MCFM for the roaring twenties

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- Apologies for the title
- In fact what I will talk about is:-
  - pp → W(→ lν) + γ process in NNLO QCD and NLO Electroweak
  - arXiv:2105.00954 with John Campbell, Giuseppe de Laurentis and Satyajit Seth, (CDES)

Why Wγ?

- I will use Wγ as a shorthand for the processes $pp \to l^- \, \bar{\nu}_l \, \gamma$ and $pp \to \nu_l \, l^+ \, \gamma$ for $l = e, \mu$ (and τ);
- LHC data will approach the accuracy where higher order QCD and EW corrections are needed;
- Wγ is the largest of all the processes involving the triple boson coupling;
- The Wγ process contains a radiation zero for SM three boson coupling, making it a sensitive probe of this coupling.

Status of W γ experimental data

Experiment	$\int L dt$ fb^{-1}	$\sqrt{s}$ [TeV]	$E^{\gamma}_{T min}$ [GeV]	Experimental cross section [pb]	Theoretical cross section [pb]
CDF [26]	0.020	1.8	7	$13.2 \pm 4.2 \pm 1.3$	18.6 ± 2.8 [32]
D0 [27]	0.0138	1.8	10	$138^{+51}_{-38} \pm 21$	112 ± 10 [33]
CDF [28]	0.200	1.96	7	18.1 ± 3.1	19.3 ± 1.4 [32, 34]
D0 [29]	0.162	1.96	8	$14.8 \pm 1.6 \pm 1.0 \pm 1.0$	16.0 ± 0.4 [32]
D0 [30]	4.2	1.96	15	$7.6 \pm 0.4 \pm 0.6$	7.6 ± 0.2 [32]
CMS [35]	0.036	7	10	$56.3 \pm 5.0 \pm 5.0 \pm 2.3$	49.4 ± 3.8 [6]
ATLAS [36]	0.035	7	15	$36.0 \pm 3.6 \pm 6.2 \pm 1.2$	36.0 ± 2.3 [6]
ATLAS [37]	1.02	7	15	$4.60 \pm 0.11 \pm 0.64$	3.70 ± 0.28 [38]
CMS [39]	5	7	15	$37.0 \pm 0.8 \pm 4.0 \pm 0.8$	31.8 ± 1.8 [8, 40]
CMS [31]	137.1	13	25	$15.58 \pm 0.05 \pm 0.73 \pm 0.15$	$15.4 \pm 1.2 \pm 0.1$ [41, 42]
					$22.4 \pm 3.2 \pm 0.1$ [9]

Table 1. Experiments at various energies with $p\bar{p}$ and pp on the $l^{\pm}\nu\gamma$ process.

Total rather than fiducial cross section

ArXiv:2102.02283

- Theoretical cross sections are those quoted by the experiments;
- Experimental errors are statistical + systematic+(luminosity)
- The most recent measurement have errors commensurate with the theory predictions→so more theoretical work needed.

Radiation Zero

- Consider photon coupling to three charged scalar particles

$$M = \sum_{i=1}^{3} \frac{A_i B_i}{C_i}$$

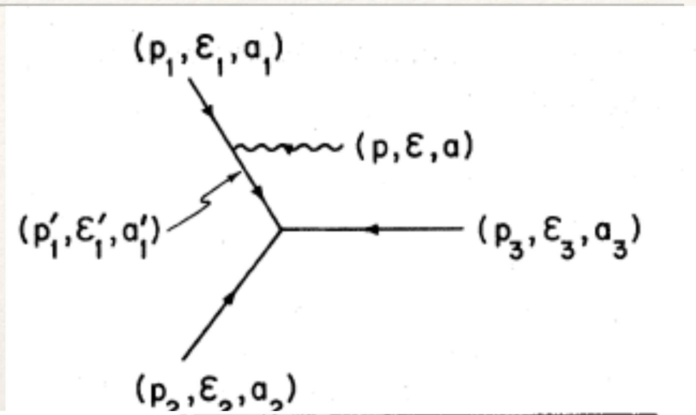


Diagram	A_i	B_i	C_i
1	Q_1	$p_1 \cdot \epsilon$	$p_1 \cdot p$
2	Q_2	$p_2 \cdot \epsilon$	$p_2 \cdot p$
3	$-(Q_1 + Q_2)$	$p_3 \cdot \epsilon$	$p_3 \cdot p$

- Amplitude decomposed as such that $\sum_{i=1}^{3} A_i = \sum_{i=1}^{3} B_i = \sum_{i=1}^{3} C_i = 0$

- So that

$$M = \frac{A_1 B_1}{C_1} + \frac{A_2 B_2}{C_2} + \frac{A_3 B_3}{C_3} = \frac{1}{C_3}(A_1 C_2 - A_2 C_1)\left(\frac{B_2}{C_2} - \frac{B_1}{C_1}\right)$$

$$M = \frac{1}{p_3 \cdot p}\left[Q_1\, p_2 \cdot p - Q_2\, p_1 \cdot p\right]\left(\frac{p \cdot \epsilon}{p_2 \cdot p} - \frac{p \cdot \epsilon}{p_2 \cdot p}\right)$$ which vanishes for $\frac{p_1 \cdot p}{p_2 \cdot p} = \frac{Q_1}{Q_2}, \cos\theta = \frac{1}{3}$

Brown, Sahdev and Mikaelian, PRD20 (1979) 1164
Goebel, Halzen and Leveille, PRD23 (1981) 2682

Precursor of BCJ relations

MCFM

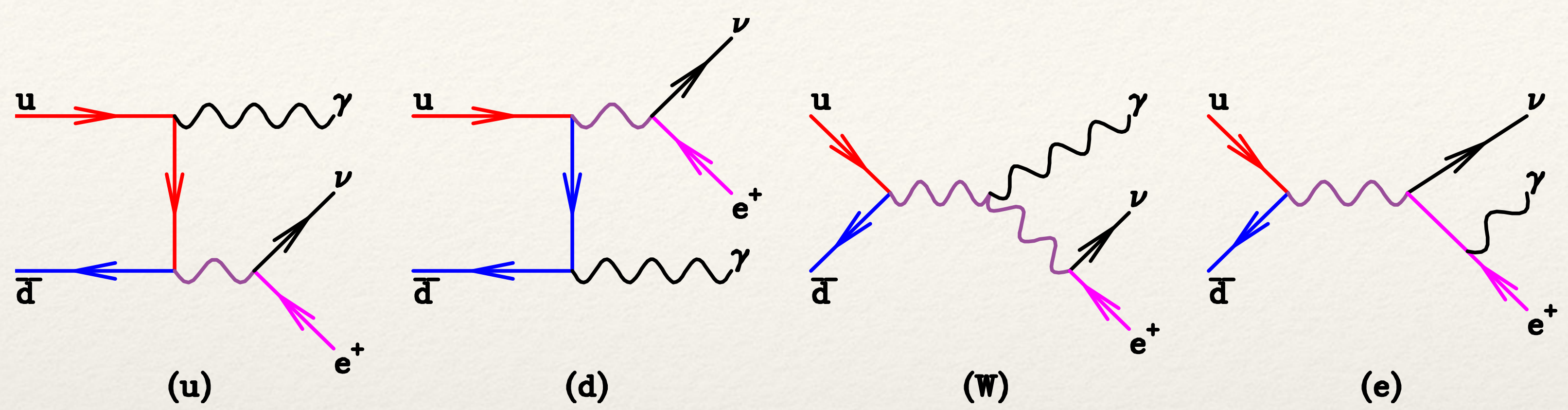
- MCFM contains about 350 processes evaluated at NLO.
- Since matrix elements are calculated using analytic formulae, one can expect better performance, in terms of stability and computer speed, than numerical codes.
- In addition MCFM contains several process evaluated at NNLO using the jetti-ness slicing scheme.
    - Process available at NNLO are W+, W-, Z, H($m_t \to \infty$), $\gamma\gamma$, W^+H, W^-H, ZH, Zγ, $W^+\gamma$, $W^-\gamma$, W^++jet, W^-+jet, Z+jet, γ+jet, H+jet($m_t \to \infty$);
    - WW, ZZ, W^+Z, W^-Z are currently being implemented.

