Systematic Uncertainties of the Inverse Amplitude Method

Abstract: *Effective theories such as HEFT, are a controllable approximation to strong dynamics only near threshold, as they miss exact unitarity. Unitarized chiral perturbation theory extends the reach of the EFTs up to the resonance region, but in general with unknown systematic uncertainties. We review the derivation of the Inverse Amplitude Method (IAM), quantifying the uncertainty introduced at each step of the method. We find that, provided a check for CDD zeroes of the amplitude, the IAM extension of the EFT can be assigned a limited (10%~20%) uncertainty in the prediction for the position of a resonance.*

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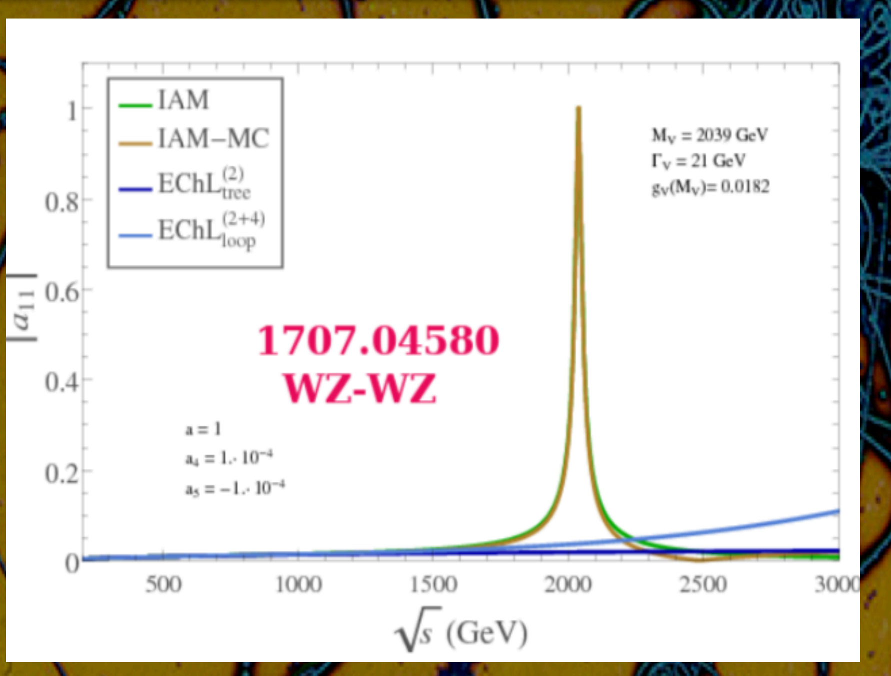
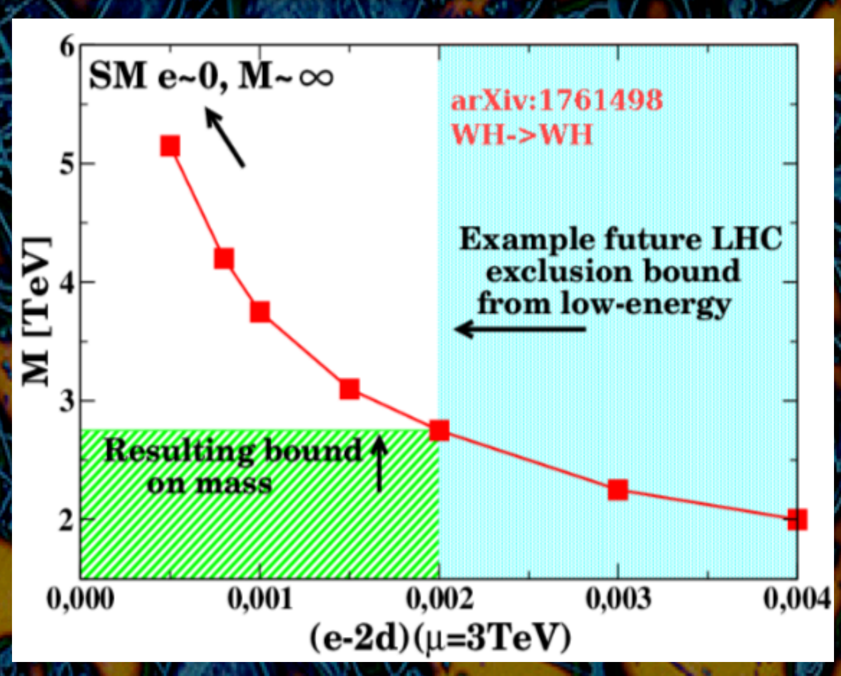
Based on the preprint: arXiv 2010.13709

Unitarization Methods are widely used for predicting BSM resonances:





But what is the precision of these predictions?

To this end we review the derivation one of the mostly used unitarization methods:

The Inverse Amplitude Method (IAM).

