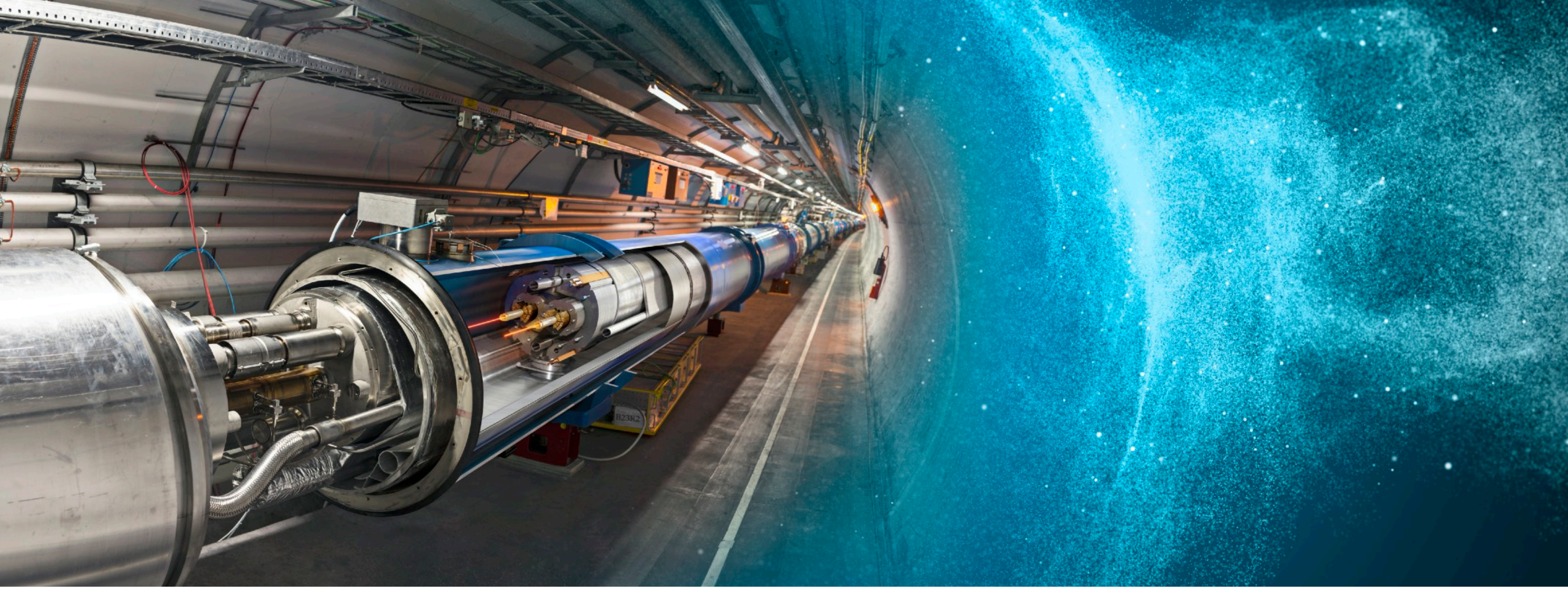
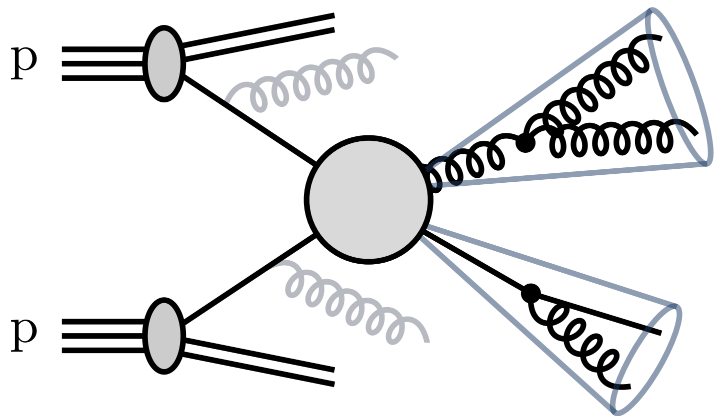


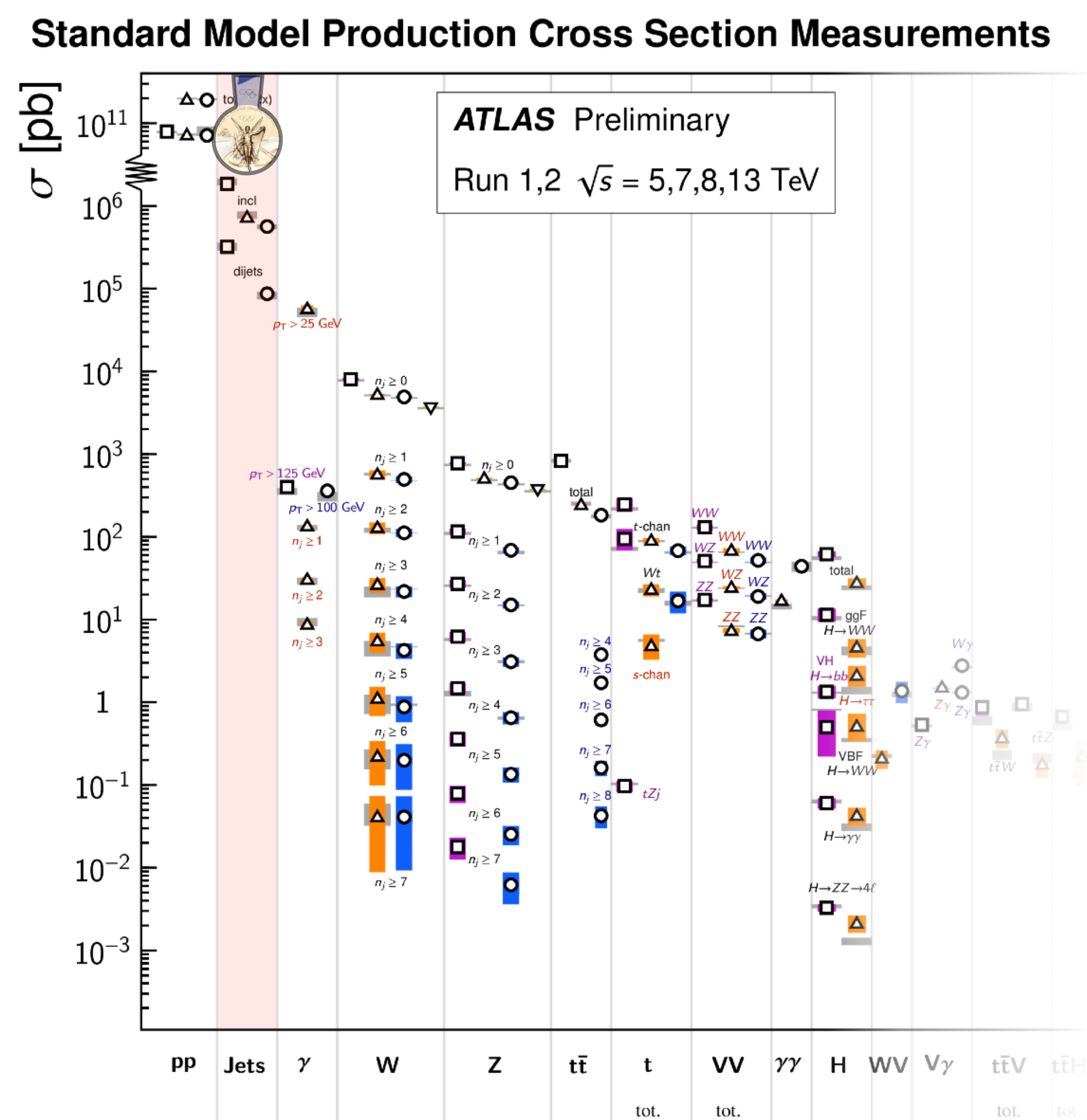
JETS AT NNLO

Alexander Huss

A Loop Summit — Cadenabbia, 26th July 2021



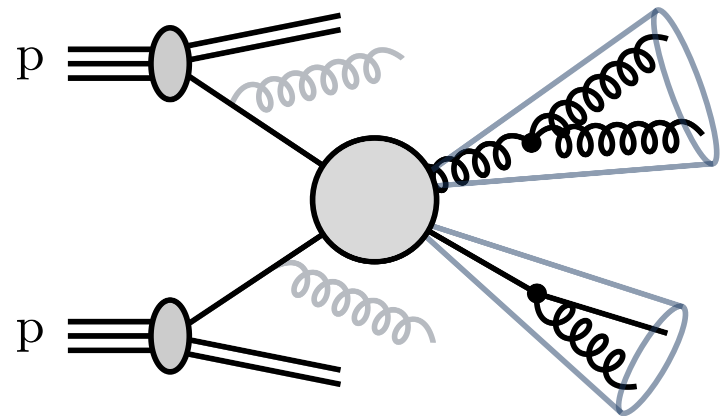
JETS ARE ...



... ideal pQCD laboratory

- simple $2 \rightarrow 2$ parton scattering

... produced in *abundance*



JETS ARE ...

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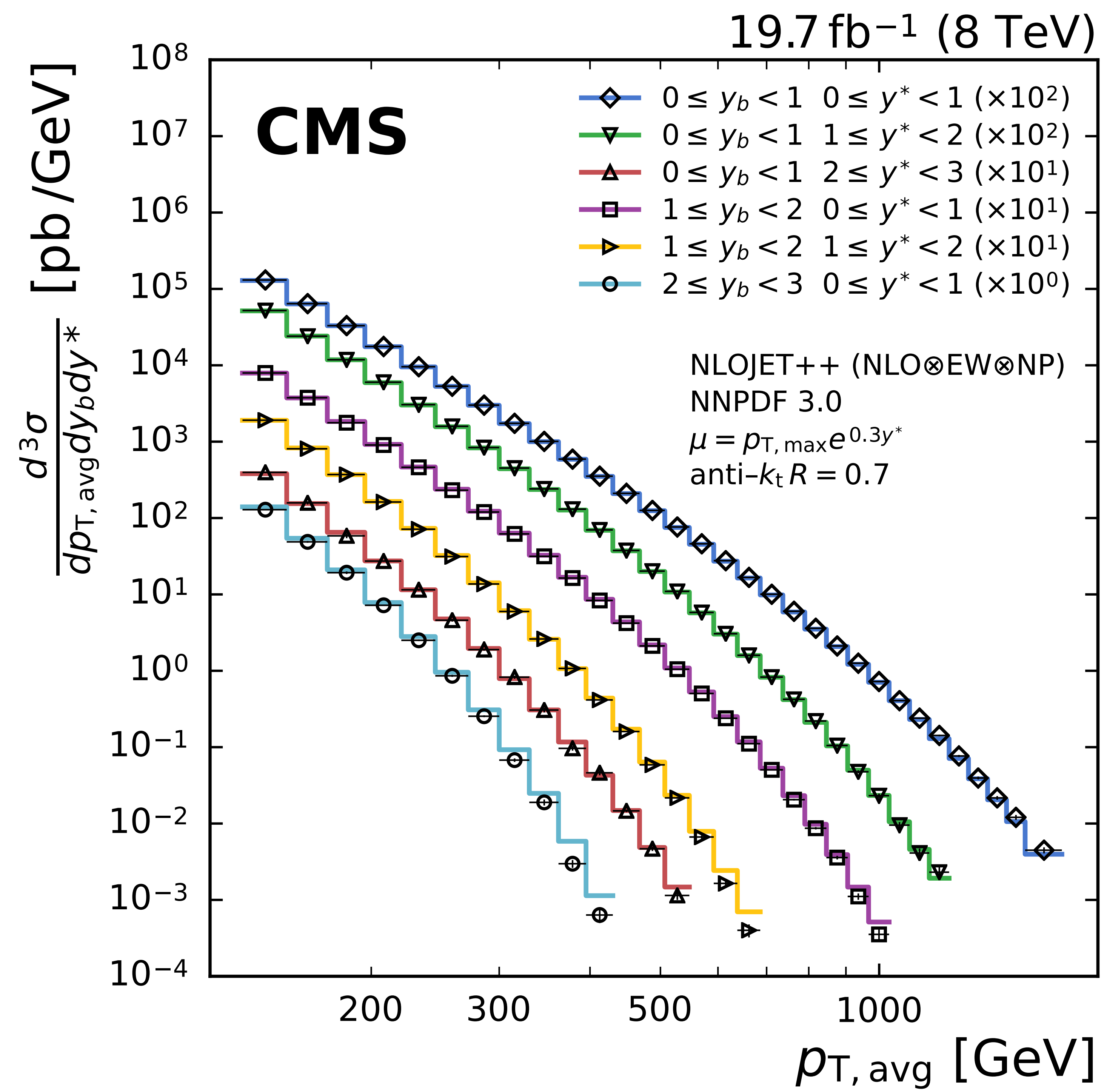
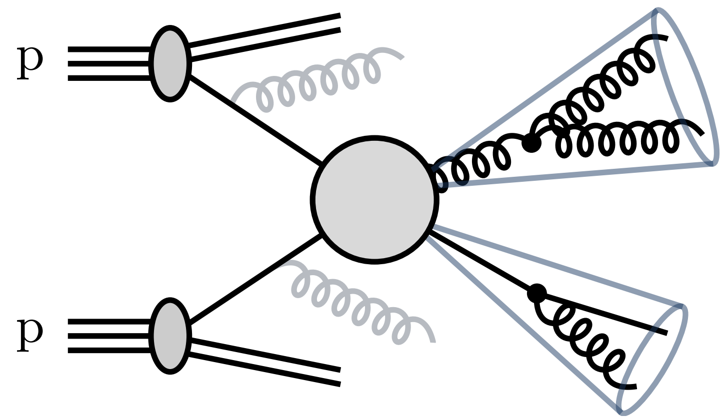
... wide kinematic range