Examples of NNLO results from MCFM

Process	nproc	τ_{cut} [GeV]	σ^{NLO}	σ^{NNLO}	fitted corr.	CPU time [h]
W^+	1	$6 \cdot 10^{-3}\, m_W$	4.221 nb	4.209 ± 0.005 nb	-27 ± 15 pb	7.6
W^-	6	$6 \cdot 10^{-3}\, m_W$	3.315 nb	3.275 ± 0.004 nb	-25 ± 10 pb	7.8
Z	31	$6 \cdot 10^{-3}\, m_Z$	885.3 pb	875.8 ± 0.9 nb	-3.5 ± 2.0 fb	13.0
H	112	$4 \cdot 10^{-3}\, m_H$	1.396 pb	1.872 ± 0.002 pb	7 ± 6 fb	9.7
$\gamma\gamma$	285	$1 \cdot 10^{-4}\, m_{\gamma\gamma}$	27.91 pb	43.54 ± 0.08 pb	0.36 ± 0.10 pb	83.2
W^+H	91	$3 \cdot 10^{-3}\, m_{W^+H}$	2.204 fb	2.262 ± 0.004 fb	0.002 ± 0.008 fb	16.0
W^-H	96	$3 \cdot 10^{-3}\, m_{W^-H}$	1.491 fb	1.526 ± 0.003 fb	-0.005 ± 0.007 fb	13.0
ZH	110	$3 \cdot 10^{-3}\, m_{ZH}$	0.753 fb	0.842 ± 0.001 fb	-0.005 ± 0.003 fb	12.5
$Z\gamma$	300	$3 \cdot 10^{-4}\, m_{Z\gamma}$	434 fb	525.5 ± 1.0 fb	4.5 ± 1.7 fb	202.5

- Benchmark NNLO cross sections for colour singlet processes.

Ingredients of the Wγ calculation



- 5 parton: $0 \to \bar{u}(p_1) + d(p_2) + \nu_e(p_3) + e^+(p_4) + \gamma(p_5)$ calculated at orders $1, g^2, g^4$

1-loop:Dixon, Kunszt & Signer

2-loop:Gehrmann & Tancredi

- 6 parton: $0 \to \bar{u}(p_1) + d(p_2) + \nu_e(p_3) + e^+(p_4) + \gamma(p_5) + g(p_6)$ calculated at orders g, g^3

Analytic 1-loop results in our paper CDES, exploiting, in part, results from Bern, Dixon and Kosower

- 7 parton: $0 \to \bar{u}(p_1) + d(p_2) + \nu_e(p_3) + e^+(p_4) + \gamma(p_5) + g(p_6) + g(p_7)$ and
$0 \to \bar{u}(p_1) + d(p_2) + \nu_e(p_3) + e^+(p_4) + \gamma(p_5) + q(p_6) + \bar{q}(p_7)$ calculated at order g^2

Full analytic results given in our paper CDES

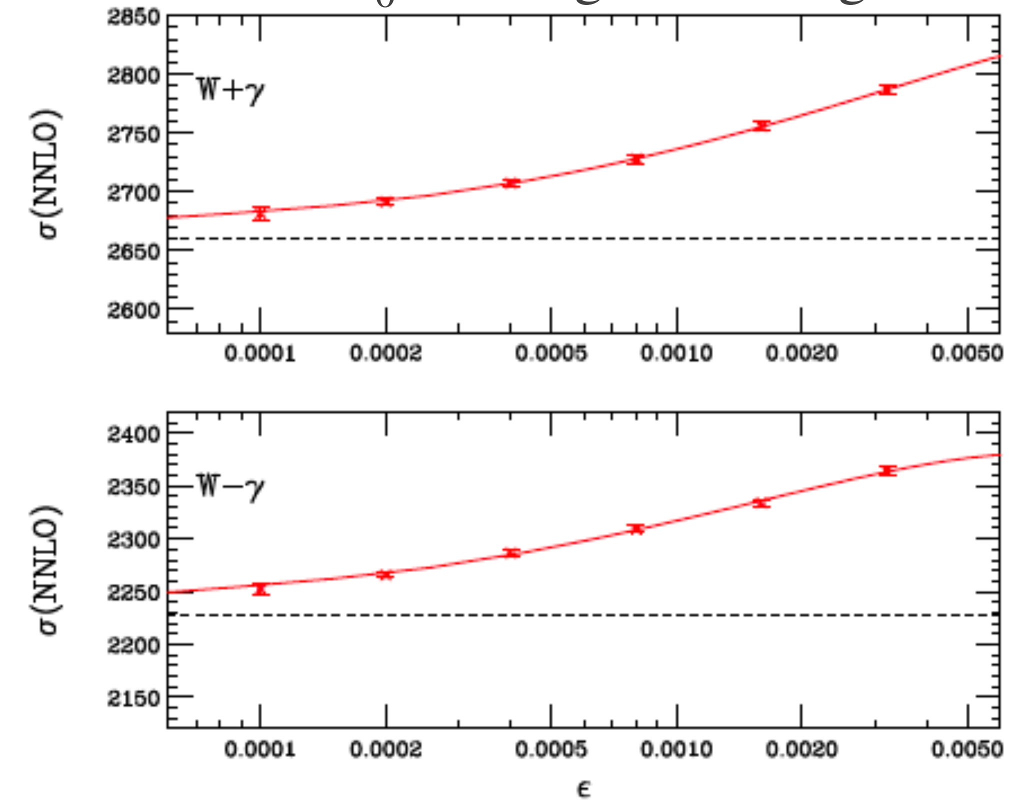
N-jettiness

- A collision of partons a and b with momentum fractions $x_{a,b}$, originating from the incoming beam protons with momenta $p_{a,b}$ can potentially produce a final state including N jets with momenta p_i
- the jettiness of parton j with momentum q_j is defined as $\tau(q_j) = \min\limits_{i=a,b,1,\ldots,N} \left\{ \dfrac{2\, p_i \cdot q_j}{P_i} \right\}$,
- We denote by E_i the jet or beam energy. P_i is a measure of the jet/beam hardness. In our numerical results we set this equal to twice the jet/beam energy, $P_i = 2E_i$
- We can now define the event jettiness, or N-jettiness, as the sum over all the M final-state parton jettiness values
- $$\tau = \sum_{j=1}^{M} \tau(q_j) = \sum_{j=1}^{M} \min_{i=a,b,1,\ldots,N} \left\{ \frac{2\, p_i \cdot q_j}{P_i} \right\} .$$
- For Leading Order (LO) events we have $q_i = p_i$ and the event jettiness is zero.
- Beyond LO extra particles are emitted ($q_i = p_i$), the event jettiness goes to zero only in the soft/collinear limit.
- The event N-jettiness can be used in a non-local subtraction approach where we can isolate the doubly unresolved region of the phase space by demanding $\tau < \tau_{cut}$

