\right)\right]$$

$$s = (p_1 + p_2)^2\,, \quad t = (p_2 - p_3)^2\,,$$
$$u = (p_1 - p_3)^2\,, \quad s + t + u = 2M^2\,.$$

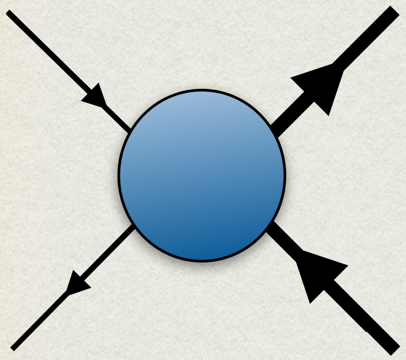
- Compute interference

$$\mathcal{M}^{(n)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\text{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(n)}\right)$$

- In the massless electron limit ($m_e^2 = 0$)
  4-point process depending on 3 scales

Algebraic decomposition

$$e^+e^- \to \mu^+\mu^-$$



$$\mathcal{A}(\alpha) = 4\pi\alpha\left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2\mathcal{A}^{(2)} + \mathcal{O}\left(\alpha^3\right)\right]$$

$$s = (p_1+p_2)^2\,, \quad t = (p_2-p_3)^2\,,$$
$$u = (p_1-p_3)^2\,, \quad s+t+u = 2M^2\,.$$

- Compute interference

$$\mathcal{M}^{(n)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\mathrm{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(n)}\right)$$

- In the massless electron limit ($m_e^2 = 0$) 4-point process depending on 3 scales

- Integrand/integral reductions

$$\mathcal{M}^{(2)}(e\mu \to e\mu) = \sum_k c_k\left(s,t,m^2,\epsilon\right) I_k^{(2)}\left(s,t,m^2,\epsilon\right)$$

O(10000) monomials

O(100) MIs

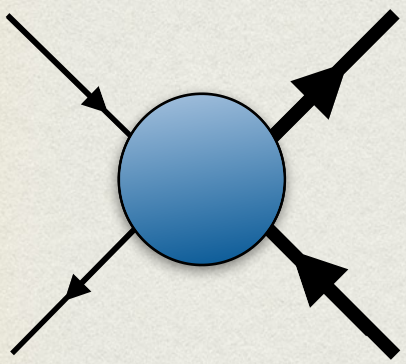
[Aida :: Mastrolia, Peraro, Primo, Ronca, W.J.T.]

+

[Reduze :: Studerus, von Manteuffel (2012)]

Algebraic decomposition

$$e^+e^- \to \mu^+\mu^-$$



$$\mathcal{A}(\alpha) = 4\pi\alpha\left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2\mathcal{A}^{(2)} + \mathcal{O}\left(\alpha^3\right)\right]$$

$$s = (p_1+p_2)^2\,, \quad t = (p_2-p_3)^2\,,$$
$$u = (p_1-p_3)^2\,, \quad s+t+u = 2M^2\,.$$

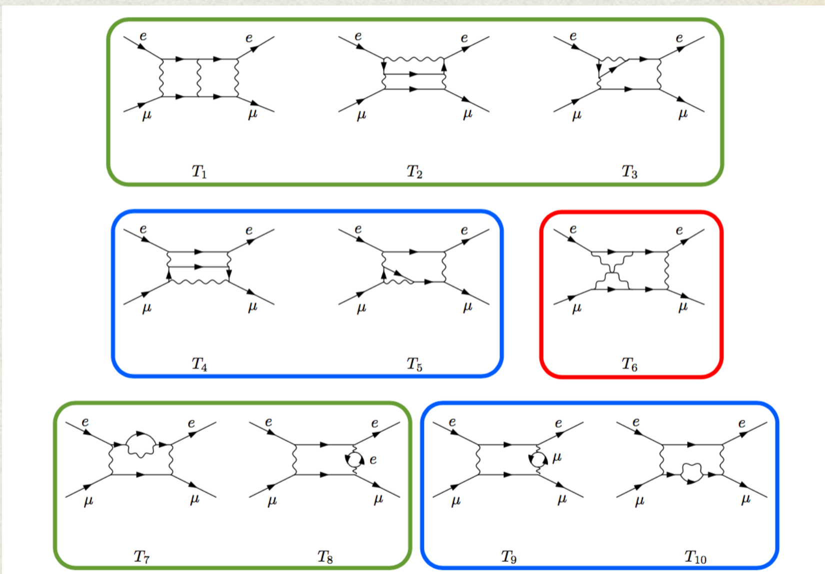
- Compute interference

$$\mathcal{M}^{(n)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\text{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(n)}\right)$$

- In the massless electron limit ($m_e^2 = 0$)
  4-point process depending on 3 scales

- Plug analytic expression of MIs

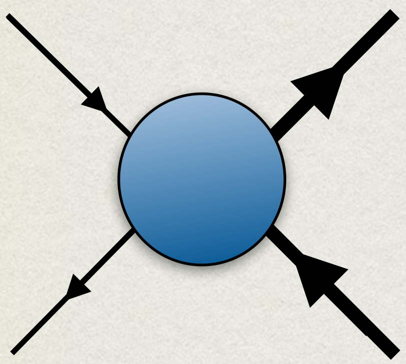
[Bonciani, Ferroglia, Gehrmann, von Manteuffel (2008-13)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]



Algebraic decomposition

$$
\begin{aligned}
\mathcal{A}^{(0)} &= \mathcal{A}_{\rm b}^{(0)} \\
\mathcal{A}^{(1)} &= \mathcal{A}_{\rm b}^{(1)} + \left(\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right) \mathcal{A}_{\rm b}^{(0)} \\
\mathcal{A}^{(2)} &= \mathcal{A}_{\rm b}^{(2)} + \left(2\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right) \mathcal{A}_{\rm b}^{(1)} \\
&\quad + \left(\delta Z_\alpha^{(2)} + \delta Z_F^{(2)} + \delta Z_f^{(2)} + \delta Z_f^{(1)} \delta Z_\alpha^{(1)}\right) \mathcal{A}_{\rm b}^{(0)} \\
&\quad + \delta Z_M^{(1)} \mathcal{A}_{\rm b}^{(1,\text{mass CT})}
\end{aligned}
$$

$$e^+e^- \to \mu^+\mu^-$$



$$\mathcal{A}(\alpha) = 4\pi\alpha \left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \mathcal{A}^{(2)} + \mathcal{O}\left(\alpha^3\right)\right]$$

$$s = (p_1+p_2)^2 \,, \quad t = (p_2 - p_3)^2 \,, \quad u = (p_1 - p_3)^2 \,, \quad s+t+u = 2M^2 \,.$$

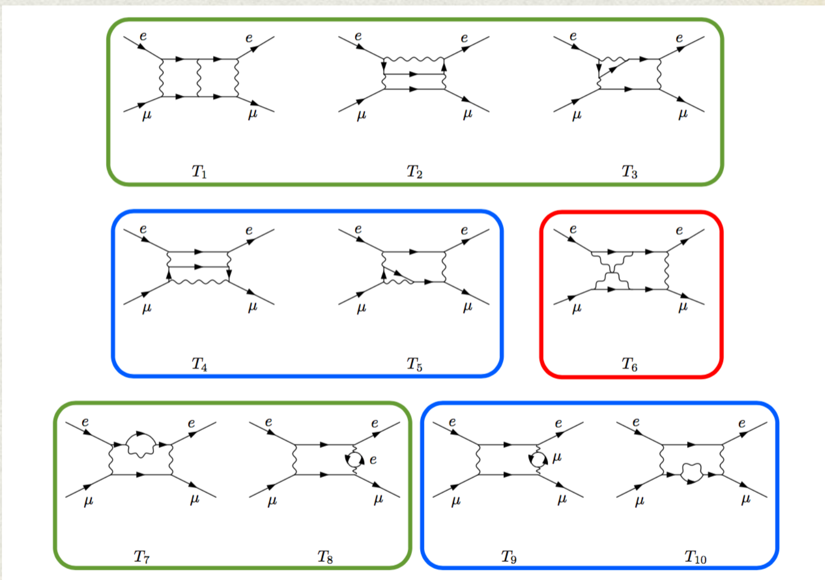
- Compute interference

$$\mathcal{M}^{(n)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\rm spins} 2{\rm Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(n)}\right)$$

