bctp Bethe Center for Theoretical Physics

UNIVERSITÄT BONN

The Drell-Yan cross section at three loops

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based on work in collaboration with
Falko Dulat and Bernhard Mistlberger

A Loop Summit
27 July 2021

QCD at the LHC

- The LHC is becoming a precision machine, where we can test the Standard Model at percent-level accuracy.

  ➡ Need theory prediction at the same level of accuracy!

- Standard approach to LHC computations: QCD factorisation + perturbation theory.

$$\sigma_{p\,p\to X}(S) = \sum_{i,j}\int_0^1 dx_1\, dx_2\, f_i(x_1,\mu_F^2)\, f_j(x_2,\mu_F^2)\,\hat{\sigma}_{ij\to X}(\hat{s},\mu_F^2) \qquad S = E_{CoM}^2 \qquad \hat{s} = x_1 x_2 S$$

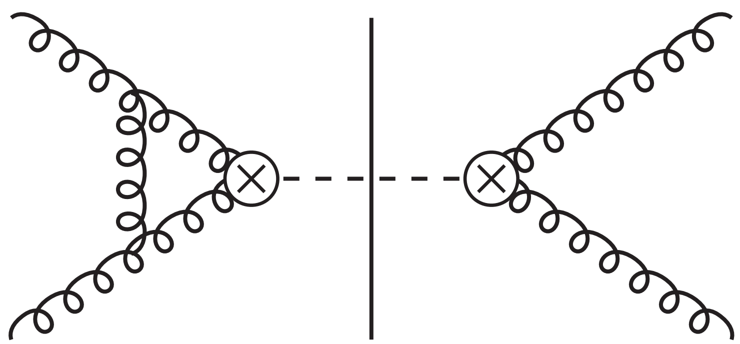
$$\hat{\sigma}_{ij\to X} = \hat{\sigma}_0 + \alpha_s(\mu_R)\,\hat{\sigma}_1 + \alpha_s(\mu_R)^2\,\hat{\sigma}_2 + \ldots \qquad \alpha_s(m_Z^2) = 0.118$$

- Naive counting: NLO $\longrightarrow$ 10% NNLO $\longrightarrow$ 1%

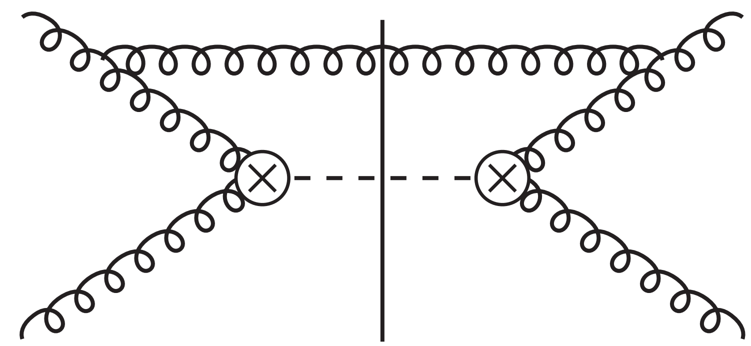
  ➡ We know several examples where this naive counting fails (e.g. Higgs production).

The cross section

- The NLO cross section:

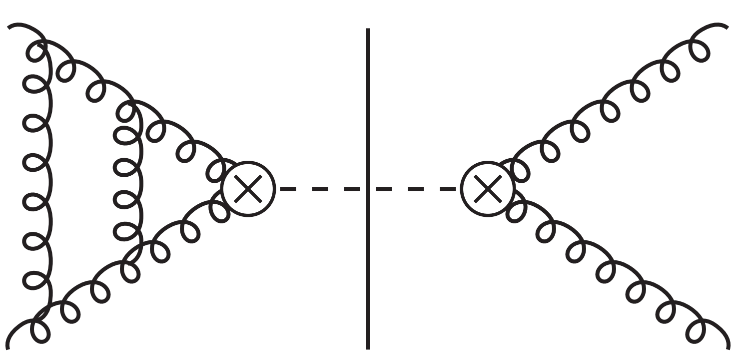


Virtual corrections ('loops')

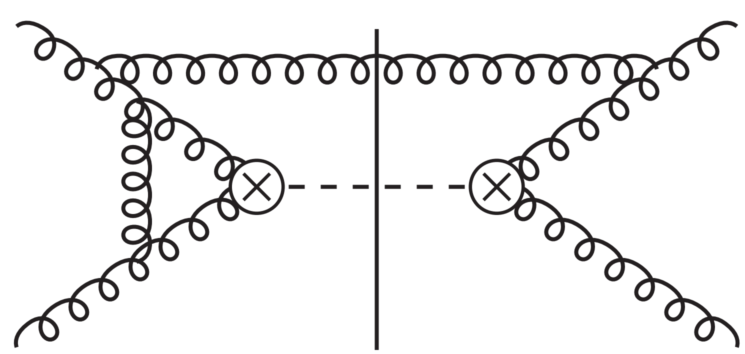


Real emission

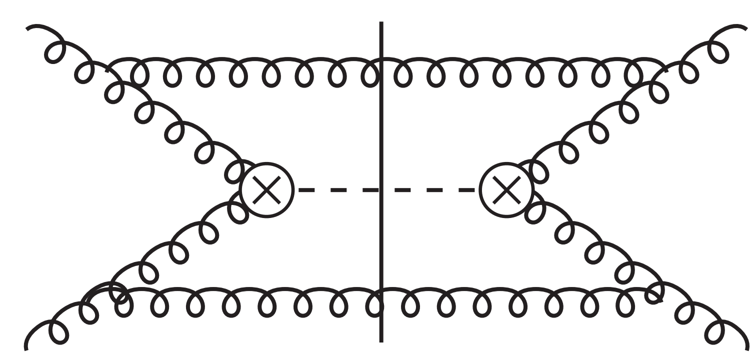
- The NNLO cross section:



Double virtual



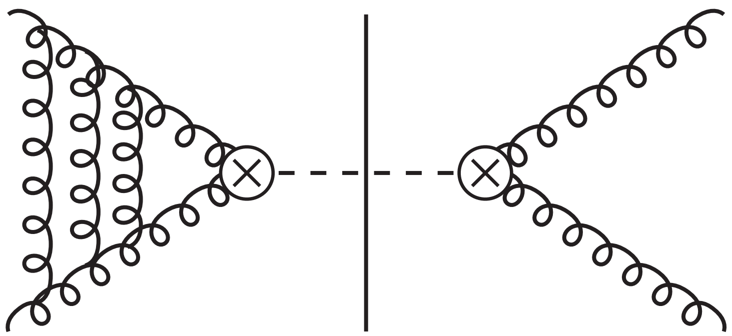
Real-virtual



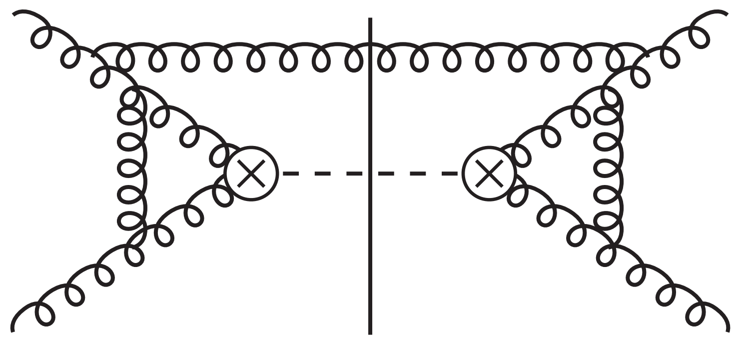
Double real

The cross section

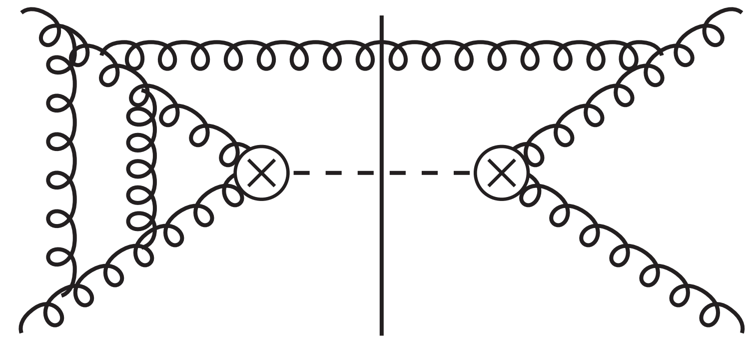
- The N3LO cross section:



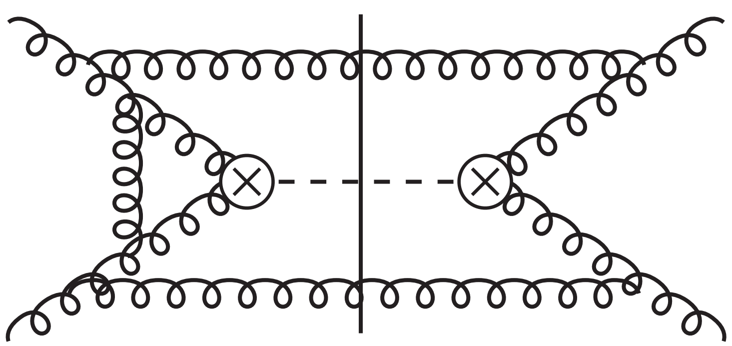
Triple virtual



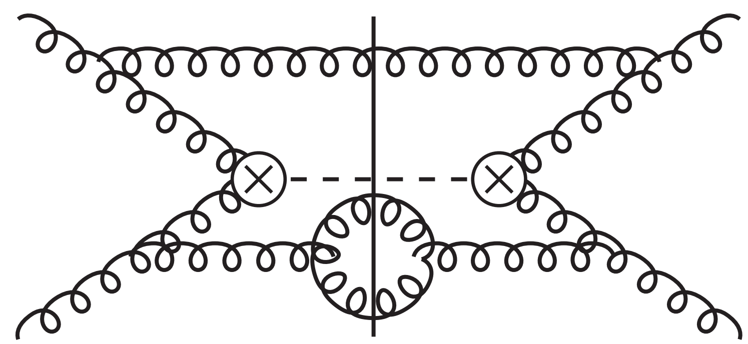
Real-virtual squared



Double virtual real



Double real virtual



Triple real

Quark-initiated processes

- A few years ago we had computed the Higgs cross section at N3LO.
- We have recently completed the computation of N3LO cross section for $b\,\bar{b} \to H$, $p\,p \to \gamma^*, p\,p \to W^{\pm}$ and $p\,p \to W^{\pm} H$.
- Twofold opportunity:
  - ➡ Understand structure of QCD at N3LO.
  - ➡ Explore pheno implications.
- Spin-off: First independent confirmation of all 3-loop splitting functions.

  [Moch, Vermaseren, Vogt]
- Remainder of this talk: Focus on phenomenology of Drell-Yan production at N3LO.