A first look at results at 13 TeV

- Results depend on the slicing parameter;
- Form of power corrections for the approach to limit is known.
- Extrapolation is performed in the dimensionless variable ε where $\tau_0^{cut} = \epsilon \times m_{\ell\nu\gamma}$ and $m^2_{\ell\nu\gamma} = (p_\ell + p_\nu + p_\gamma)^2$
- Results on this slide have MATRIX cuts.

$$\sigma = \sigma_0 + a\epsilon \log^3 \epsilon + b\epsilon \log^2 \epsilon \,.$$



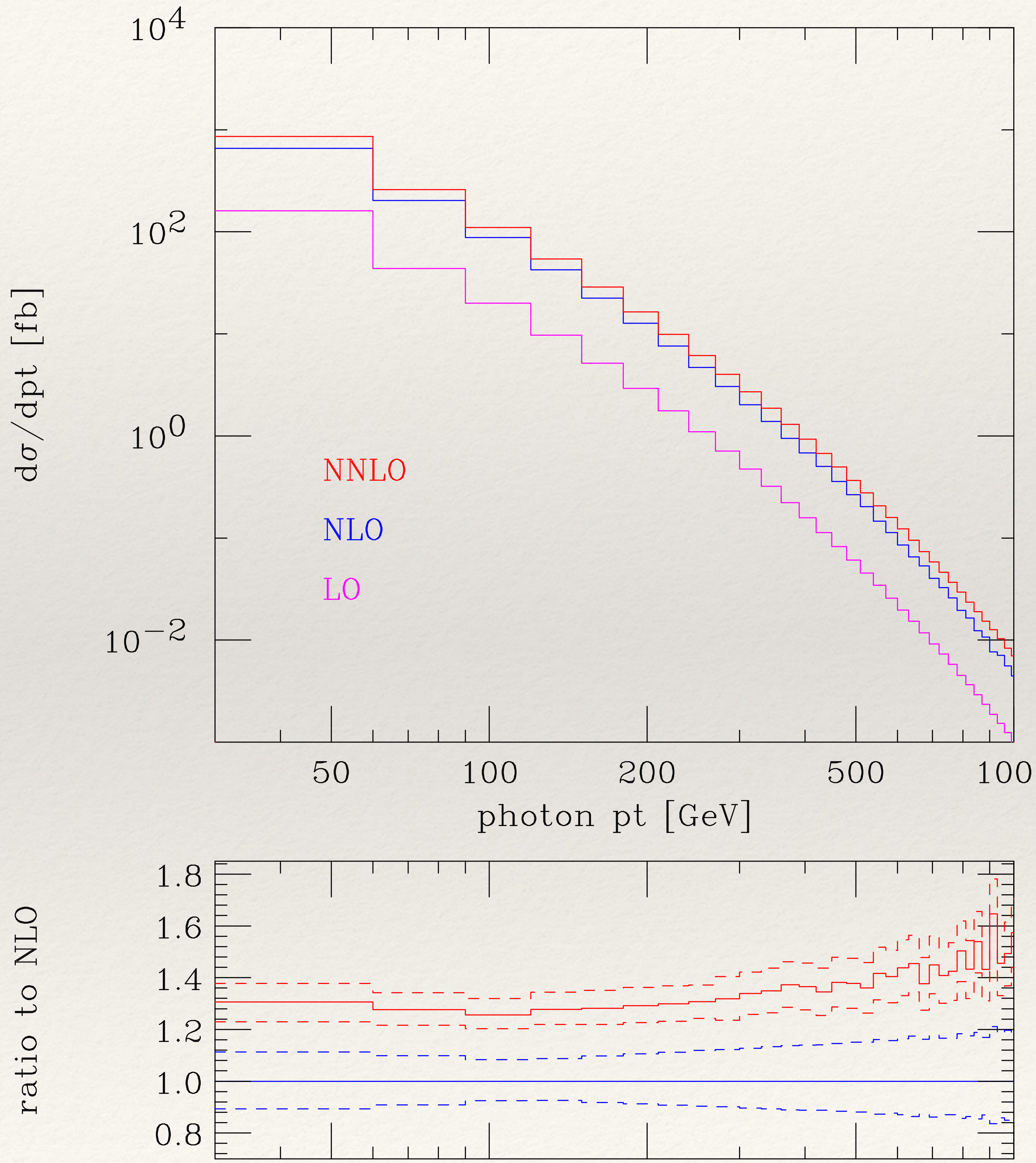
- Cross sections are in agreement with earlier results from MATRIX when using the same parameters.
- Note the progression LO :NLO:NNLO ~ 1 : 2.5 : 3.0
- At NLO gq and $g\bar{q}$ initial states contribute 39% of the cross section; indicating that normal perturbative ordering is disrupted.

Process	σ_{LO}(fb)	σ_{NLO}(fb)	$\sigma^{\epsilon=10^{-4}}_{NNLO}$(fb)	$\sigma^{\rm fit}_{NNLO}$(fb)
$pp \to e^+\nu_e\gamma$ (no CKM)	861.6	$2187^{+6.6\%}_{-5.3\%}$	2689(5)	$2668(8)^{+3.9\%}_{-3.7\%}$
$pp \to e^+\nu_e\gamma$ (with CKM)	854.6	$2181^{+6.6\%}_{-5.3\%}$	2681(5)	$2661(8)^{+3.9\%}_{-3.7\%}$
$pp \to e^-\bar{\nu}_e\gamma$ (no CKM)	726.2	$1849^{+6.6\%}_{-5.3\%}$	2260(4)	$2240(7)^{+3.7\%}_{-3.5\%}$
$pp \to e^-\bar{\nu}_e\gamma$ (with CKM)	720.1	$1843^{+6.6\%}_{-5.3\%}$	2252(4)	$2228(7)^{+3.7\%}_{-3.5\%}$

Table 4. Cross-section results with the cuts of Table 3. The theoretical error is estimated by a 7-point scale variation. Parentheses indicate the residual error resulting from numerical integration of the NNLO result (this error is beyond the indicated number of digits at NLO).