☑ **All bare amplitudes —> Computed!**

- In the massless electron limit ($m_e^2 = 0$) 4-point process depending on 3 scales

- Plug analytic expression of MIs



[Bonciani, Ferroglia, Gehrmann, von Manteuffel (2008-13)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]

UV Renormalisation

- Fields

$$\psi_{\rm b} = \sqrt{Z_2}\psi\,, \quad A^\sigma_{\rm b} = \sqrt{Z_3}A^\sigma\,, \quad M_{\rm b} = Z_M M$$

- QED interaction vertex ⟶ Fixed from QED Ward id'

$$\mathcal{L}_{\rm int} = e_{\rm b}\,\bar\psi_{\rm b}\,\not{A}_{\rm b}\,\psi_{\rm b} = e\,\bar\psi\,\not{A}\,\psi$$

- Scheme :: On-shell + MSbar

$$Z_{2,e} = Z_{2,e}^{\rm OS}\,, \quad Z_{2,\mu} = Z_{2,\mu}^{\rm OS}\,, \quad Z_M = Z_M^{\rm OS}\,, \quad Z_\alpha^{\overline{\rm MS}} = 1/Z_3^{\overline{\rm MS}}$$

$$Z_j = 1 + \left(\frac{\alpha}{\pi}\right)\delta Z_j^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \delta Z_j^{(2)} + \mathcal{O}\left(\alpha^3\right)$$

- UV Renormalised amplitudes

$$\begin{aligned}
\mathcal{A}^{(0)} &= \mathcal{A}_{\rm b}^{(0)} \\
\mathcal{A}^{(1)} &= \mathcal{A}_{\rm b}^{(1)} + \left(\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right)\mathcal{A}_{\rm b}^{(0)} \\
\mathcal{A}^{(2)} &= \mathcal{A}_{\rm b}^{(2)} + \left(2\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right)\mathcal{A}_{\rm b}^{(1)} \\
&\quad + \left(\delta Z_\alpha^{(2)} + \delta Z_F^{(2)} + \delta Z_f^{(2)} + \delta Z_f^{(1)}\delta Z_\alpha^{(1)}\right)\mathcal{A}_{\rm b}^{(0)} + \delta Z_M^{(1)}\mathcal{A}_{\rm b}^{(1,\text{mass CT})}
\end{aligned}$$

UV Renormalisation

- Fields

$$\psi_{\rm b} = \sqrt{Z_2}\psi\,, \quad A^\sigma_{\rm b} = \sqrt{Z_3}A^\sigma\,, \quad M_{\rm b} = Z_M M$$

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- Scheme :: On-shell + MSbar

$$Z_{2,e} = Z^{\rm OS}_{2,e}\,, \quad Z_{2,\mu} = Z^{\rm OS}_{2,\mu}\,, \quad Z_M = Z^{\rm OS}_M\,, \quad Z^{\overline{\rm MS}}_\alpha = 1/Z^{\overline{\rm MS}}_3$$

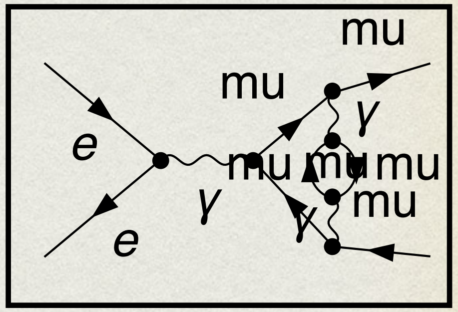
$$Z_j = 1 + \left(\frac{\alpha}{\pi}\right)\delta Z^{(1)}_j + \left(\frac{\alpha}{\pi}\right)^2\delta Z^{(2)}_j + \mathcal{O}\left(\alpha^3\right)$$

- UV Renormalised amplitudes

$$\mathcal{A}^{(0)} = \mathcal{A}^{(0)}_{\rm b}$$

$$\mathcal{A}^{(1)} = \mathcal{A}^{(1)}_{\rm b} + \left(\delta Z^{(1)}_\alpha + \delta Z^{(1)}_F\right)\mathcal{A}^{(0)}_{\rm b}$$

$$\mathcal{A}^{(2)} = \mathcal{A}^{(2)}_{\rm b} + \left(2\delta Z^{(1)}_\alpha + \delta Z^{(1)}_F\right)\mathcal{A}^{(1)}_{\rm b} + \left(\delta Z^{(2)}_\alpha + \delta Z^{(2)}_F + \delta Z^{(2)}_f + \delta Z^{(1)}_f\delta Z^{(1)}_\alpha\right)\mathcal{A}^{(0)}_{\rm b} + \delta Z^{(1)}_M\,\mathcal{A}^{(1,\text{mass CT})}_{\rm b}$$



$$\delta Z^{(2)}_f = n_h\left(\frac{1}{16\epsilon} - \frac{5}{96} + \ln\frac{\mu^2}{8}\right)$$

[QCD :: Czakon, Mitov, Moch (2007)]

UV Renormalisation

- Fields

$$\psi_{\rm b} = \sqrt{Z_2}\psi\,, \quad A^\sigma_{\rm b} = \sqrt{Z_3}A^\sigma\,, \quad M_{\rm b} = Z_M M$$

- QED interaction vertex ⟶ Fixed from QED Ward id'

$$\mathcal{L}_{\rm int} = e_{\rm b}\,\bar\psi_{\rm b}\,\not{A}_{\rm b}\,\psi_{\rm b} = e\,\bar\psi\,\not{A}\,\psi$$

- Scheme :: On-shell + MSbar

$$Z_{2,e} = Z^{\rm OS}_{2,e}\,, \quad Z_{2,\mu} = Z^{\rm OS}_{2,\mu}\,, \quad Z_M = Z^{\rm OS}_M\,, \quad Z^{\overline{\rm MS}}_\alpha = 1/Z^{\overline{\rm MS}}_3$$

$$Z_j = 1 + \left(\frac{\alpha}{\pi}\right)\delta Z^{(1)}_j + \left(\frac{\alpha}{\pi}\right)^2\delta Z^{(2)}_j + \mathcal{O}\left(\alpha^3\right)$$

- UV Renormalised amplitudes

$$\mathcal{A}^{(0)} = \mathcal{A}^{(0)}_{\rm b}$$

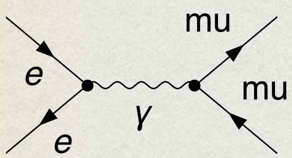
$$\mathcal{A}^{(1)} = \mathcal{A}^{(1)}_{\rm b} + \left(\delta Z^{(1)}_\alpha + \delta Z^{(1)}_F\right)\mathcal{A}^{(0)}_{\rm b}$$