Phenomenology at N3LO

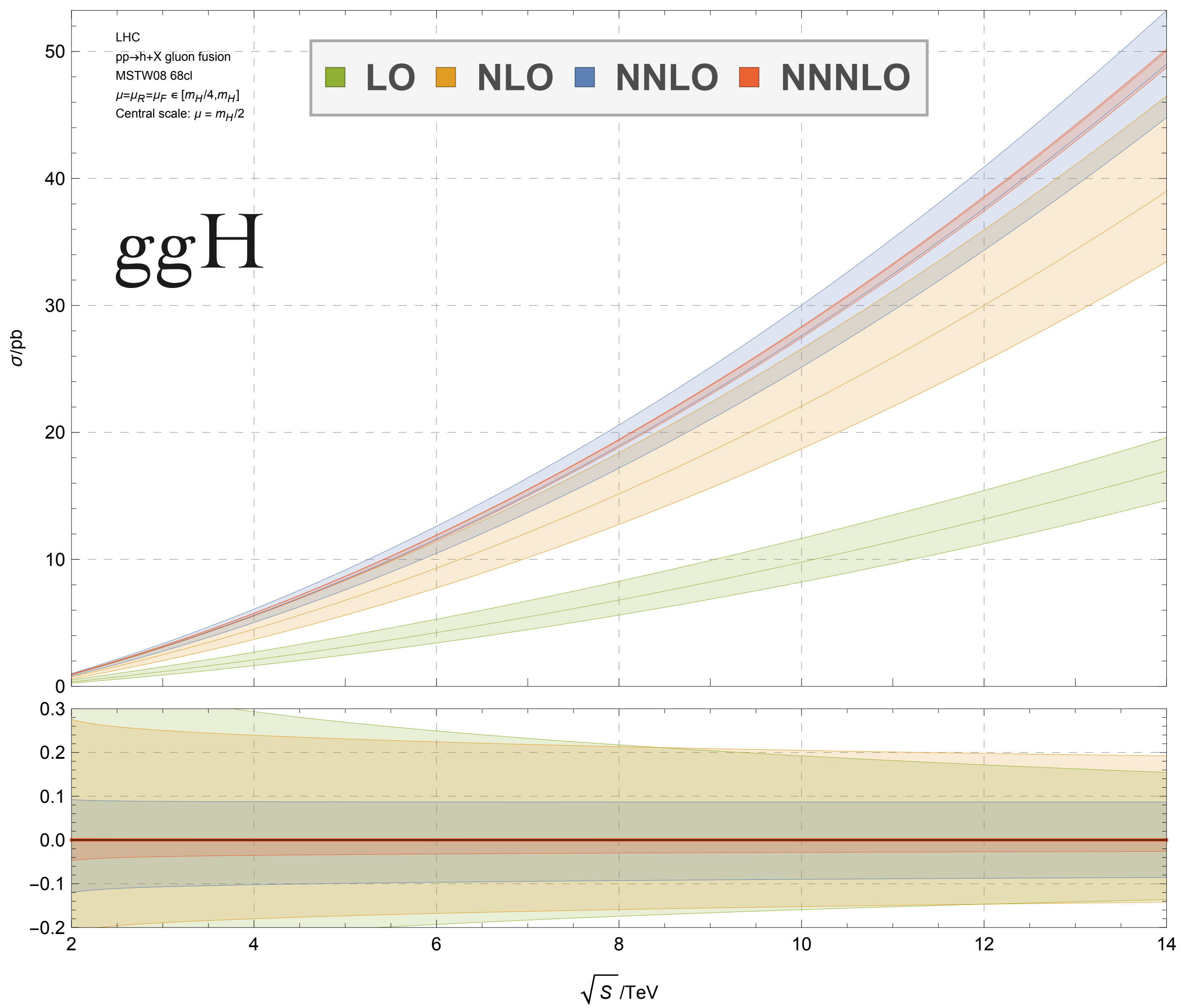
Convergence of the perturbative series

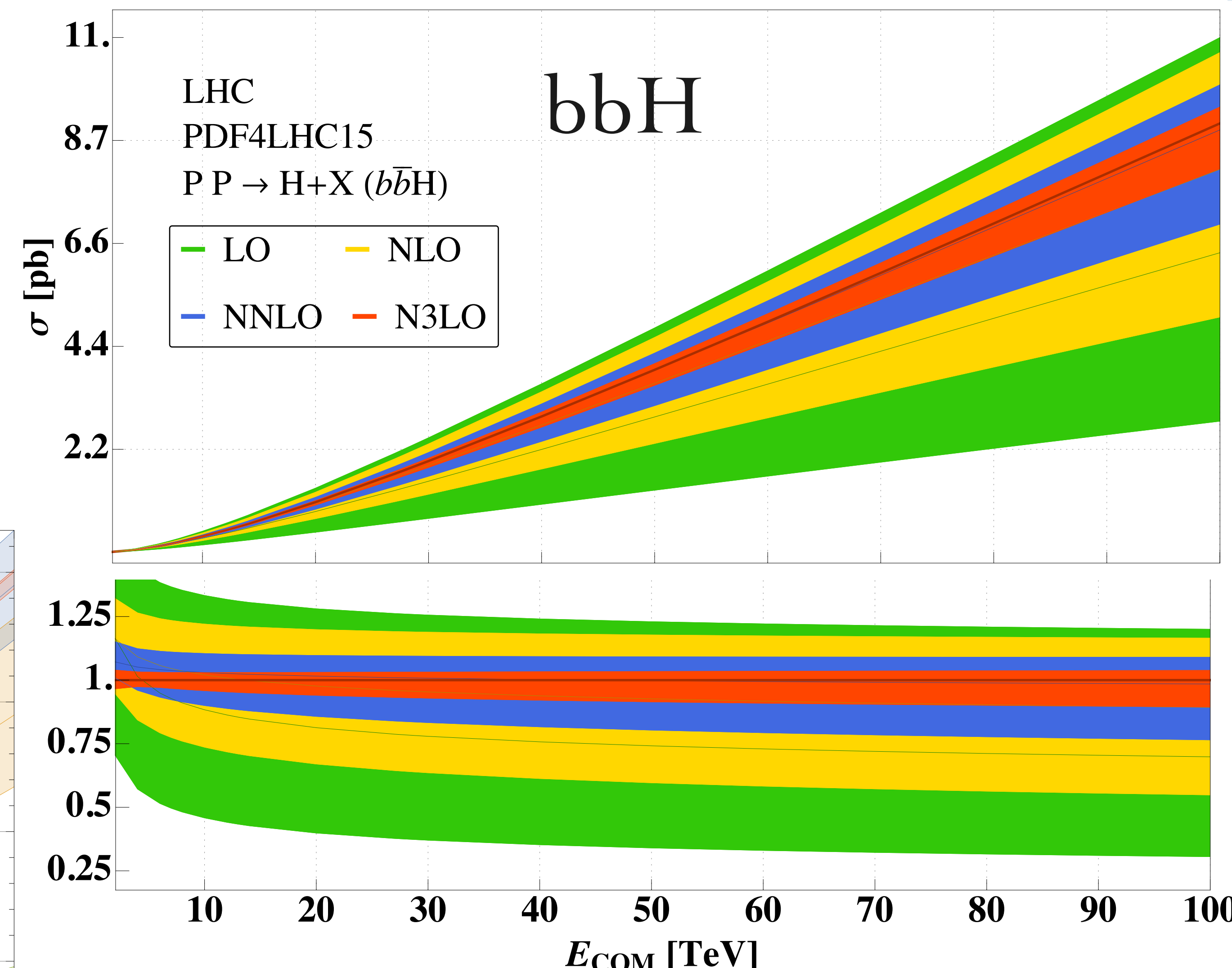
PDF effects

Energy variation

Higgs production:

Nice convergence of perturbative expansion.



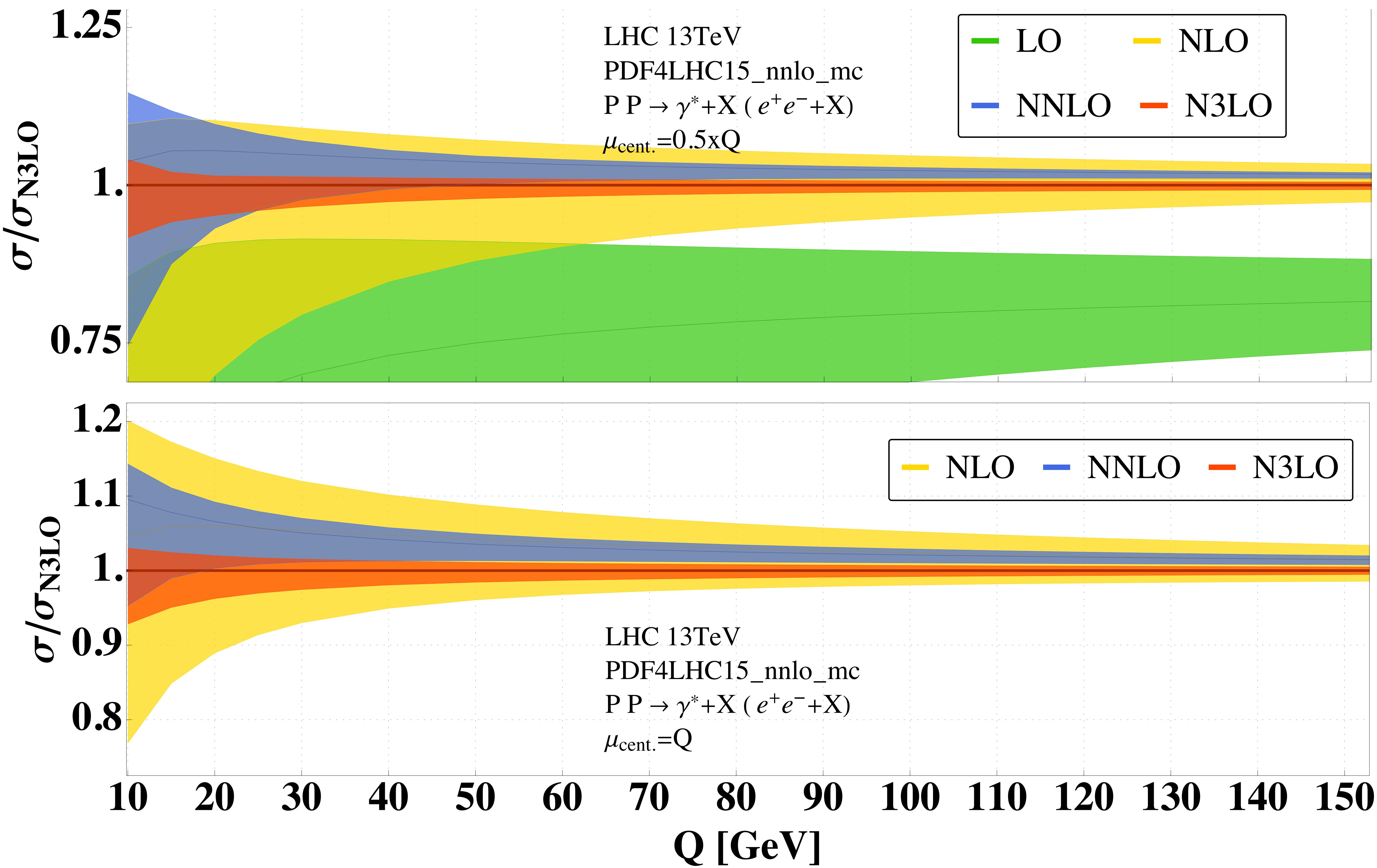


Choice of central scales:

