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DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

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NCBJ

based on: **T. Binder, T. Bringmann, M. Gustafsson and AH**
1706.07433, 2103.01944

EPS-HEP 2021, DESY, Hamburg (online)

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Thermal Relic Density

STANDARD SCENARIO

(chemical and kinetic)

T ↑			time ↓
	$\Gamma_{\text{ann}} > H$	DM in full equilibrium	
	$\Gamma_{\text{ann}} \sim H$	chemical decoupling	
	$\Gamma_{\text{ann}} < H$	freeze-out	
	$\Gamma_{\text{scatt}} \sim H$	kinetic decoupling	

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

Thermal Relic Density

standard approach

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., 1409.3049

⇓ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\,\sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\; f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

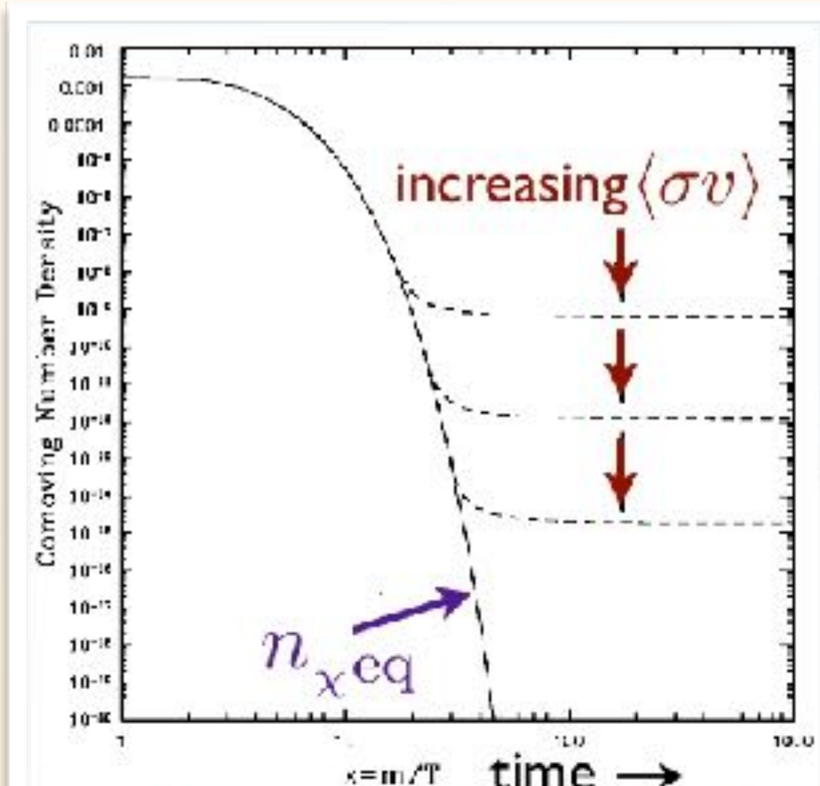


Fig.: Jungman, Kamionkowski & Griest, PR'96

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilations

Two possible approaches:

fBE ↙ ↘ cBE

solve numerically for full $f_\chi(p)$

- have insight on the distribution
- no constraining assumptions
- numerically challenging
- often an overkill

consider system of equations for moments of $f_\chi(p)$

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

- partially analytic/much easier numerically
- manifestly captures all of the relevant physics
- finite range of validity
- no insight on the distribution

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact

DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium, Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]**

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling *this talk!*

Prediction for the DM
phase space distribution
(see talk by K. Dienes!)

Late kinetic decoupling
and impact on cosmology
see e.g., 1202.5456

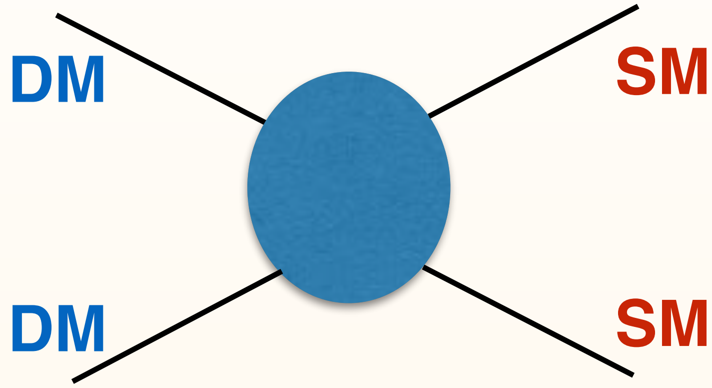
…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

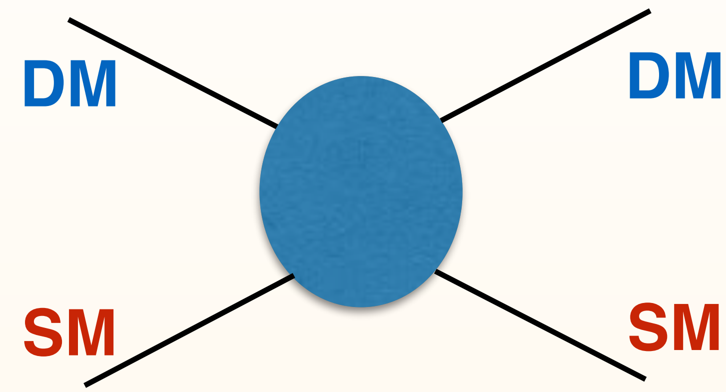
FREEZE-OUT *VS.* DECOUPLING

annihilation



crossing sym.

(elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically
still kinetically coupled to the plasma

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Recall: in *standard* thermal relic density calculation:

Critical assumption:

kinetic equilibrium at chemical decoupling

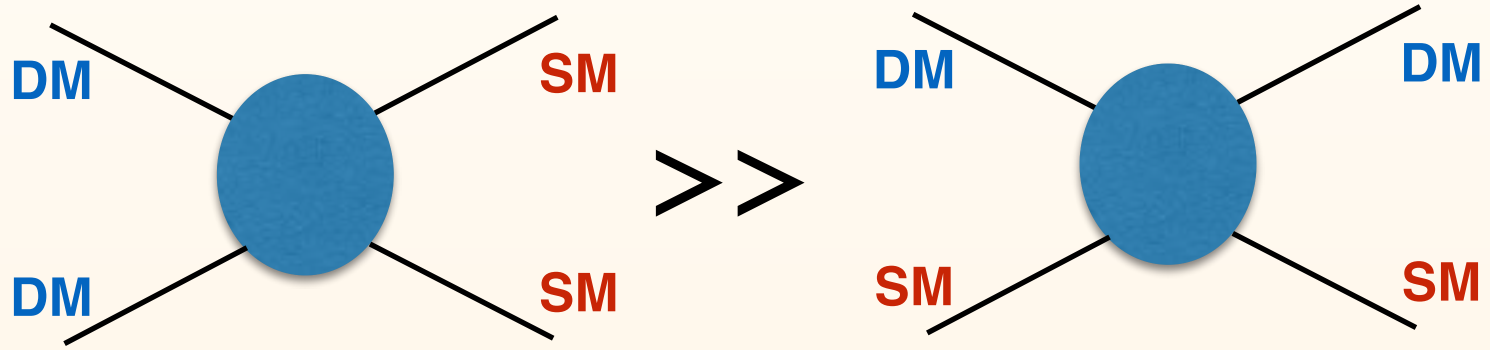
$$f_\chi \sim a(\mu) f_\chi^{\text{eq}}$$

EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

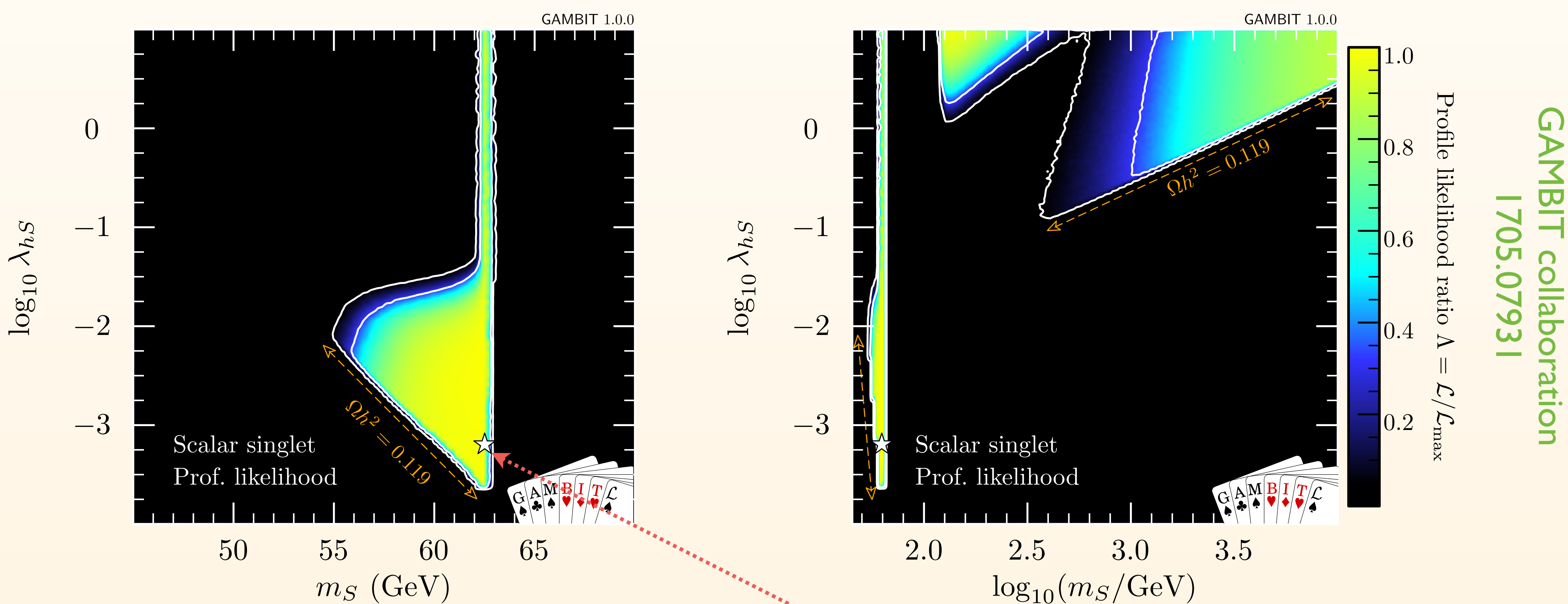
e.g., semi-annihilation, 3 to 2 models,…