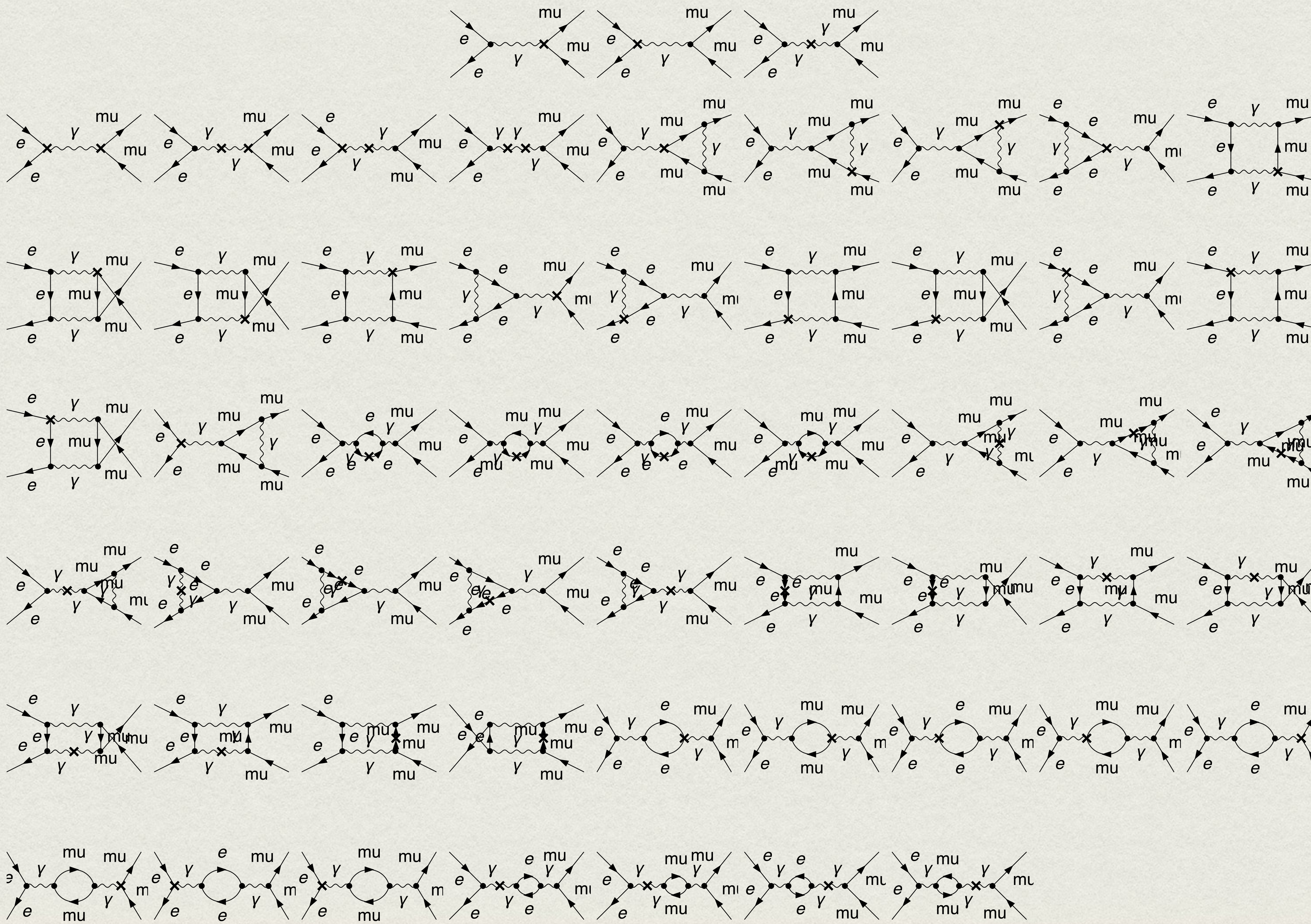
$$\mathcal{A}^{(2)} = \mathcal{A}^{(2)}_{\rm b} + \left(2\delta Z^{(1)}_\alpha + \delta Z^{(1)}_F\right)\mathcal{A}^{(1)}_{\rm b} + \left(\delta Z^{(2)}_\alpha + \delta Z^{(2)}_F + \delta Z^{(2)}_f + \delta Z^{(1)}_f\delta Z^{(1)}_\alpha\right)\mathcal{A}^{(0)}_{\rm b} + \delta Z^{(1)}_M\mathcal{A}^{(1,\text{mass CT})}_{\rm b}$$

UV Renormalisation

$$
\begin{aligned}
\mathcal{A}^{(2)} = {} & \mathcal{A}_{\rm b}^{(2)} + \left(2\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right)\mathcal{A}_{\rm b}^{(1)} \\
& + \left(\delta Z_\alpha^{(2)} + \delta Z_F^{(2)} + \delta Z_f^{(2)} + \delta Z_f^{(1)}\delta Z_\alpha^{(1)}\right)\mathcal{A}_{\rm b}^{(0)} \\
& + \delta Z_M^{(1)}\mathcal{A}_{\rm b}^{(1,\text{mass CT})}
\end{aligned}
$$