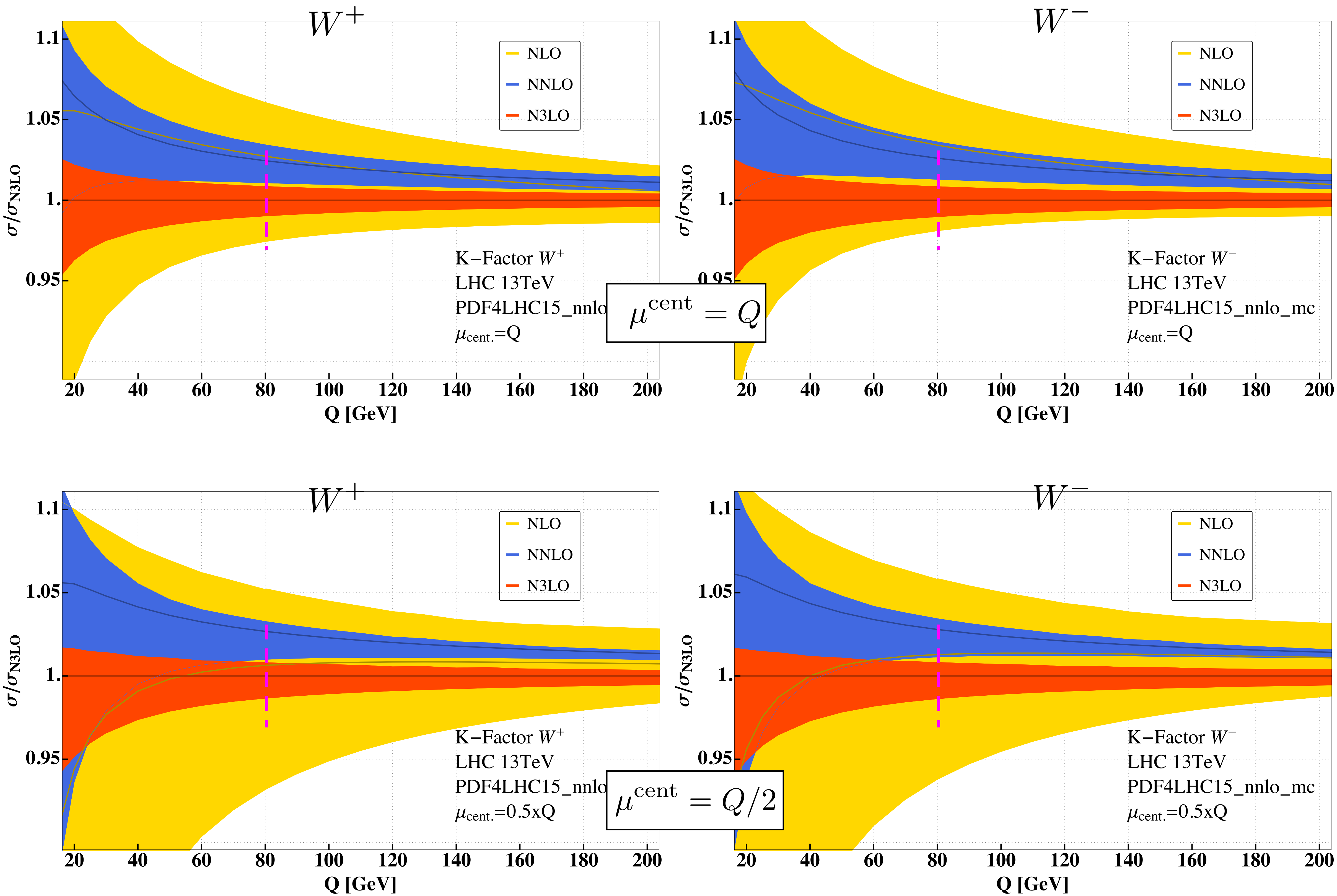
ggH: $\mu_F = \mu_R = m_H/2$

bbH: $\mu_F = m_H/4\,,\ \ \mu_R = m_H$

Q-variation (DY)

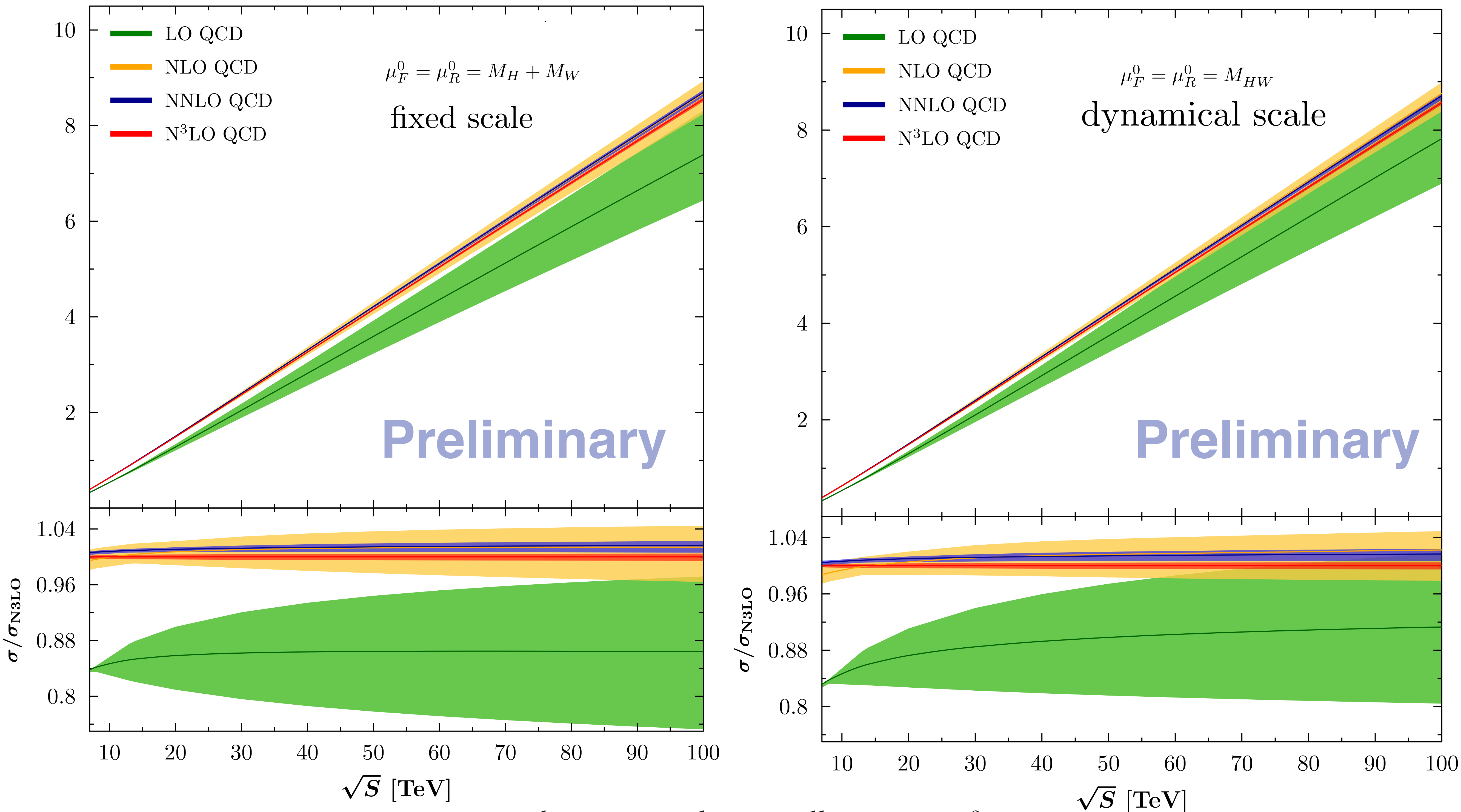


Q-variation (W)