EXAMPLE A
SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



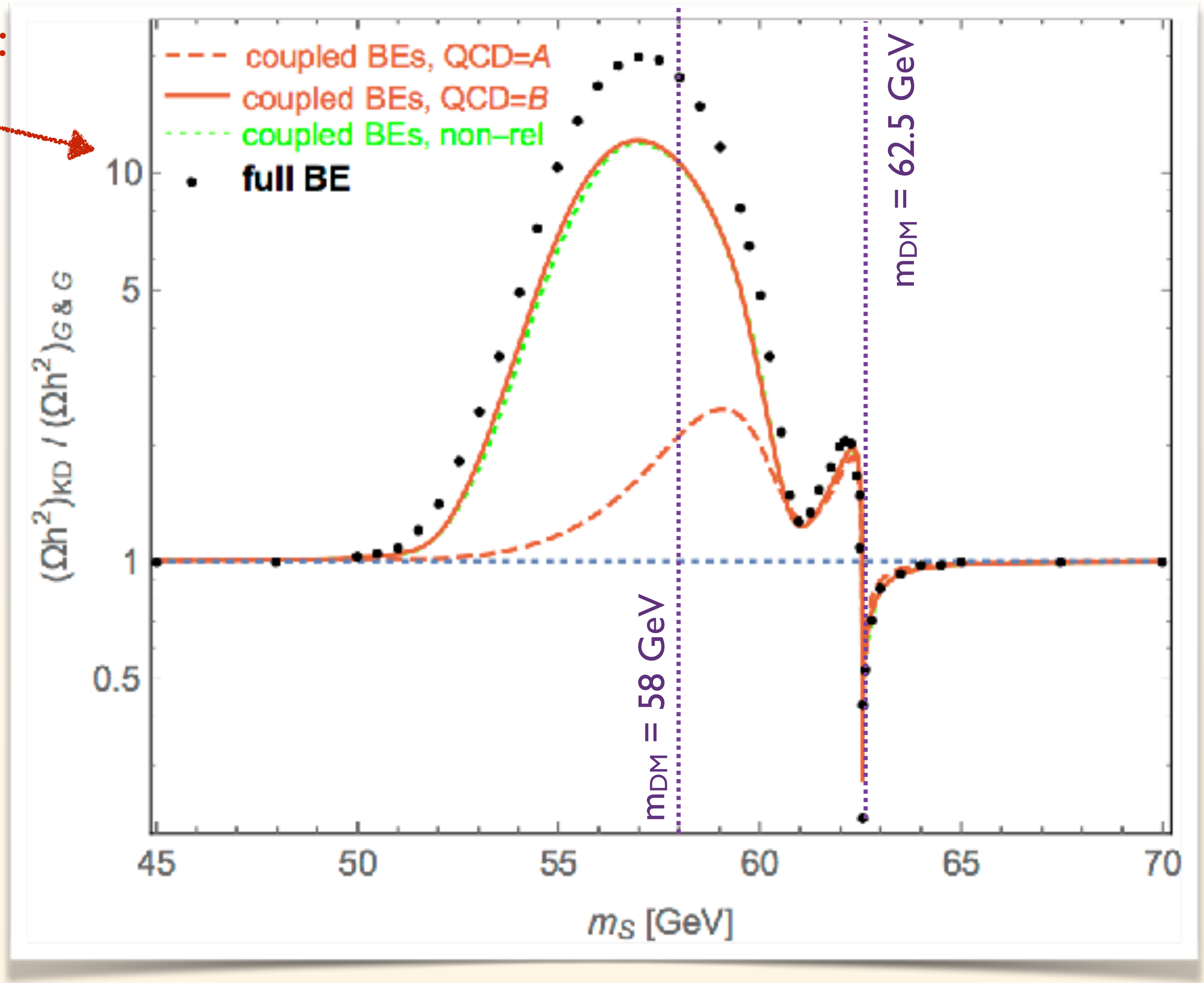
Most of the parameter space excluded, but… even such a simple model is hard to kill

→ best fit point hides in the resonance region!

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to O(~10)



[… Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

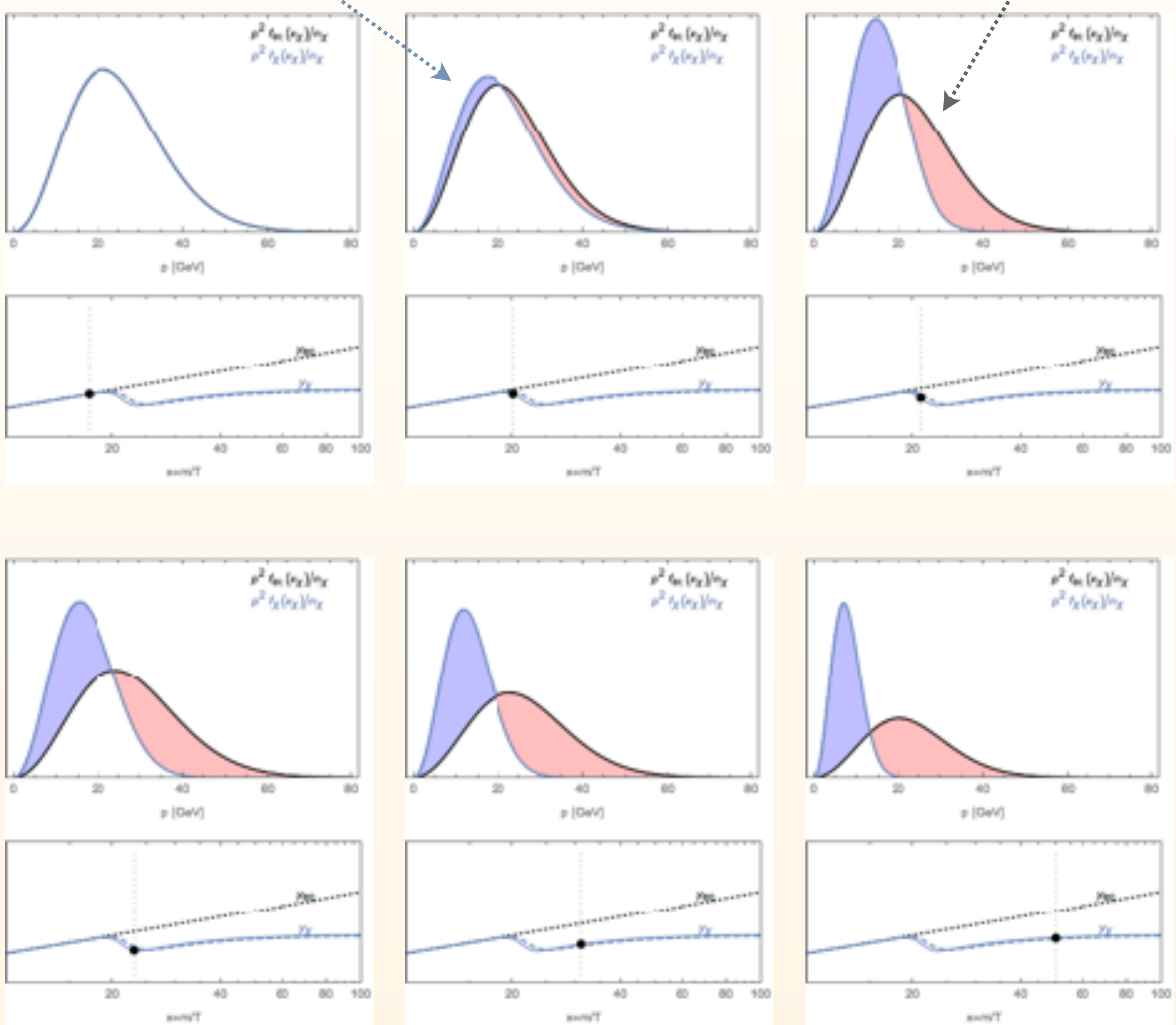
two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV
QCD = B - only light quarks contribute to scattering and only down to $4T_c$ …]