JETS ARE …

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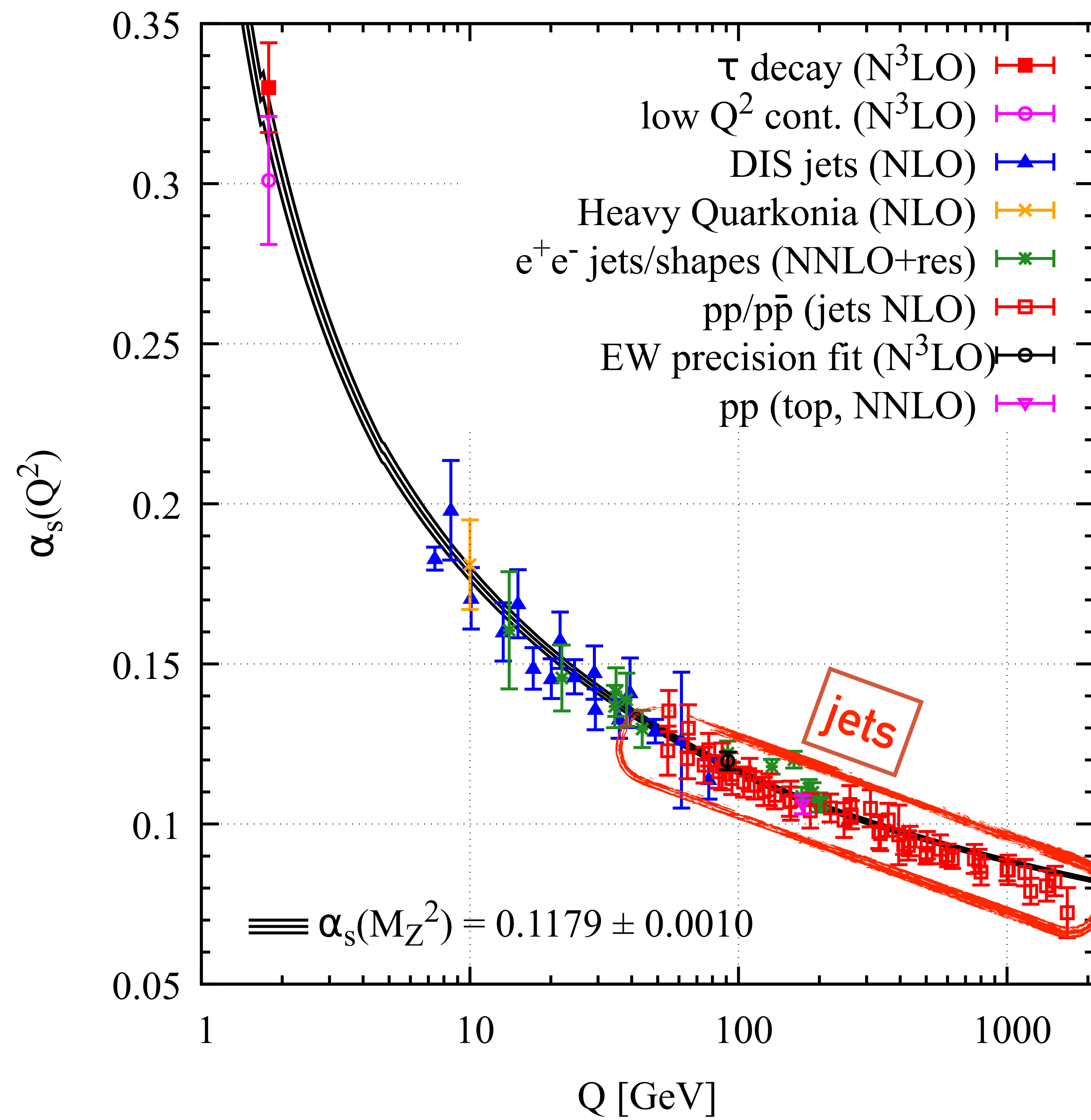
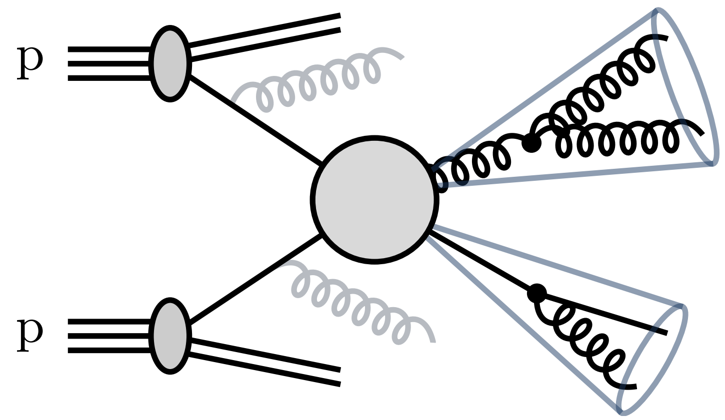
… produced in *abundance*

… wide kinematic range

… α_s & running





Jets are ...

... ideal pQCD laboratory

- simple $2 \to 2$ parton scattering

... produced in *abundance*

... wide kinematic range

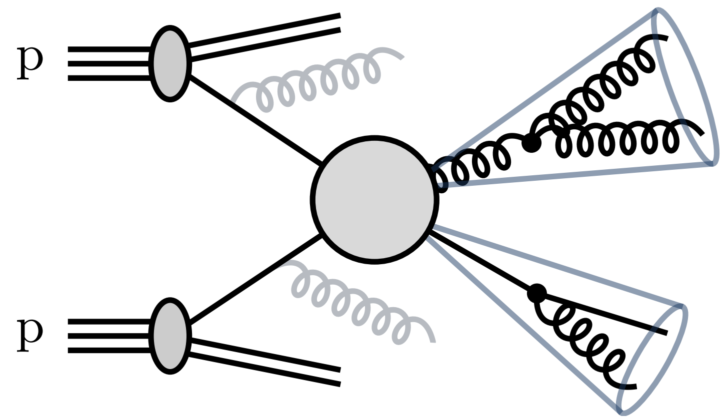
... α_s & running

... constrain PDFs (high-x g)

$$x = \frac{p_T}{\sqrt{s}} \left(e^{\pm y_j} + e^{\pm y_{j'}} \right)$$

- @ LO: 3 variables (p_T, y_j, $y_{j'}$)
- inclusive jet[2] (some smearing)

- di-jet[3] (reconstructible: 3-D)



Jets are …

… ideal pQCD laboratory

- simple $2 \to 2$ parton scattering

… produced in *abundance*

… wide kinematic range

… α_s & running

… constrain PDFs (high-x g)

… BSM searches

…

NNLO { antenna [Currie, Glover, Pires '16]
stripper [Czakon, van Hameren, Mitov, Poncelet '19]

$$x = \frac{p_T}{\sqrt{s}} \left(e^{\pm y_j} + e^{\pm y_{j'}} \right)$$

- @ LO: 3 variables $(p_T, y_j, y_{j'})$
- inclusive jet[2] (some smearing)

- di-jet[3] (reconstructible: 3-D)

The Plan.

- Antenna Subtraction
  - & the NNLOJet Framework
- Inclusive Jets
  - — IR Sensitivity & Scale Choices
- Di-Jet Production
  - — 3D Measurements
- Impact on PDFs
  - & Interpolation Tables

The Plan.

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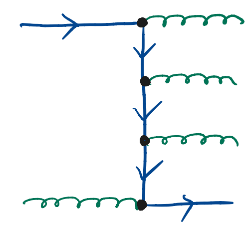
Anatomy of NNLO Calculations

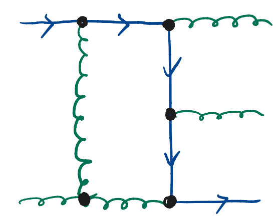
$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \mathrm{d}\sigma^{\text{RR}}_{\text{NNLO}}$$



[Mangano, Parke '91; ...]

- single-unresolved
- double-unresolved

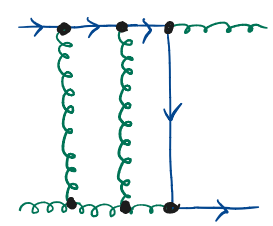
$$+ \int_{\Phi_{Z+2}} \mathrm{d}\sigma^{\text{RV}}_{\text{NNLO}}$$



[Bern, Dixon, Kosower '93, '95; Kunszt, Signer, Trocsanyi '94; ...]

- single-unresolved
- $1/\epsilon^2$, $1/\epsilon$

$$+ \int_{\Phi_{Z+1}} \mathrm{d}\sigma^{\text{VV}}_{\text{NNLO}}$$



[Anastasiou, Glover, Oleari, Tejeda-Yeomans '01, '03;
Bern, De Freitas, Dixon '02, '03, '04; ...]

- $1/\epsilon^4$, $1/\epsilon^3$, $1/\epsilon^2$, $1/\epsilon$

$\sum$ finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

NNLO using Subtraction

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \left(\mathrm{d}\sigma^{\mathrm{RR}}_{\text{NNLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+2}} \left(\mathrm{d}\sigma^{\mathrm{RV}}_{\text{NNLO}} - \mathrm{d}\sigma^{\mathrm{T}}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+1}} \left(\mathrm{d}\sigma^{\mathrm{VV}}_{\text{NNLO}} - \mathrm{d}\sigma^{\mathrm{U}}_{\text{NNLO}} \right)$$

$$\sum \qquad \text{finite} \qquad - 0$$

- $\mathrm{d}\sigma^{\mathrm{S}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{T}}_{\text{NNLO}}$:
  mimic $\mathrm{d}\sigma^{\mathrm{RR}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{RV}}_{\text{NNLO}}$
  in unresolved limits
- $\mathrm{d}\sigma^{\mathrm{T}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{U}}_{\text{NNLO}}$:
  analytic cancellation of poles in $\mathrm{d}\sigma^{\mathrm{RV}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{VV}}_{\text{NNLO}}$

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

NNLO using Subtraction

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \left(d\sigma^{\text{RR}}_{\text{NNLO}} - d\sigma^{\text{S}}_{\text{NNLO}}\right)$$

$$+ \int_{\Phi_{Z+2}} \left(d\sigma^{\text{RV}}_{\text{NNLO}} - d\sigma^{\text{T}}_{\text{NNLO}}\right)$$

$$+ \int_{\Phi_{Z+1}} \left(d\sigma^{\text{VV}}_{\text{NNLO}} - d\sigma^{\text{U}}_{\text{NNLO}}\right)$$

$$\sum \qquad \text{finite} \qquad -0$$

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+1}(\ldots,i,j,k,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\text{ unresolved}} \underbrace{X^0_3(i,j,k)}_{\text{antenna function}} \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{K},\ldots)|^2}_{\text{reduced ME}}$$

$+$ mapping
$\{p_i, p_j, p_k\} \to \{\widetilde{p}_I, \widetilde{p}_K\}$

- captures *multiple limits* and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}} P_{ij}(z)$	$\widetilde{p}_I \to (p_i + p_j),\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}} P_{kj}(z)$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to (p_j + p_k)$

* c.f. dipoles: $X^0_3(i,j,k) \sim \mathcal{D}_{ij,k} + \mathcal{D}_{kj,i}$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+2}(\ldots,i,j,k,l,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\ \&\ k \text{ unresolved}} \underbrace{\boxed{X^0_4(i,j,k,l)}}_{\text{antenna function}} \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{L},\ldots)|^2}_{\text{reduced ME}}$$

+ mapping $\{p_i,p_j,p_k,p_l\} \to \{\widetilde{p}_I,\widetilde{p}_L\}$

- captures *multiple limits* and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}} P_{ij}(z)$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}} P_{kj}(z)$

- double soft: $j,k \to 0$
- triple-collinear:
  $(i \parallel j \parallel k)$ & $(j \parallel k \parallel l)$
- double collinear: $(i \parallel j),(k \parallel l)$
- soft–collinear:
  $(i \parallel j), k \to 0$ & $(k \parallel l), j \to 0$
- single-unresolved

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues, J. Pires, D. Walker, J. Whitehead

Processes computed using the antenna subtraction method

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ NNLO
  $\hookrightarrow V \to \ell\bar{\ell}$ $(V = \mathrm{Z}/\gamma^*,\ \mathrm{W}^\pm)$
- $\mathrm{pp} \to$ jets (inc. jets, 2j) @ NNLO
- $\mathrm{pp} \to \gamma + j$ @ NNLO
- $\mathrm{ep} \to 1j$ @ N^{3}LO
- $\mathrm{ep} \to 2j$ @ NNLO
- $\mathrm{e}^+\mathrm{e}^- \to 3$ jets @ NNLO
- $\mathrm{pp} \to \mathrm{H}$ (ggH) @ N^{3}LO
- $\mathrm{pp} \to \mathrm{H} + j$ (ggH) @ NNLO
- $\mathrm{pp} \to \mathrm{H} + 2j$ (VBF) @ NNLO
  $\hookrightarrow \mathrm{H} \to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$
- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  $\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder,
T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues,
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Processes computed using the antenna subtra... m...

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ ...
  - $\hookrightarrow V \to \ell\bar{\ell}$ (V...
- $\mathrm{pp} \to$ jets
- $\mathrm{pp} \to \gamma$ + ...
- ep ...
- ep ... @ NNLO
- $\mathrm{e}^+\mathrm{e}^-$ ...s @ NNLO

... @ N^{3}LO

... @ NNLO

... (VBF) @ NNLO

... $\to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$

- $\mathrm{pp} \to VH$ @ NNLO
  - $\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...