WH production

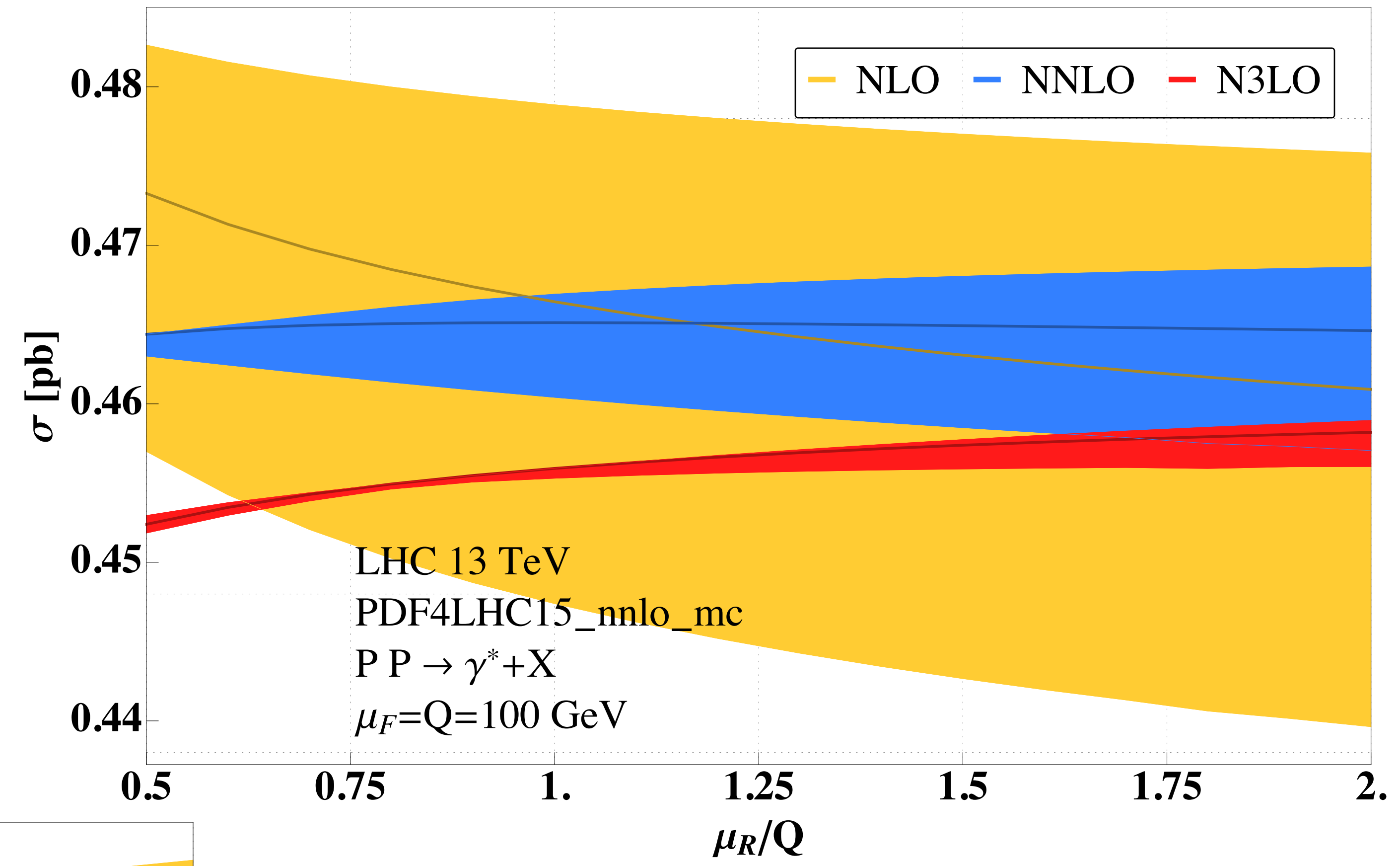
$$p\,p \to W^+ H, \text{pdf4lhc}$$

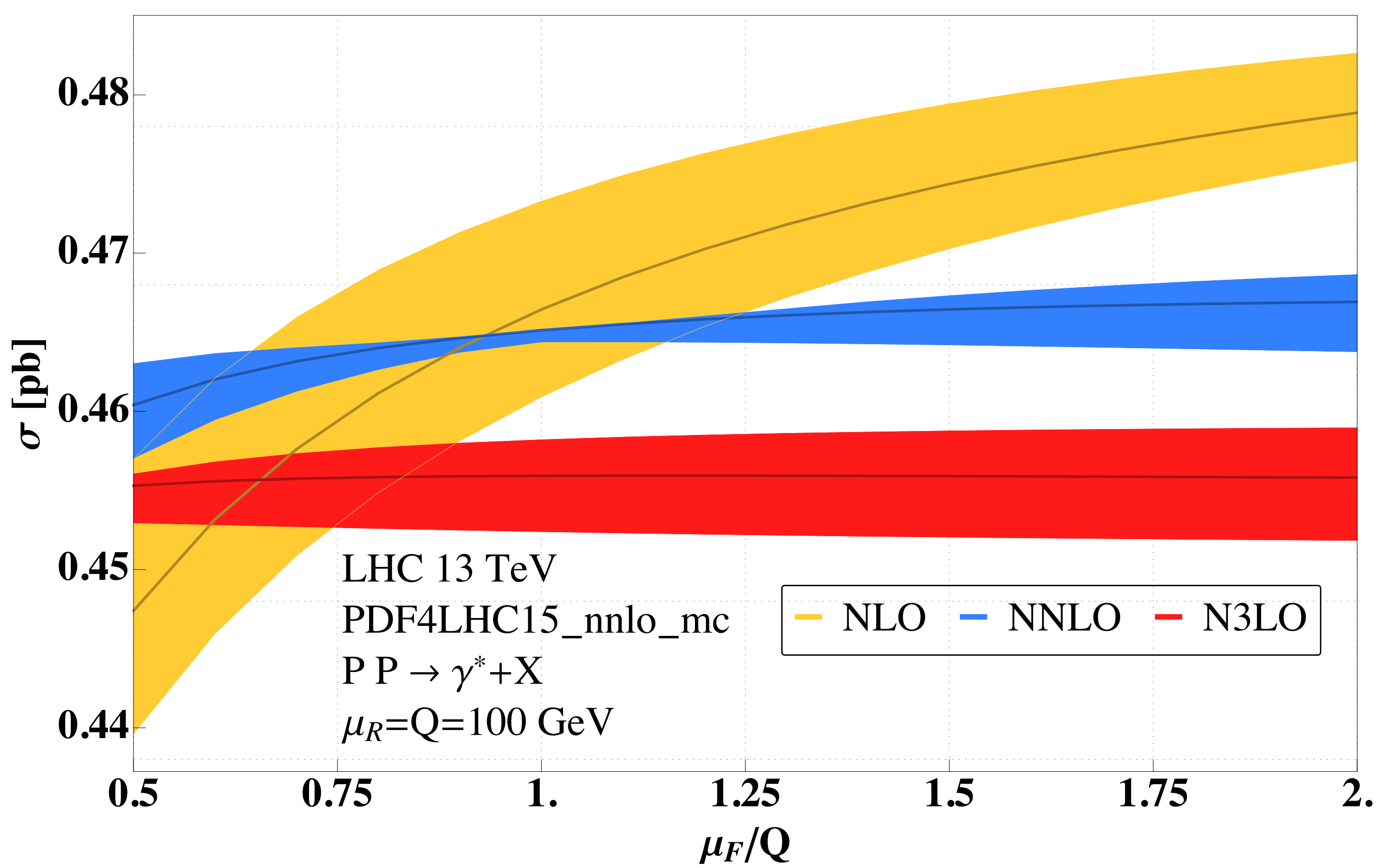


[Baglio, CD, Dulat, Mistlberger, Szafron]

Scale dependence (DY)