QCD corrections

- Scale variation of NLO does not encompass NNLO prediction.
- "NNLO" is effectively NLO for gluon initiated part of the cross section.
- We use a hybrid photon isolation scheme, 1904.01044

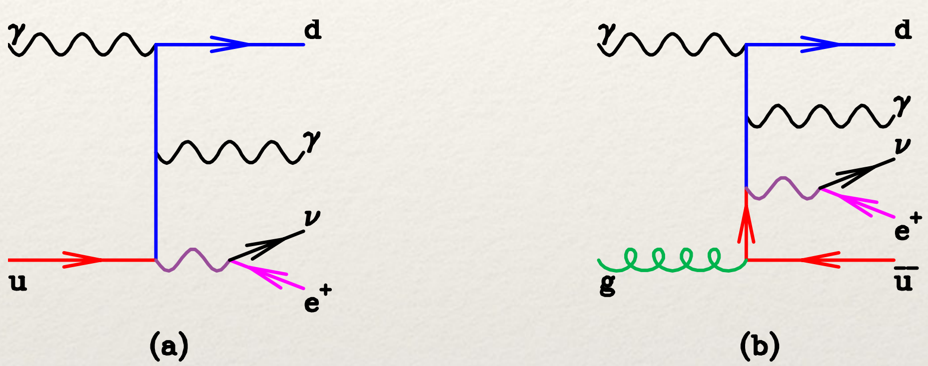


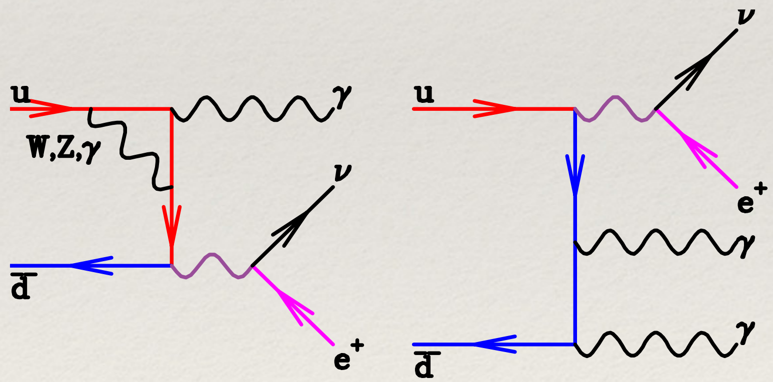
Electroweak corrections

- We have seen that the sequence of the perturbative series is disrupted by the radiation zero
- At NLO:
  - $u + \bar{d} \to \nu + e^+ + \gamma$ and $u + \bar{d} \to \nu + e^+ + g + \gamma$ and related 60%;
  - $u + g \to \nu + e^+ + \gamma + d$ and related 40%;
  - $W\gamma$ would be a prime candidate for a complete mixed QCD-electroweak calculation (hard, but cf W+4 partons, Hartanto et al (1906.11862) and also for an QCD N^{3}LO calculation.
  - We shall consider the gq process with a jet cut.
- The numerical results for the virtual electroweak corrections have been obtained using the RECOLA library (1605.01090,1711.07388)

Electroweak corrections (continued)

- Processes involving initial-state photons, and associated terms needed to remove initial state collinear singularities;
- Important to re-examine because of increased information on photon distribution functions (post LUX)
- Virtual electroweak corrections to the basic processes and real corrections associated with the emission of extra photons, together with the counterterms needed to remove singularities from soft and collinear photon emission.





Choice of α electromagnetic

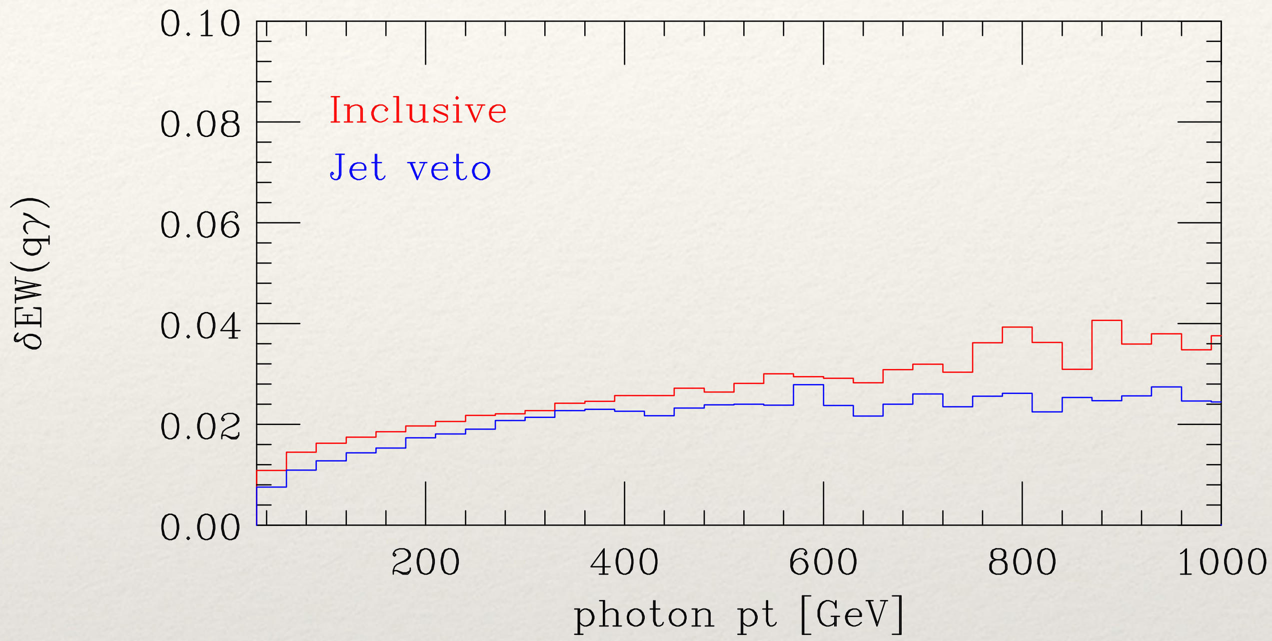
- The Recola library supplies results in three different renormalization schemes,
  - the $\alpha(G_\mu)$ scheme, $\alpha = \sqrt{2}G_\mu/\pi M_W^2(1 - M_W^2/M_Z^2)$ which includes universal terms associated with the renormalization of the weak mixing angle;
  - the $\alpha(0)$ scheme, where α is fixed by the measured value at $p^2 = 0$;
  - the $\alpha(M_Z)$ scheme, where α is fixed by the value at $p^2 = M_Z^2$, taking into account the running from $p^2 = 0$ to $p^2 = M_Z^2$, which at low p^2 is inadequately treated in perturbation theory.

Electroweak corrections - total cross section

- Executive summary
  - Decrease $\alpha(0)/\alpha(M_Z^2) \approx 0.965$
  - Incoming γq process, +0.9 to+1.3% depending on photon pt cut and/or jet veto.
  - Virtual and real photon emission corrections $q\bar{q}$ process, -1.3%
  - Situation changes at large pT.