NNLO subtraction set up for

$\mathrm{p\,p} \to$ "colour neutral" $+\ 0{,}1{,}2$ jets

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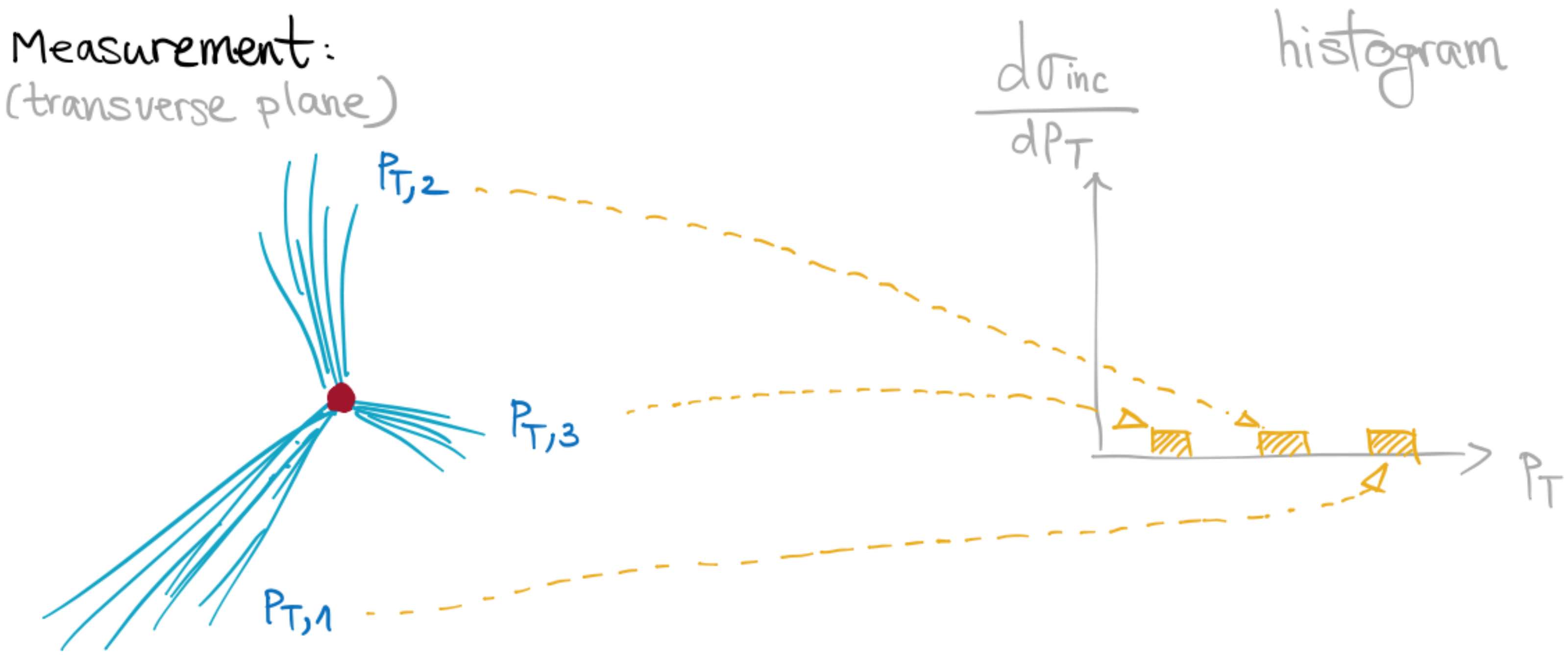
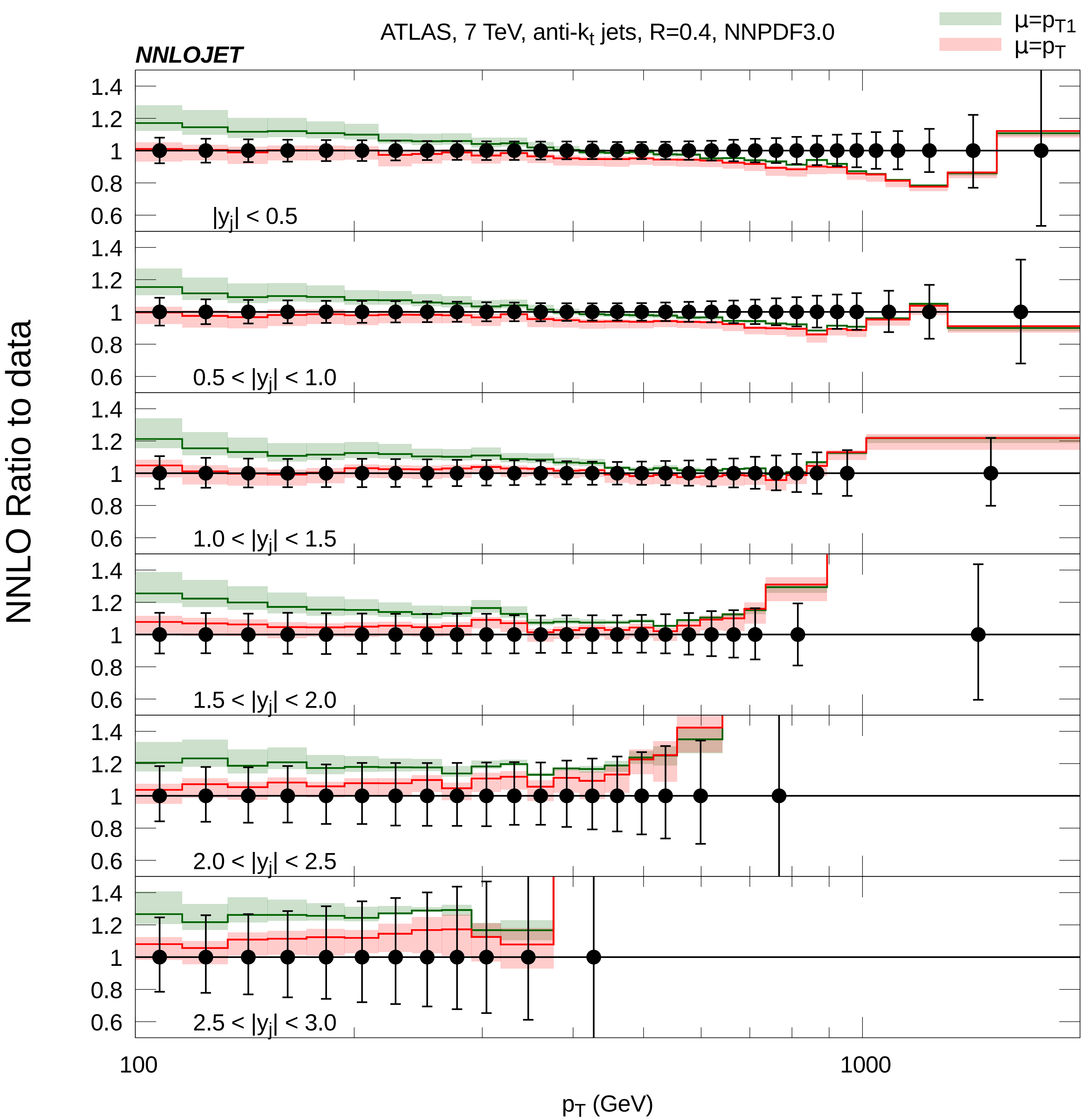
Inclusive Jet Production



$$\left\{ n \text{ reconstructed jets in the event} \right\} \longleftrightarrow \left\{ n \text{ binnings to the histogram} \right\} \qquad \frac{d\sigma_{\rm inc}}{dp_T^j} = \frac{d\sigma}{dp_T^{j_1}} + \frac{d\sigma}{dp_T^{j_2}} + \frac{d\sigma}{dp_T^{j_3}} + \cdots$$

- many scale choices possible:
  - "event based" — $p_{T,\max}$, $\langle p_T \rangle$, $\hat{H}_T$, …
  - "jet based" — p_T , …

Inclusive Jet Production — Scales p_T v.s. $p_{T,1}$

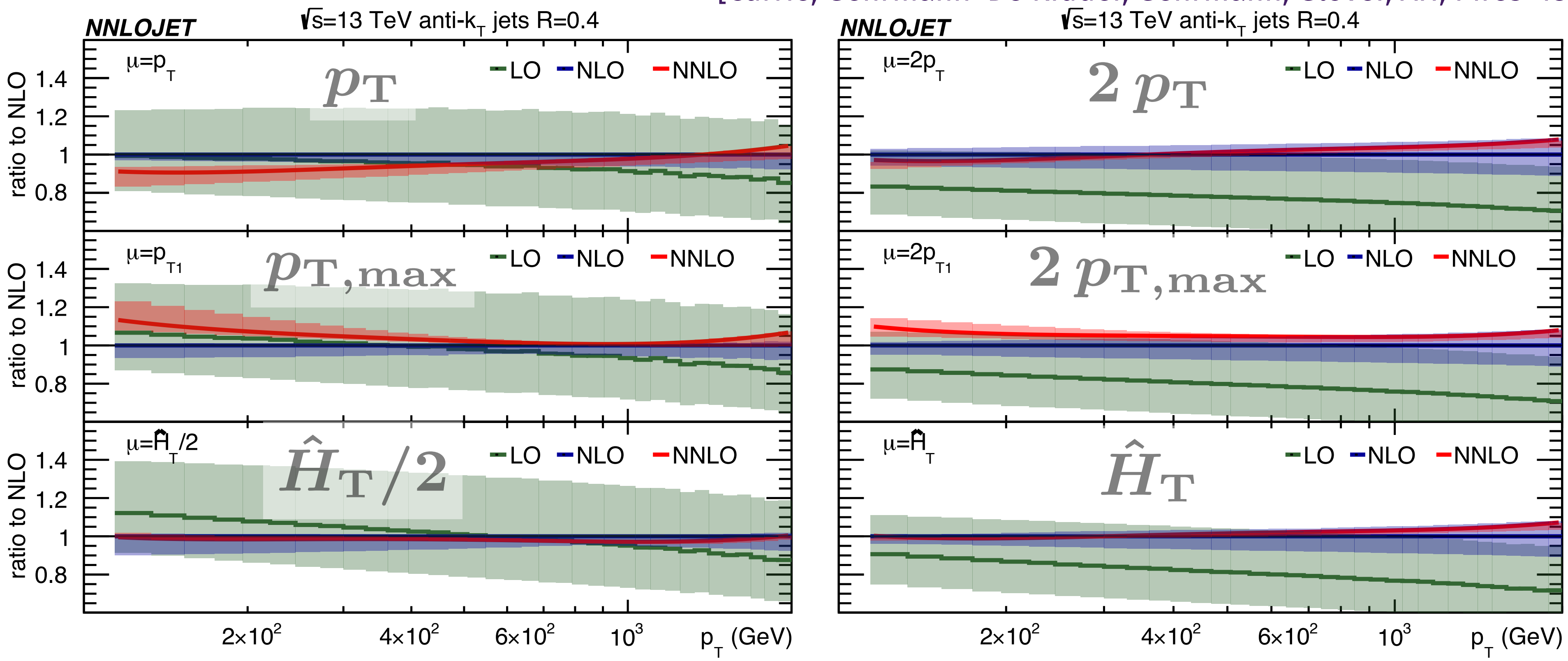


- NNLO (anti-k_T R=0.4)
  - $\mu_0 = p_{T,1}$
  - $\mu_0 = p_T$
- high-p_T ✓
  - both scales coincide
- low-p_T ✗
  - significant differences 15–20%
  - non-overlapping bands NLO → NNLO (larger)
  - p_T closer to data

Large effects from scales ambiguity!

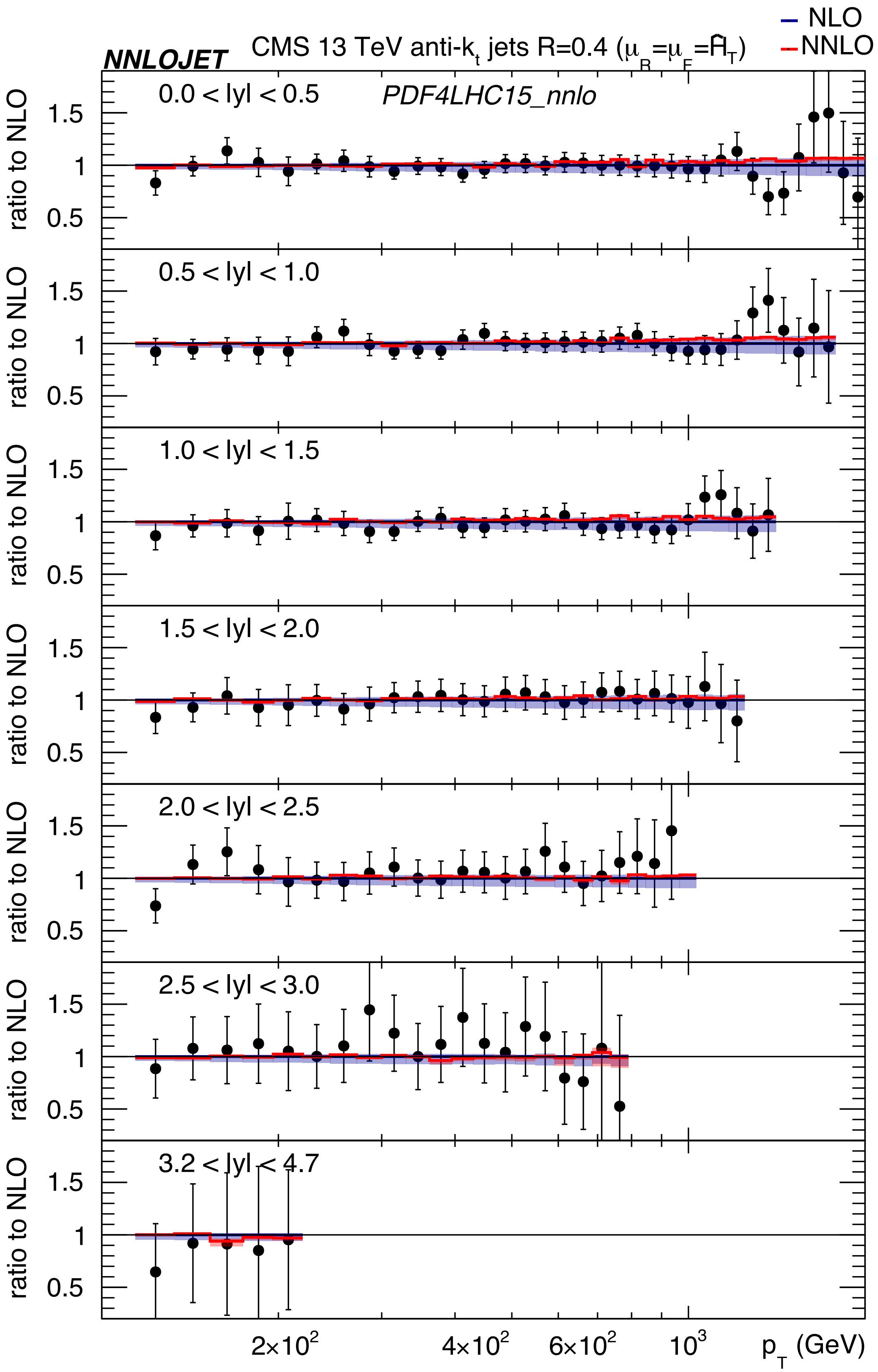
INCLUSIVE JET PRODUCTION — SCALE CHOICES (R=0.4)

[Currie, Gehrmann–De Ridder, Gehrmann, Glover, AH, Pires '18]