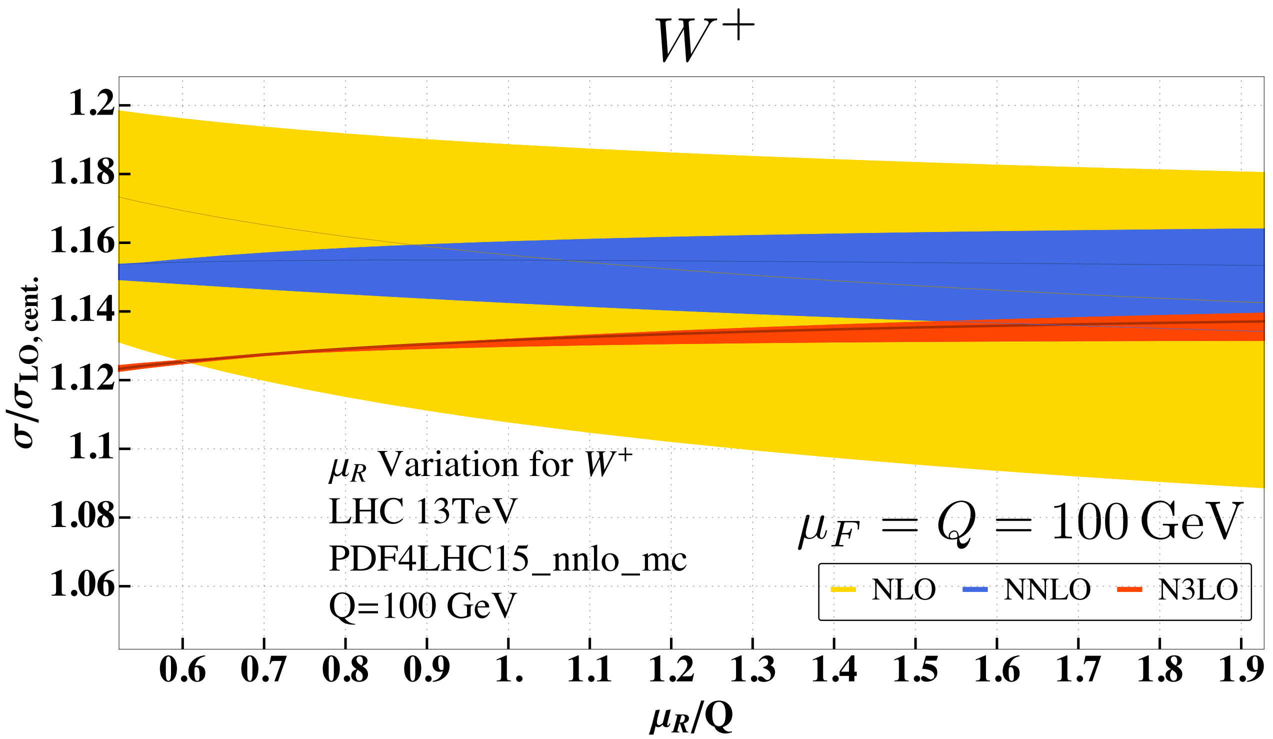
$$\mu_F = Q = 100\,\text{GeV}$$

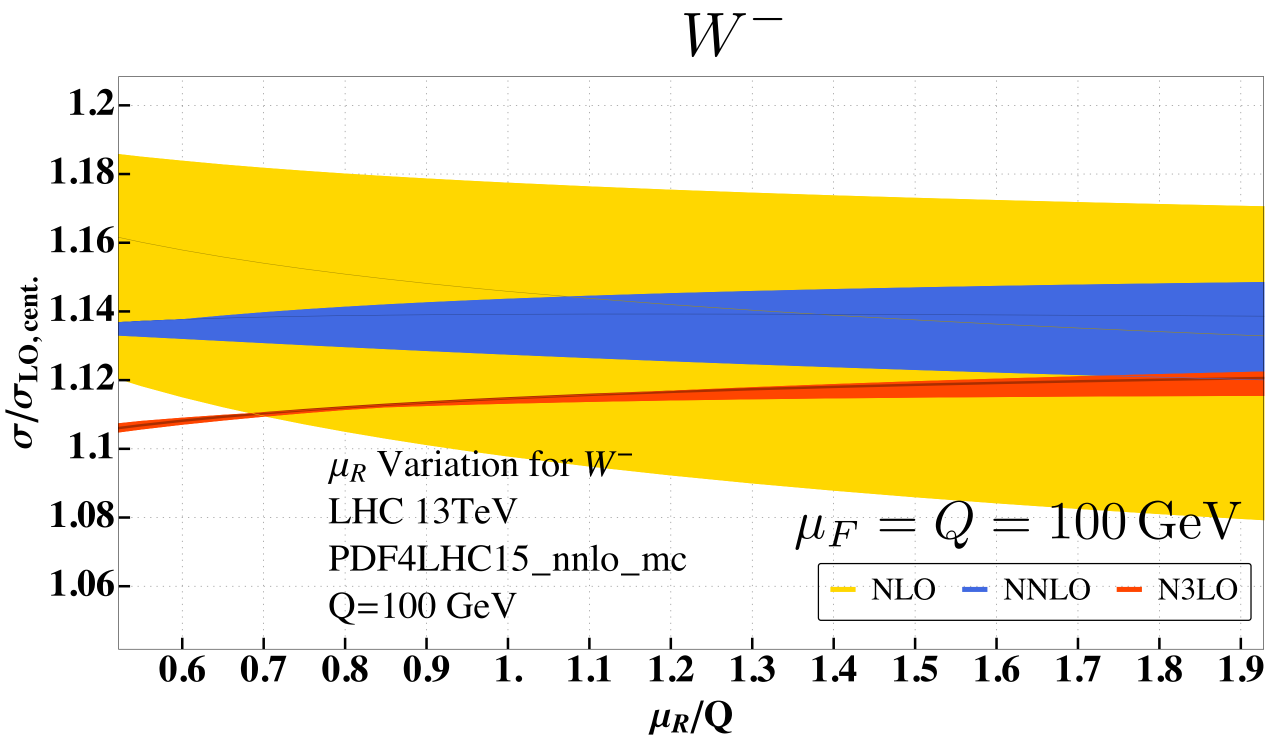


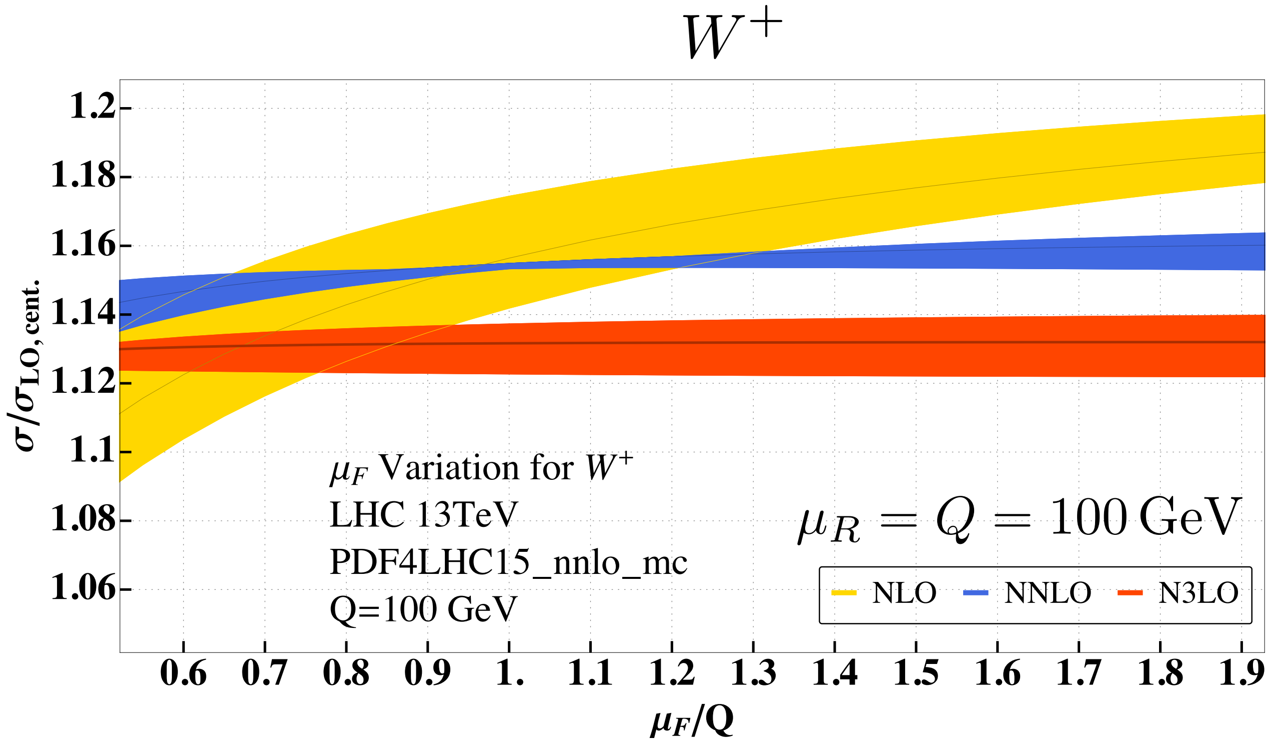


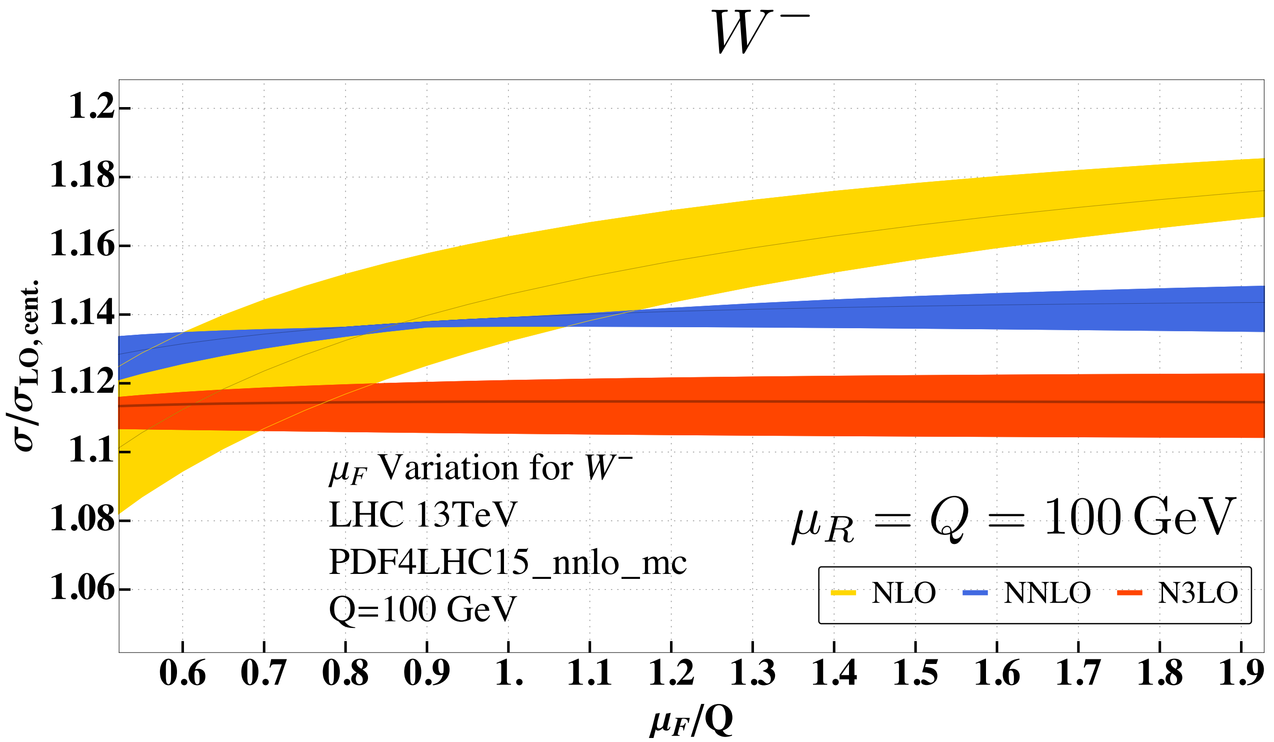
$$\mu_R = Q = 100\,\text{GeV}$$

Scale dependence (W)

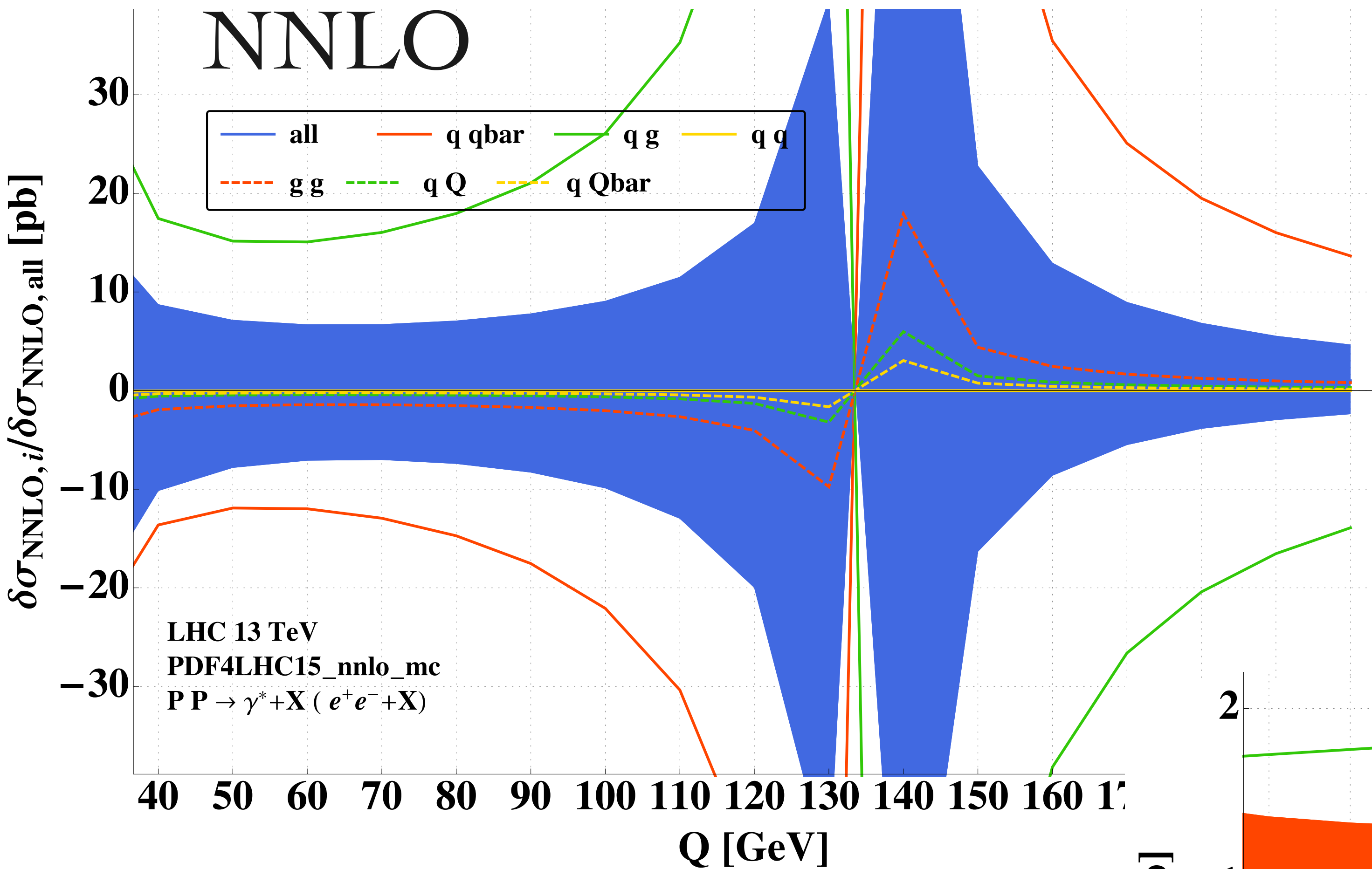






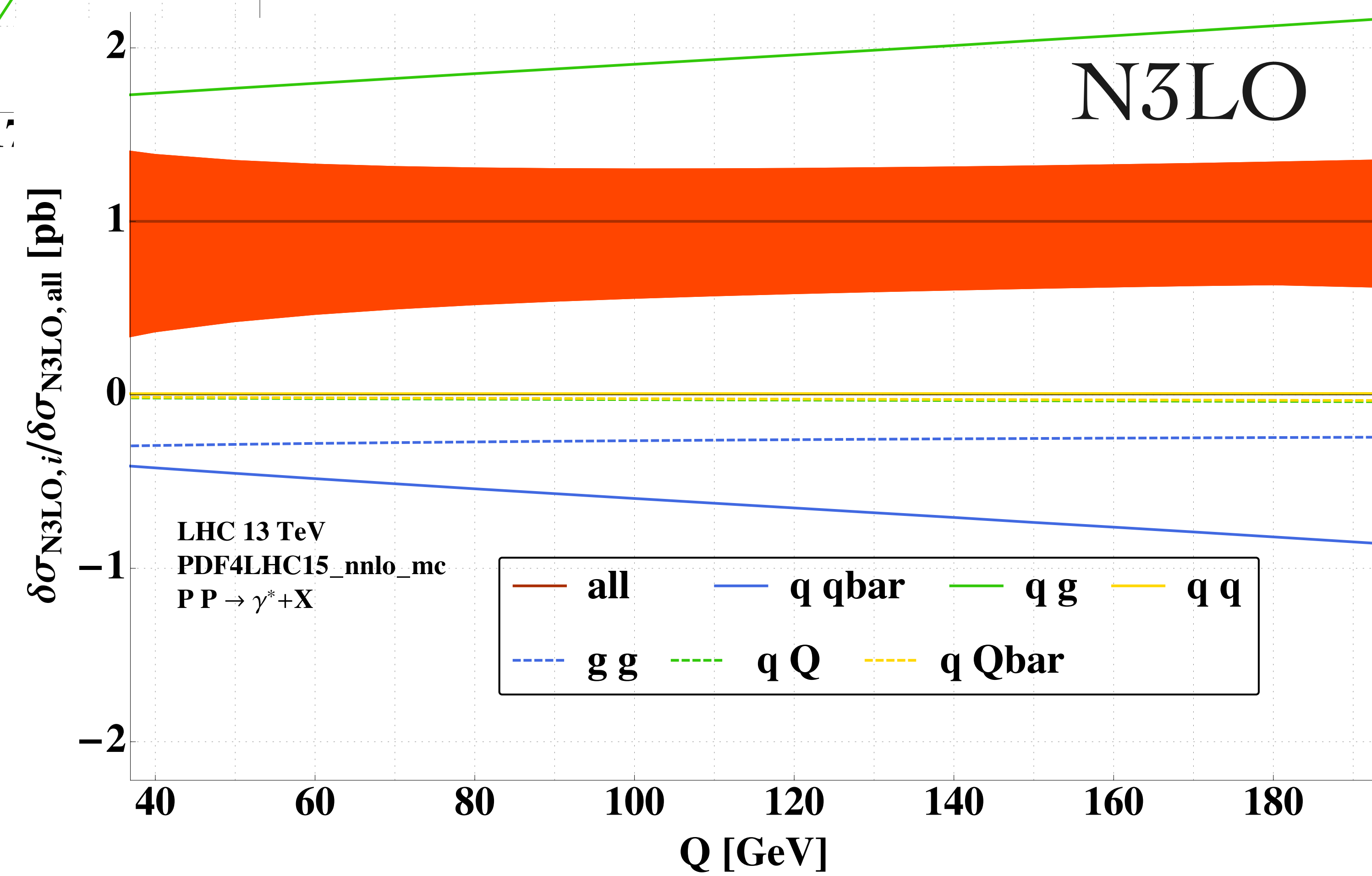


Variation per channel (DY)



Very large cancellations between channels.

Already present at NNLO!



Similar cancellations for W.

There are no such cancellations for Higgs production.

Scale dependence

- For Higgs (ggH & bbH): Scale bands overlap very well (for smallish μ_F).
- For DY (photon recently confirmed by Chen, Gehrmann, Glover, Huss, Yang): Scale bands do not overlap over a large range of virtualities.
  - ➡ Difference in central values: few %.
  - ➡ For both μ_F and μ_R .
  - ➡ All results obtained with `pdf4lhc_nnlo_mc` (more later).
- Observation: Large cancellation between channels for DY at NNLO and N3LO (both photon and W).
  - ➡ No cancellation for Higgs.