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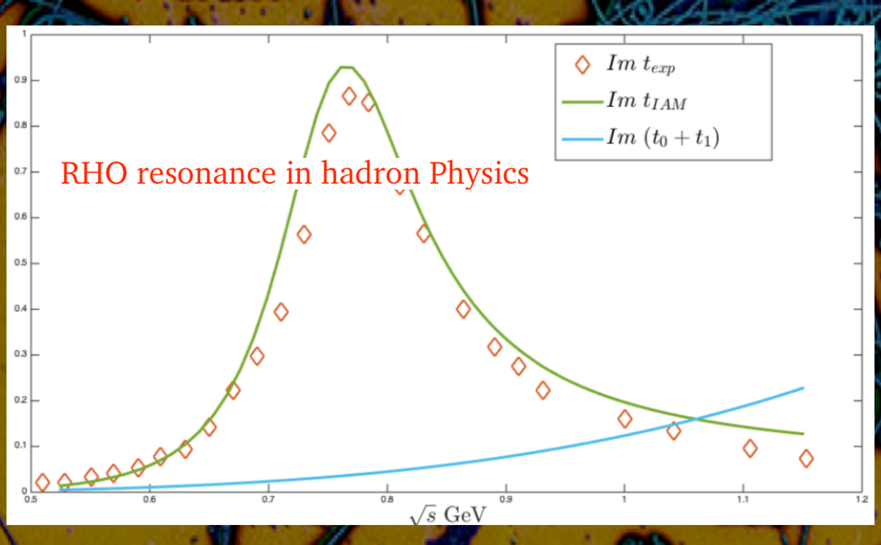
EFTs often fail near threshold, whereas the inverse amplitude reproduces resonant behaviour only using threshold data:



$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty}(2J+1)t_{IJ}(s)P_J(\cos\theta_s)$$

Typical EFT expansion of Partial wave

$$t_{IJ}(s) \simeq \underbrace{t_0}_{O(s)} + \underbrace{t_1}_{O(s^2)} + \dots$$

This is so because the EFT expansion only satisfies unitarity order by order:

$$\mathrm{Im}\, t_1(s) = \sigma(s)|t_0(s)|^2$$

The inverse amplitude satisfies unitarity exactly at NLO:

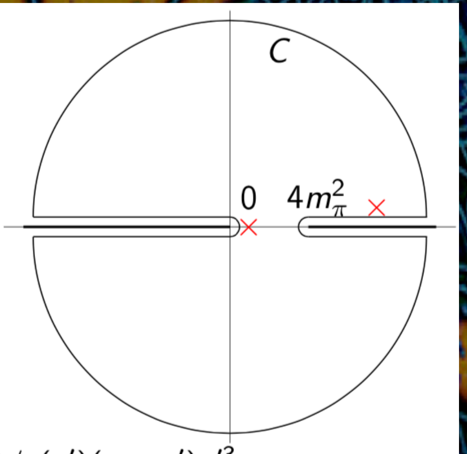
$$\frac{1}{t} \simeq \frac{1}{t_0+t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \implies \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0-t_1}}$$

Advantage: for $s > s_{th}$,

$$\mathrm{Im}\frac{1}{t_{IJ}(s)} = -\sigma(s) \simeq -1$$

We adopt the dispersive derivation of the IAM to assess its uncertainties:

- Causality $\implies$ analiticity
- Large circumference convergence $G \propto e^{-s}$ (1912.08747)
- Can apply Cauchy's theorem



to the function $t_0^2(s')/t(s')(s-s')s'^3$.

$$G(s) \equiv \frac{t_0^2(s)}{t(s)}$$

$$\implies G(s) = G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + PC(G) + \frac{s^3}{\pi}\int_{RC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)} + \frac{s^3}{\pi}\int_{LC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)}$$

To obtain the IAM amplitude one takes the approximations:

$$G(s) = \overbrace{G(0) + G'(0)s + \frac{1}{2}G''(0)s^2}^{\text{NLO subtraction constants}} + \overbrace{PC(G)}^{\text{Neglected}} + \underbrace{\frac{s^3}{\pi}\int_{RC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)} + \frac{s^3}{\pi}\int_{LC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)}}_{\text{NLO imaginary part Im G} \longrightarrow \text{-Im t1}}$$

Contributions from poles of G

Integrals over cuts

$$\implies \boxed{t \simeq t_0^2/(t_0 - t_1) = t_{IAM}}$$

Sources of uncertainty of the IAM and how to bound them:

- Neglected pole contributions of the inverse coming from zeroes of the amplitude:
  - Adler zeroes: Amplitudes are very small below threshold. They affect little to the position of the resonance (one can use the modified IAM: 0109056)
  - CDD zeroes: They signal new physics. IAM must be modified to include them.
- Inelasticities:
  - One can use the coupled channel IAM to estimate the 2-body inelastic contribution (KK in pi-pi scattering in Hadron Physics or *hh* for *ww* scattering in HEFT).
  - 4-body inelasticities (and higher) are heavily suppressed by phase space.
- NLO subtraction constants: Estimated including NNLO corrections in Resonance Effective Theory.
- Left-Cut uncertainty: can be estimated for different energy regimes.

Source of uncertainty	Behavior	Pole displacement at $\sqrt{s} = m_\rho$	Can it be improved?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	10^{-3} - 10^{-4}	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0 - $\mathcal{O}(1)$	Yes: extract zero
Inelastic 2-body	$(m_\rho/f_\pi)^4$	10^{-3}	Yes: matrix form
Inelastic 4...-body	$(m_\rho/f_\pi)^8$	10^{-4}	Partially
$O(p^4)$ truncation	$(m_\pi^2 m_\rho^4)/f_\pi^6$	10^{-2}	Yes: $O(p^6)$ IAM
Approximate Left Cut	$(m_\rho/f_\pi)^6$	0.17	Partially

Further reading: 2010.13709
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