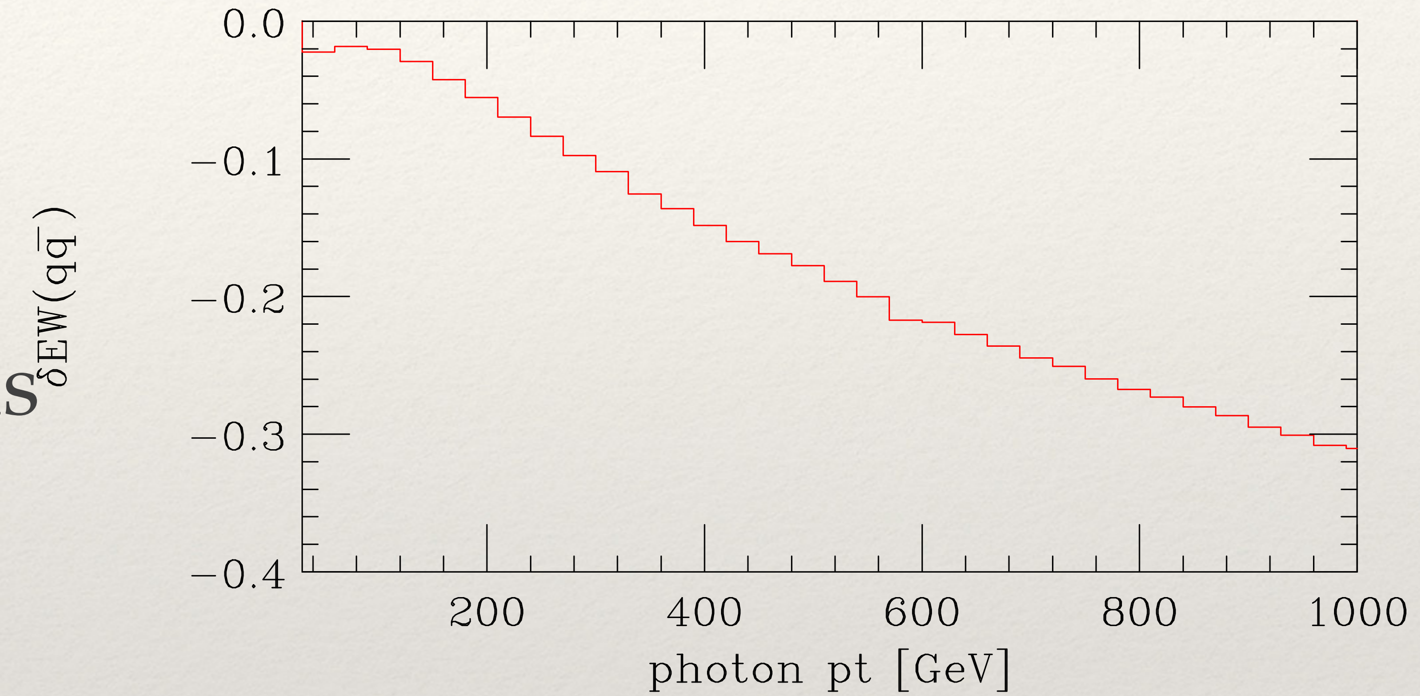
EW corrections to processes with initial photons

- $(\gamma + q \to \nu + e^+ + \gamma + d)$+all related γq processes;
- The impact of the particular EW correction is shown as a factor relative to the NNLO process;
- overall $\delta_{EW}(q\gamma) = +1.3\,\%$;
- rising to a 4% effect at p_T^γ=1 TeV, smaller with jet cut;
- Size of these corrections, smaller than pre-LUX estimates.



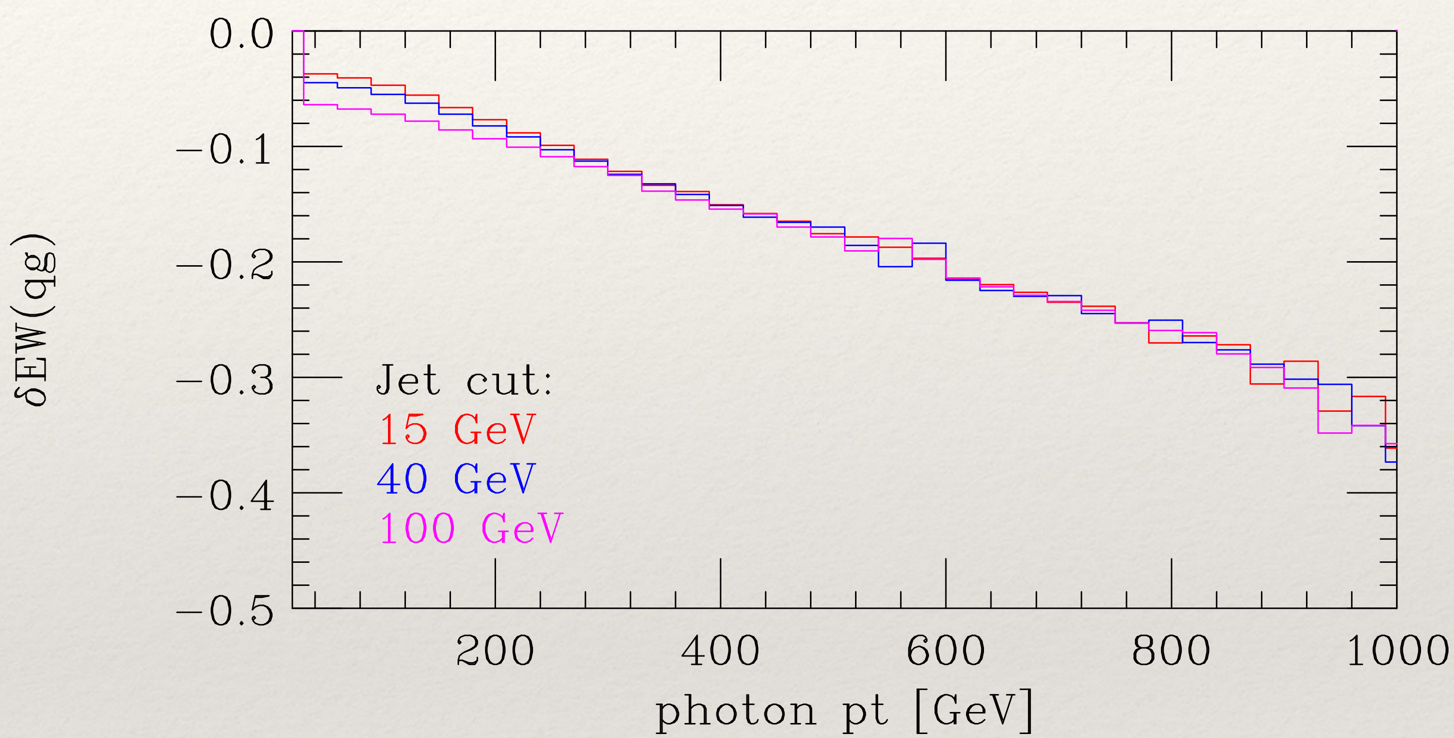
EW corrections to $u + \bar{d} \to \nu + e^{+} + \gamma$

- $(u + \bar{d} \to \nu + e^{+} + \gamma)$ +all related $q\bar{q}$ processes;
- The impact of the particular EW correction is shown as a factor relative to the LO process.
- overall $\delta_{EW}(q\bar{q}) = -1.3\ \%$
- larger effect in the tail.