FULL PHASE-SPACE EVOLUTION

blue - full solution for f_{DM} at T_{DM}

m_{DM} = 58 GeV

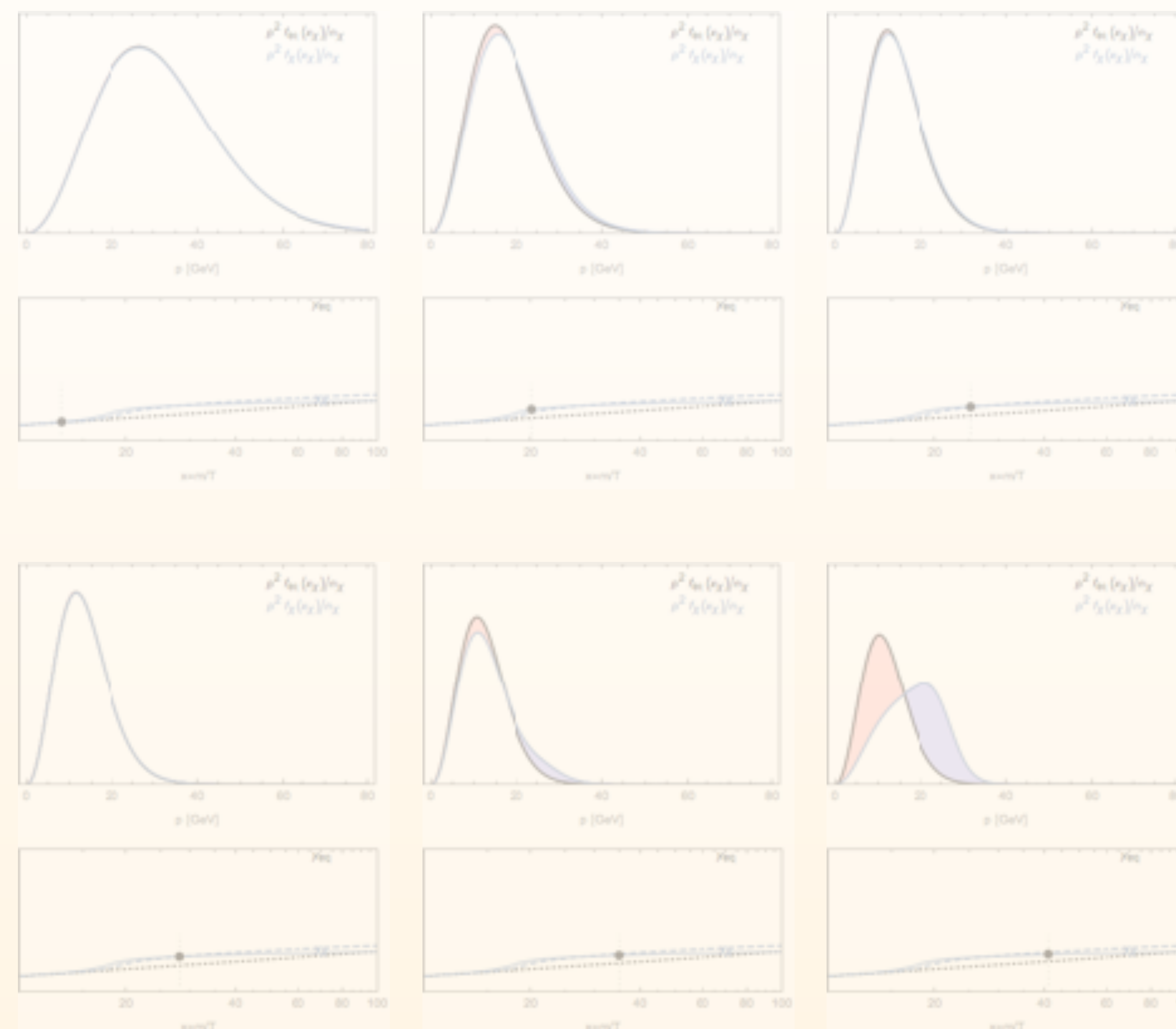
black - equilibrium at T_{DM}



significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV



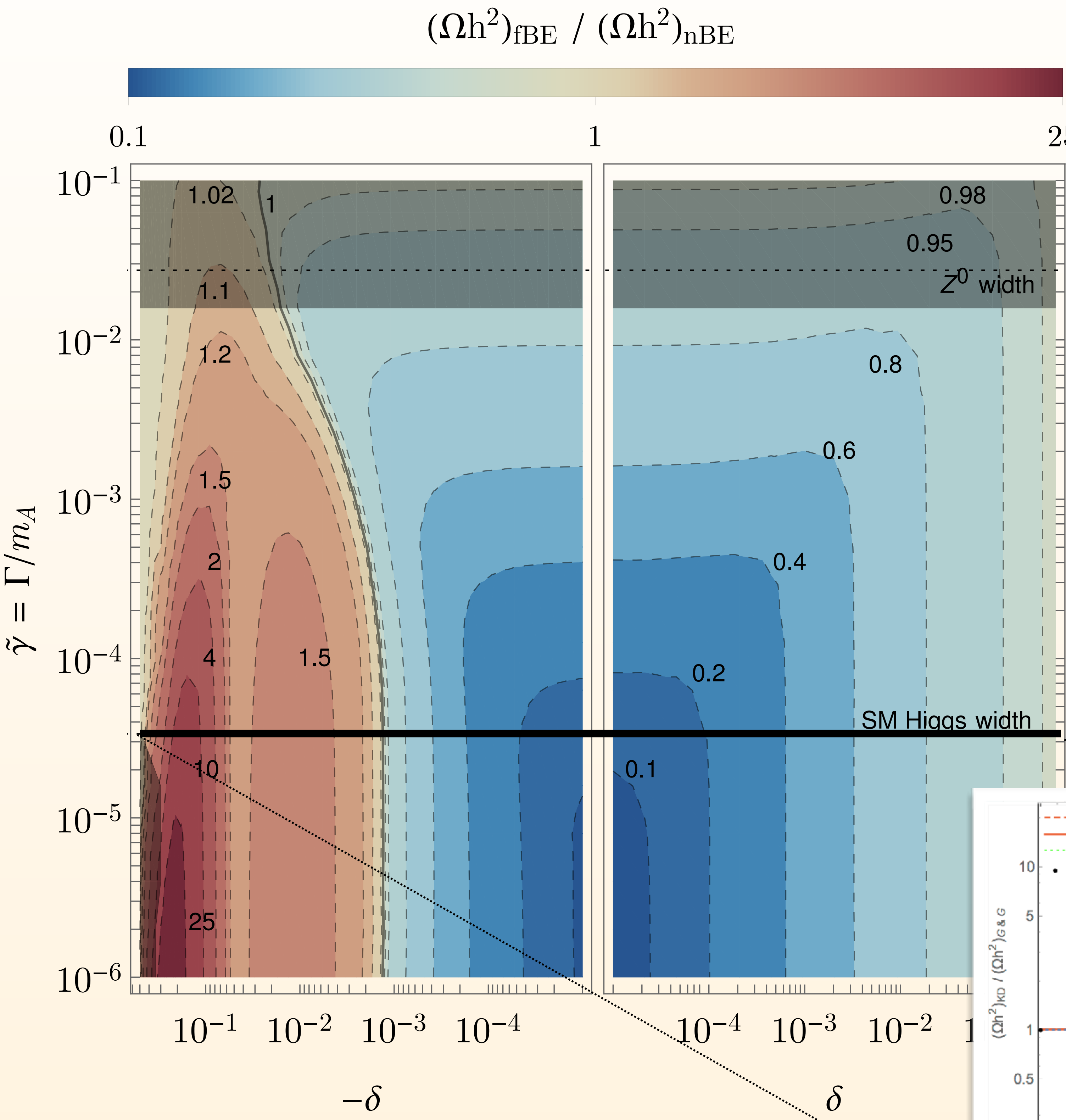
large deviations only at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

