Generalizing the Ryu-Takayanagi formula to probe entanglement shadows of BTZ black holes

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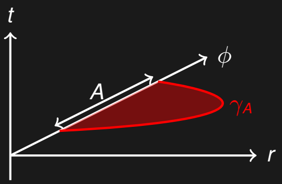
EPS-HEP Conference 2021

July, 29 2021

[arXiv:1910.05352, arXiv:2105.01097]

Motivation

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A) = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$ [Ryu, Takayanagi '06]



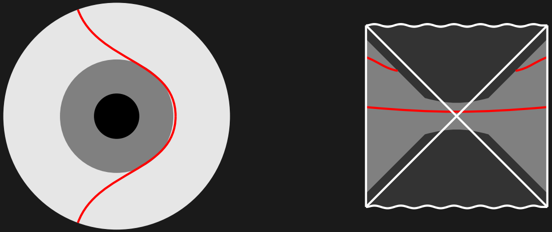
Generalizations in a number of directions:

- time-dependent states [Hubeny, Rangamani, Takayanagi '07]
- quantum corrections [Faulkner, Lewkowycz, Maldacena '13], [Engelhardt, Wall '15]
- "island formulas" to solve the black hole information paradox [Penington '20], [Almheiri, Mahajan, Maldacena, Zhao '20]

In this talk: generalization to also describe entanglement between different fields of the theory

Entanglement shadows

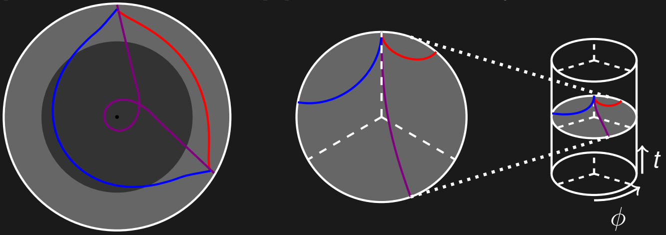
- Finite regions of spacetime around naked singularities or black hole horizons not probed by RT surfaces



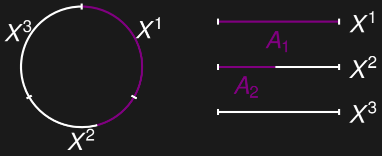
- How does this fit together with the "entanglement builds geometry" idea in AdS/CFT [Van Raamsdonk '10], [Swingle '12]?
- Previous conjecture: entanglement is not enough to probe wormhole interiors [Susskind '16]
- Other possible resolution: use generalized entanglement measures to probe entanglement shadows

Entwinement

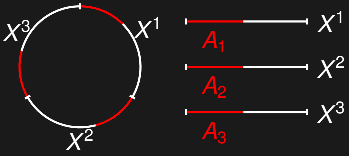
Previous studies for conical defects in AdS_3 under the name of "entwinement" [Balasubramanian, Chowdhury, Czech, Boer '15], [Balasubramanian et al. '16], [Balasubramanian, Craps, De Jonckheere, Sárosi '19]



Entanglement for non-spatial DoF = length of winding geodesic in conical defect



entwinement



entanglement entropy

D1/D5 system [David, Mandal, Wadia '02]

$1/G_N \sim c$

- orbifold point
- supergravity point

$T_{\text{string}} \sim g_{\text{CFT}}$

large strings, weakly coupled CFT

small strings, strongly coupled CFT

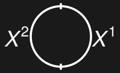
The S_N orbifold theory

- Take N copies of a seed CFT, identify copies under the S_N permutation symmetry
- Twisted sectors: boundary conditions

$$X^i(\phi + 2\pi) = X^{g(i)}(\phi) \quad \forall i \in 1, ..., N \text{ and } g \in S_N$$

Example: $X^1(\phi + 2\pi) = X^2(\phi)\,, \quad X^2(\phi + 2\pi) = X^1(\phi)$

X^1 and X^2 joined together into a single field,



- States in different twisted sectors are orthogonal to each other, thermal density matrix is block diagonal

$$\Rightarrow \rho(\beta) = \frac{e^{-\beta H}}{Z(\beta)} = \bigoplus_C p_C \rho_C$$

Generalized entanglement entropy

- "Ordinary" entanglement entropy (used in RT formula):

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

for $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$, $\rho_A = \mathrm{Tr}_{A^c}(\rho)$.