- *most common* choice: $\mu = p_{\mathrm{T}}$ & $\mu = p_{\mathrm{T,max}}$
  $\hookrightarrow$ worst perturbative behaviour
- harder scales preferred: $\mu = 2\,p_{\mathrm{T}}$ & $\mu = \hat{H}_{\mathrm{T}}$
  $\hookrightarrow$ show good properties
- origin: infrared sensitivity of the inclusive-jet observable
  $\hookrightarrow$ driven by 2$^{\mathrm{nd}}$ leading jet distribution $p_{\mathrm{T}}^{j_2}$ (very small @ NLO)
  $\hookrightarrow$ mismatch between real & virtual corrections (alleviated with larger R)

Inclusive Jet Production — Scale $\hat{H}_T$



[Currie, Gehrmann–De Ridder, Gehrmann, Glover, AH, Pires '18]

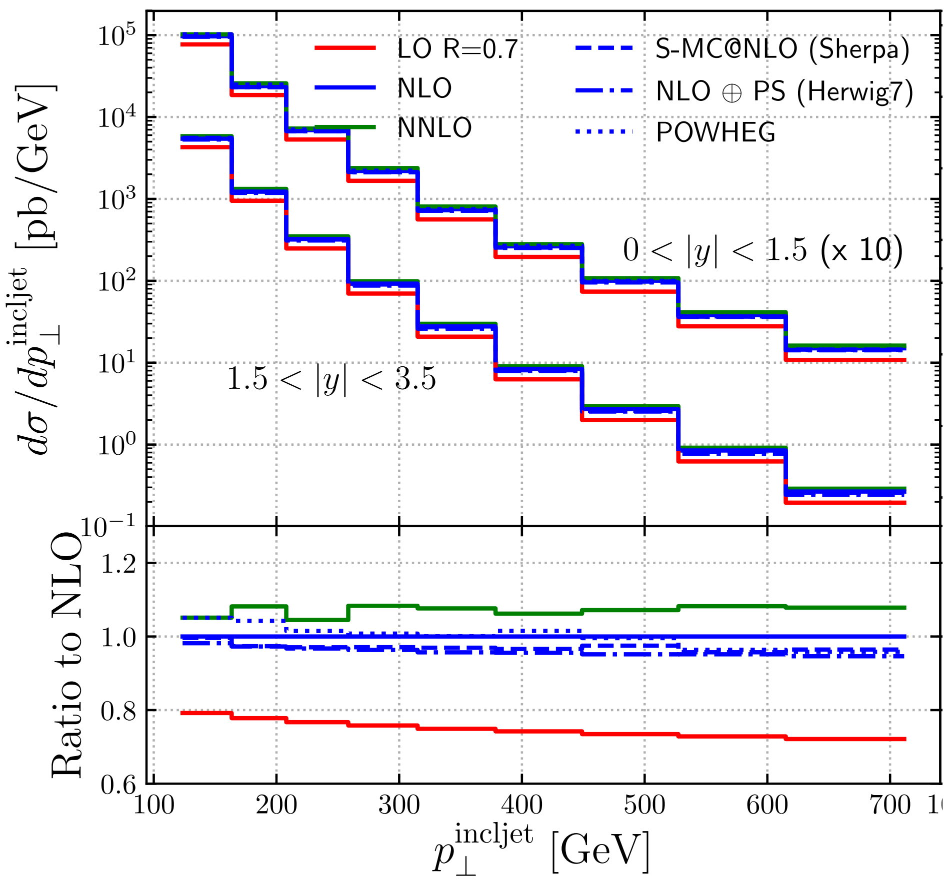
NLO

NNLO

- desirable properties
  - small NNLO corrections ✓
  - overlapping bands ✓
  - convergence of individual spectra ✓
    (especially stability of $p_{T,2}$)
- good description of exp. data

Inclusive Jet Production — Les Houches

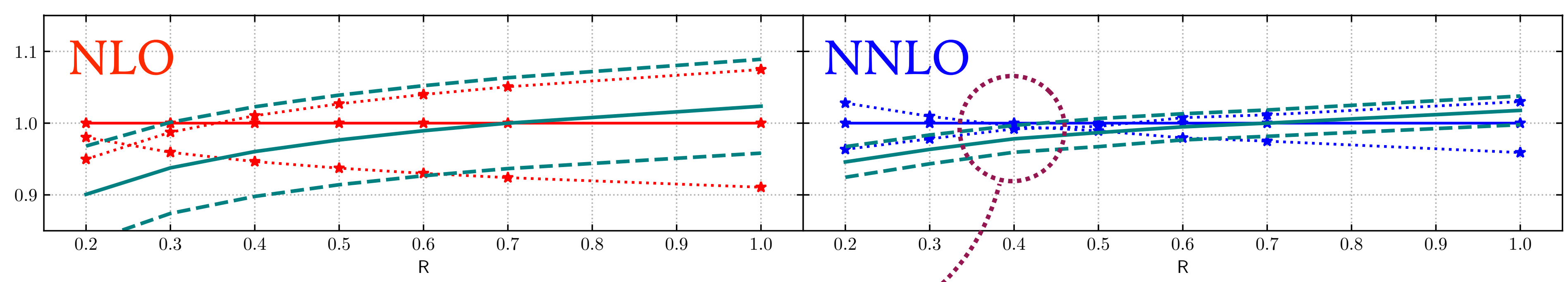
[Bellm at al.`19]



NNLO
NLO
LO

- fixed order v.s. PS
  - NLO+PS (Sherpa, Herwig, Powheg) $\simeq$ NLO
  - $R \sim 0.7$ found to be a *sweet spot*
  - "hard" NNLO not captured by PS

- *R* dependence:



  - $p_\mathrm{T} > 196\,\mathrm{GeV}$
    $\mu_0 = \hat{H}_\mathrm{T}$
  - *accidental* $\delta_{\mathrm{scl}} \sim 0$ @ $R \sim 0.4$ (depends on p_T, μ_0, …)
  - small R $\rightsquigarrow$ resummation [Dasgupta, Dreyer, Salam, Soyez '16] [Liu, Moch, Ringer '17 '18]

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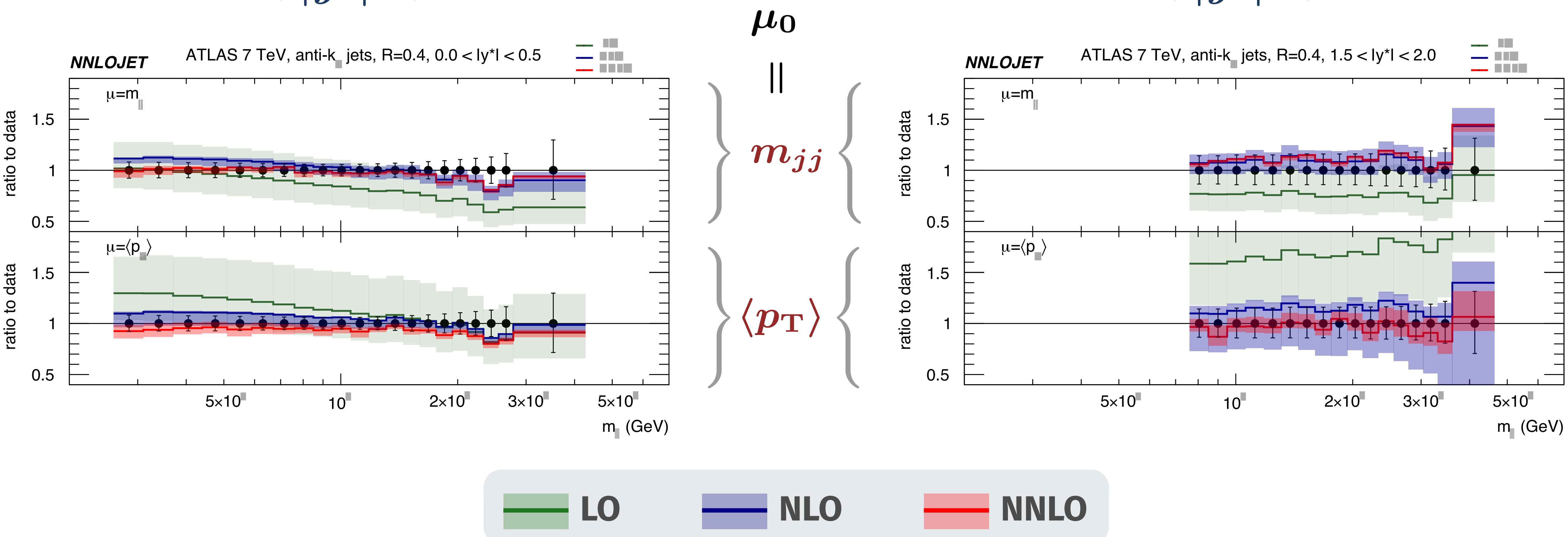
Di-Jet Production — Scales m_{jj} v.s. $\langle p_{\mathrm{T}} \rangle$

[Currie, Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '17]

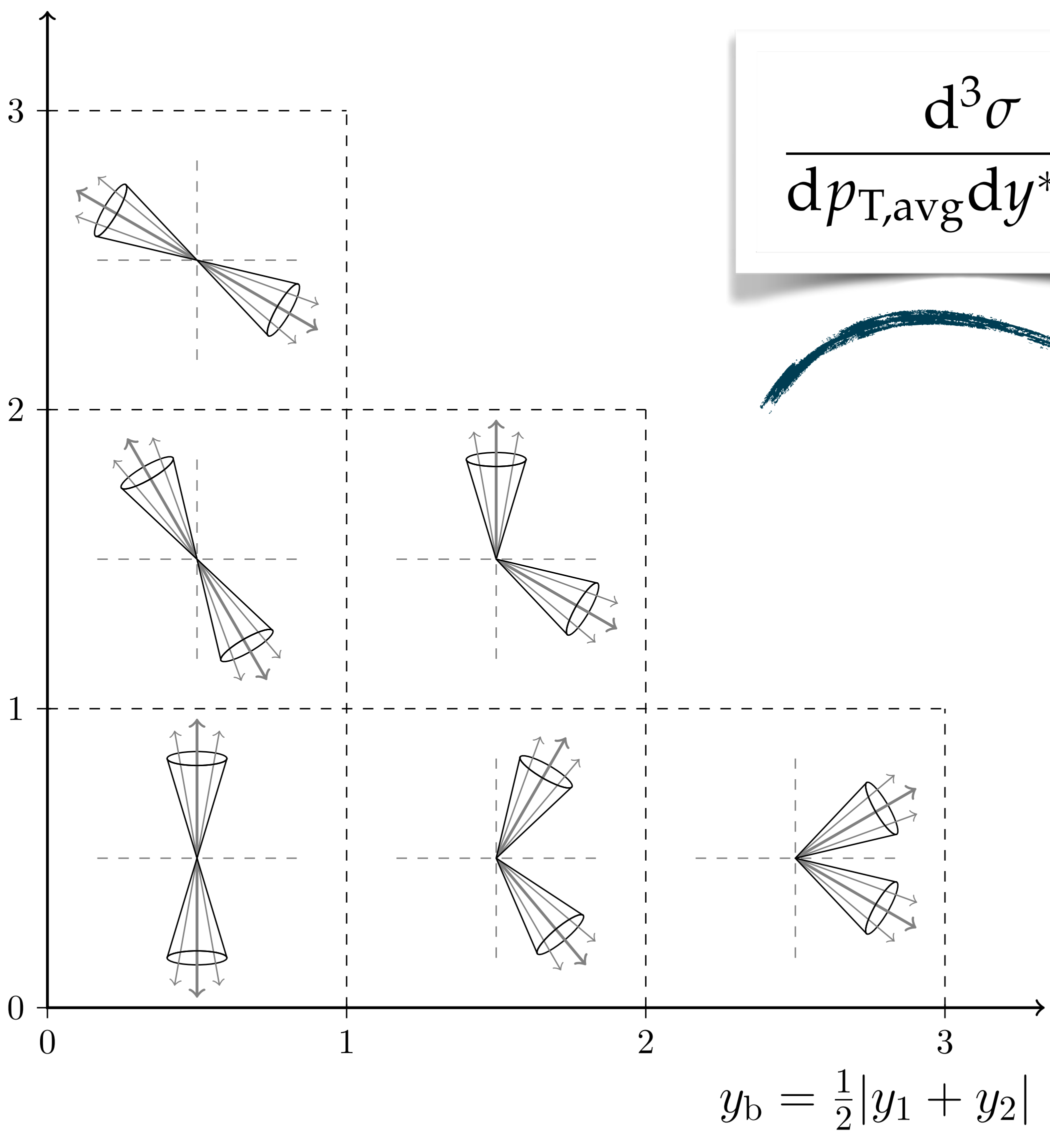
$0.0 < |y^*| < 0.5$ $\quad$ $1.5 < |y^*| < 2.0$



- central — both scales good behaviour
- forward — $\langle p_{\mathrm{T}} \rangle$ larger NLO corr. & unc. (neg. for $|y^*| > 2$)

At NNLO scales mutually compatible!

