Factorization and resummation at high energy beyond the leading power

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A Loop Summit - new perturbative results and methods in precision physics:
Quantum Field Theory, Collider Physics and Gravity
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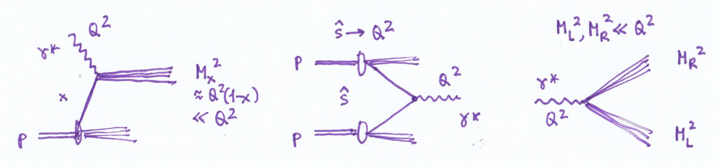
- NLP log resummation for $2 \to 1$, $1 \to 2$ processes
- Factorization of Drell-Yan production at NLP near threshold, endpoint divergences
- Off-diagonal channels and soft quarks, the "soft quark Sudakov factor"
- Off-diagonal DIS at NLP LL, splitting function and the Bernoulli series

MB, Garny, Jaskiewicz, Szafron, Wang, Vernazza **2008.04943**; + Broggio **1809.10631**; MB, Broggio, Jaskiewicz, Vernazza **1912.01585**, and MB, Garny, Szafron, Wang 1712.04416, 1712.07462, 1808.04724, 1907.05463

[Nice figures courtesy of S. Jaskiewicz and R. Szafron]

Motivation

- Kinematic thresholds of $2 \to 1$ / $1 \to 2$ processes



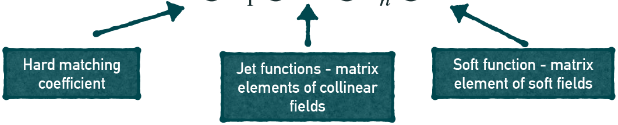
- All-order resummation often required/useful (threshold, event shapes, jets, semi-inclusive observables ...), $\tau \ll 1$
  - Leading power IR logs $\alpha_s^n \left[\frac{\ln^m \tau}{\tau}\right]_+$ by now standard: LL, NLL, NNLL, N3LL, ...
  - Where there are logs, there are powers, and powers times logs $\to$ next-to-leading power (NLP) resummation
  - Structure of NLP Logs $[\alpha_s^n \ln^m \tau]$, $m \leq 2n - 1$ not well known

Method: SCET

$$p^\mu = n_+ \cdot p \, \frac{n_-^\mu}{2} + p_\perp^\mu + n_- \cdot p \, \frac{n_+^\mu}{2}$$

- Strict expansion in $\lambda \sim p_\perp / n_+ p \equiv \sqrt{1-x}, \sqrt{1-z}, \sqrt{\tau}$. NLP is $\mathcal{O}(\lambda^2)$
- Factorize into **single scale** ("homogeneous") objects, which have **gauge-invariant operator definitions: hard, jet/collinear and soft functions**

$$\sigma \sim H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

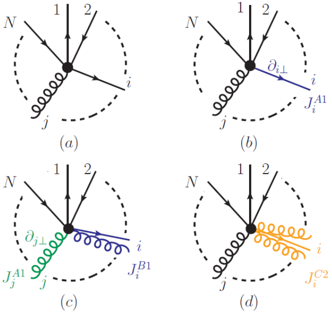


- IR logs in QCD are UV logs in SCET. **Resummation is an operator renormalization / mixing + renormalization group problem**

SCET basics - hard process

Hard sub-processes are represented by N-jet light-ray operators of gauge-invariant quark and gluon "jet" fields, which scale $\mathcal{O}(\lambda)$.
General form of the operator:

$$\mathcal{O}(0) = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i}\, C(\{t_{ik_i}\}) \times \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \ldots t_{i_{n_i}})$$



Collinear building blocks for J_i:

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i, \quad \mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$$

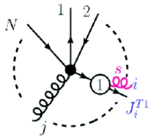
- LP: $J_i = \chi_i(t_i n_{i+})$ or $\mathcal{A}^\mu_{\perp i}(t_i n_{i+})$
- For this talk, at NLP, only need

$$\int dt_1 dt_2 C^{B1}(t_1, t_2) \chi(t_1 n_+) \mathcal{A}^\mu_\perp(t_2 n_i)$$

SCET basics - Lagrangian at NLP

Position-space SCET formalism [MB, Chapovsky, Diehl, Feldmann, 2002]
Lagrangian describes collinear splittings and interactions of (separate) collinear and soft fields

$$\mathcal{L}^{(0)}_{\rm SCET} = \sum_{i=1}^{N} \mathcal{L}^{(0)}_{c_i} + \mathcal{L}_{\rm soft}$$



