Mixed QCD-EW Corrections To Drell-Yan Processes

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Plan of the talk

Brief Introduction: why Drell-Yan?
Theoretical Predictions: literature
The calculation of Mixed corrections
A couple of plots ...

Introduction

- NC and CC Drell-Yan processes, $pp(\bar{p}) \rightarrow l^- l^+$ and $pp(\bar{p}) \rightarrow l\nu$ are of fundamental importance for an accurate check of the SM at hadron colliders. Sizable cross sect and high sensitivity to the properties of the gauge bosons!
- NC DY is important for the detector calibration, accurate determination of input SM parameters, as $\sin^2 \theta_{eff}^{lep}$ (permille level at Tevatron and LHC)
- CC DY production is important for the determination of the W mass (via transverse mass and lepton transverse momentum distributions using electron-neutrino and muon-neutrino final states), that was measured with $\delta m_W \sim 16$ MeV (arXiv:1204.0042) and at LHC with similar accuracy. Global fit: $\delta m_W \sim 8$ MeV. This requires an accurate theoretical control on the distributions
- NC and CC Drell-Yan processes contitute the ``SM background" for processes of new physics as Z' or W' production. Tail of kinematic distributions (high transverse momentum, high invariant mass distributions)
- Drell-Yan processes are important for the constraint on the PDFs
- Mixed QCD-EW corrections important also for the stabilization of the scale dipendence





Drell-Yan TH predictions in the Literature

- DY was one of the first hadronic processes for which perturbative corrections were calculated
- Dominant Perturbative Contributions come from QCD:



- ✤ Total cross section at NLO and NNLO
 - G. Altarellí, R. K. Ellís, G. Martínellí, Nucl.Phys.B 157 (1979) 461 R. Hamberg, W. van Neerven and T. Matsuura, Nucl.Phys.B 359 (1991) 343
- ✦ Differential cross section at NNLO
 - C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. Lett. 91 (2003) 182002 C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. D 69 (2004) 094008 K. Melnikov and F. Petriello, Phys. Rev. D 74 (2006) 114017
 - S. Catani, L. Cieri, G, Ferrera, D. De Florian and M. Grazzini, Phys. Rev. Lett. 103 (2009) 082001
- ✤ Total cross section at NNNLO
 - C. Duhr, F. Dulat and B. Místlberger, Phys. Rev. Lett. 125 (2020) 172001 C. Duhr, F. Dulat and B. Místlberger, JHEP 11 (2020) 143
- Lept-pair less inclusive at NNNLO, fiducial cross section and rapidity distribution S. Camarda, L. Cieri and G. Ferrera, 2103.04974
 X. Chen, T. Gehrmann, N. Glover, A. Huss, T. Yang and H. X. Zhu, 2107.09085
- Resummation 1987 up to NNNLL S. Moch, A. Vogt, 2005; V. Ravindran, 2006; S. Cataní, L. Cierí, D. de Florían, G. Ferrera, M. Grazzíní, 2014; ...

Drell-Yan TH predictions in the Literature

• DY was one of the first hadronic processes for which perturbative corrections were calculated $\boxed{\alpha_S^2 \sim \alpha}$

NLO EW Contributions



S. Dittmaier and M. Kraemer, Phys. Rev. D 65 (2002) 073007
U. Baur and D. Wackeroth, Phys. Rev. D 70 (2004) 073015
V. Zykunov, Phys. Atom. Nucl. 69 (2006) 1522
A. Arbuzov at al., Eur. Phys. J. C 46 (2006) 407

C. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 12 (2006) 016

♦ NC Drell-Yan

U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, Phys. Rev. D 65 (2002) 033007
V. Zykunov, Phys. Rev. D 75 (2007) 073019
C. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 10 (2007) 109
Arbuzov at al., Eur. Phys. J. C 54 (2008) 451
S. Dittmaier and M. Huber, JHEP 01 (2009) 060

Differential NLO corrections both QCD and EW

MCFM, HORACE, POWHEG, MADGRAPH, MC@NLO

Drell-Yan TH predictions in the Literature

Mixed QCD-EW NNLO Corrections

- ♦ On-Shell Z/W production
 - Mixed QCD-QED corrections to the inclusive prod of an on-shell Z D. de Florian, M. Der and I. Fabre, Phys. Rev. D 98 (2018) 094008
 - Fully differential mixed QCD-QED corrections to the prod of an on-shell Z
 - M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, JHEP 01 (2019) 043
 - Mixed QCD-EW corrections to the inclusive prod of an on-shell Z (analytic) B., F. Buccioni, N. Rana and A. Vicini, Phys. Rev. Lett. 125 (2020) 232004
 - Fully differential mixed QCD-EW corrections to the inclusive prod of an on-shell Z and W