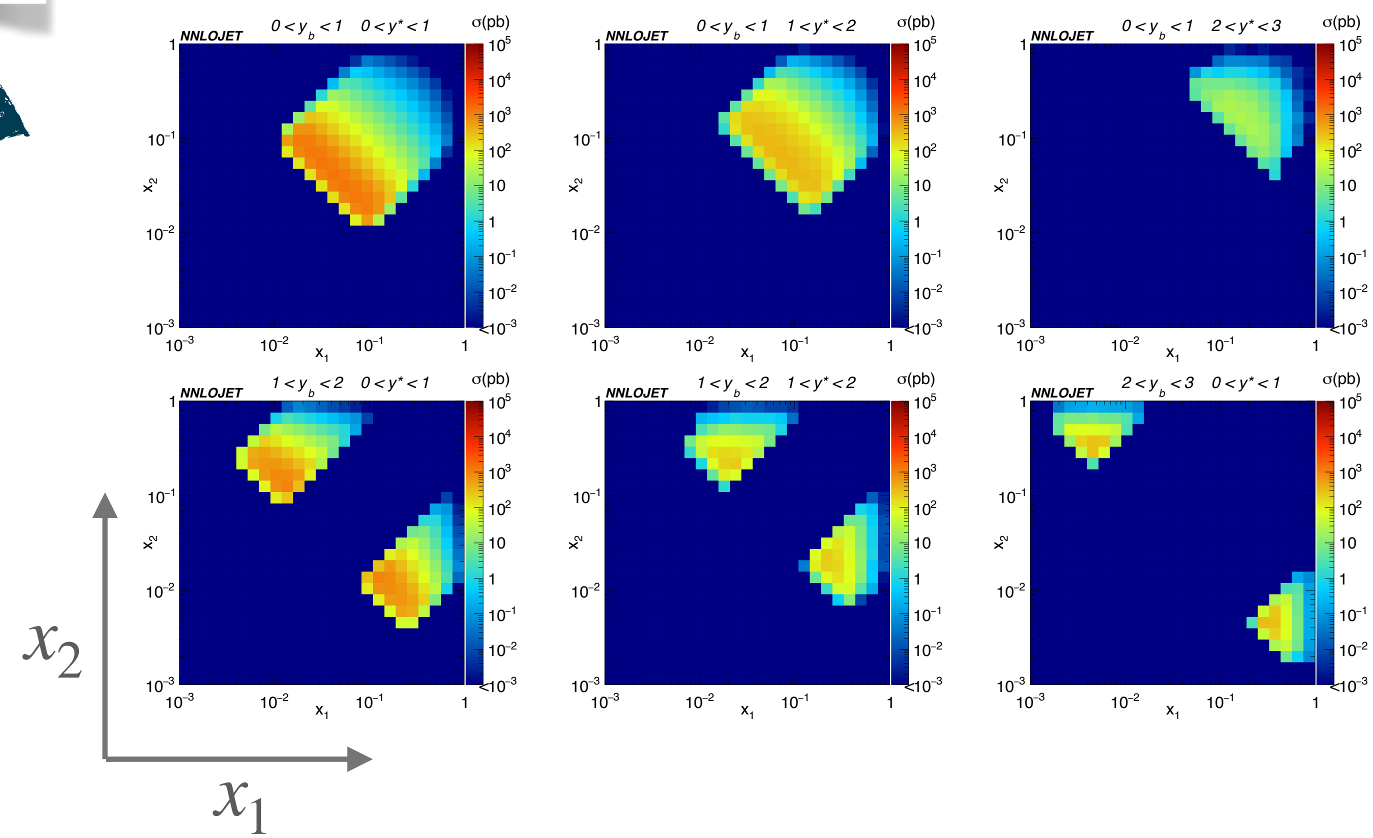
Triple-Differential Di-Jet Cross Section



[CMS, Eur. Phys. J. C77, 746 '17]

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}p_{\mathrm{T,avg}}\mathrm{d}y^*\mathrm{d}y_\mathrm{b}}$$

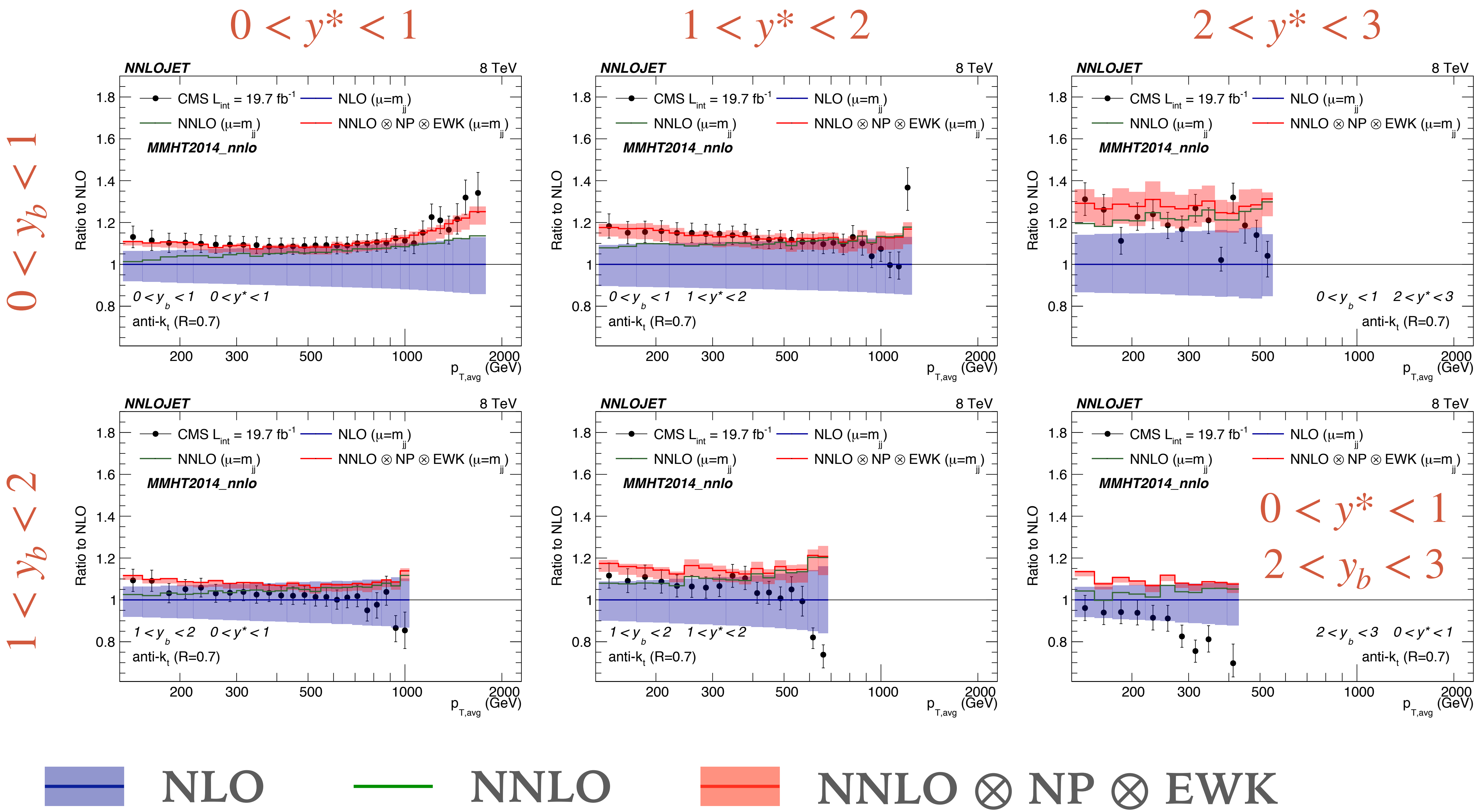
$$x_{1,2} = \frac{2p_{\mathrm{T,avg}}}{\sqrt{s}}\mathrm{e}^{\pm y_b}\cosh(y^*)$$



- ➤ study different kinematic regimes
- ➤ disentangle momentum fractions x_1 & x_2

Triple-Differential Di-Jet Cross Section @ NNLO

[Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '19]



improved description of data & reduced uncertainties!

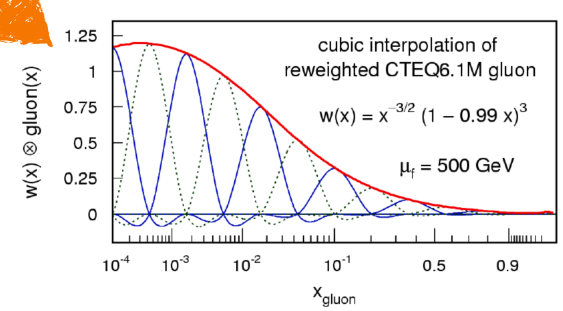
FAST INTERPOLATION GRIDS: APPLFAST

[APPLgrid, fastNLO, NNLOJET '19]

➤ NNLO calculation: O(100k) CPUh $\Rightarrow$ prohibitive in PDF & α_s fits!

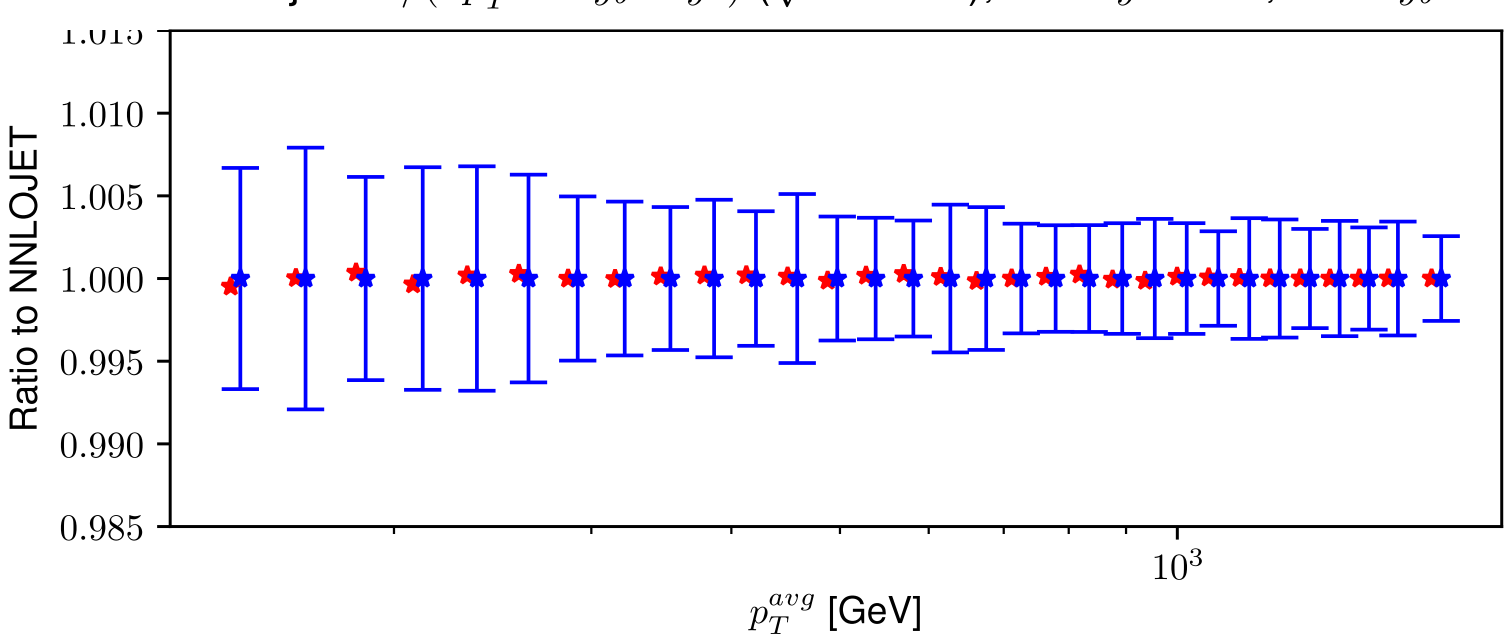
sum: cheap!

$$\sigma = \int_0^1 \mathrm{d}x \; f_a(x) \; \alpha_s^n \; \hat{\sigma}_a(x) \simeq \sum_i f_a(x^{(i)}) \; \alpha_s^n \underbrace{\left[\int_0^1 \mathrm{d}x \; \hat{\sigma}_a(x) \; E^{(i)}(x) \right]}_{\hat{\sigma}_a^{(i)}}$$



$$f_a(x) \simeq \sum_i f_a(x^{(i)}) \; E^{(i)}(x)$$

CMS di-jet $\mathrm{d}\sigma/(\mathrm{d}p_T^{avg} \cdot \mathrm{d}y_b \cdot \mathrm{d}y^*)$ ($\sqrt{s}$=8 TeV), 0.0< y^* <1.0, 0.0< y_b <1.0