GENERIC RESONANT ANNIHILATION

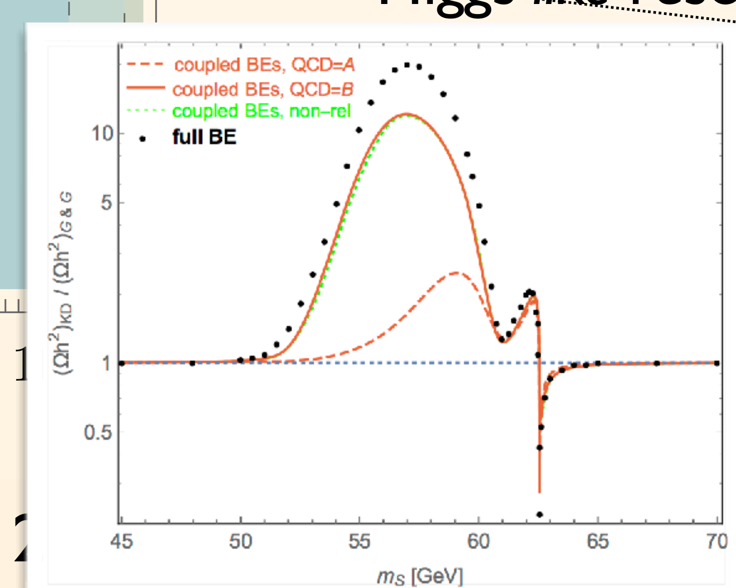
EXAMPLE EFFECT OF EARLY KD ON RELIC DENSITY

resonance **width** normalized to m_{DM}



can reach O(10%) even for *Z-like* resonance

very large effect for Higgs-*like* resonance

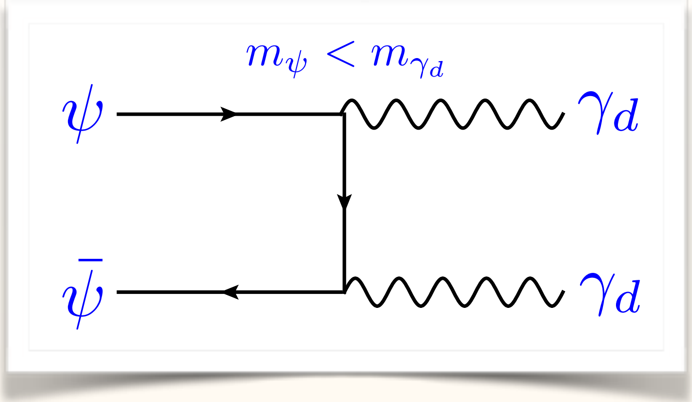


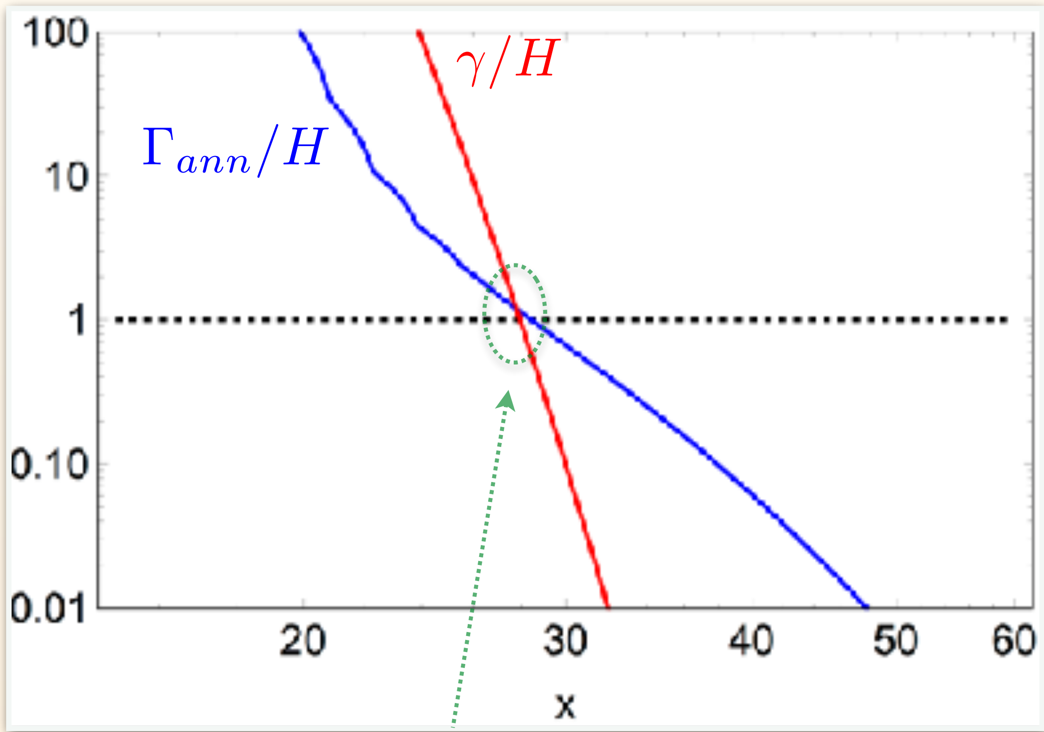
distance in mass from the exact condition 2

Example B
Forbidden Dark Matter

DM is a thermal relic that annihilates <u>only</u> to heavier states
(forbidden in zero temperature)