- diagram-by-diagram approach

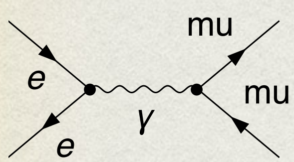


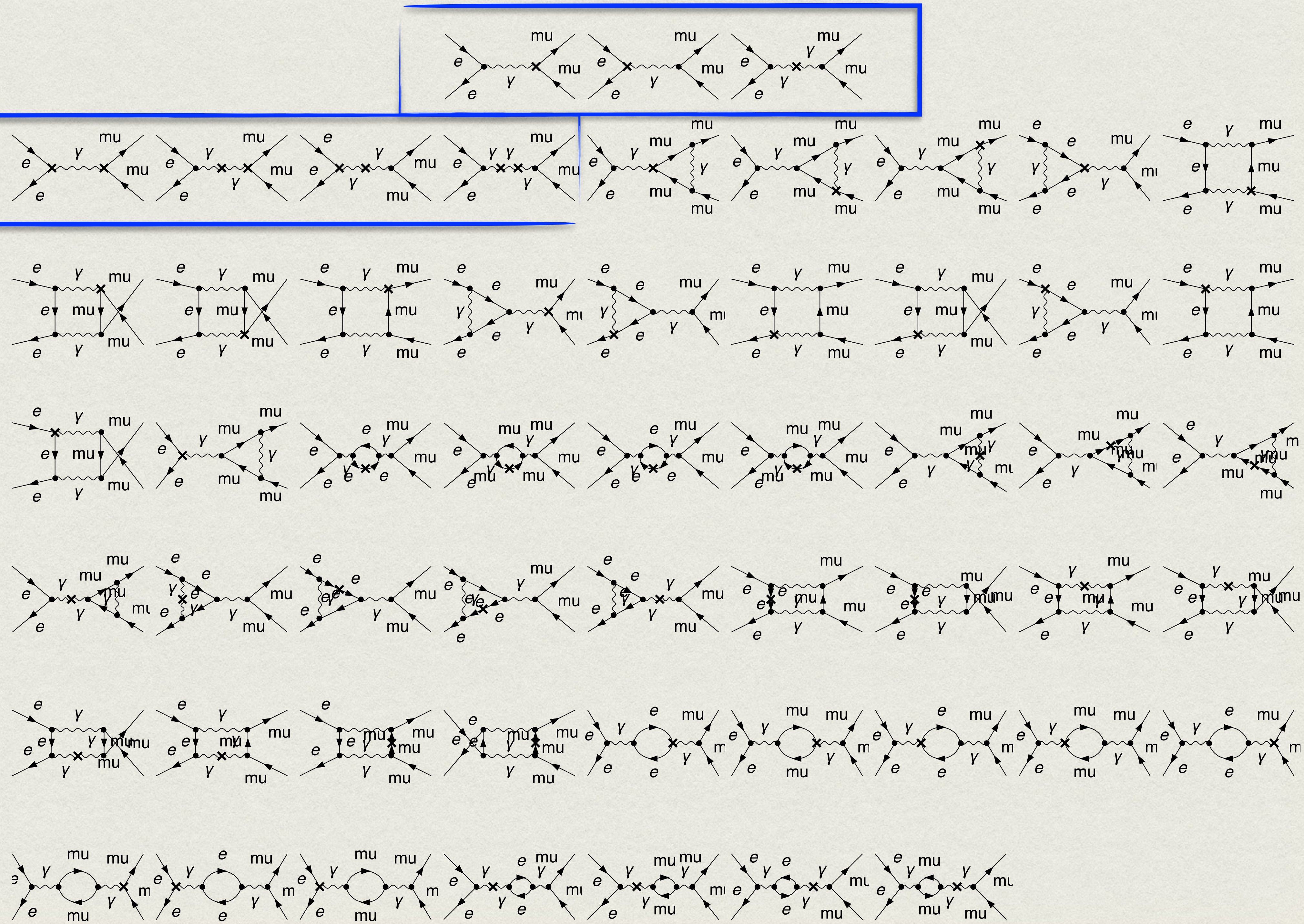


UV Renormalisation

$$\mathcal{A}^{(2)} = \mathcal{A}_{\rm b}^{(2)} + \left(2\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}\right)\mathcal{A}_{\rm b}^{(1)} + \left(\delta Z_\alpha^{(2)} + \delta Z_F^{(2)} + \delta Z_f^{(2)} + \delta Z_f^{(1)}\delta Z_\alpha^{(1)}\right)\mathcal{A}_{\rm b}^{(0)} + \delta Z_M^{(1)}\mathcal{A}_{\rm b}^{(1,\text{mass CT})}$$

- diagram-by-diagram approach

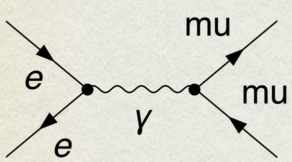


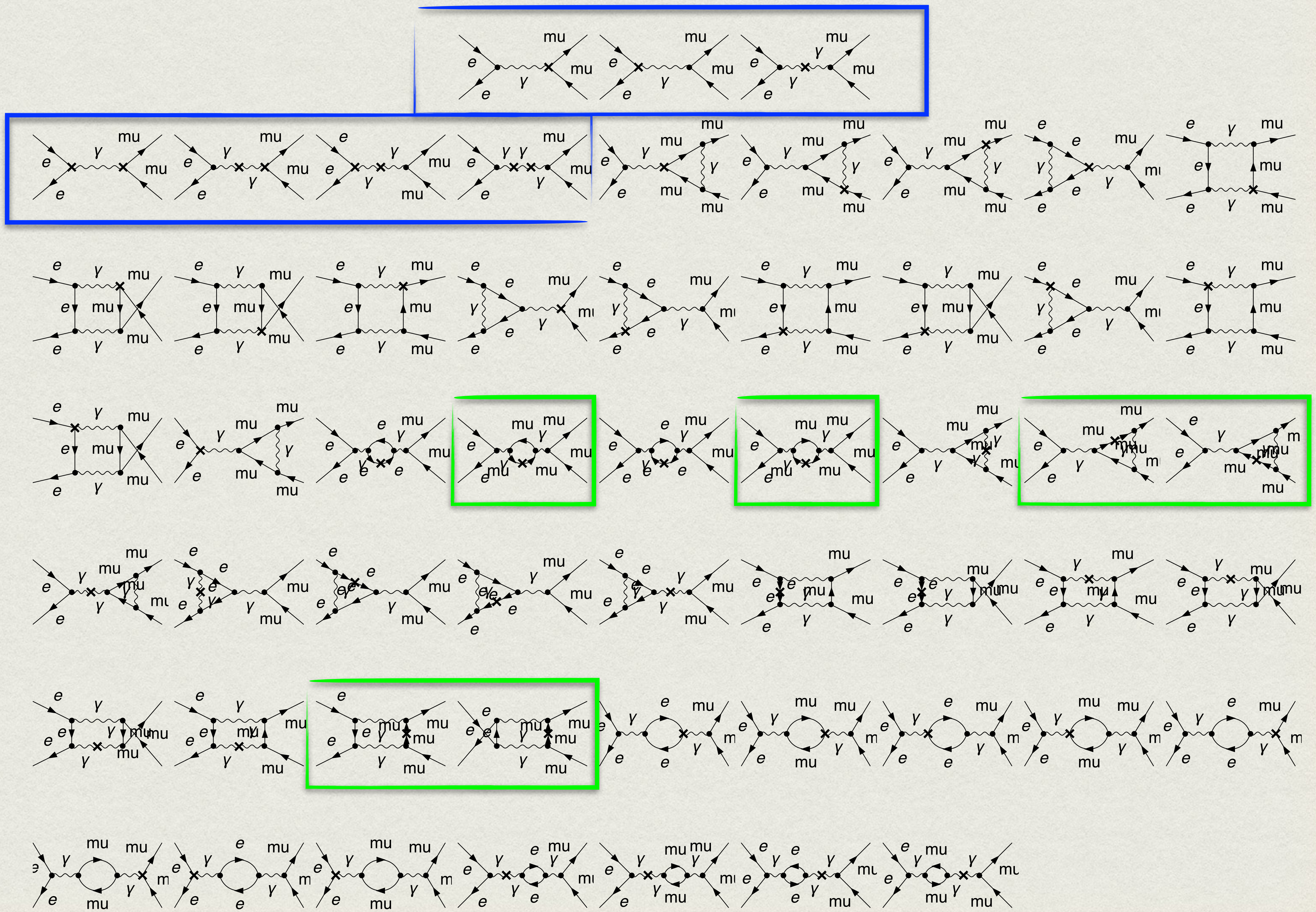


UV Renormalisation

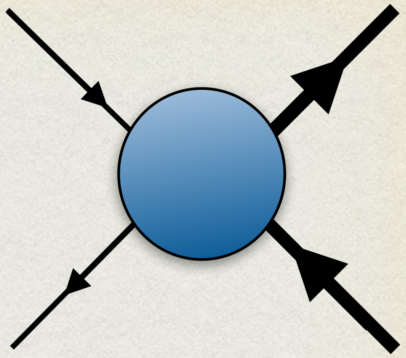
$$\mathcal{A}^{(2)} = \mathcal{A}_{\mathrm{b}}^{(2)} + \left(2\delta Z_{\alpha}^{(1)} + \delta Z_{F}^{(1)}\right)\mathcal{A}_{\mathrm{b}}^{(1)} + \left(\delta Z_{\alpha}^{(2)} + \delta Z_{F}^{(2)} + \delta Z_{f}^{(2)} + \delta Z_{f}^{(1)}\delta Z_{\alpha}^{(1)}\right)\mathcal{A}_{\mathrm{b}}^{(0)} + \delta Z_{M}^{(1)}\mathcal{A}_{\mathrm{b}}^{(1,\text{mass CT})}$$

- diagram-by-diagram approach





Results



$$\mathcal{M}^{(n)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\text{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(n)}\right)$$

$$\mathcal{M}^{(1)} = A^{(1)} + \boldsymbol{n_l}\, B_l^{(1)} + \boldsymbol{n_h}\, C_h^{(1)}$$
$$\mathcal{M}^{(2)} = A^{(2)} + \boldsymbol{n_l}\, B_l^{(2)} + \boldsymbol{n_h}\, C_h^{(2)} + \boldsymbol{n_l^2}\, D_l^{(2)} + \boldsymbol{n_h n_l}\, E_{hl}^{(2)} + \boldsymbol{n_h^2}\, F_h^{(2)}$$