Cross section ratios

$$R_{XY}(Q) = \frac{\sigma_X(Q)}{\sigma_Y(Q)}\,, \qquad X,Y \in \{W^\pm, \gamma^*\}\,.$$

- Prescriptions for ratios:
  - ➡ A: Ratio of expansions, correlated scales.
  - ➡ B: Ratio of expansions, uncorrelated scales.
  - ➡ A': Expansion of ratio, correlated scales.
  - ➡ B': Expansion of ratio, uncorrelated scales.
  - ➡ C: Progression of series, correlated scales:

$$\delta(\text{pert.}) = \pm\left|1 - \frac{R_{XY}^{(n)}(Q)}{R_{XY}^{(n-1)}(Q)}\right| \times 100\%$$

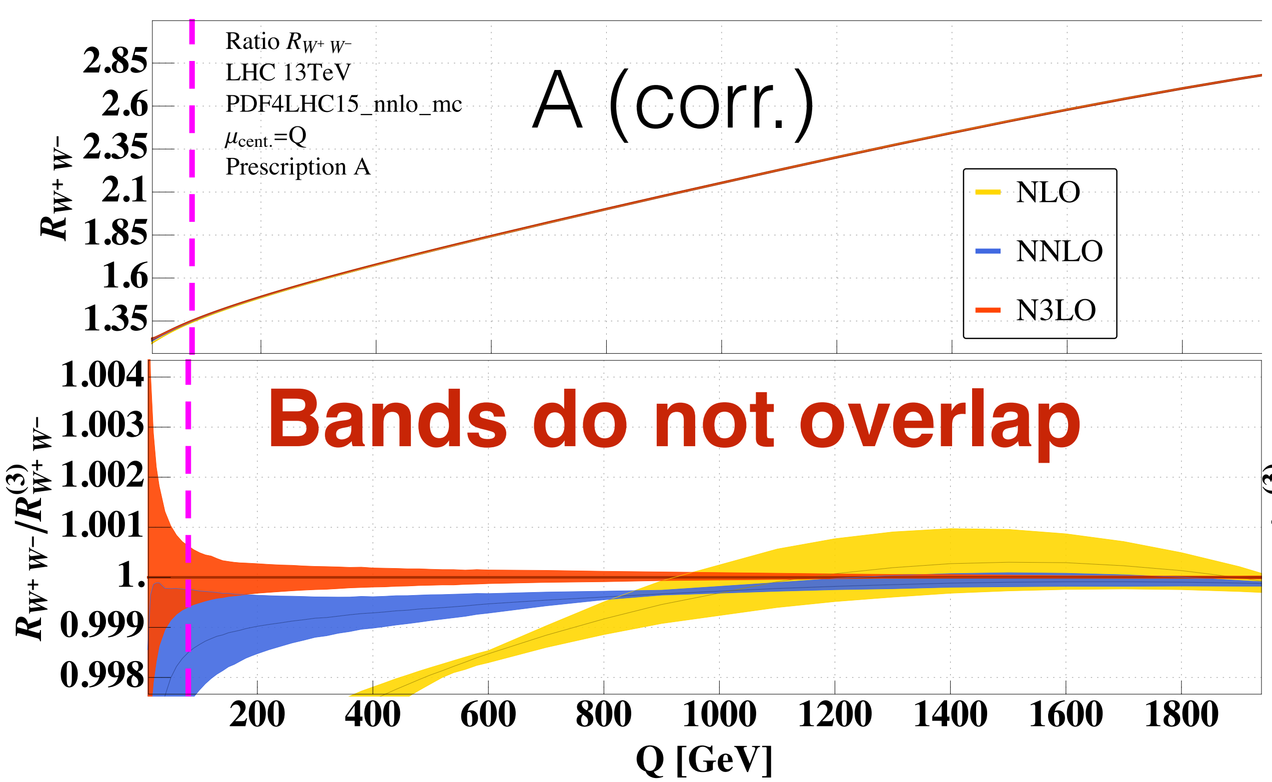
Cross section ratios

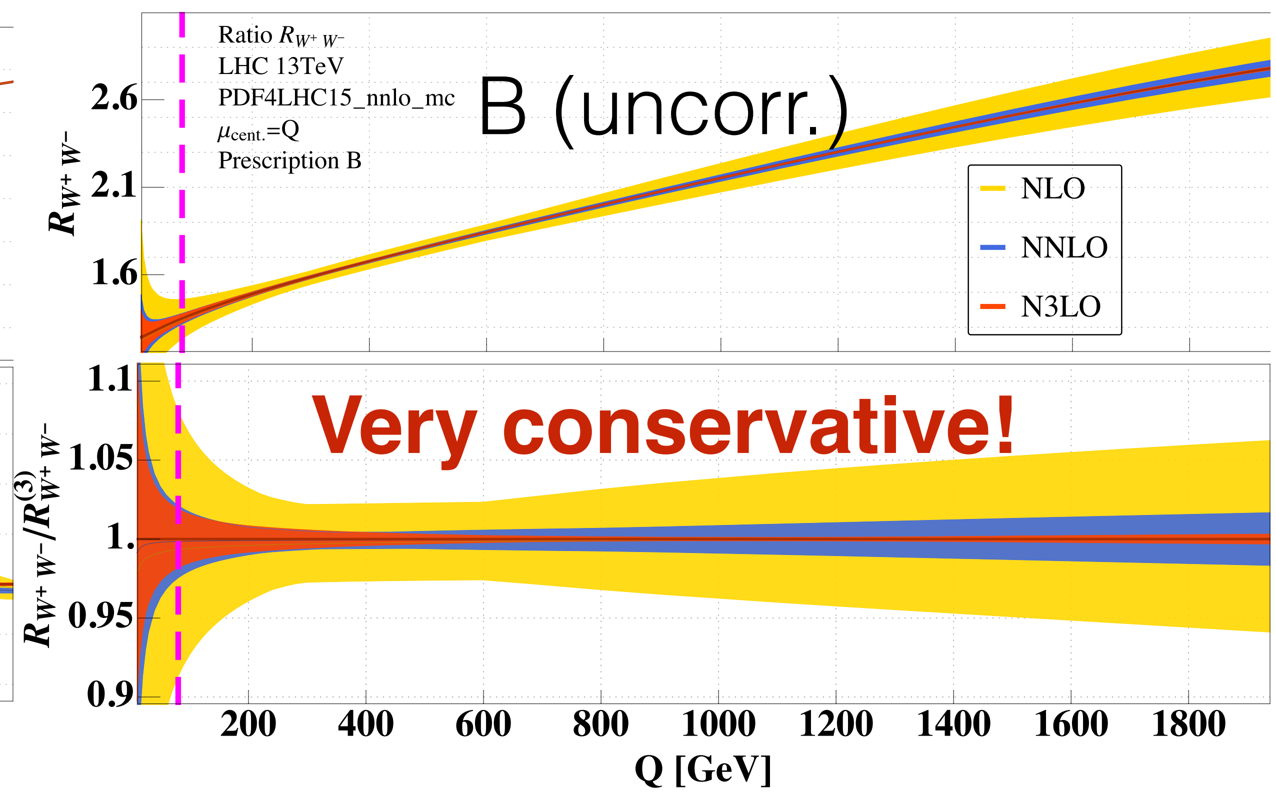
$$R_{W^+W^-}(Q = m_W)$$

	NLO		NNLO		N^{3}LO	
μ^{cent}	m_W	$m_W/2$	m_W	$m_W/2$	m_W	$m_W/2$
A	$1.342^{+0.10\%}_{-0.08\%}$	$1.342^{+0.07\%}_{-0.05\%}$	$1.348^{+0.12\%}_{-0.10\%}$	$1.349^{+0.15\%}_{-0.11\%}$	$1.350^{+0.05\%}_{-0.06\%}$	$1.350^{+0.04\%}_{-0.05\%}$
A$'$	$1.343^{+0.13\%}_{-0.16\%}$	$1.344^{+0.10\%}_{-0.21\%}$	$1.349^{+0.13\%}_{-0.09\%}$	$1.351^{+0.33\%}_{-0.13\%}$	$1.350^{+0.02\%}_{-0.03\%}$	$1.350^{+0.01\%}_{-0.09\%}$
B	$1.342^{+8.82\%}_{-8.08\%}$	$1.342^{+12.9\%}_{-11.4\%}$	$1.348^{+2.26\%}_{-2.31\%}$	$1.349^{+2.24\%}_{-2.27\%}$	$1.350^{+2.21\%}_{-2.14\%}$	$1.350^{+2.21\%}_{-2.14\%}$
B$'$	$1.343^{+5.28\%}_{-7.40\%}$	$1.344^{+8.09\%}_{-8.97\%}$	$1.349^{+1.85\%}_{-2.63\%}$	$1.351^{+2.21\%}_{-2.24\%}$	$1.350^{+2.60\%}_{-2.25\%}$	$1.350^{+4.65\%}_{-2.70\%}$
C	$1.342^{+0.99\%}_{-0.99\%}$	$1.342^{+0.58\%}_{-0.58\%}$	$1.349^{+0.52\%}_{-0.52\%}$	$1.349^{+0.53\%}_{-0.53\%}$	$1.350^{+0.15\%}_{-0.15\%}$	$1.350^{+0.11\%}_{-0.11\%}$

- ➡ Almost no difference between “expansion of ratio” or “ratio of expansions” already at lower orders.
- ➡ Large difference for scale variation between correlated and uncorrelated.
- ➡ Ratio is extremely stable in perturbation theory.