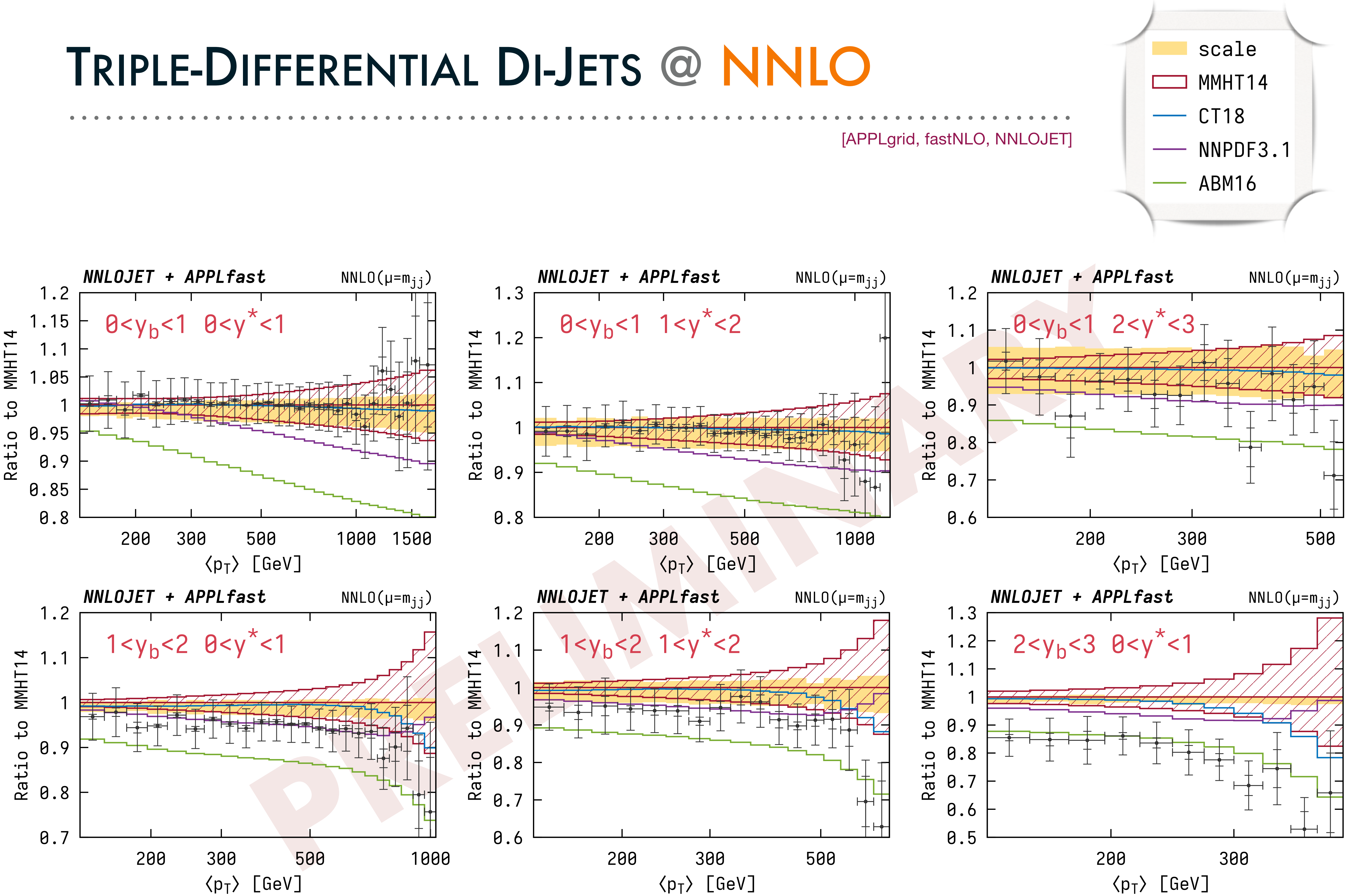
APPLfast -vs- "vanilla" NNLOjet

➤ stat. error: $\lesssim$ 0.5—1 %

➤ interp. bias: $\lesssim$ 0.05 %

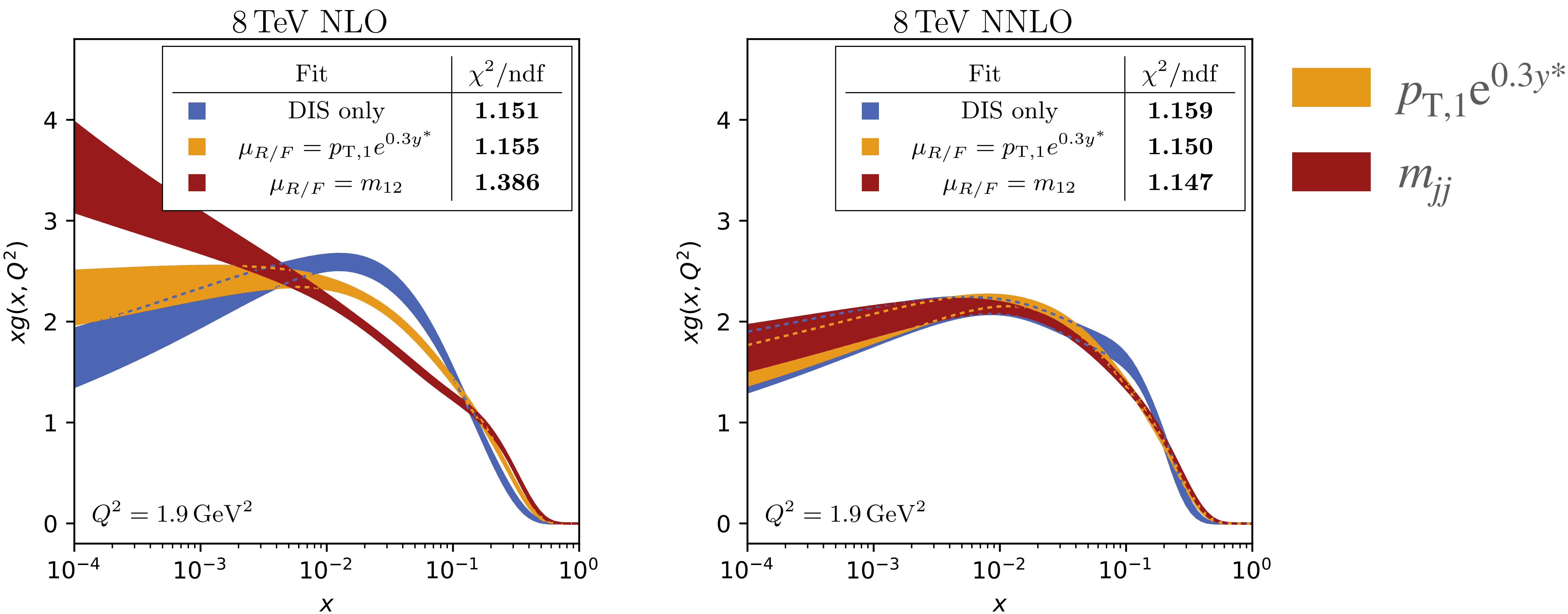
Triple-Differential Di-Jets @ NNLO

[APPLgrid, fastNLO, NNLOJET]



Triple-Differential Di-Jets ⊕ HERA Data

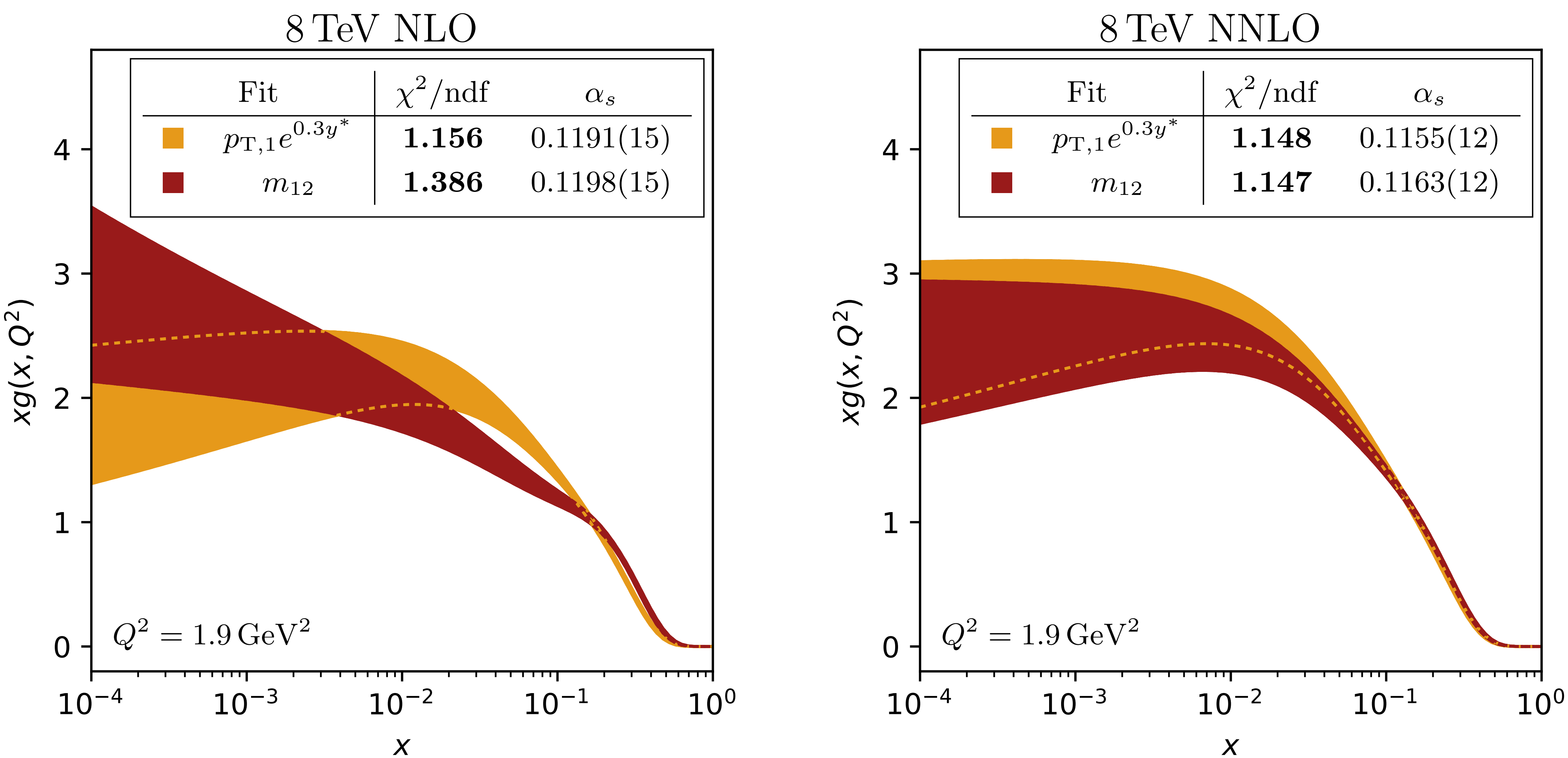
[Jakob Stark (master thesis), Klaus Rabbertz (supervisor) '21]



- NLO — large differences between the two scales
- NNLO — nice agreement between the scales
  - gluon *suppressed* @ intermediate x, *enhanced* @ high x

Triple-Differential Di-Jets — PDF & α_s

[Jakob Stark (master thesis), Klaus Rabbertz (supervisor) '21]



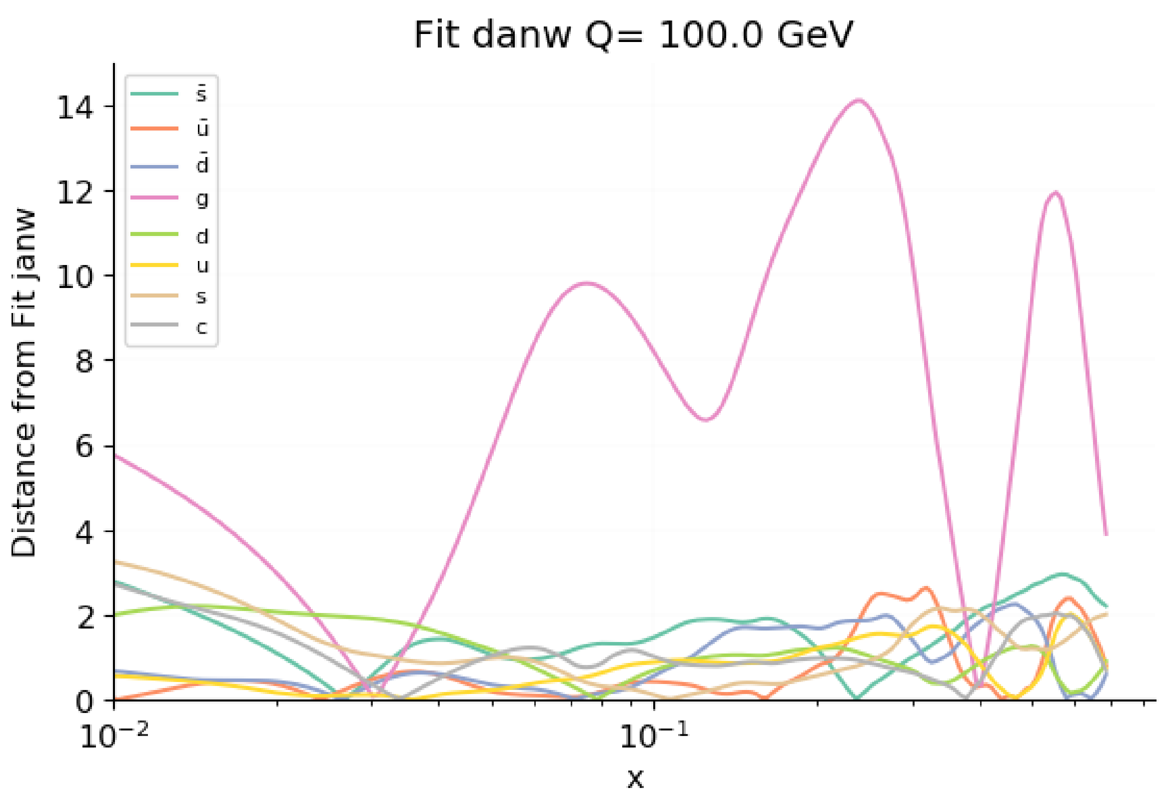
- NNLO — as before, good agreement between scale choices

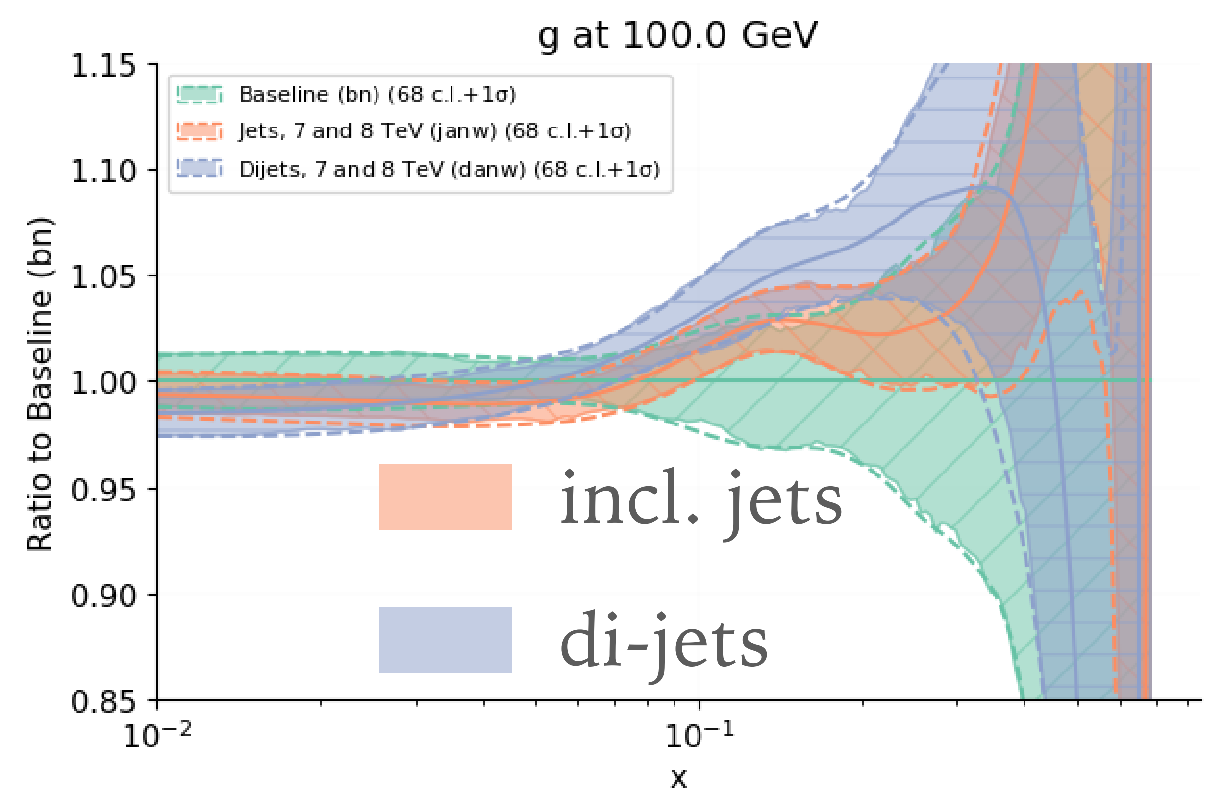
$$\alpha_s(M_z) = 0.1155 \pm 0.0012(\mathrm{exp})\,^{+0.0008}_{-0.0017}(\mathrm{scale}) \quad p_{\mathrm{T},1}\mathrm{e}^{0.3y*}$$

$$\alpha_s(M_z) = 0.1163 \pm 0.0013(\mathrm{exp})\,^{+0.0010}_{-0.0004}(\mathrm{scale}) \quad m_{jj}$$