- Evaluation @ $s/M^2 = 5\,, t/M^2 = -5/4\,, \mu = M.$

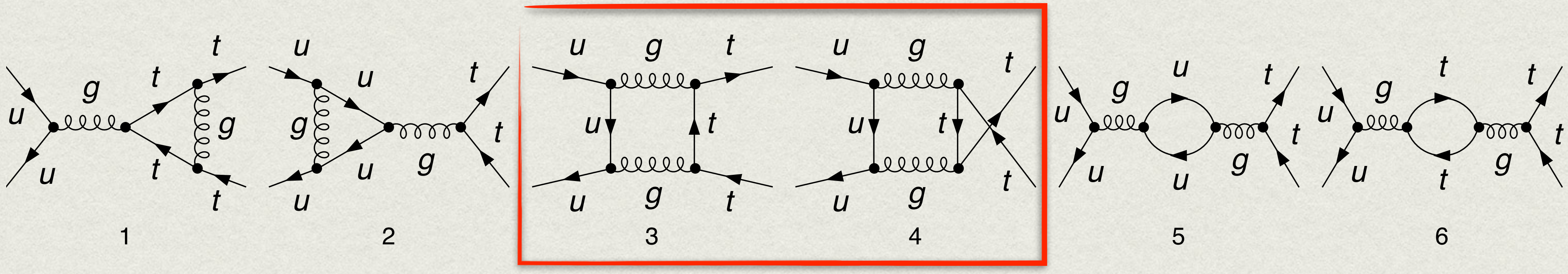
[Ginac :: Vollinga, Weinzierl (2004)]
[HandyG :: Naterop, Signer, Y. Ulrich (2019)]

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}	ϵ
$\mathcal{M}^{(0)}$	-	-	-	-	$\frac{181}{100}$	-2
$A^{(1)}$	-	-	$-\frac{181}{100}$	1.99877525	22.0079572	-11.7311017
$B_l^{(1)}$	-	-	-	-	-0.069056030	4.94328573
$C_h^{(1)}$	-	-	-	-	-2.24934027	2.54943566
$A^{(2)}$	$\frac{181}{400}$	-0.499387626	-35.4922919	19.4997261	48.8842283	-
$B_l^{(2)}$	-	$-\frac{181}{400}$	0.785712779	-16.1576674	-3.75247701	-
$C_h^{(2)}$	-	-	1.12467013	-9.50785825	-25.8771503	-
$D_l^{(2)}$	-	-	-	-	-3.96845688	-
$E_{hl}^{(2)}$	-	-	-	-	-4.88512563	-
$F_h^{(2)}$	-	-	-	-	-0.158490810	-

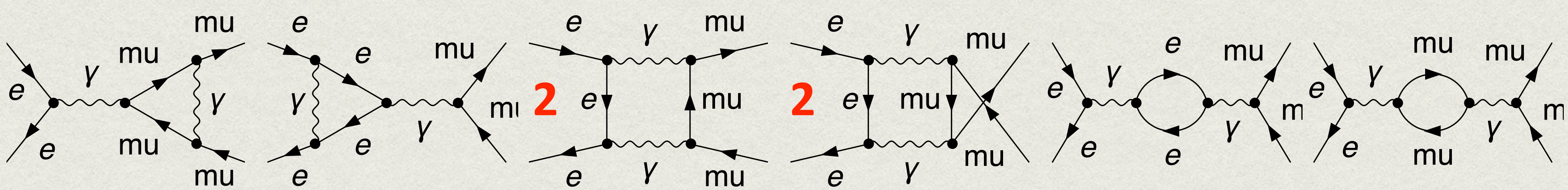
[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, W.J.T., Tramontano (2021)]

Checks

- QED -> QCD @ 1L

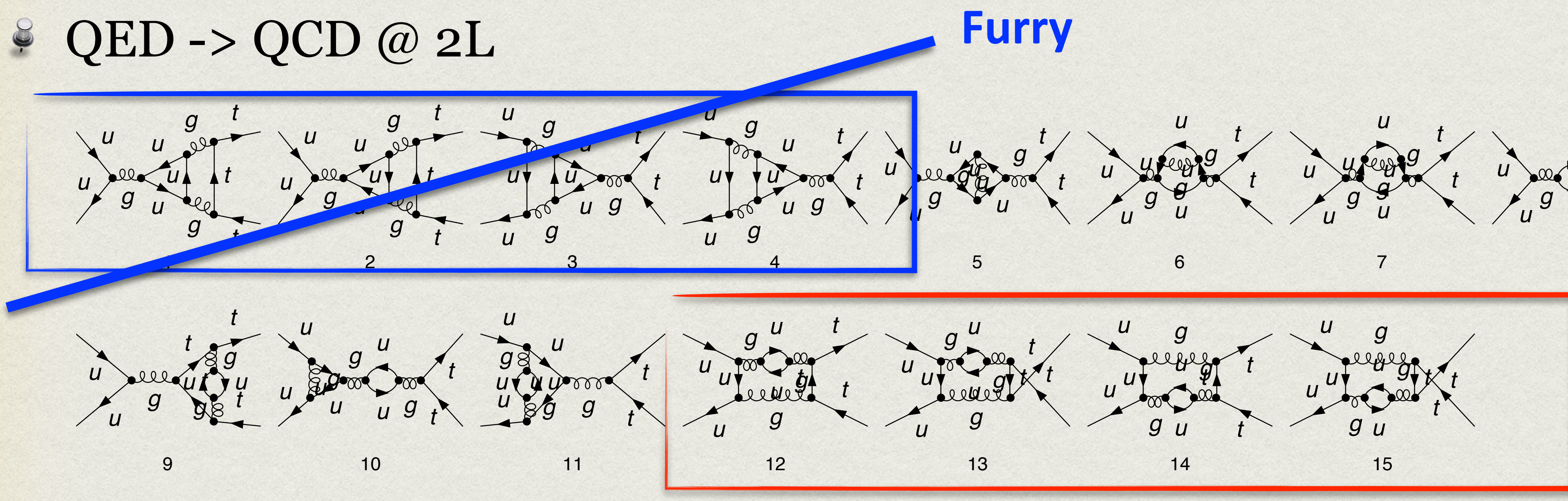


$$\frac{1}{8}\left(\mathrm{N1}^2 - 1\right)\left(\mathrm{diag}(3)\,\mathrm{N1} + \frac{-\mathrm{diag}(1) - \mathrm{diag}(2) - 2\,\mathrm{diag}(3) - 2\,\mathrm{diag}(4)}{\mathrm{N1}} + \mathrm{diag}(5) + \mathrm{diag}(6)\right)$$