Soft interactions

$$\mathcal{L}^{\rm gluon}_{sc}(x) = \bar{\xi}\Big[\underbrace{g_s n_- A_s(x_-)}_{\text{LP, eikonal}} + \underbrace{x_\perp^\mu n_-^\nu\, W_c\, g_s F^{s}_{\mu\nu}(x_-) W_c^\dagger}_{\text{NLP}}\Big]\frac{\not{n}_+}{2}\,\xi + \mathcal{O}(\lambda^2)$$

$$\mathcal{L}^{\rm quark}_{sc}(x) = \underbrace{\bar{q}(x_-)\, W_c^\dagger\, i\not{D}_{\perp c}\,\xi}_{\text{starts at NLP}} + \text{h.c.} + \mathcal{O}(\lambda^2)$$

$$iD_c = i\partial + g_s A_c\,, \qquad x_-^\mu = \frac{1}{2} n_+ \cdot x\, n_-^\mu$$

- LP eikonal soft interaction can be decoupled via soft Wilson line field redefinition $\xi \to Y(x_-)\xi^{(0)}$ [Bauer, Pirjol, Stewart, 2001]
- Multipole expansion of the soft field around x_- in collinear interactions.
- No purely collinear subleading interactions. At least one soft field in every vertex.

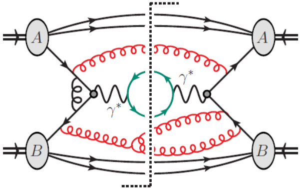
LP resummation [Sterman, 1987; Catani, Trentadue 1989; Korchemsky, Marchesini, 1993; SCET: Idilbi et al. (2005/06); Becher, Neubert, Xi, 2007]

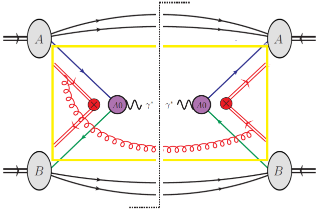
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, z\, \frac{\hat\sigma_{ab}(z)}{z}$$

$$\hat\sigma(z) = H(Q^2)\, Q S_{\rm DY}(Q(1-z))$$

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{i\Omega x^0/2}\, \widetilde{S}_0\left(x^0\right)$$

$$\widetilde{S}_0(x) = \frac{1}{N_c}{\rm Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x) Y_-(x)\right] \mathbf{T}\left[Y_-^\dagger(0) Y_+(0)\right]|0\rangle$$





- Hard and soft only. No hard-collinear functions.
- Soft function consists only of Wilson lines.

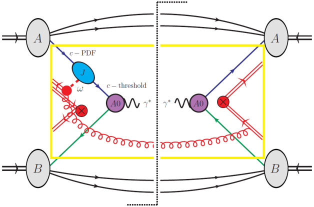
Drell-Yan production near threshold - NLP

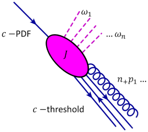
NLP LL resummation [1808.10631] and NLP factorization [1912.01585]

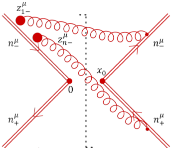
$$\Delta^{dyn}_{\rm NLP}(z) = -\frac{2}{(1-\epsilon)}\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma}$$
$$\times \int d(n_+p)\, C^{A0,A0}(n_+p, x_b n_- p_B)\, C^{*A0,A0}(x_a n_+ p_A, x_b n_- p_B)$$
$$\times \sum_{i=1}^{5} \int \{d\omega_j\}\, J_{i,\gamma\beta}(n_+p, x_a n_+ p_A; \{\omega_j\})\, S_i(\Omega; \{\omega_j\}) + \text{h.c.}\,,$$

$$i\int d^4z\, \mathbf{T}\Big[\chi_{c,\gamma f}(tn_+)\, \mathcal{L}^{(2)}(z)\Big] = 2\pi \sum_i \int \frac{d\omega}{2\pi} \int \frac{dn_+p}{2\pi}\, e^{-i(n_+p)t} \int \frac{dn_+p_a}{2\pi}$$
$$\times J_{i;\gamma\beta,\mu,fbd}(n_+p, n_+p_a; \omega)\, \hat{\chi}^{\rm PDF}_{c,\beta b}(n_+p_a) \int dz_-\, e^{-i\omega z_-}\, \mathfrak{s}_{i;\mu,d}(z_-)\,,$$

$$S_1(x^0; z_-) = \frac{1}{N_c}\, \text{Tr}\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0) Y_-(x^0)\right] \mathbf{T}\left(\left[Y_-^\dagger(0) Y_+(0)\right] \frac{i\partial_\perp^\nu}{in_-\partial} \mathcal{B}^+_{\nu_\perp}(z_-)\right)|0\rangle$$