Cross section ratios

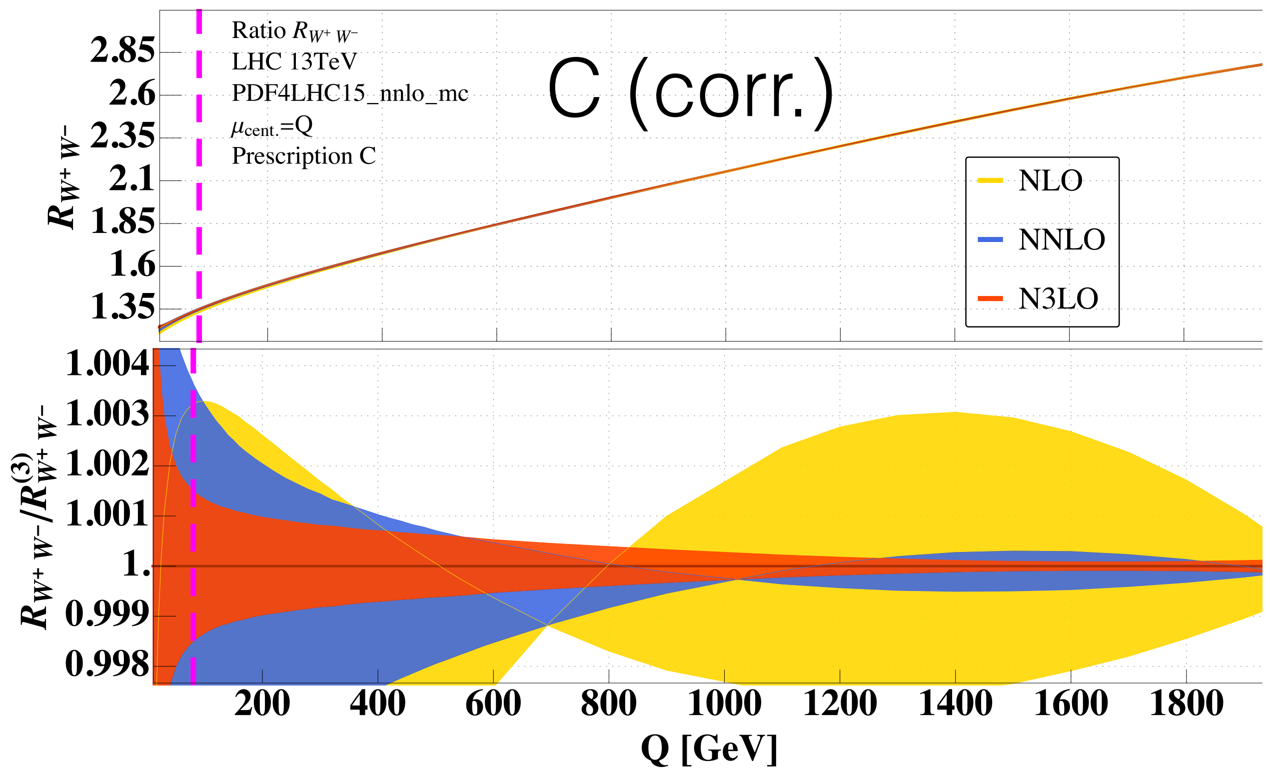




$$R_{W^+W^-}(Q)$$

$$\mu^{\text{cent.}} = Q$$

pdf4lhc_nnlo_mc

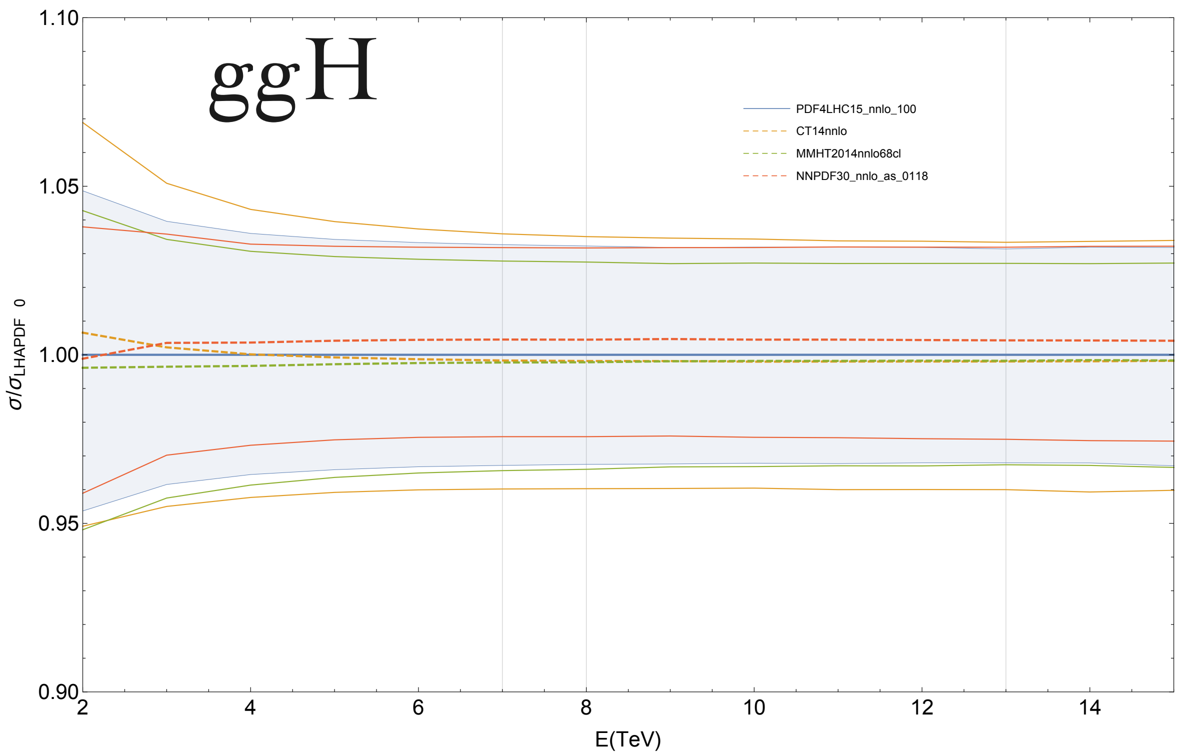


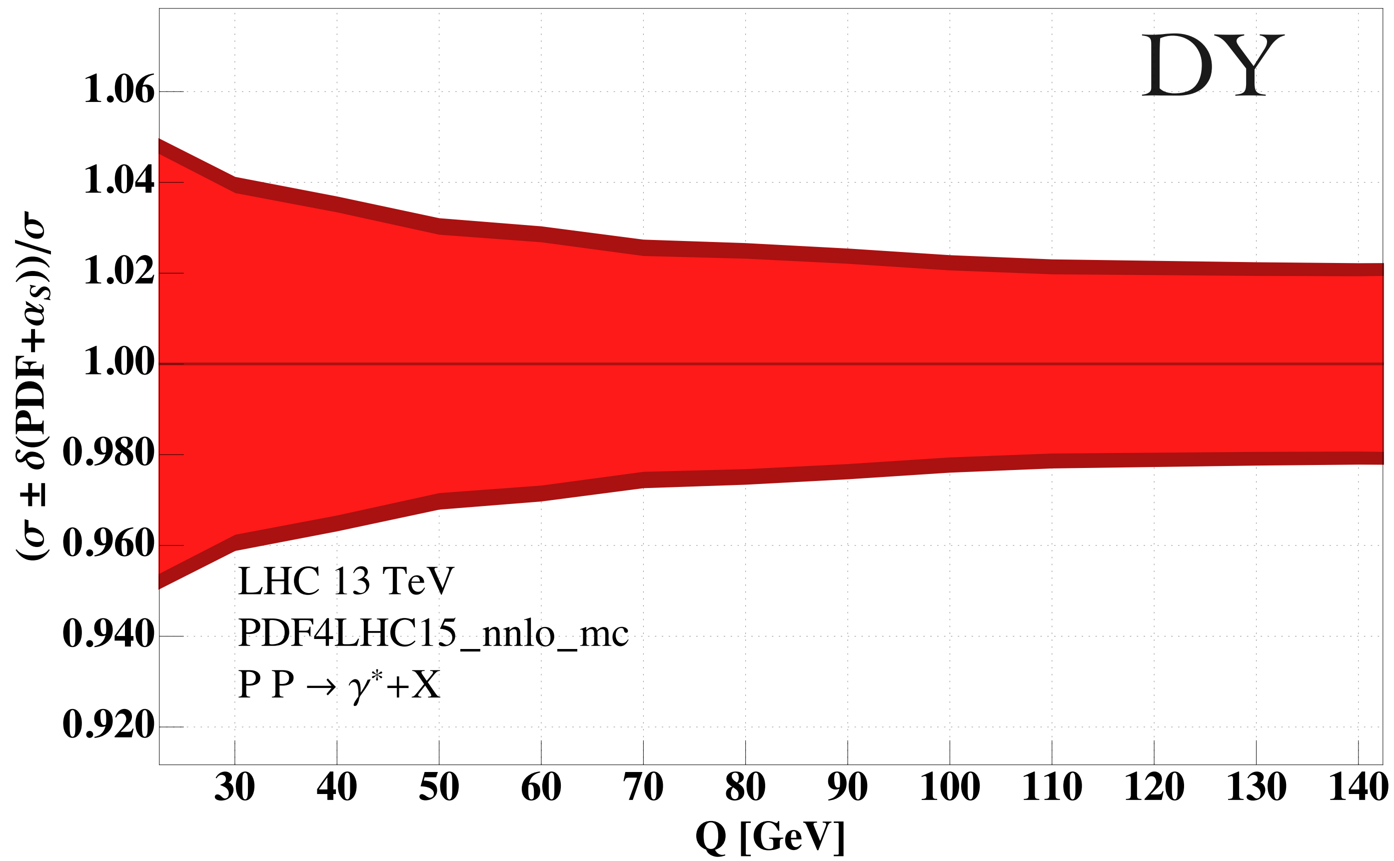
Phenomenology at N3LO

Convergence of the perturbative series

PDF effects

PDF+ α_s -uncertainty





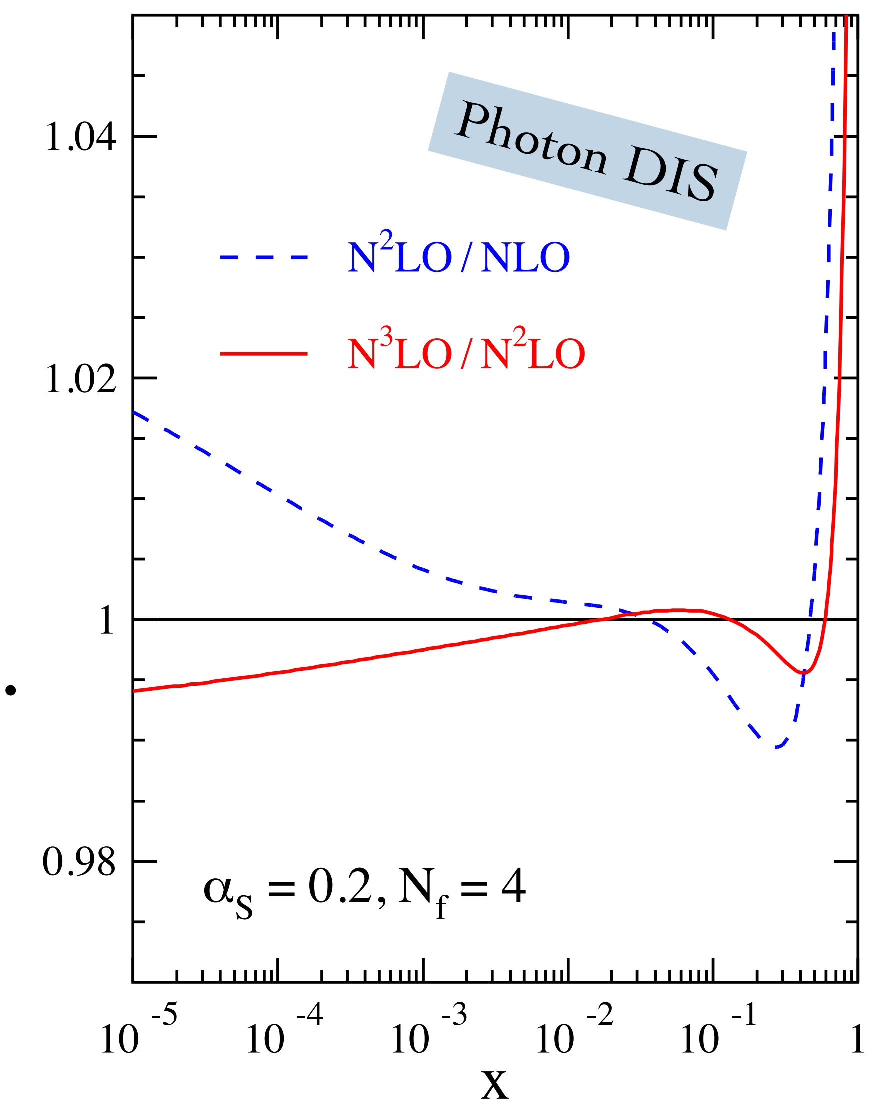
- Dependence of the cross on PDF+ α_s : ~2—9% at LHC energies.
  - ➡ Central set: pdf4lhc_nnlo_mc.
  - ➡ Uncertainty band obtained following PDF4LHC recommendation.