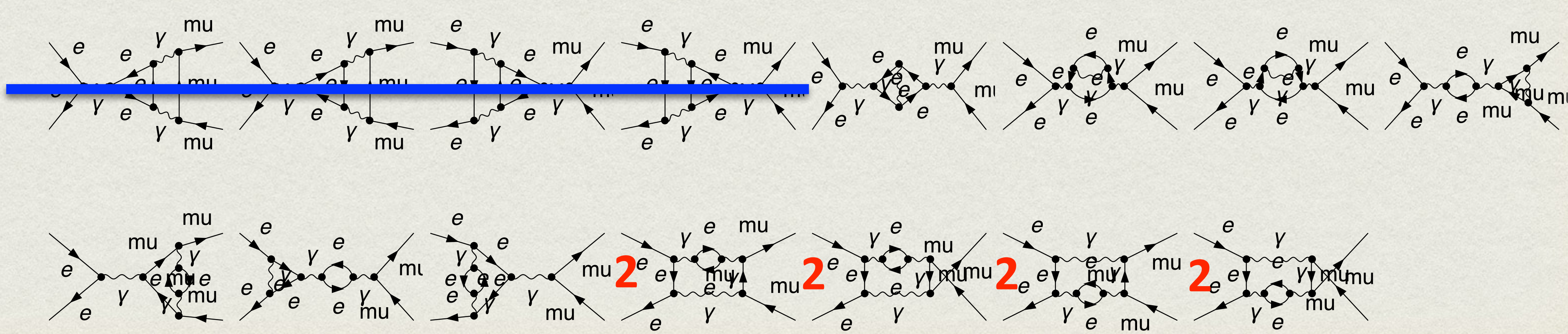


Checks

- QED -> QCD @ 2L



$$\frac{1}{16}\left(\mathrm{N1}^2-1\right)\left((\mathrm{diag}(2)+\mathrm{diag}(3)+\mathrm{diag}(6)+\mathrm{diag}(7)+\mathrm{diag}(12)+\mathrm{diag}(14))\,\mathrm{N1}+\frac{1}{\mathrm{N1}}(-2\,\mathrm{diag}(1)-2\,\mathrm{diag}(2)-2\,\mathrm{diag}(3)-2\,\mathrm{diag}(4)-\mathrm{diag}(5)-\mathrm{diag}(6)-\mathrm{diag}(7)-\mathrm{diag}(8)-\mathrm{diag}(9)-\mathrm{diag}(10)-\mathrm{diag}(11)-2\,\mathrm{diag}(12)-2\,\mathrm{diag}(13)-2\,\mathrm{diag}(14)-2\,\mathrm{diag}(15))\right)$$



Checks :: IR Structure w/ massive particles in the loop

- tree- & one-loop contributions —> two-loop IR poles after UV renormalisation

[Becher, Neubert (2009)]
[Hill (2017)]

$$\mathcal{M}^{(1)}\Big|_{\text{poles}} = \frac{1}{2} Z_1^{\text{IR}} \mathcal{M}^{(0)}\Big|_{\text{poles}}$$

$$\mathcal{M}^{(2)}\Big|_{\text{poles}} = \frac{1}{8}\left[\left(Z_2^{\text{IR}} - \left(Z_1^{\text{IR}}\right)^2\right)\mathcal{M}^{(0)} + 2Z_1^{\text{IR}}\mathcal{M}^{(1)}\right]\Big|_{\text{poles}}$$

- Anomalous dimension —> IR structure

$$\Gamma = \gamma_{\text{cusp}}(\alpha)\ln\left(-\frac{s}{\mu^2}\right) + 2\gamma_{\text{cusp}}(\alpha)\ln\left(\frac{t-M^2}{u-M^2}\right) + \gamma_{\text{cusp},M}(\alpha,s) + \gamma_h(\alpha,s) + \gamma_l(\alpha,s)$$

- IR renormalisation factor

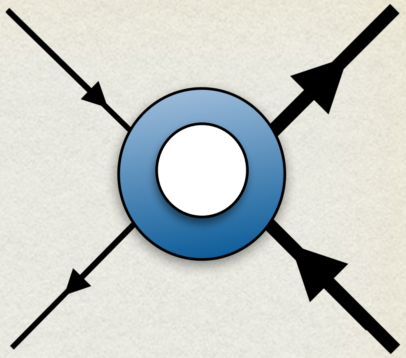
$$\ln Z^{\text{IR}} = \frac{\alpha}{4\pi}\left(\frac{\Gamma_0'}{4\epsilon^2} + \frac{\Gamma_0}{2\epsilon}\right) + \left(\frac{\alpha}{4\pi}\right)^2\left(-\frac{3\beta_0\Gamma_0'}{16\epsilon^3} + \frac{\Gamma_1' - 4\beta_0\Gamma_0}{16\epsilon^2} + \frac{\Gamma_1}{4\epsilon}\right) + \mathcal{O}\left(\alpha^3\right)$$

$$\Gamma' = \frac{\partial}{\partial \ln\mu}\Gamma(\alpha)$$

☑ Full agreement of the IR poles structure obtained by direct calculation of the two-loop diagrams

real & imaginary part!

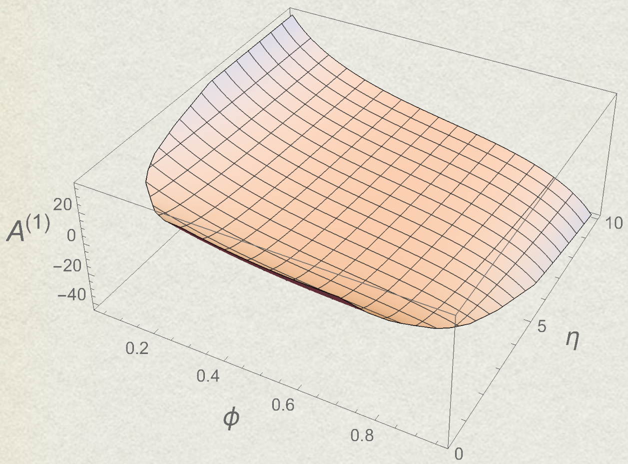
Finite reminders

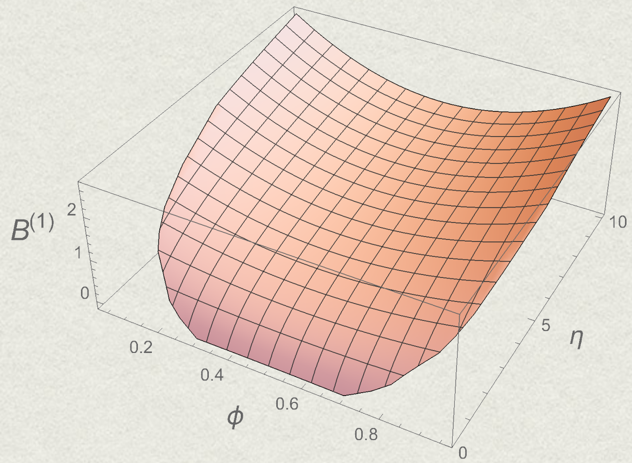


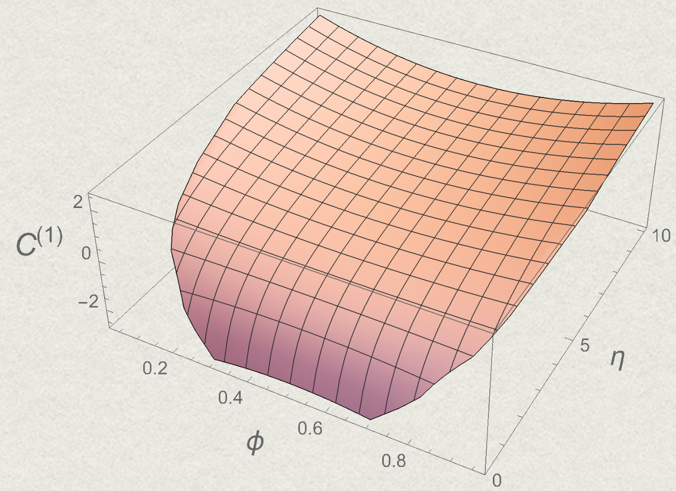
$$\mathcal{M}^{(1)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\text{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(1)}\right)$$

