Dimensional regularization and γ_5 — no-compromise* approach to the BMHV scheme

Dominik Stöckinger

TU Dresden

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Collaborators: Bélusca-Maïto, Ilakovac, Kühler, Mađor-Božinović

* or: traditional/old-fashioned/stubborn...

γ_5 and DReg: well-known problem

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \quad (1)$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \quad (2)$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1)\,. \quad (3)$$

Inconsistent in $D \neq 4$ (can prove that trace=0).

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Give up at least one $\Rightarrow$ many proposals!

- "Naive" anticommuting? Reading point? ...Many alternatives!
- Often limited range of applicability
- BMHV (non-anticommuting, very complicated, breaks gauge inv. But unitary, consistent)

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Our idea: No-compromise approach to BMHV — apply it and accept/deal with its difficulties! So far 1-loop YM and 2-loop abelian

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Study motivated by need for precision and general effort on regularization schemes
($\leadsto$ reports "To d or not to d" and "May the four be with you")

BMHV scheme — non-anticommuting γ_5

QFT consistent, unitary; breaks symmetries, complicated

- "*D*-dim space" split into pure 4-dim space $\oplus$ (-2ϵ)-dim space

$$
\begin{aligned}
X^\mu &= \bar{X}^\mu + \hat{X}^\mu \\
\gamma_5 &= i\gamma_0\gamma_1\gamma_2\gamma_3 \\
\{\gamma_5, \bar{\gamma}^\mu\} &= 0 \\
[\gamma_5, \hat{\gamma}^\mu] &= 0
\end{aligned}
$$

Goals:

- Take seriously, apply to 1-loop, 2-loop . . . EW calculations
- Technical questions: restore gauge invariance (here)
- Conceptual: relation to other schemes, RGEs (future)
- Progress will feed back to other schemes

Preview: Main technical result

symmetry-restoring counterterms for general YM+fermions+scalars:

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020]

earlier result without scalars: C.P. Martin, Sanchez-Ruiz '99

$$
\begin{aligned}
S^1_{\text{fct,restore}} = {} & \\
\frac{\hbar}{16\pi^2}\Bigg\{ & g^2 \frac{S_2(R)}{6}\left(5S_{GG} + S_{GGG} - \int d^4x\, G^{a\,\mu}\partial^2 G^a_\mu\right) + \frac{Y_2(S)}{3}\overline{S_{\Phi\Phi}} \\
& + g^2 \frac{(T_R)^{abcd}}{3}\int d^4x\, \frac{g^2}{4} G^a_\mu G^{b\,\mu} G^c_\nu G^{d\,\nu} - \frac{(\mathcal{C}_R)^{ab}_{mn}}{3}\int d^4x\, \frac{g^2}{2} G^a_\mu G^{b\,\mu}\Phi^m\Phi^n \\
& + g^2\left(1 + \frac{\xi - 1}{6}\right) C_2(R) S_{\overline{\psi}\psi} - \frac{\left((Y^m_R)^* T_{\overline{R}}{}^a Y^m_R\right)_{ij}}{2}\int d^4x\, g\overline{\psi}_i \not{G}^a P_{\mathrm{R}}\psi_j \\
& - g^2 \frac{\xi C_2(G)}{4}\left(S_{\bar{R}c\psi_R} + S_{Rc\overline{\psi_R}}\right)\Bigg\} ,
\end{aligned}
$$

Finite, NON-evanescent counterterms. Not gauge invariant!
Modify all self-energies and some interactions!
But rather compact, universal, could be implemented e.g. in FeynArts

Preview: Main technical result

symmetry-restoring counterterms for general YM+fermions+scalars:

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020]

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$$S^1_{\mathrm{fct,restore}} =$$