F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, Phys. Lett. B 811 (2020) 135969 A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, PRD 103 (2021) 013008

♦ Beyond On-Shell production

- Dominant QCD-EW corrections in resonant region, neutral and charged DY S. Díttmaier, A. Huss and C. Schwinn, Nucl. Phys. B 885 (2014) 318
 - S. Dittmaier, A. Huss and C. Schwinn, Nucl. Phys. B 904 (2016) 216
- Mixed QCD-QED corrections to neutrino pair production L. Cieri, D. de Florian, M. Der and J. Mazzitelli, JHEP 09 (2020) 155
- Mixed QCD-EW corrections to helicity amplitudes for lepton pair prod M. Heller, A. Von Manteuffel, R. M. Schabinger and H. Spiesberger, JHEP 05 (2021) 216

• Mixed QCD-EW corrections to $h_1 + h_2 \rightarrow l\bar{\nu} + X$ and $h_1 + h_2 \rightarrow l^+l^- + X$

L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini and F. Tramontano, 2102.12539

B., L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini, 2106.11953

Theoretical framework: Perturbative QCD At LHC hadronic collisions $h_1 + h_2 \to l^+ l^- + X$ we rely on Factorization Theorem

PDFs: Universal Part Evolution with Fact scale predicted by the theory

Partonic CS: Process-dep Part Calculation in PT Theory

 $\sigma_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{i,h_1}(x_1,\mu_F) f_{j,h_2}(x_2,\mu_F) \hat{\sigma}_{ij}(\hat{s},m^2,\alpha_S(\mu_R),\mu_F,\mu_R)$

NNLO

NNNLO

S. Moch, J. Vermaseren, A. Vogt, Nucl. Phys. B688 (2004) 101 A. Vogt, S. Moch, J. Vermaseren, Nucl. Phys. B691 (2004) 129

S. Moch, B. Ruíjl, T. Ueda, J. Vermaseren, A. Vogt, Phys. Lett. B782 (2018) 627 J. Davis, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, Nucl. Phys. B915 (2017) 335 S. Moch, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, JHEP 10 (2017) 041

NNLO mixed D. de Florian, G. F. R. Sborlini and G. Rodrigo, Eur. Phys. J. C 76 (2016) 282



• The three sets are separately divergent: for a differential cross section we need IR counterterms in a subtraction scheme: we used Qt suibtr scheme

S. Cataní and M. Grazzíní, Phys. Rev. Lett. 98 (2007) 222002

The Qt subtraction formalism was used to derive the IR counter term for top-antitop production at NNLO and further extended to the production of a lepton-neutrino pair.

S. Cataní, S. Devoto, M. Grazzíní, S. Kallweit, J. Mazzítellí and H. Sargsyan, Phys. Rev. D 99 (2019) 051501 L. Buonocore, M. Grazzíní, S. Kallweit, C. Savoíní and F. Tramontano, 2102.12539

need no collinear div in the final state -> massive leptons

- However: Collinear divergences cancel in the amplitude when we add s-t and s-u box contributions (we explicitly checked the cancellation of the highest pole). Remaining collinear divergences come from factored one-loop diagrams —> easy to compute
- Therefore, we can set the leptonic mass to zero in the most complicated diagrams (the ones with many scales)