$$\mathcal{M}^{(1)} = A^{(1)} + n_l\, B_l^{(1)} + n_h\, C_h^{(1)}$$

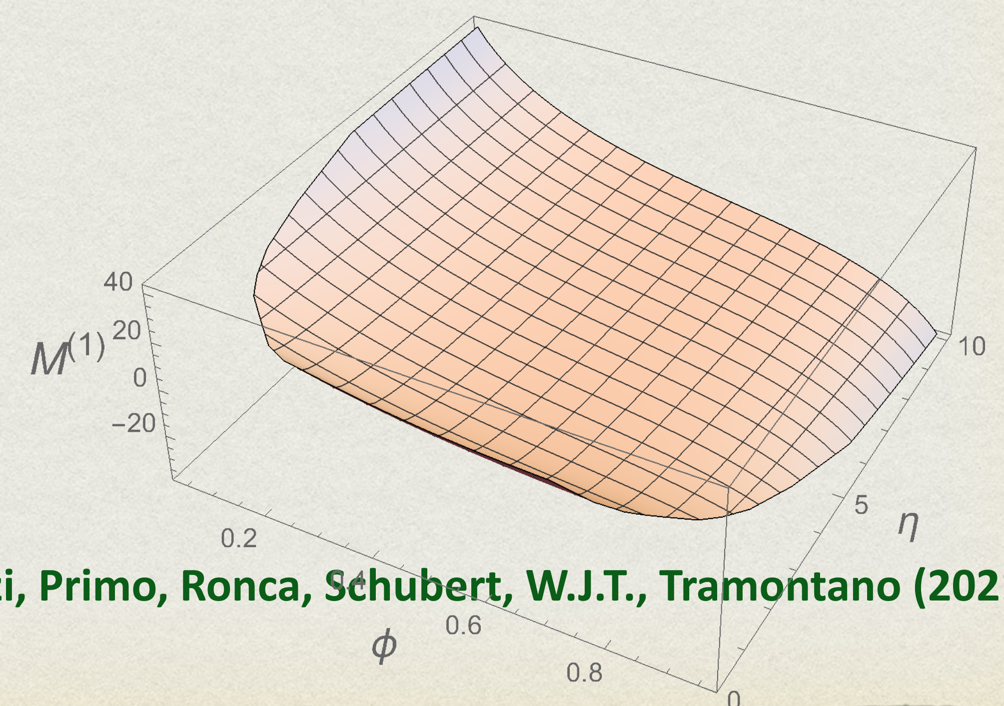
$$\eta = \frac{s}{4M^2} - 1\,, \phi = -\frac{t-m^2}{s}\,,$$
$$\frac{1}{2}\left(1-\sqrt{\frac{\eta}{1+\eta}}\right) \le \phi \le \frac{1}{2}\left(1+\sqrt{\frac{\eta}{1+\eta}}\right)$$





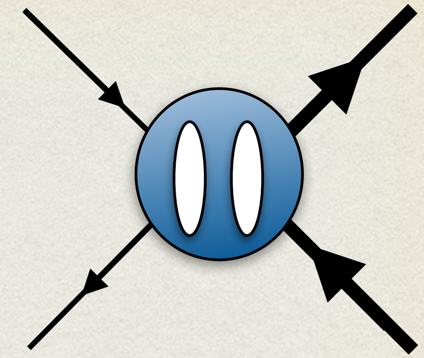


- Full one-loop contribution $n_l = n_h = 1$



[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, W.J.T., Tramontano (2021)]

Finite reminders

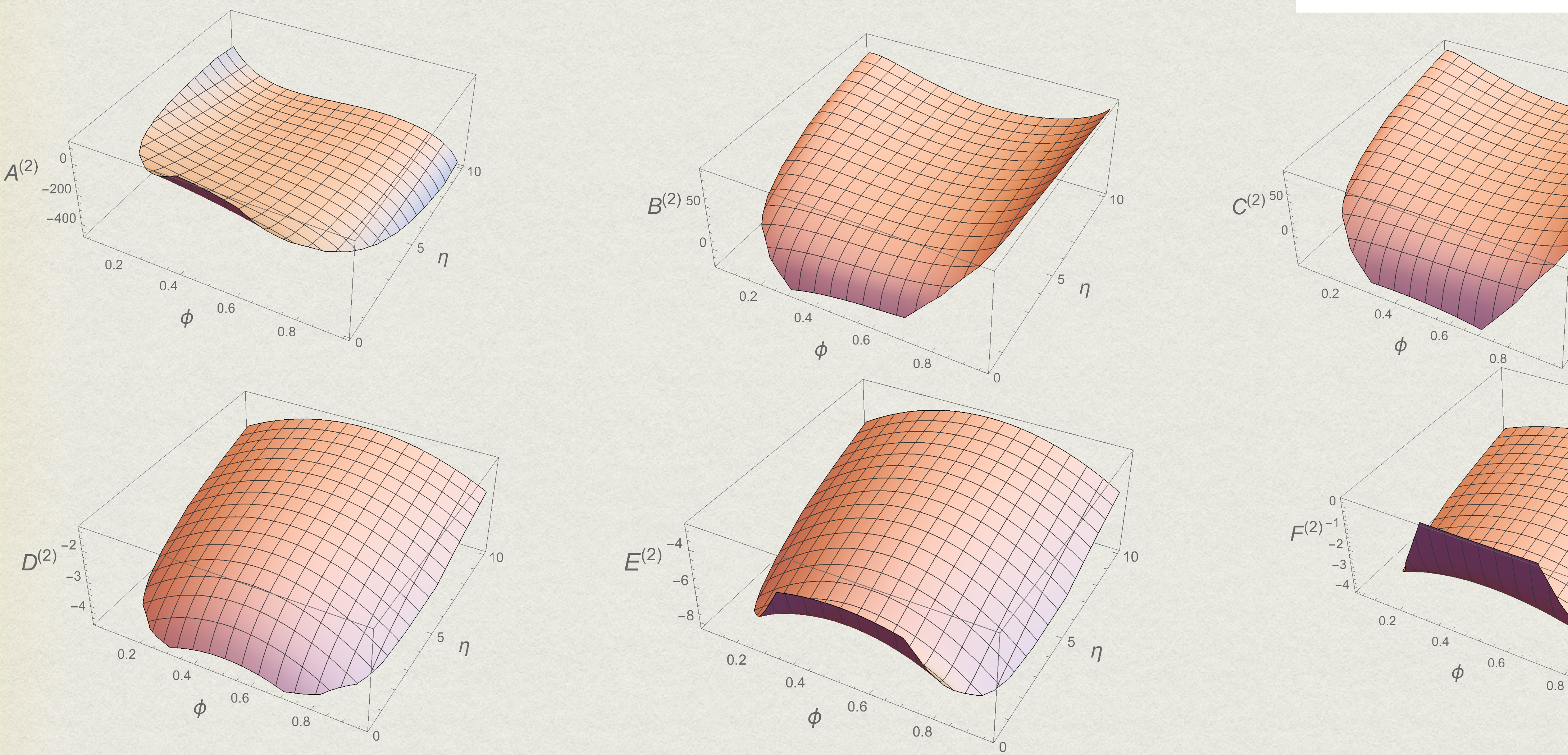


$$\mathcal{M}^{(2)}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{1}{4}\sum_{\text{spins}} 2\text{Re}\left(\mathcal{A}^{(0)*}\mathcal{A}^{(2)}\right)$$

$$\mathcal{M}^{(2)} = A^{(2)} + \boldsymbol{n_l}\, B_l^{(2)} + \boldsymbol{n_h}\, C_h^{(2)} + \boldsymbol{n_l^2}\, D_l^{(2)} + \boldsymbol{n_h n_l}\, E_{hl}^{(2)} + \boldsymbol{n_h^2}\, F_h^{(2)}$$

$$\eta = \frac{s}{4M^2} - 1\,, \phi = -\frac{t-m^2}{s}\,,$$

$$\frac{1}{2}\left(1-\sqrt{\frac{\eta}{1+\eta}}\right) \leq \phi \leq \frac{1}{2}\left(1+\sqrt{\frac{\eta}{1+\eta}}\right)$$