…, D’Agnolo, Ruderman ’15, …





kinetic and chemical
decoupling close

Annihilation threshold → velocity dependence

”heavy” SM particle → scattering rate low

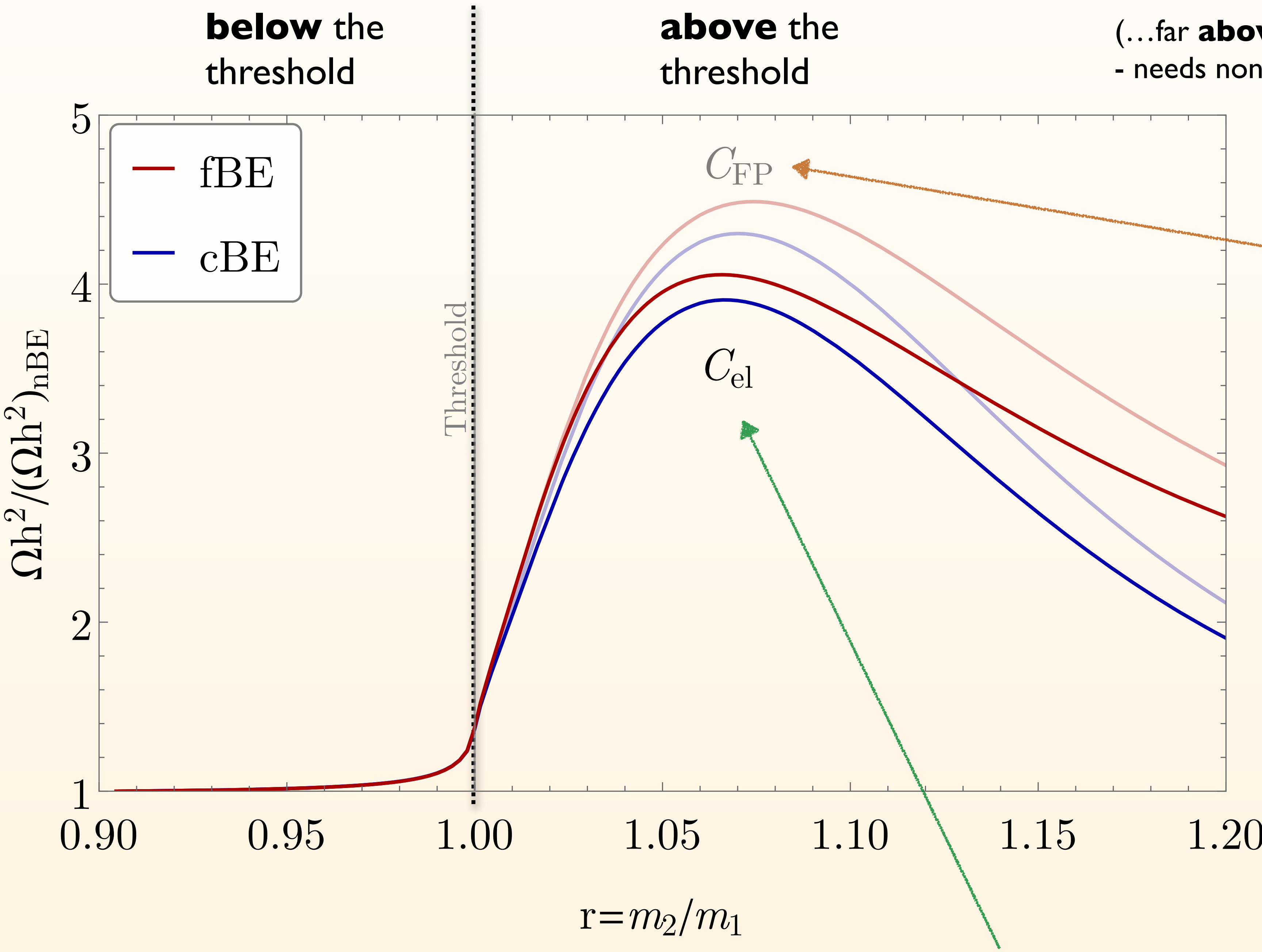
Forbidden Dark Matter

Example effect of early KD on relic density

below the threshold

above the threshold

(…far **above** the threshold - needs non-pert. coupling)



Fokker-Planck approx. for the scattering term

effect on relic density: up to O(~few)