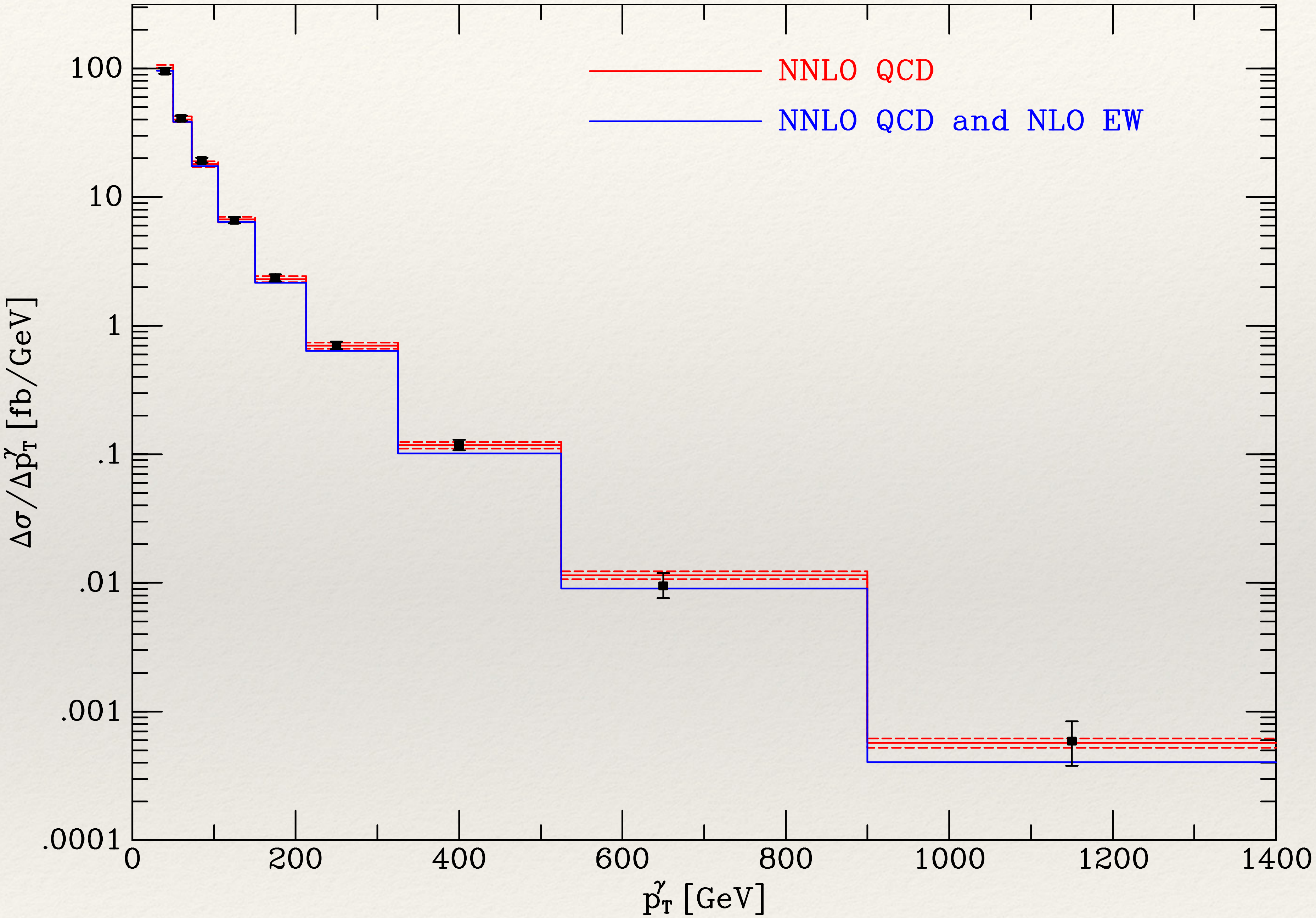


Relative correction to $u + g \to \nu + e^+ + \gamma + d$

- $(u + g \to \nu + e^+ + \gamma + d)$ +all related $qg, \bar{q}g$ processes;
- important in view of the sizable contribution of this channel to the NLO rate.
- We cannot estimate the effect of this correction on the total cross section.
- Instead we display the result for 3 different jet cuts; above $p_T^\gamma = 300$ GeV the result is independent of the jet cuts.
- Contribution is substantial and needs to be taken into account.



Comparison with data at $\sqrt{s} = 13$ TeV



All $\sqrt{s}$ = 13TeV data for distributions taken from CMS-PAS-SMP-20-005

Total cross section/Scale error

- Expt cross section inclusive over all jets
  $\sigma = 3705^{+218}_{-212}$ fb

- Our theory result
  $\sigma = 3799 \pm 351$ fb.

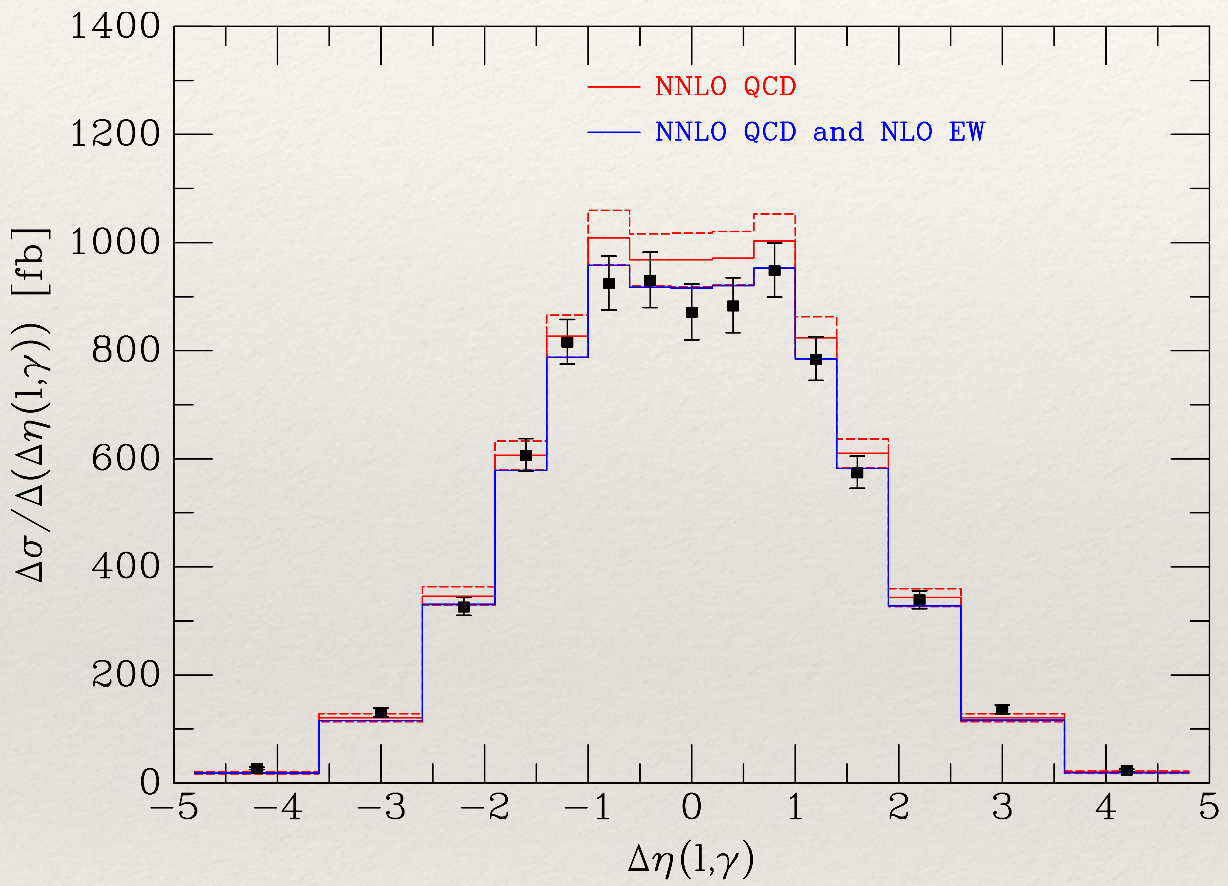
- QCD scale error at 5% is bigger than EW corrections at all pT. At large pT where the EW corrections are ~4%, experimental errors are also large.