Therefore we actually computed the following diagrams

QCDxQED -> massive leptons

One- and Two-boson diagrams -> massless lept

Two-Loop Amplitude

The UV-renormalised amplitude has the following IR structure

$$\begin{split} |\mathcal{M}^{(1,1)}\rangle &= \frac{1}{\epsilon^4} \left\{ \frac{1}{4} e_q^2 \, C_F \, |\mathcal{M}^{(0)}\rangle \right\} \\ &+ \frac{1}{\epsilon^3} \left\{ \frac{1}{2} C_F \left[\left(\frac{3}{2} + i\pi \right) e_q^2 - \Gamma_t^{(0,1)} \right] \, |\mathcal{M}^{(0)}\rangle \right\} \\ &- \frac{1}{2\epsilon^2} \left\{ e_q^2 |\mathcal{M}_{fin}^{(1,0)}\rangle + C_F |\mathcal{M}_{fin}^{(0,1)}\rangle + C_F \left[\left(\frac{7}{12}\pi^2 - \frac{9}{8} - \frac{3}{2}i\pi \right) e_q^2 + \left(\frac{3}{2} + i\pi \right) \Gamma_t^{(0,1)} \right] |\mathcal{M}^{(0)}\rangle \right\} \\ &- \frac{1}{2\epsilon} \left\{ \left[\left(\frac{3}{2} + i\pi \right) e_q^2 - 2\Gamma_t^{(0,1)} \right] \, |\mathcal{M}_{fin}^{(1,0)}\rangle + C_F \left(\frac{3}{2} + i\pi \right) \, |\mathcal{M}_{fin}^{(0,1)}\rangle \right. \\ &+ \frac{C_F}{8} \left[\left(\frac{3}{2} - \pi^2 + 24\zeta_3 + \frac{2}{3}i\pi^3 \right) e_q^2 - \frac{3}{2}\pi^2\Gamma_t^{(0,1)} \right] |\mathcal{M}^{(0)}\rangle \right\} \\ &+ |\mathcal{M}_{fin}^{(1,1)}\rangle \end{split}$$

$$\Gamma_t^{(0,1)} = -\frac{1}{4} \left\{ (e_{l^-}^2 + e_{l^+}^2)(1 - i\pi) + \sum_{i=1,2; \ j=3,4} e_i e_j \ln\left(\frac{(2p_i \cdot p_j)^2}{Q^2 m_l^2}\right) + 2e_{l^-} e_{l^+} \left[\frac{1}{2v} \ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v} + 1\right)\right] \right\}$$

 $v = \sqrt{1 - \left(\frac{2m_l^2}{Q^2 - 2m_l^2}\right)^2}$

S.Cataní, M. Grazzíní and A. Torre, Nucl. Phys. B890 (2014) 518 L. Buonocore, M. Grazzíní and F. Tramontano, Eur. Phys. J. C 80 (2020) 254

Computation of the Amplitude ... goes through the "usual" steps:

- Generation of the Feynman diagrams: QGRAF
 - P. Nogueira, J. Comput. Phys. 105 (1993) 279
- We computed directly the Interference with the tree-level: FORM B. Ruijl, T. Ueda and J. A. M. Vermaseren, 1707.06453
- We used a naive anticommuting γ_5 D. Kreimer, Phys. Lett. B 237 (1990) 59
- We renormalized in the G_{μ} scheme, keeping complex boson masses

S.Dittmaier, A. Huss and C. Schwinn, Nucl. Phys. B 885 (2014) 318

 The Dim-Reg Scalar integrals were reduced to the MIs using Integration-by-Parts Identites implemented in KIRA and Reduze2

> P. Maierhoefer, J. Usovitsch and P. Uwer, Comp. Phys. Commun. 230 (2018) 99 J. Klappert, F, Lange, P. Maierhoefer and J. Usovitsch, Comput.Phys.Commun. 266 (2021) 108024 A. Von Manteuffel and C. Studerus

Master Integrals

• We basically divide the computation in two subsets

♦ Massive final state MIs

B., A. Ferroglía, T. Gehrmann, D. Maítre and C. Studerus, JHEP 07 (2008) 129 B., A. Ferroglía, T. Gehrmann and C. Studerus, JHEP 08 (2009) 067 P. Mastrolía, M. Passera, A. Prímo and U. Schubert, JHEP 11 (2017) 198

For the massive part of the calculation, all the MIs could be expressed in terms of GPLs.

♦ Massless final state MIs

B., S. Di Vita, P. Mastrolia and U. Schubert, JHEP 09 (2016) 091 M. Heller, A. Von Manteuffel and R. M. Schabinger, Phys. Rev. D 102 (2020) 016025 S. M. Hasan and U. Schubert, JHEP 11 (2020) 107

We rely on JHEP 09 (2016) 091 — All the MIs (with the exception of 5 two-mass BOXES) could be expressed in terms of GPLs. The 5 boxes still have a numeric int.



Differential Equations

The MIs were computed using the Differential Equations Method

$$\frac{\partial}{\partial x_i} f(x,\epsilon) = A_{x_i}(x,\epsilon) f(x,\epsilon)$$

V. Kotikov, Phys. Lett. B 254 (1991) 158
Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B 412 (1994) 751
E. Remiddí, Nuovo Cím. A 110 (1997) 1435
T. Gehrmann and E. Remiddí, Nucl. Phys. B 580 (2000) 485

- Massless final state MIs were evaluated analytically using the canonical form for the system $df(x,\epsilon) = \epsilon \, dA(x) \, f(x,\epsilon)$ J. M. Henn, Phys. Rev. Lett. 110(2013) 251601 M. Argeri, S. Di Vita, P. Mastrolia, E. Mirabella, J, Schlenk, U. Schubert and L. Tancredi, JHEP 03 (2014) 082
- However, the alphabet contains squared roots that could not be linearised simultaneously. Not all of the masters in GPLs —> remaining numeric integration 5. Caron-Huot and J. M. Henn, JHEP 06 (2014) 114

The numeric evaluation of the remaining integration is not optimal, in particular for the analytic continuation What to do?