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$$+g^2\frac{(T_R)^{abcd}}{3}\int d^4x\,\frac{g^2}{4}G^a_\mu G^{b\,\mu}G^c_\nu G^{d\,\nu}-\frac{(\mathcal{C}_R)^{ab}_{mn}}{3}\int d^4x\,\frac{g^2}{2}G^a_\mu G^{b\,\mu}\Phi^m\Phi^n$$

$$+g^2\left(1+\frac{\xi-1}{6}\right)C_2(R)S_{\overline{\psi}\psi}-\frac{\left((Y^m_R)^* T_{\overline{R}}{}^a Y^m_R\right)_{ij}}{2}\int d^4x\, g\overline{\psi}_i \not{G}^a P_{\mathrm{R}}\psi_j$$

$$-g^2\frac{\xi C_2(G)}{4}\left(S_{\bar{R}c\psi_R}+S_{Rc\overline{\psi_R}}\right)\Bigg\}\,,$$

Q: How do we derive this? At 2-loop order?
see also: SUSY in DRed (3-loop) [DS’05, Hollik,DS’05, DS,Unger’18]

Plan here: chiral "QED" (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac,Kühler Mađor-Božinović, DS, in progress]

Abelian theory like $U(1)_Y$-part of SM, only ψ_{Ri} interact

$$\mathcal{L} = i\overline{\psi_R}_i \not{D} \psi_{Ri} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \text{g.fix, ghost, source terms}$$

$$D^\mu = \partial^\mu - ieA^\mu \mathcal{Y}_R \,,$$

Description of symmetry: gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is requirement on renormalized theory:

$$S(\Gamma_{\text{ren}}) = 0$$

combines well-known STI/WIs between 1PI Green functions e.g.

- photon self energy = transverse
- Ward identity between fermion self energy and vertex function, etc

Plan here: chiral "QED" (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac,Kühler Mađor-Božinović, DS, in progress]

1. Define D-dimensional Lagrangian compute symmetry breaking
2. Determine 1-loop UV divs $\rightsquigarrow \mathcal{L}_{\text{sct}}$
3. Determine 1-loop violation of Slavnov-Taylor identity
4. Determine 1-loop symmetry-restoring counterterms $\rightsquigarrow \mathcal{L}_{\text{fct}}$
5. Repeat at 2-loop new features?

1. Define D-dimensional Lagrangian

What should we take: $\mathcal{L}$ in D-dim?

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} \gamma^\mu P_R A_\mu \psi + \ldots$$

- γ^μ must be D-dimensional (else: propagator not regularized)
- $\gamma^\mu P_R$ or $P_L \gamma^\mu P_R$ or $\bar{\gamma}^\mu P_R$?
- in all cases: gauge invariance broken.

- here: simplest choice
  future: can study different options, also DRED, FDH, Larin…

1. Define *D*-dimensional Lagrangian

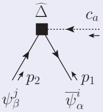
Our choice has *D*-dim kinetic, 4-dim interaction term

$$\mathcal{L}_{\text{fermions}} = i\overline{\psi}_i \not{\partial} \psi_i + e\mathcal{Y}_{Ri}\overline{\psi_{Ri}}\not{A}\psi_{Ri}\,.$$

It breaks *D*-dim gauge/BRST invariance

⇒ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x\,(e\mathcal{Y}_{Ri})c\left\{\overline{\psi}_i\left(\overleftarrow{\widehat{\not{\partial}}}P_{\mathrm{R}} + \overrightarrow{\widehat{\not{\partial}}}P_{\mathrm{L}}\right)\psi_i\right\}.$$



$$= (e\mathcal{Y}_{Ri})\left(\widehat{\not{p}}_1 P_{\mathrm{R}} + \widehat{\not{p}}_2 P_{\mathrm{L}}\right)_{\alpha\beta}$$

This is the core of the difficulties. Can be written as a local Feynman rule

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^1_{\rm sct} = S^1_{\rm sct,inv} + S^1_{\rm sct,break}\,,$$