Missing N3LO PDFs

- We do not have N3LO PDFs
- This introduces a mismatch in our calculation.
- Estimate of the uncertainty:

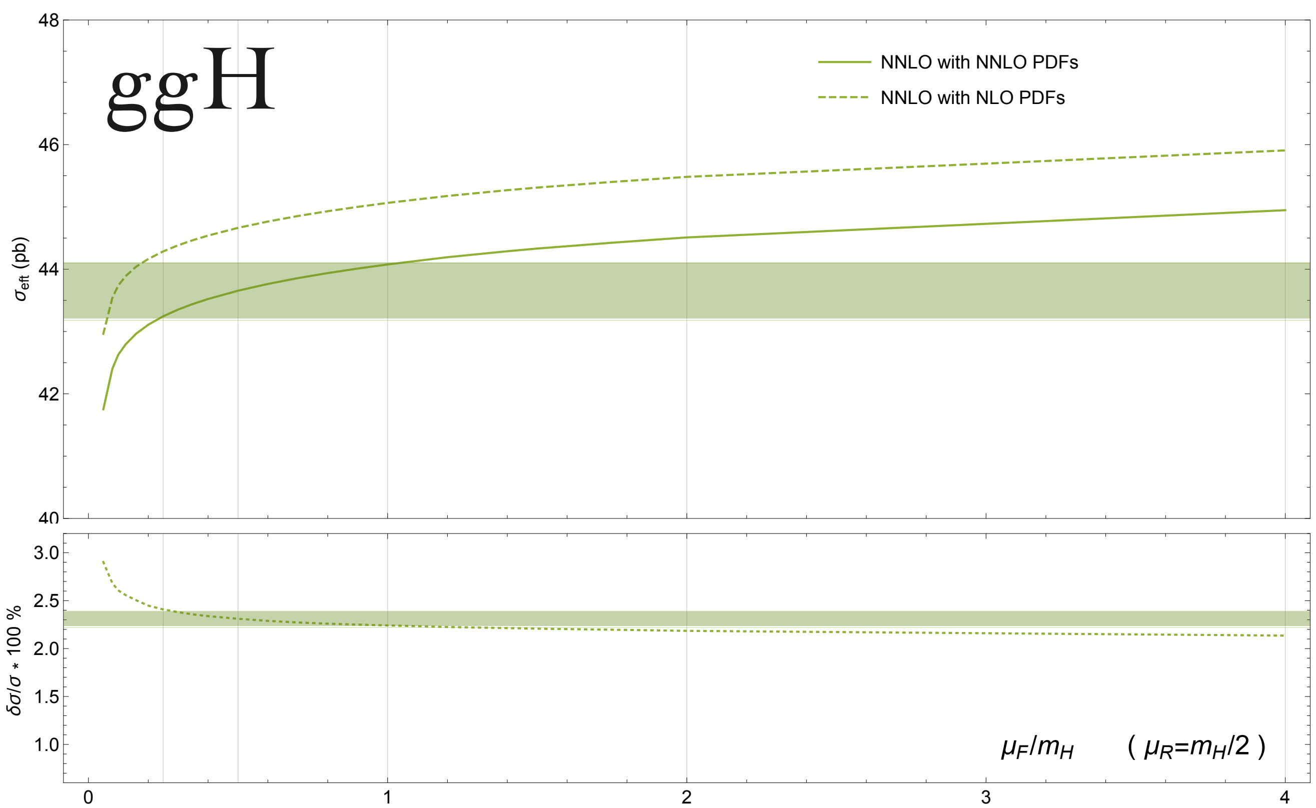
$$\delta^{\mathrm{N^3LO}}_{\mathrm{PDF}} = \frac{1}{2}\left|\frac{\sigma^{\mathrm{NNLO}}_{\mathrm{NNLO\text{-}PDFs}} - \sigma^{\mathrm{NNLO}}_{\mathrm{NLO\text{-}PDFs}}}{\sigma^{\mathrm{NNLO}}_{\mathrm{NNLO\text{-}PDFs}}}\right|$$

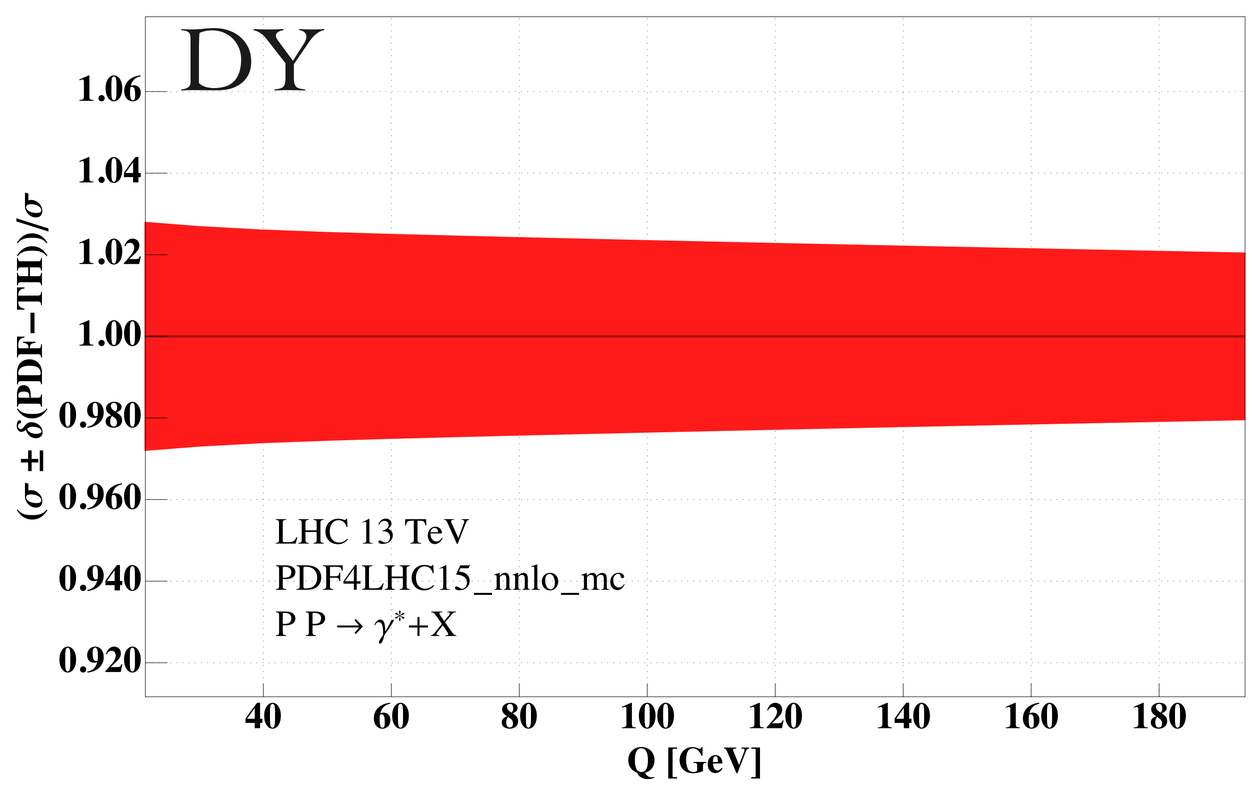
- The factor 1/2 takes into account that this estimate is most likely overly conservative.
  - ➡ cf. convergence pattern of DIS.



[Moch, Vermaseren, Vogt]

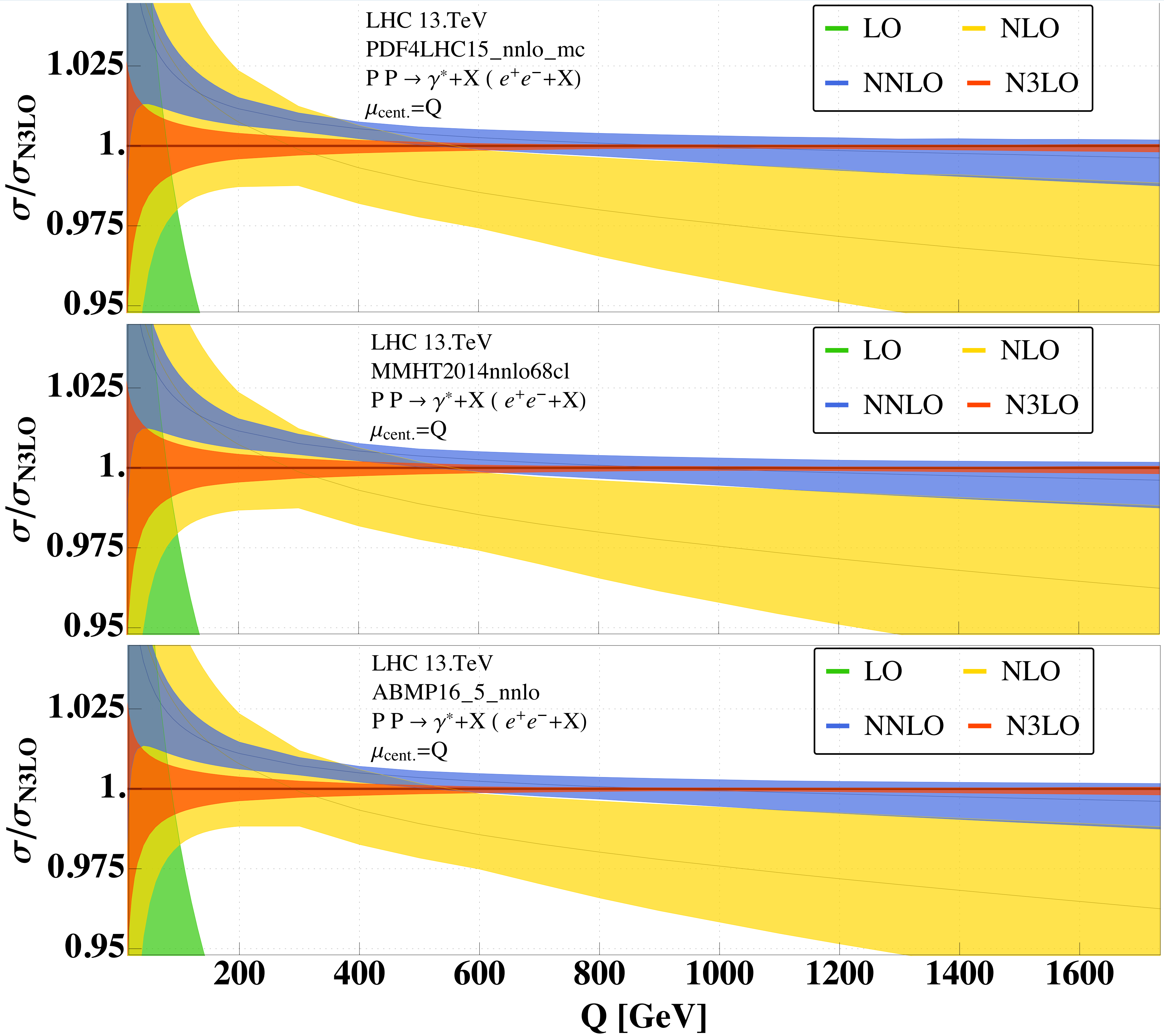
Missing N3LO PDFs





- In all cases we observe $\delta_{\mathrm{PDF}}^{\mathrm{N^3LO}} \sim 1-3\%$.
- N.B.: Current PDF sets use NNLO Drell-Yan data in the fits.
  - ➡ Some higher-order corrections fitted into PDFs?
  - ➡ Impact on scale variation for DY?

Impact of PDF choice



No qualitative differences between different sets!

Summary

	Q[GeV]	$\mathrm{K}_{\mathrm{QCD}}^{\mathrm{N^3LO}}$	δ_{scale}
ggH		1.04	$^{+0.21\%}_{-2.37\%}$
bbH		0.978	$^{+3.0\%}_{-4.8\%}$
DY/W	30	0.952	$^{+1.5\%}_{-2.5\%}$
	90	0.978	$^{+0.75\%}_{-0.89\%}$
	150	0.985	$^{+0.50\%}_{-0.54\%}$
HW+		0.984	$^{+0.58\%}_{-0.30\%}$
HW-		0.994	$^{+0.33\%}_{-0.43\%}$
ggHH		1.03	$^{+0.66\%}_{-2.8\%}$
VBF (DIS, 14 TeV)		0.999	$^{+0.05\%}_{-0.05\%}$

Typical K-factors and scale dependence:

$$\mathrm{K}_{\mathrm{QCD}}^{\mathrm{N^3LO}} = \frac{\mathrm{N^3LO}}{\mathrm{NNLO}} \sim 0.95 - 1.05$$

$$\delta_{\mathrm{scale}} \sim \text{few } \%$$

Typical PDF uncertainty:

$$\delta_{\mathrm{PDF}} \sim 2 - 9\%$$

Missing N3LO PDFs:

$$\delta_{\mathrm{PDF}}^{\mathrm{N^3LO}} \sim 1 - 3\%$$