Jet Data in a Global PDF fit

[NNLOJET + NNPDF '20]





- main impact on gluon PDFs (reduced uncertainties)
  - *suppressed* @ $0.01 \lesssim x \lesssim 0.1$ *enhanced* @ $0.1 \lesssim x \lesssim 0.4$
- qualitatively similar effect — incl. jets v.s. di-jets
  - *di-jets:* stronger pull, better pert. behaviour, (slightly) better fit
- no deterioration of other data

Conclusions

- precision for jet observables
  - ↪ crucial in the full exploitation of the (HL-)LHC
  - ⇝ jets @ NNLO (2 calcs) & recently pp $\rightarrow$ 3 jets [Czakon, Mitov, Poncelet '21]
- *inclusive jets* & *di-jets* closely related but can exhibit different features
  - *both* generally well behaved for larger R, but for smaller cone sizes…
  - *inclusive jets* — R/V mismatch, scale ambiguities, IR sensitivity, …
  - *di-jets* — plagued with issues at NLO but very robust at NNLO
- jet data provides genuine new information to fits
  - *mutually* compatible as well as with *other global data*
  - *shift* & *reduced uncertainties* of gluon distribution

Conclusions

- precision for jet observables
  - ↪ crucial in the full exploitation of the (HL-)LHC
  - ⇜ jets @ NNLO (2 calcs) & recently pp $\to$ 3 jets [Czakon, Mitov, Poncelet '21]
- *inclusive jets* & *di-jets* closely related but can exhibit different features
  - *both* generally well behaved for larger R, but for smaller cone sizes…
  - *inclusive jets* — R/V mismatch, scale ambiguities, IR sensitivity, …
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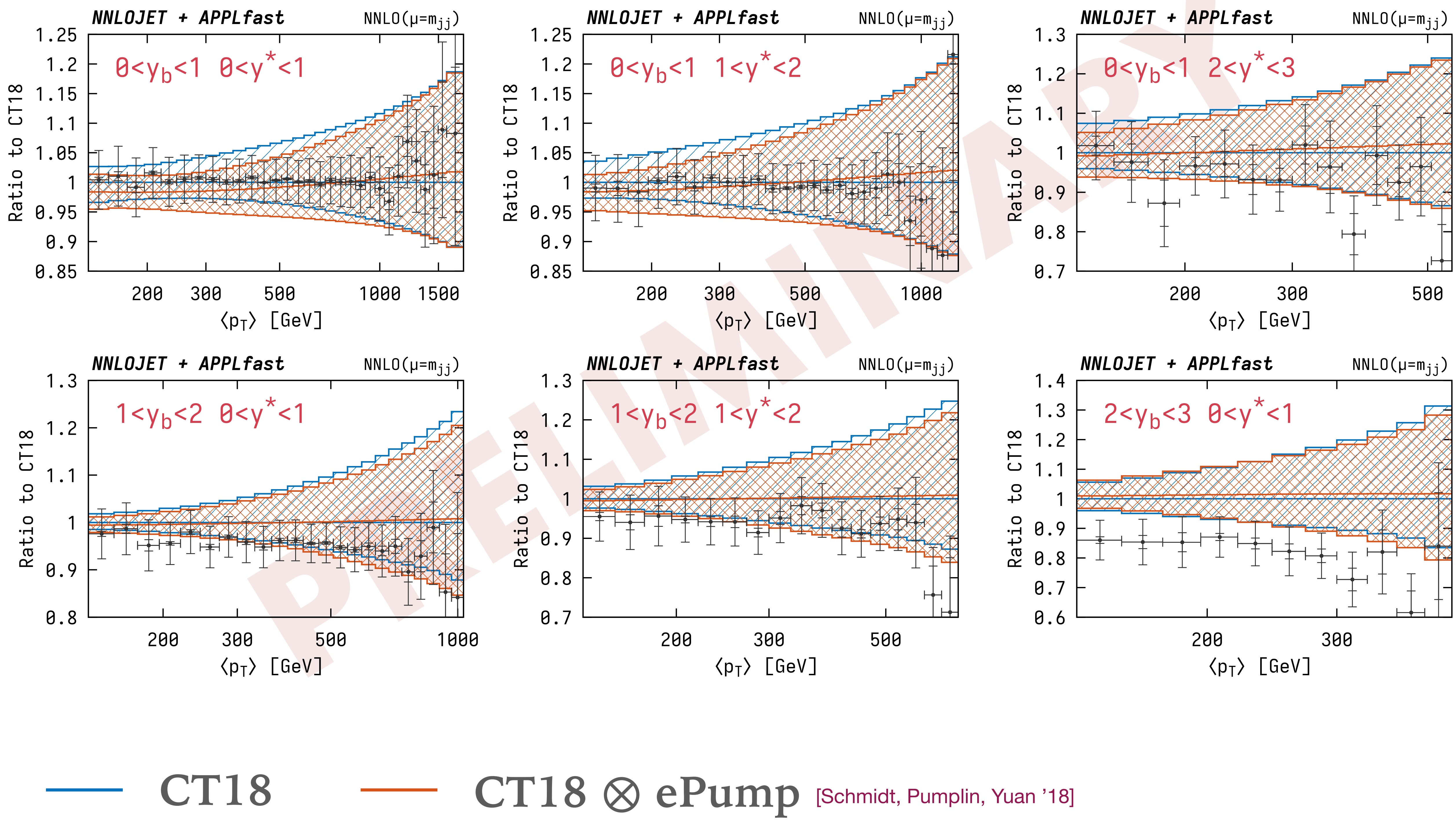
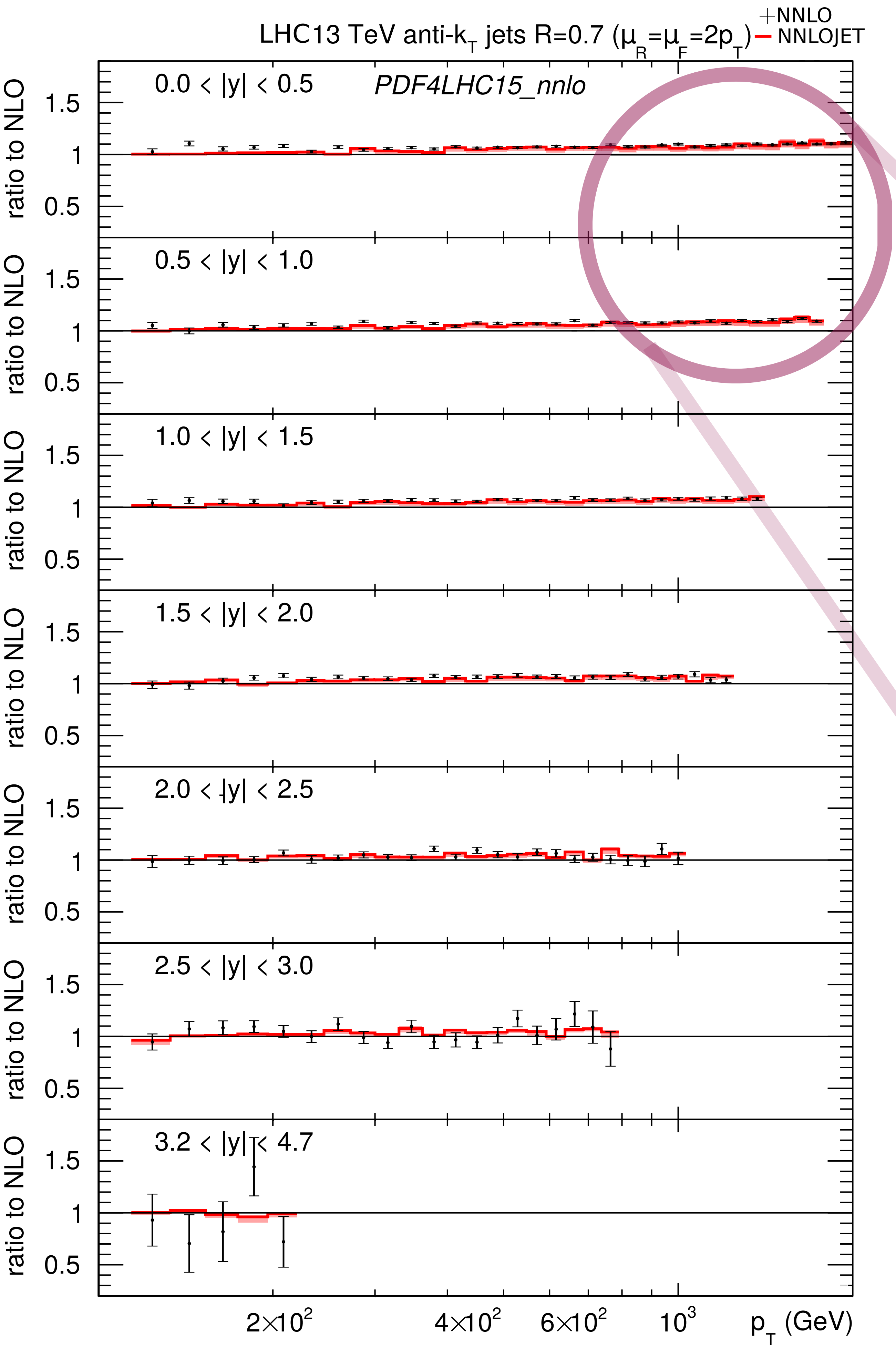
Thank you!

Backup.

TRIPLE-DIFFERENTIAL DI-JETS @ NNLO



[Schmidt, Pumplin, Yuan '18]

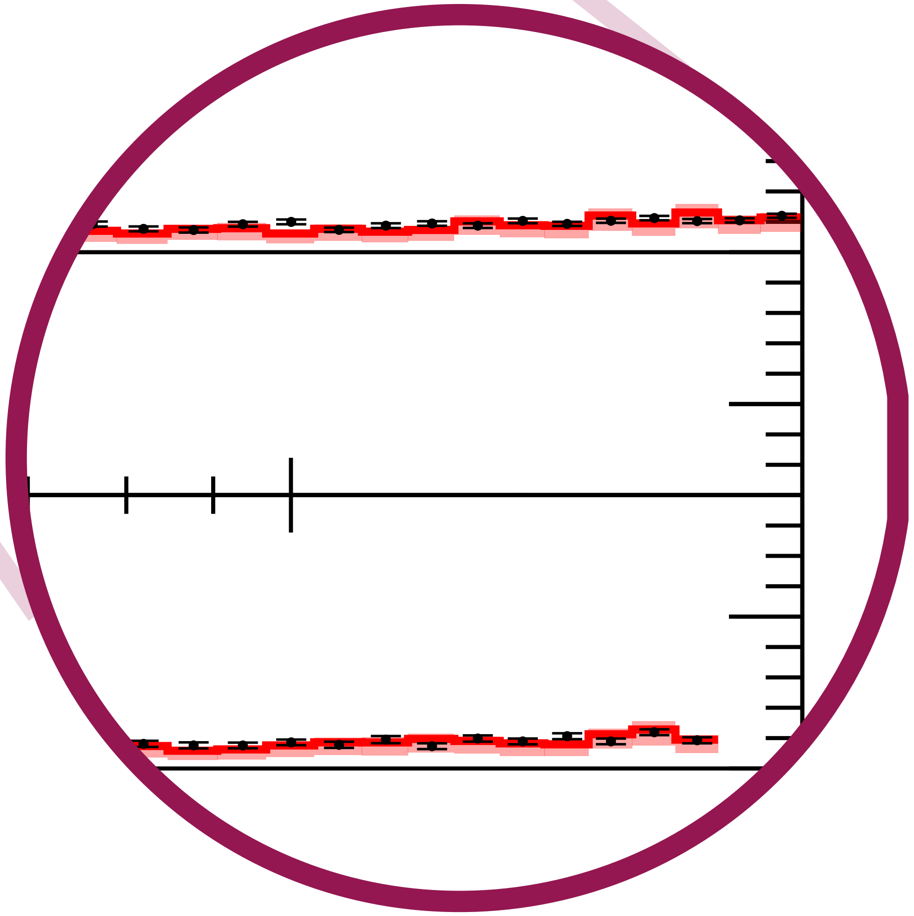


Two Calculations!

NNLOJET [Currie, Glover, Pires '16]

STRIPPER [Czakon, van Hameren, Mitov, Poncelet '19]

- excellent agreement
- sub-leading colour negligible (missing in NNLOJET)



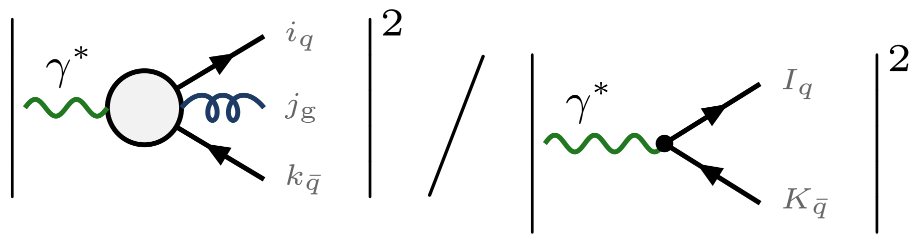
Antenna Subtraction — Building Blocks

- $X(\ldots)$ based on physical matrix elements $\quad X = \overbrace{A,\ B,\ C}^{q\bar{q}}, \overbrace{D,\ E}^{qg}, \overbrace{F,\ G,\ H}^{gg}$

$$X_3^0(i,j,k) = \frac{|\mathcal{A}_3^0(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2}, \qquad X_4^0(i,j,k,l) = \frac{|\mathcal{A}_4^0(i,j,k,l)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{L})|^2},$$

$$X_3^1(i,j,k) = \frac{|\mathcal{A}_3^1(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2} - X_3^0(i,j,k)\frac{|\mathcal{A}_2^1(\widetilde{I},\widetilde{K})|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2},$$

$$A_3^0(i_q, j_{\mathrm{g}}, k_{\bar{q}}) =$$



- integrating the antennae $\longleftrightarrow$ phase-space factorization

$$\begin{aligned} &\mathrm{d}\Phi_{m+1}(\ldots,p_i,p_j,p_k,\ldots) \\ &\qquad = \mathrm{d}\Phi_m(\ldots,\widetilde{p}_I,\widetilde{p}_K,\ldots)\,\mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\widetilde{p}_I+\widetilde{p}_K) \end{aligned}$$

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna Subtraction — Building Blocks

$q\bar{q}$ qg gg

All building blocks known!