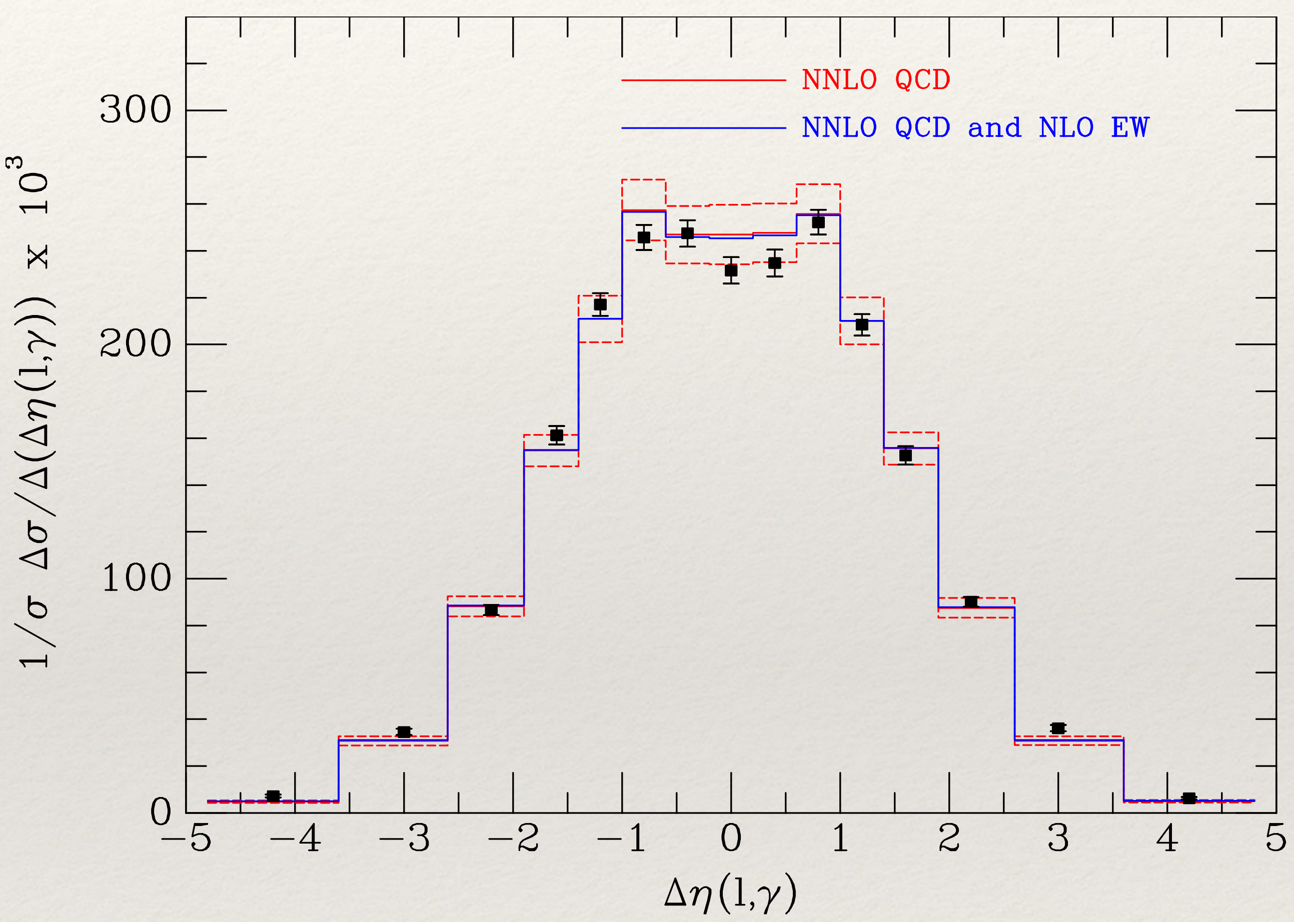
Experimental data from SMP-20-005-PAS

Number of jets (GeV)	Best fit	Stat	Syst
		σ(fb)	
$=0$	1403^{+73}_{-70}	$^{+11}_{-11}$	$^{+73}_{-69}$
$=1$	1254^{+64}_{-61}	$^{+11}_{-10}$	$^{+63}_{-60}$
≥ 2	1048^{+81}_{-81}	$^{+10}_{-10}$	$^{+80}_{-81}$

p_T [GeV]	σ [fb]	Scale error %
40.	96.5	5.1
60.	38.5	5.2
85	17.4	5.1
125	6.4	5.2
175	2.2	5.2
250	0.64	5.6
400	0.10	6.2
650	9.17E-03	7.1
1150	4.0E-04	8.3