Differential Equations: Semi-analytic evaluation

In some cases it is difficult to find closed-form solutions for the differential equations What can be done is a solution of the relative differential equation in series expansion



- The differential equation and the solution are expanded in series around the singular points Every series depends on two arbitrary constants. Imposing the matching we express all of them in terms of the two constants
- ♦ Imposing initial conditions we fix the two constants. One can construct a numerical routine that evaluates F(x) for every value of x with arbitrary precision !!
- ♦ The convergence can be improved adding series expansions in intermediate regular points

U. Aglietti, B., L. Grassi and E. Remiddi, Nucl. Phys. B 789 (2008) 45 R. N. Lee, A. V. Smirnov and V. A. Smirnov, JHEP 03 (2018) 008 B., G. Degrassi, P. P. Giardino and R. Groeber, Comp. Phys. Comm. 241 (2019) 122

Differential Equations: Semi-analytic evaluation

- The examples above are one-dimensional, but the approach can be generalised to more dimesions and used for a general system of differential equations for the MIs
- The differential equations in s and t are combined and a one-dim diff eq is recovered and solved along a contour connecting two fixed points in the s-t plain

 (s_0, t_0)

(s,t)

The method is quite efficient and enables to compute fast a point in the phase space with arbitrary precision

 Analitycal continuation is done expanding in the singular point and matching the series using Feynman prescription for the invariants
 F. Moriello, JHEP 01 (2020) 150

Recently this method was implemented in a Mathematica code: DiffExp M. Hidding, 2006.05510

 ALL the results were checked against pySecDec, FIESTA (or numeric evaluation of analytical expressions done with GiNaC)
 S. Borowka, G. Heinrich, S. Jahn, S. P. Jones, M. Kerner, J. Schlenk and T. Zirke, Comp. Phys. Commun. 222 (2018) 313
 A. V. Smirnov, Comp. Phys. Commun. 204 (2016) 189
 J. Vollinga and S. Weinzierl, Comput. Phys. Commun. 167 (2005) 177

The Computation of the IR subtracted amplitude

- We used a grid in \sqrt{s} and $\cos\theta$ of 3250 points from 50 GeV to 3 TeV with smaller intervals around the Z peak
- The entire grid is evaluated in O(1h) on a cluster of 64 cores
- The grid was then interpolated to use it in the MC integrator



 $\frac{2\Re \langle \mathcal{M}^{(0)} | \mathcal{M}_{fin}^{(1,1)} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle}$

Differential distributions

PDFs NNPDF31_nnlo_as_0118_luxqed Selection cuts $p_{T,\mu^{\pm}} > 25 \,\text{GeV}\,, \qquad |y_{\mu^{\pm}}| < 2.5\,, \quad m_{\mu\mu} > 50 \,\text{GeV}$ Scales $\mu_R = \mu_F = m_Z$

Exact results are compared with

 $\sqrt{s} = 14 \,\mathrm{TeV}$

Finite part of the 2-loop in Pole ApproximationFactorized Approximation for QCD and EW

The antí-muon pt

14

-4

+20

-40

-60

 $\mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T},\mu^+}\left[\mathrm{pb}/\mathrm{GeV}\right]$

do/dolo [%]

The muon-pair inv mass





Conclusions

- We presented the calculation of the mixed QCD-EW corrections to the inclusive production of a lepton pair in proton proton collisions at 14 TeV
- We calculated the two-loop virtual amplitude and combined with the real radiation. IR subtraction was done in the Qt scheme.
- We find that the mixed corrections to the fiducial cross section amount to +0.5% with respect to the LO (the other corrections up to NNLO QCD basically vanish, amounting to -0.1% of the LO)
- For the anti-muon pt distribution we find that the PA is in perfect agreement with the exact result all over the kinematic range
- For the di-muon invariant mass distribution we find that the PA is in good agreement with the exact result with small differences in the peak region and in the high inv mass region where the PA undershoot the exact result by about 30%