- Hard-collinear functions at amplitude level
- Generalized soft functions with field insertions [MB, Campanario, Mannel, Pecjak, 2004]
- Convolution $J(\omega) \otimes S(\omega)$
- Validation: One-loop collinear fns [1912.01585] and two-loop soft [Broggio, Jaskieiwcz, Vernazza, 2021] reproduce diagrammatic NNLO results and beyond [Bonocore et al., 2016]

NLP LL resummation and divergent convolutions

- The LL resummed partonic cross section for $q\bar{q} \to \gamma^* + X$ is very simple.

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \exp\left[4S^{\rm LL}(\mu_h,\mu) - 4S^{\rm LL}(\mu_s,\mu)\right] \times \frac{-8C_F}{\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}$$

  Similar result for Higgs production [1910.12685].

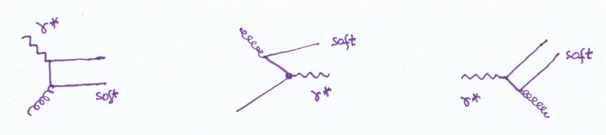
- However, at NLL a convolution in $J \otimes S$ appears, that exists in d dimensions, but does not for $\epsilon \to 0$.

$$\int_0^\Omega d\omega \, \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}} \underbrace{\frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}}$$

- Do not have a renormalized factorization theorem for the partonic cross section. Have to refactorize the parton distributions for $x \to 1$ as well from NLP.

"Off-diagonal" channels and soft quarks

- Soft quarks appear first at NLP. Recent interest from various directions: $B \to \mu\mu$ [MB, Bobeth, Szafron, 2017], mass-suppressed form factors [Penin, Liu, 2017] and $H \to \gamma\gamma$ [Penin, Liu, 2019; Neubert et al., 2019].
- $1 \to 2$ / $2 \to 1$ off-diagonal high-energy scattering (threshold) [2008.04943]



- Two intriguing observations:
  - Off-diagonal parton splitting kernel [Vogt et al., 2010]

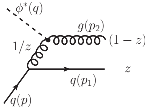
$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

  - Quark Sudakov exponentiation conjecture for the $\gamma^* \to g + (q\bar{q})$ amplitude for $s_{q\bar{q}} \ll Q^2$ [Moult et al. 2019] contains

$$\frac{\alpha_s}{4\pi}\ln N \exp\left(-\alpha_s C_F/\pi \ln^2 N\right) \times \frac{e^{-a}-1}{a}$$

$q + \phi^* \to X$ DIS for $M_X^2 \ll Q^2$

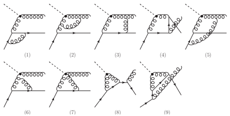
Virtual correction to the structure function



$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^\epsilon \mathcal{P}_{qg}(s_{qg}, z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}$$

$$\mathcal{P}_{qg}(s_{qg}, z)\big|_{\text{tree}} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z} + \mathcal{O}(\epsilon, \lambda^2)$$

Endpoint divergent, gives NLP LL at $\mathcal{O}(\alpha_s)$.



One-loop correction, double pole part ($\leftrightarrow$ NLP LL)

$$\mathscr{P}_{qg}(z)\,|_{1-\text{loop}} = \mathscr{P}_{qg}(z)\,|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left\{T_1\cdot T_0\left(\frac{\mu^2}{zQ^2}\right)^\epsilon + T_2\cdot T_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon + T_1\cdot T_2\left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon\right]\right\}$$

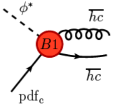
Formally single pole

Promoted to leading pole after integration

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1 - z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3}$$

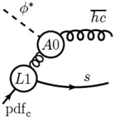
- Expansion in ϵ before integration gives wrong $-1/\epsilon^3$. No leading pole for soft gluon emission.
- Resummation of terms singular as $z \to 0$ is required to all orders, z^ϵ counts as $\mathcal{O}(1)$
- zQ^2 as an emergent new scale

$$\int_0^1 dz C^{B1}(Q,z) \quad \times$$



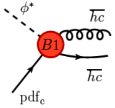
z-SCET interpretation and refactorization

$z \ll 1$ region: $\displaystyle \int_0^1 dz C^{B1}(Q,z) \quad \times$



z-SCET interpretation and refactorization

$$\int_0^1 dz\, C^{B1}(Q,z) \quad \times$$



For $z \ll 1$ the C^{B1} matching coefficient is a two-scale object and must be resummed. Construct an "auxiliary" z-SCET containing z-soft and z-anti-softcollinear modes.