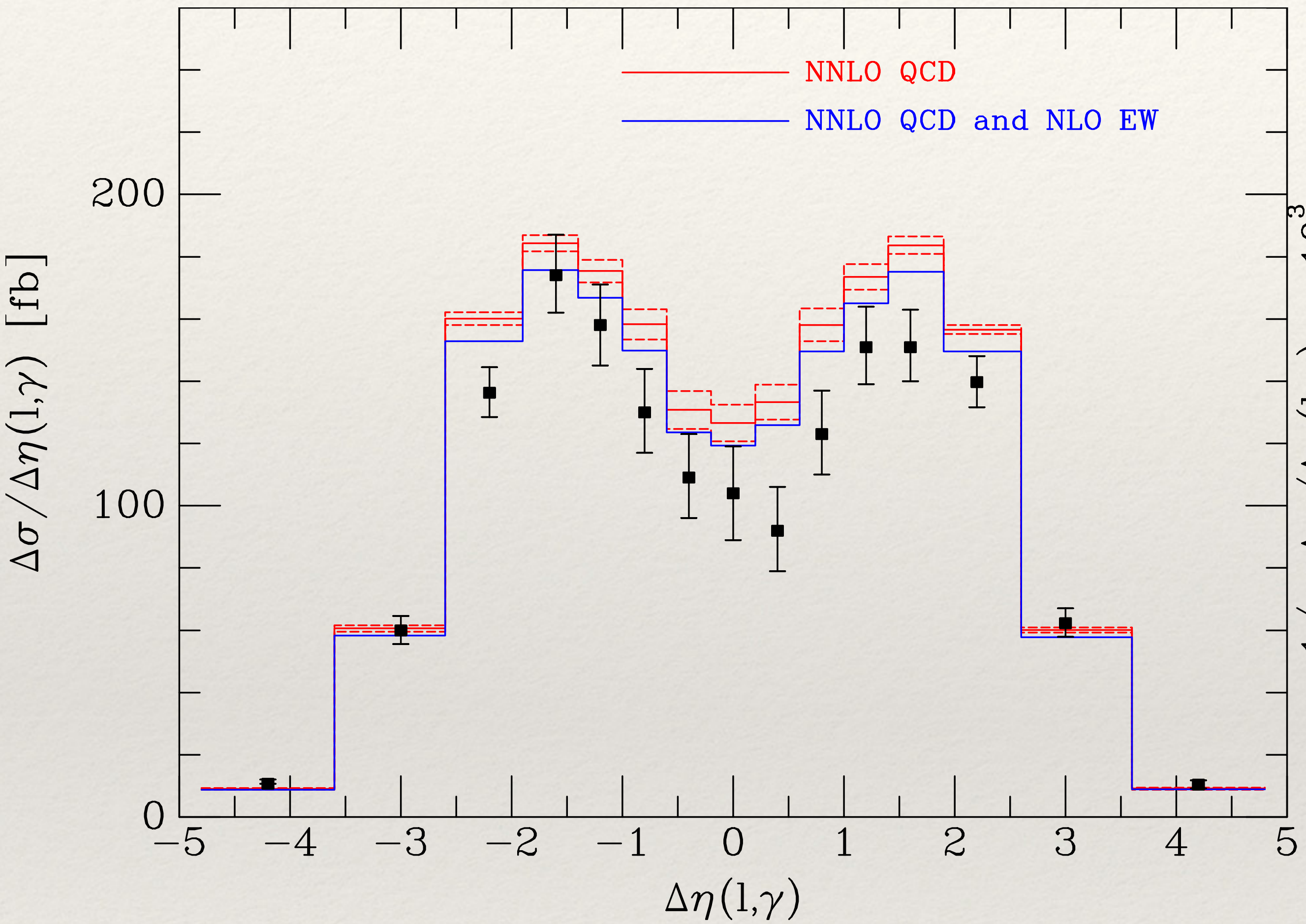
Lepton-photon pseudo rapidity separation

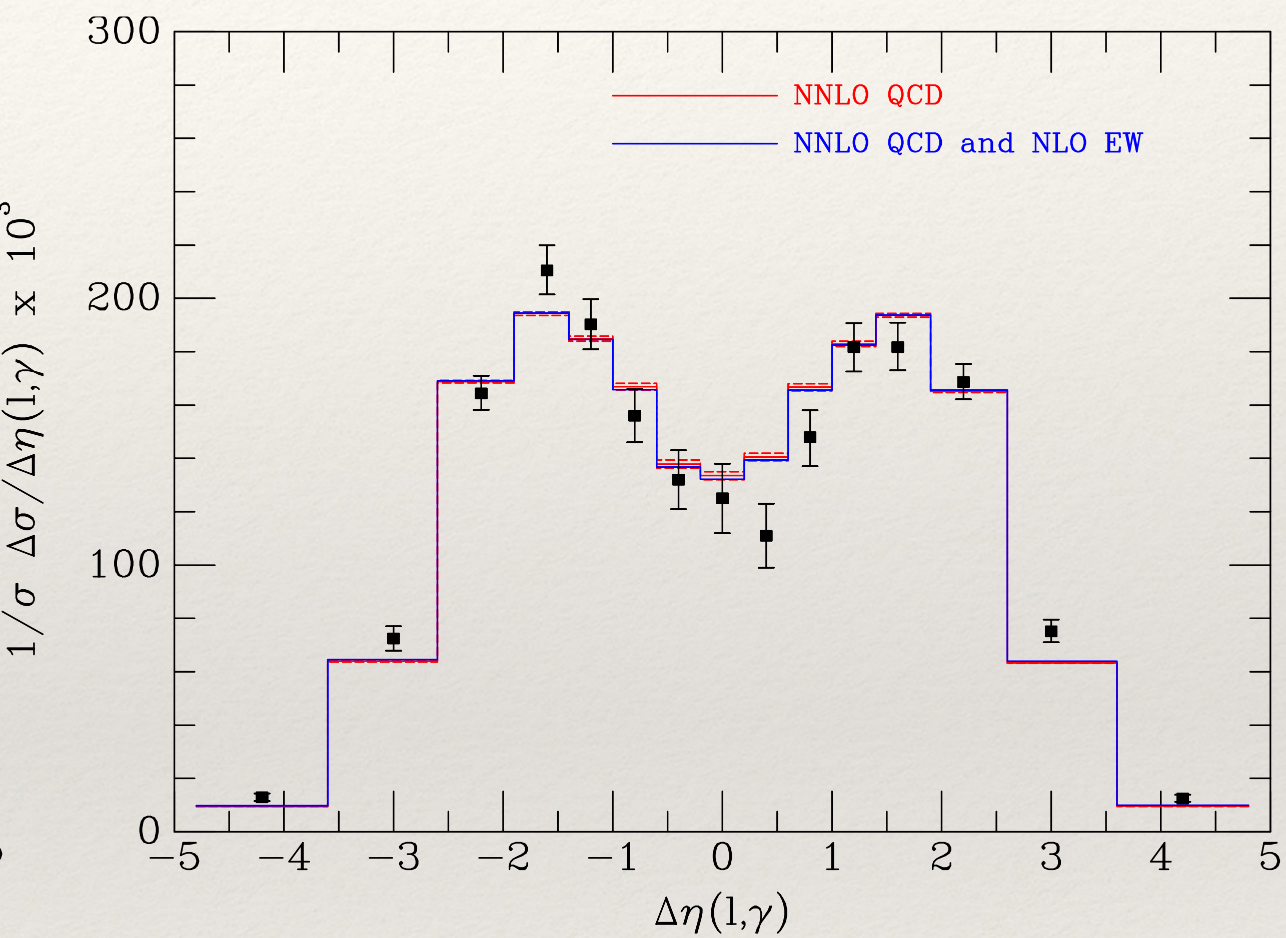




- Radiation zero is almost completely obscured.

Lepton-photon pseudorapidity separation with jet veto





- Perform cut $m_T^{cluster} > 150$ GeV and jet veto $p_T > 30$ GeV and $|\eta| < 2.5$
- Radiation zero is made manifest.

Conclusions

- NNLO QCD and NLO EW calculation for $W\gamma$;
- First look at EW corrections for important gq process;
- Choice of α_{EM} changes results by 3.4%, other EW corrections will be hard to observe, with a NNLO calculation and associated scale error;
- Good agreement with CMS paper CMS-PAS-SMP 20-005