First part as usual

$$S^1_{\rm ct,inv} = \frac{\delta Z^1_A}{2} L_A + \frac{\delta Z^1_c}{2} L_c + \frac{\delta Z^1_{\psi_R}}{2} \overline{L_{\psi_R}} + \frac{\delta e^1_A}{e_A} L_{e_A}\,,$$

second part is special for BMHV, sym-breaking and "evanescent"

$$S^1_{\rm sct,break} = \frac{-\hbar\, e^2_A}{16\pi^2\epsilon} \frac{\mathrm{Tr}(\mathcal{Y}^2_R)}{3} \left(2(\overline{S}_{AA} - S_{AA}) + \int \mathrm{d}^d x\, \frac{1}{2} \bar{A}^\mu \widehat{\partial}^2 \bar{A}_\mu \right)\,.$$

Divergences for evanescent operators with independent coefficients, beyond the usual field/parameter renormalization

well-known in DRed: needed for unitarity/finiteness at higher orders

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

$$\Gamma^{(1)}_{\text{DReg}} = \Gamma^{(1)} + S^1_{\text{sct}} + S^1_{\text{fct}} \,,$$

Evaluate STI at 1-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(1)}_{\text{DReg}}) = \underbrace{\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}}} + \mathcal{S}_d S^1_{\text{fct}}$$

Left term means: breaking of regularized STI; must be computed.

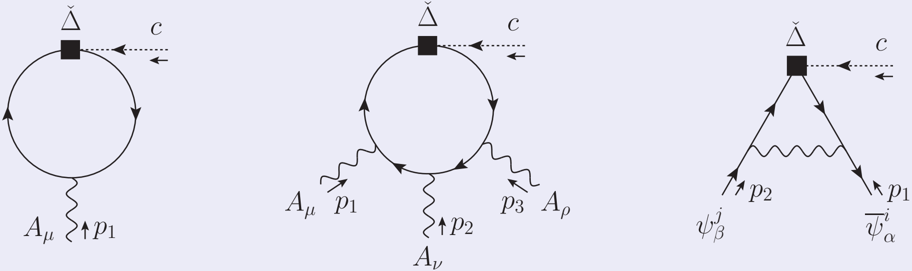
In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

But can be simplified by using quantum action principle (BM)

$$\mathcal{S}_d(\Gamma^{(1)}) = \widehat{\Delta} \cdot \Gamma^{(1)} \,,$$

Bonneau (1980): only power-counting divergent diagrams matter!

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

4. Determine symmetry-restoring counterterms

$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \stackrel{!}{=} 0$$

Requiring this renormalized STI to hold leads to the result

$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2}\int d^4x \left\{ \frac{-\operatorname{Tr}(\mathcal{Y}_R^2)}{6}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \operatorname{Tr}(\mathcal{Y}_R^4)}{12}(\bar{A}^2)^2 + \left(\frac{5+\xi}{6}\right)(\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i \bar{\not{\partial}} P_R \psi_j\right) \right\}.$$

This is the full 1-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Modify both self-energies and A^4 interaction

2. Determine UV divs at 2-loop

Many 2-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^2_{\text{sct}} = S^2_{\text{sct,inv}} + S^2_{\text{sct,break}}\,,$$

First part as usual $\sim$ field and parameter renormalization
second part is special for BMHV, "sym-breaking" (partially non-evan.)

$$S^2_{\text{sct,break}} = -\frac{e^4}{256\pi^4\epsilon}\frac{\text{Tr}(\mathcal{Y}_R^4)}{3}\left(2(\overline{S}_{AA} - S_{AA}) + \left(\frac{1}{2\epsilon} - \frac{17}{24}\right)\int d^d x\, \frac{1}{2}\bar{A}^\mu \widehat{\partial}^2$$
$$- \frac{e^4}{256\pi^4}\frac{(\mathcal{Y}_R^j)^2}{3\epsilon}\left(\frac{5}{2}(\mathcal{Y}_R^j)^2 - \frac{2}{3}\text{Tr}(\mathcal{Y}_R^2)\right)\overline{S^j_{\bar{\psi}\psi_R}}$$