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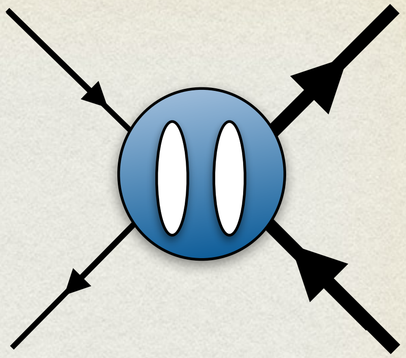

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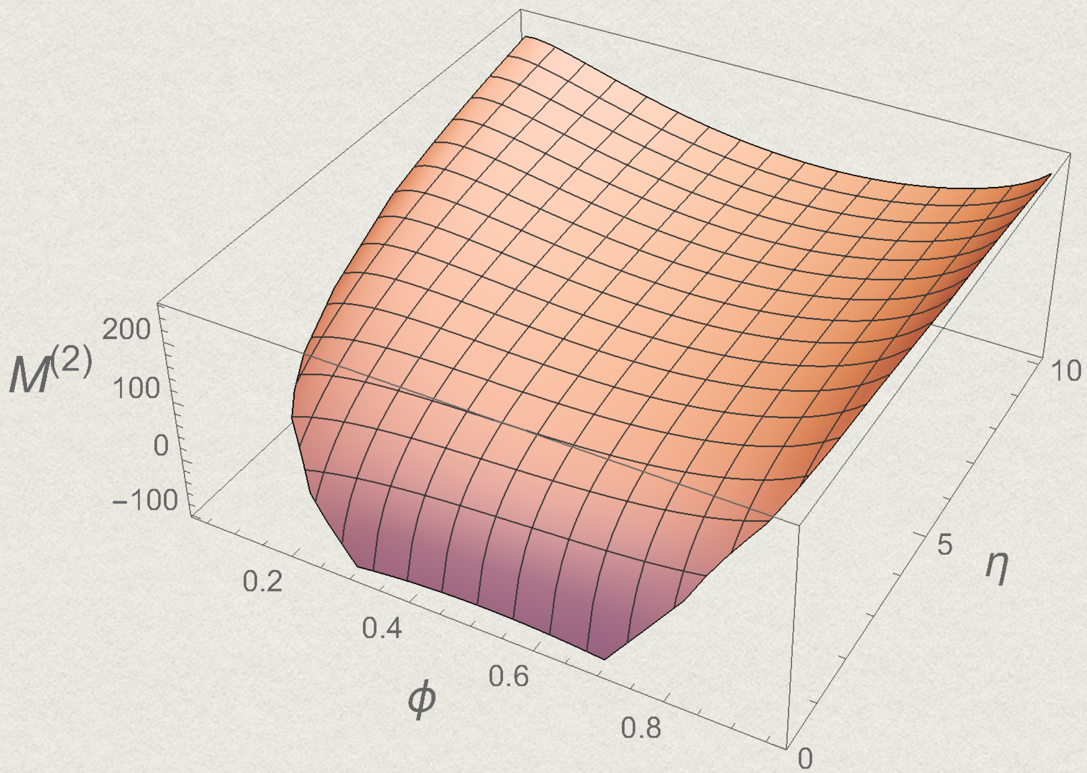

[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, W.J.T., Tramontano (2021)]

Conclusions/Outlook

- ☑ Integrand decomposition methods —> @2 loops Automated (AIDA)
- ☑ Complete analytic UV renormalised two-loop contribution to di-muon production
- ☑ Straightforward numerical evaluation—> Finite reminders
- ☑ Full agreement w/ IR-pole prediction
- ☑ Theoretical support to MUonE experiment

- ☐ Provide full NNLO calculation
- ☐ Compute MIs for in the $m_{\rm e}/m_\mu$ expansion [Heller (2021)]
- ☐ Extend our approach to QCD

[Barnreuther, Czakon, Fiedler (2013)]
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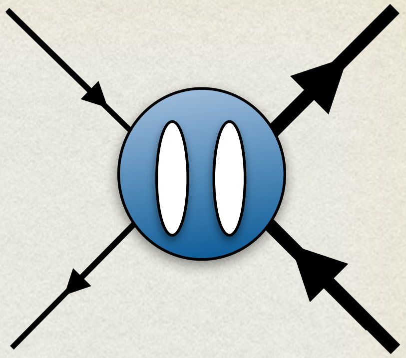
Thanks

Extra slides

Finite reminders



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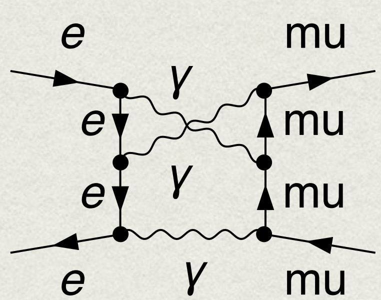
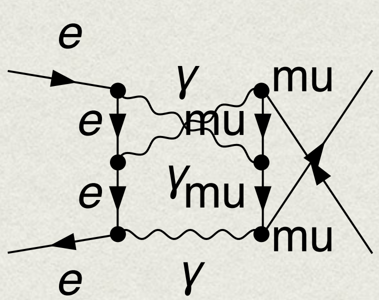
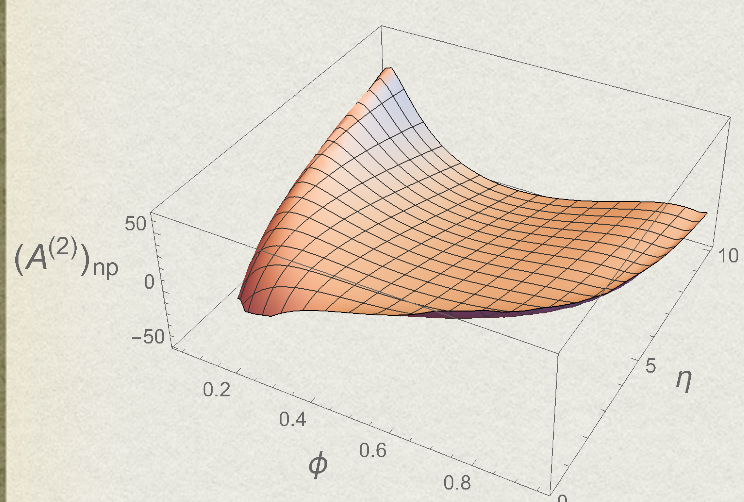
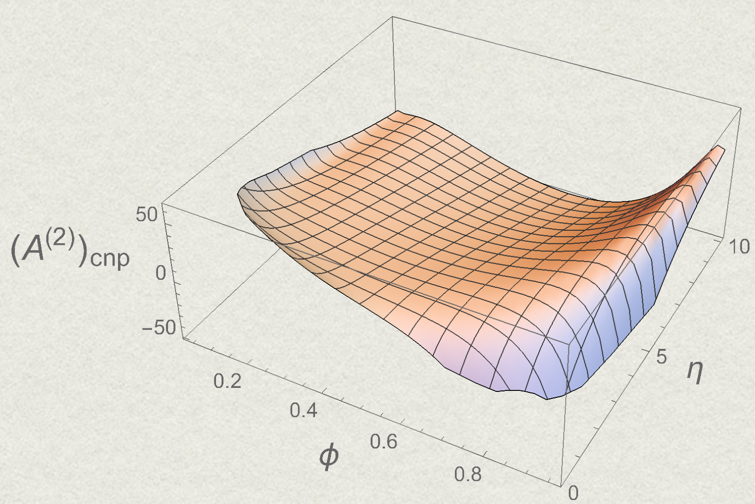
- Evaluation of non-planar diagrams









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