full (numerical) scattering term

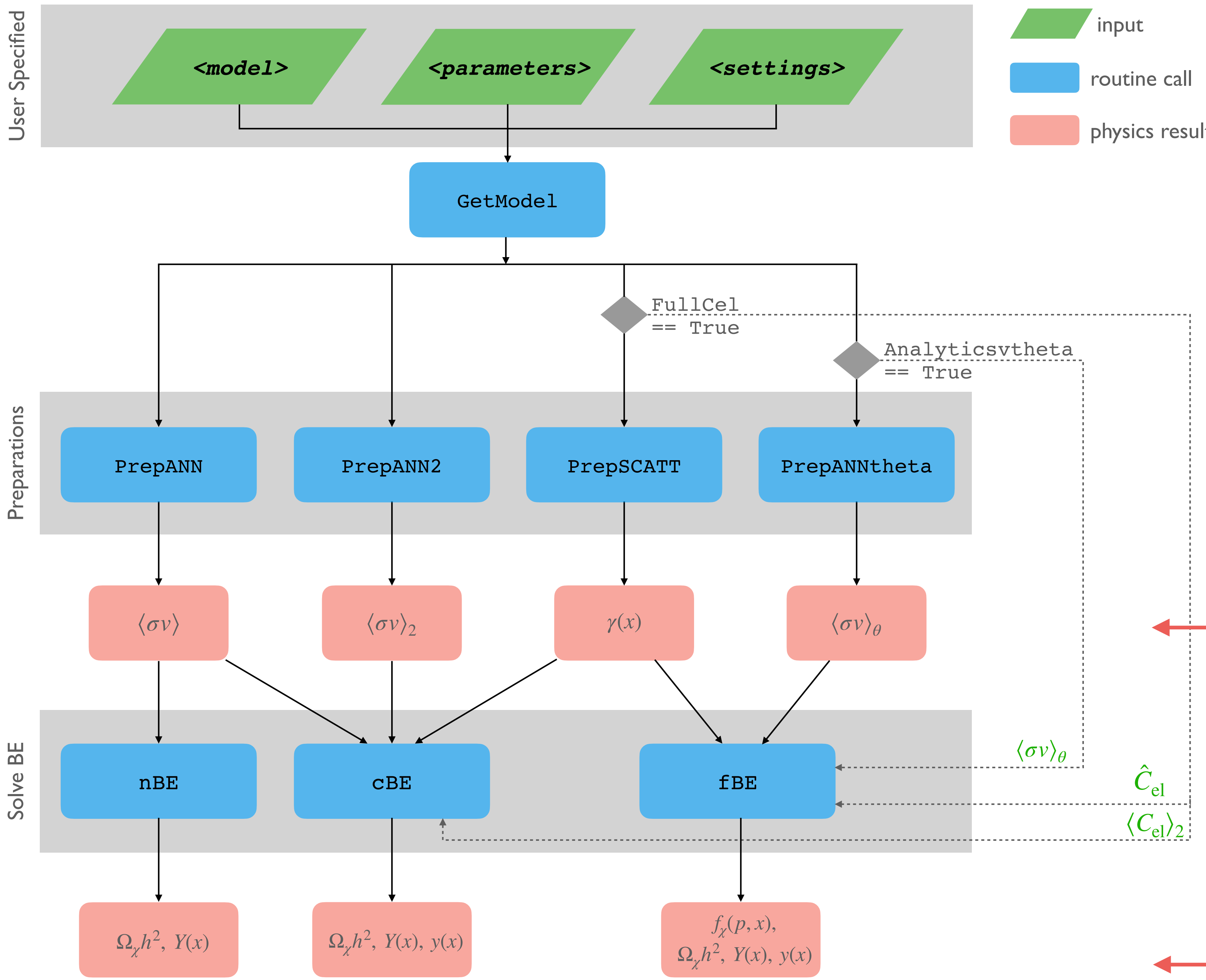
DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org

FEW WORDS ABOUT THE CODE

written in *Wolfram Language*, lightweight, modular and simple to use both via script and front end usage



Input: model file (few models implemented, user extendable), parameter values and settings choices

Preparation phase: all needed thermal averages etc.