Multi-scale object

$$C^{B1}(Q,z) J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2) \int d^4x\, T\left\{ J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z-\overline{sc}}}(x) \right\} = C^{A0}(Q^2)\, D^{B1}(zQ^2,\mu^2)\, J^{B1}_{z-\overline{sc}}$$

Single-scale objects

Renormalization group equations

$$\frac{d}{d\ln\mu} C^{A0}(Q^2,\mu^2) = \frac{\alpha}{\pi} C_A \ln\frac{Q^2}{-\mu^2} C^{A0}(Q^2,\mu^2)$$

$$\frac{d}{d\ln\mu} D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha}{\pi}\left(C_F - C_A\right) \ln\frac{zQ^2}{-\mu^2} D^{B1}\left(zQ^2,\mu^2\right)$$

The solution gives the conjectured exponentiated "soft quark Sudakov factor" (after integration over z)

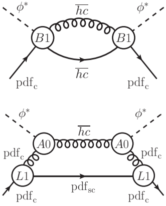
Factorization of hadronic tensor in terms of unrenormalized (“bare”) partonic cross sections and PDFs.

$$W = \sum_i W_{\phi,i} f_i$$

$$\sum_i (W_{\phi,i} f_i)^{NLP} = W^{NLP}_{\phi,q} f^{LP}_q + W^{NLP}_{\phi,\bar{q}} f^{LP}_{\bar{q}} + W^{NLP}_{\phi,g} f^{LP}_g + W^{LP}_{\phi,g} f^{NLP}_g \, .$$

TH
- *Hard*, $p^2 = Q^2$
- *Anti-hardcollinear*, $p^2 = Q^2\lambda^2 = Q^2/N$

PDF
- *Collinear*, $p^2 = \Lambda^2$
- *Soft-Collinear*, $p^2 = \Lambda^2\lambda^2 = \Lambda^2/N$



Consistency relations: Each region contribution different dependence on N,Q and ϵ. Impose pole cancellation on the general Ansatz

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c^{(n)}_{kj}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon}$$

Solution

Consistency relations $$\sum_{k=0}^{n}\sum_{j=0}^{n} j^r k^s c_{kj}^{(n)}(\epsilon=0)=0 \qquad \text{for } s+r<2n-1,\ r,s\geq 0$$

Only three undetermined coefficients at each n. Two are trivial: $c_{n0}^{(n)} = c_{00}^{(n)} = 0$, while $c_{n1}^{(n)}$ is the soft quark Sudakov factor derived before!
Booststrap the full solution algebraically from consistency.

Renormalized d-dimensional PDFs, coefficient functions and $\overline{\text{MS}}$ splitting functions:

$$\tilde{f}_k = Z_{ki} f_i, \qquad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki} \Rightarrow W_{\phi,i} f_i = \tilde{C}_{\phi,k} \tilde{f}_k$$

$$\tilde{C}_{\phi,q}^{NLP,LL}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{\mathbb{C}_F}{\mathbb{C}_F - C_A}\left(\mathcal{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N \ln\frac{\mu^2}{Q^2}\right)\right] - \exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N\ln\frac{\mu^2}{Q^2}\right)\right]\right),$$

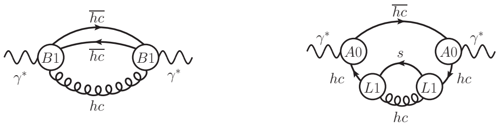
$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n \text{ and } a = \frac{\alpha_s}{\pi}(C_F - C_A)$$

$$\gamma_{gq}^{NLP,LL}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\left[F_{\text{pole}}(w,a) - w\frac{d}{da}F_{\text{pole}}(w,a)\right] = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a)$$

The Bernoulli function arises as a consequence of $\overline{\text{MS}}$ factorization. Proves Vogt's conjecture for the all-order series.

Extensions and remarks

- Same framework applies to thrust in the 2-jet region.
  No Bernoulli numbers, since PDFs are not factorized



- By crossing and reinterpetation of modes, one obtains a result for the NLP LLs in the off-diagonal DY cross section near threshold. Here the soft-quark Sudakov factor applies to the collinear function in the refactorized convolution $J \otimes S$ of the collinear and soft function.

- Provides insight into why for the off-diagonal channel even the splitting function contains double logs. Soft gluon emission does not cause this effect, because the collinear particle before and after emission has the same colour charge. "Stopping charges" causes the unconventional Sudakov exponential. Also in QED.