X_3^0, X_4^0, X_3^1 and integrated counterparts $\mathcal{X}_3^0$, $\mathcal{X}_4^0$, $\mathcal{X}_3^1$

$\forall$ configurations relevant at hadron colliders:

$\hookrightarrow$ final–final .... $\mathrm{e^+e^-}$

[Gehrmann-De Ridder, Gehrmann, Glover '05]

$\hookrightarrow$ initial–final .... $\mathrm{e^+p}$

[Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, Maitre '06,'09,'12]

$\hookrightarrow$ initial–initial .... pp

[Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Maitre, et al. '10,'11,'12]

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna Subtraction @ NLO — $q\bar{q} \to \mathrm{ggZ}$

$\mathrm{d}\hat{\sigma}^T$: $\qquad \boxed{\mathrm{d}\hat{\sigma}^T \sim \boldsymbol{J}_n^{(1)} M_n^0}$

$\mathrm{d}\hat{\sigma}^S$: $\qquad \boxed{\mathrm{d}\hat{\sigma}^S \sim X_3^0 M_n^0}$

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+2}} \Big\{ \left|\mathcal{A}_4^0(1_q, 3_{\mathrm{g}}, 4_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\Phi_{\mathrm{Z+2}})$$

$$- d_3^0(1_q, 3_{\mathrm{g}}, 4_{\mathrm{g}}) \left|\mathcal{A}_3^0(\widetilde{1}_q, \widetilde{(34)}_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}})$$

$$- d_3^0(2_{\bar{q}}, 4_{\mathrm{g}}, 3_{\mathrm{g}}) \left|\mathcal{A}_3^0(1_q, \widetilde{(34)}_{\mathrm{g}}, \widetilde{2}_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}}) \Big\} + (3 \leftrightarrow 4)$$

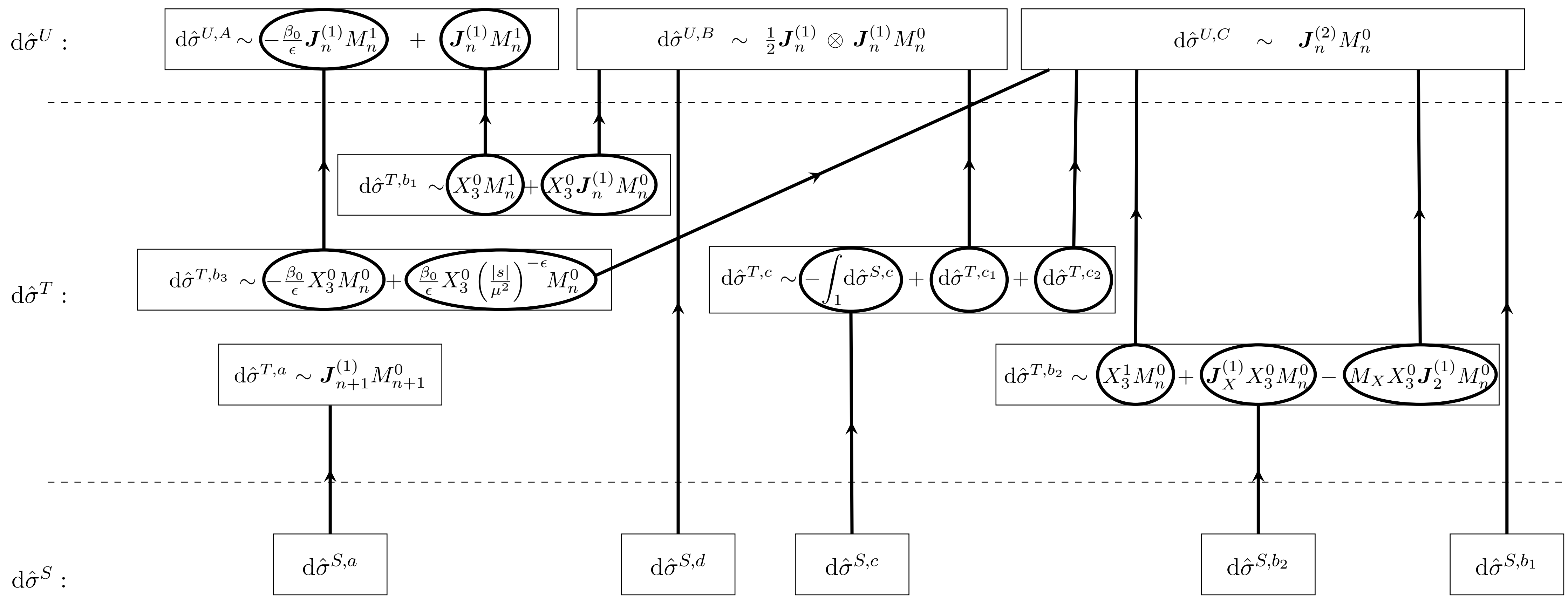
$$\int \left\{ \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+1}} \Big\{ \left|\mathcal{A}_3^1(1_q, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z})\right|^2$$

$$+ \frac{1}{2} \left[\mathcal{D}_3^0(s_{13}) + \mathcal{D}_3^0(s_{23})\right] \left|\mathcal{A}_3^0(1_q, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \Big\} \mathcal{J}(\Phi_{\mathrm{Z+1}})$$

ANTENNA SUBTRACTION @ NNLO

[J. Currie , E.W.N. Glover, S. Wells '13]



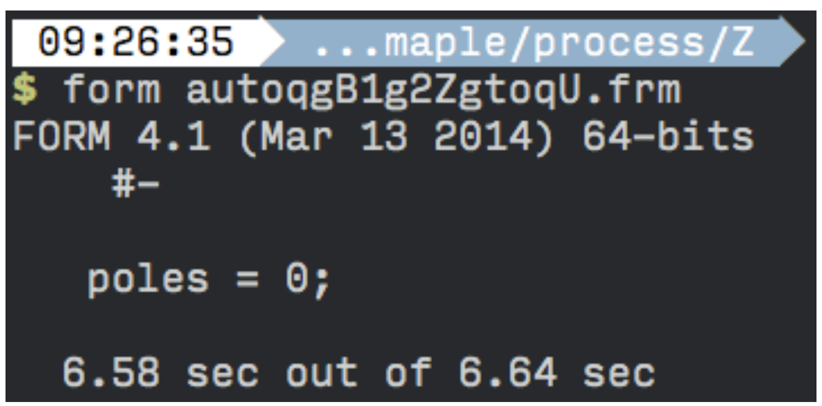
- double real: $\mathrm{d}\sigma^{\mathrm{S}} \sim X_3^0 \; |\mathcal{A}_{m+1}^0|^2, \quad X_4^0 \; |\mathcal{A}_m^0|^2, \quad X_3^0 \; X_3^0 \; |\mathcal{A}_m^0|^2$
- real–virtual: $\mathrm{d}\sigma^{\mathrm{T}} \sim \mathcal{X}_3^0 \; |\mathcal{A}_{m+1}^0|^2, \quad X_3^0 \; |\mathcal{A}_m^1|^2, \quad X_3^1 \; |\mathcal{A}_m^0|^2$
- double virtual: $\mathrm{d}\sigma^{\mathrm{U}} =$ (collect rest) $\sim \mathcal{X} \; |\mathcal{A}_m^{0,1}|^2$

ANTENNA SUBTRACTION — CHECKS OF THE CALCULATION

Analytic pole cancellation

- Poles $\left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{T}}\right) = 0$
- Poles $\left(\mathrm{d}\sigma^{\mathrm{VV}} - \mathrm{d}\sigma^{\mathrm{U}}\right) = 0$

DimReg: $D = 4 - 2\epsilon$

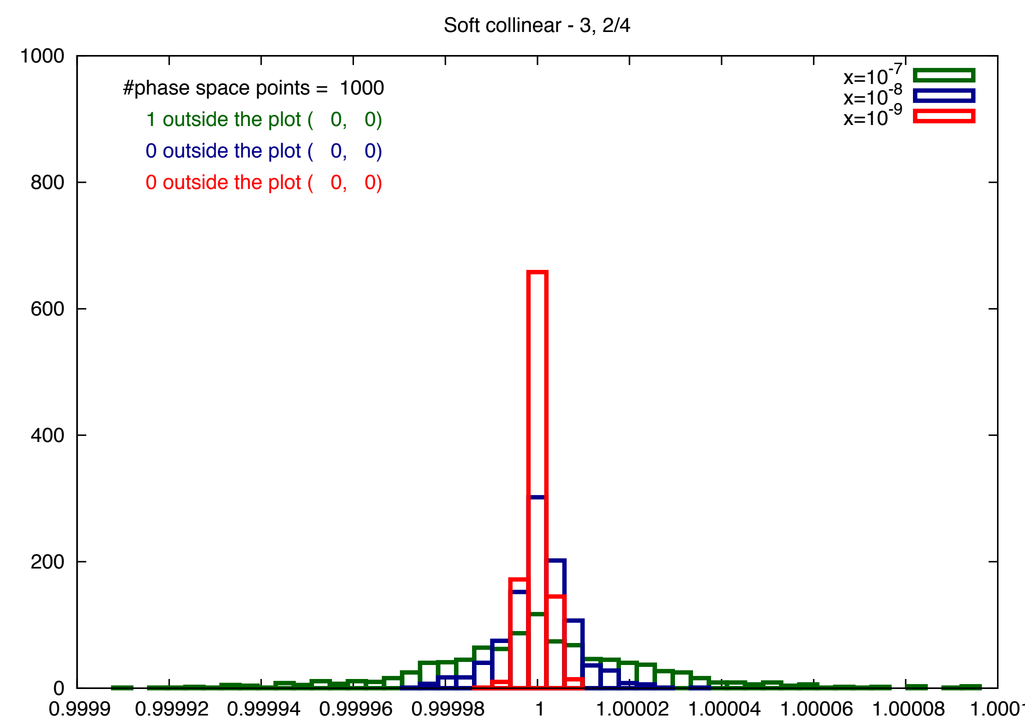


Unresolved limits

- $\mathrm{d}\sigma^{\mathrm{S}} \to \mathrm{d}\sigma^{\mathrm{RR}}$ (single- & double-unresolved)
- $\mathrm{d}\sigma^{\mathrm{T}} \to \mathrm{d}\sigma^{\mathrm{RV}}$ (single-unresolved)

bin the ratio: $\mathrm{d}\sigma^{\mathrm{S}}/\mathrm{d}\sigma^{\mathrm{RR}} \xrightarrow{\text{unresolved}} 1$

$q\,\bar{q} \to \mathrm{Z} + \mathrm{g}_3\ \mathrm{g}_4\ \mathrm{g}_5$ (g_3 soft & $\mathrm{g}_4 \parallel \bar{q}$)



(approach singular limit: $x_i = 10^{-7},\ 10^{-8},\ 10^{-9}$)

What about Angular Terms?!

- Antenna subtraction: $X_n^l \, |\mathcal{A}_m|^2 \quad \leftrightarrow \quad$ spin averaged!
- angular terms in gluon splittings:

$$P_{\mathrm{g}\to q\bar{q}} = \frac{2}{s_{ij}} \left[-g^{\mu\nu} + 4z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$$

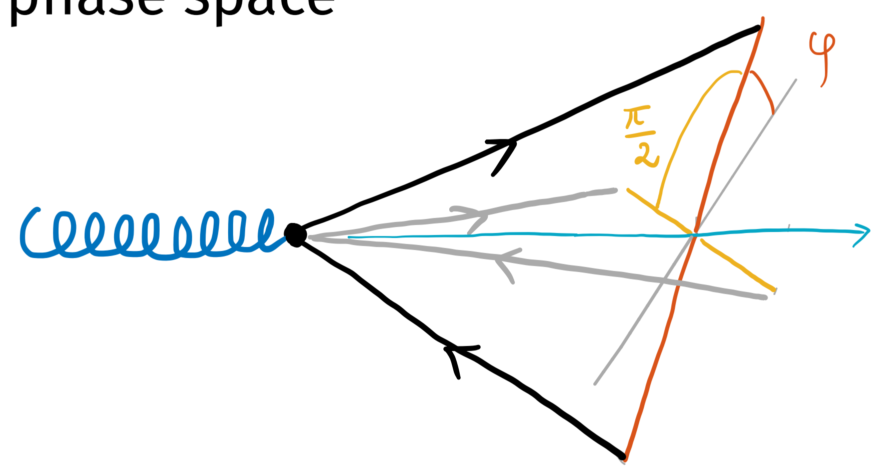
$\hookrightarrow$ subtraction non-local in these limits!
$\hookrightarrow$ vanish upon azimuthal-angle (φ) average ($\Rightarrow$ do not enter $\mathcal{X}$)

sol. 1: supplement angular terms in the subtraction

sol. 2: exploit φ dependence & average in the phase space

$$\mathcal{A}_\mu^* \, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \, \mathcal{A}_\nu \sim \cos(2\varphi + \varphi_0)$$

$\Rightarrow$ add φ & $(\varphi + \pi/2)$!



$$\vec{r} \longrightarrow \mathrm{PS}_{\text{gen.}} \longrightarrow \begin{bmatrix} \{p_i, & p_j, & \ldots\} \\ \{p_i', & p_j', & \ldots\} \end{bmatrix} \xrightarrow{(i\|j)} \begin{bmatrix} \{p_i^{\varphi}, & p_j^{\varphi}, & \ldots\} \\ \{p_i^{\varphi+\pi/2}, & p_j^{\varphi+\pi/2}, & \ldots\} \end{bmatrix}$$