3. Determine 2-loop violation of Slavnov-Taylor id.

Ultimate structure at 2-loop (fct to be determined)

$$\Gamma^{(2)}_{\text{DReg}} = \Gamma^{(2)} + S^2_{\text{sct}} + S^2_{\text{fct}}\,,$$

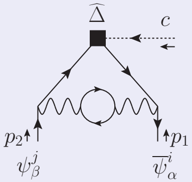
Evaluate STI at 2-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(2)}_{\text{DReg}}) = \mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}}$$

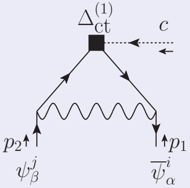
Left term (breaking of regularized STI) must be computed, use q.a.p.

$$\mathcal{S}_d(\Gamma^{(2)}) = \widehat{\Delta} \cdot \Gamma^{(2)} + \Delta^1_{\text{ct}} \cdot \Gamma^{(1)}$$

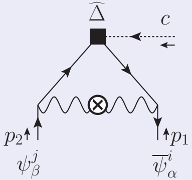
2-loop Slavnov-Taylor breaking — many diagrams of four types:

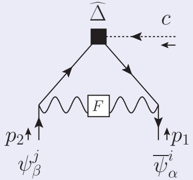


2-loop insertion of $\widehat{\Delta}$



1-loop insertion of Δ^1_{ct}





insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Sum gives $\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}}$ =local. Can cancel by local counterterms

4. Determine sym-restoring counterterms at 2-loop

$$\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}} \stackrel{!}{=} 0$$

Requiring this renormalized STI to hold leads to the result

$$S^2_{\text{fct}} = \frac{e^4}{(16\pi^2)^2}\int d^4x \left\{ \text{Tr}(\mathcal{Y}_R^4)\frac{11}{48}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2\frac{\text{Tr}(\mathcal{Y}_R^6)}{8}(\bar{A}^2)^2 \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36}(\mathcal{Y}_R^j)^2 - \frac{1}{27}\text{Tr}(\mathcal{Y}_R^2)\right)\left(\bar{\psi}_j i\bar{\partial\!\!\!/} P_\text{R}\,\psi_j\right) \right\}$$

This is the full 2-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Same structure as at 1-loop

Summary and outlook

- BMHV breaks gauge invariance already in $\mathcal{L}_D$
- Symmetry-restoring counterterms: 1-loop YM, 2-loop abelian
- Outlook: 2-loop YM, 2-loop SM ...
- Method established, result has compact simple structure
- Conceptual: non-$\overline{\text{MS}}$ cts, evanescent cts $\rightsquigarrow$ RGEs: 1-loop ok
- Outlook: 2-loop? (Compare: "ϵ-scalar mass" treatment.)
- Outlook: alternative $\mathcal{L}_D$, schemes (Larin, FDH, DRed, etc)

Summary and outlook

- 1. Define *D*-dimensional Lagrangian

  made simplest choice, Outlook: alternatives to be explored. Here: breaking is just a single term/Feynman rule

- 2. Compute Green functions

  obtain div-counterterms, split into “standard” plus “breaking” (vital for unitarity at $\geq$ 2-loop)

- 3. Determine 1-/2-loop violation of Slavnov-Taylor identity

  simplified using q.a.p., method works equally at 2-loop. Outlook: apply to YM/SM

- 4. Determine symmetry-restoring counterterms

  read off “by eye”. Structure of the result: finite modif. of all self energies and many interactions. Same structure at 1-loop/2-loop. Outlook: apply to YM/SM

- 5. Determine β-functions, other conceptual questions

  1-loop: β-functions, non-$\overline{\text{MS}}$-counterterms: ok. Outlook: More difficult at 2-loop. Compare: “ϵ-scalar mass” treatment. Outlook: Larin, other schemes. . .