[slowest part (~ few sec - some min), can be bypassed by user own routines/results…]

← (intermediate) results for rates

Final phase: tailored solvers for Boltzmann eq.

[~ few seconds]

← (final) results for $\Omega h^2, f_{DM}(p)$

Dark matter Relic Abundance beyond Kinetic Equilibrium
https://drake.hepforge.org

SNAPSHOTS FROM AN EXAMPLE NOTEBOOK

1. Load DRAKE

```
Needs["DRAKE`"]
```

2. Initialize model

```
GetModel["WIMP", "bm1", "settings_bm1"]
----------------- Model: WIMP-like toy model ----------------
{-------- card: bm1 mDM=100.   gDM=1   sv0=1.6877e-9   xkd=25.--------}
```

3. Run

nBE

```
PrepANN;
nBE
Oh2nBE = 0.12
```

cBE

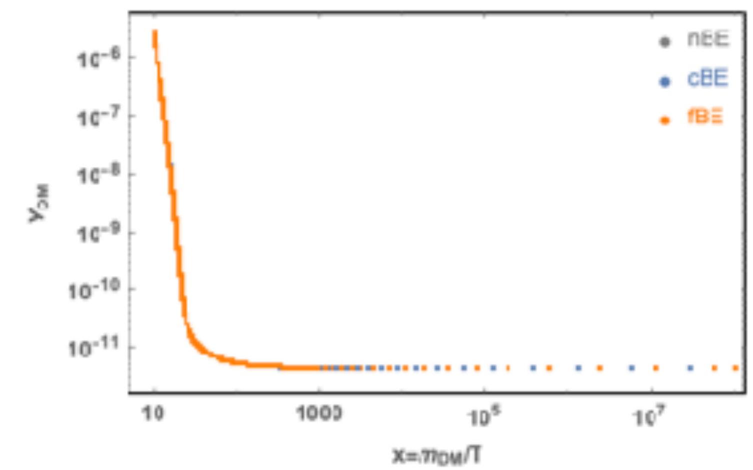
```
If[! scatttype == "gamma(x)" && ! FullCel, PrepSCATT];
(* PrepANN; *) (* uncomment if not called earlier *)
PrepANN2;
cBE
Oh2cBE = 0.120013
```

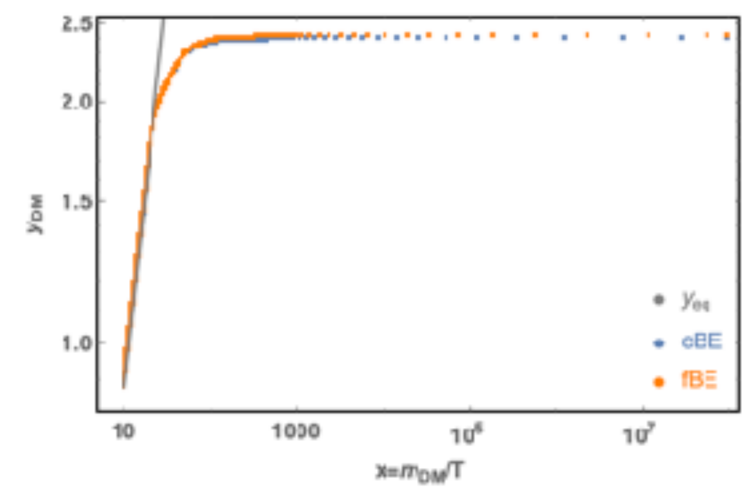
fBE

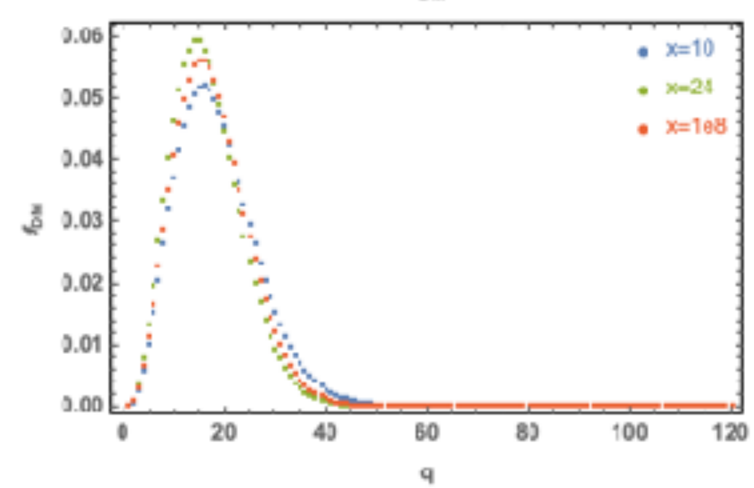
```
PrepANNtheta; RegArrayGen[tsvtheta];
fBE
Oh2fBE = 0.120037
```

4. Print plots

```
(* Print out result plots *)
MakePlots
```







SUMMARY

1. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…
…while it is not always warranted!

2. Introduced coupled system of Boltzmann eqs. for 0th and 2nd moments (cBE) allows for much more accurate treatment while the full phase space Boltzmann equation (fBE) can be also successfully solved for higher precision and/or to obtain result for $f_{\mathrm{DM}}(p)$

3. We introduced DRAKE a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium

4. Future developments and applications:

new processes (e.g., freeze-in, semi-annihilations), imprint on power spectrum, …