Each field X^i localized in the same subregion A.

Now: new ingredients

1. Consider a subset $\{C_i\}$ of the twisted sectors

$$\rho(\beta) = p_{\{C_i\}}\rho_{\{C_i\}} \oplus p_{\{C_i\}^c}\rho_{\{C_i\}^c}$$

2. Non-spatial entanglement: fields X^i localized in different subregions A^i

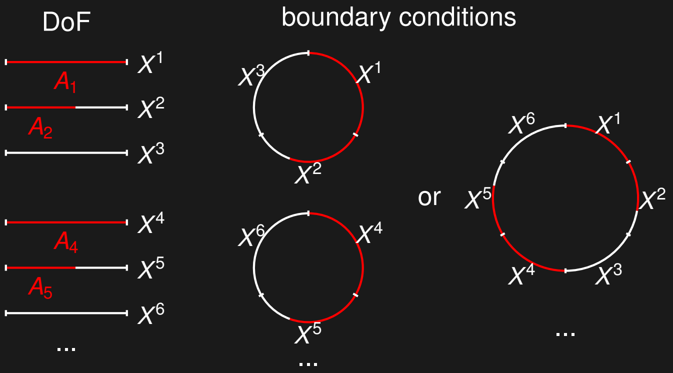
$$\rho_{\{A_i\},\{C_i\}} = \mathrm{Tr}_{\{A_i\}^c}(\rho_{\{C_i\}})$$

$$\Rightarrow S_{\{A_i\},\{C_i\}} = -\mathrm{Tr}(\rho_{\{A_i\},\{C_i\}} \log \rho_{\{A_i\},\{C_i\}})$$

Example: $S_{N/n}$ subsets

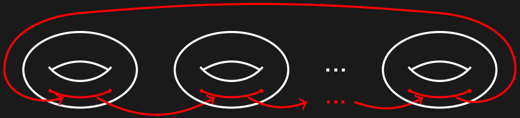
- Example: choose twisted sectors containing only cycles whose length is a multiple of some fixed $n \in \mathbb{N}$
- choose A^i to calculate “single interval” entanglement: A^i continuously connected by the twisted boundary conditions

Example: $n = 3$



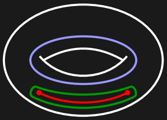
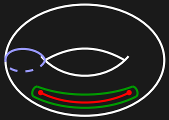
Computation of the entanglement entropy

- Replica trick: $S_A = -\lim_{\alpha\to 1} \frac{1}{1-\alpha} \log \mathrm{Tr}[(\rho_A)^\alpha]$
- $\mathrm{Tr}[(\rho_A)^\alpha]$ obtained from partition function on replica surface: take α copies, glue fields together along entangling interval



- Z_{replica} decomposes into conformal blocks, dominated by single conformal block (up to e^{-c} corrections)

$$Z_{\text{replica}} = \sum_{p,q} a_{p,q} \mathcal{F}(h_p, h_q) \bar{\mathcal{F}}(\bar{h}_p, \bar{h}_q)$$

 $T \ll 1,$  $T \gg 1$

Computation of the generalized entanglement entropy

Same as for the ordinary entanglement entropy, except for:

- different h_q dominates due to projection onto subset $\{C_i\}$ of twisted sectors,
- different choice of branch cuts on the replica surface due to the non-spatial DoF $\{A_i\}$,

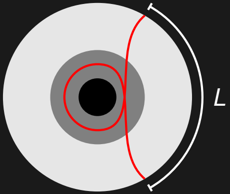
Conformal blocks obtained from monodromy method following [Zamolodchikov '87], [Hartman '13].

Entanglement entropy results

- Small intervals and high temperatures:

$$S_{\{A_i\},\{C_i\}} = \frac{c}{3n} \log \left[\frac{\beta}{2\pi\epsilon} \sinh\left(\frac{2\pi^2(L+w)}{\beta} \right) \right]$$

geodesic in BTZ geometry with opening angle $2\pi L$ and winding number w

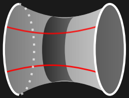


Entanglement entropy results

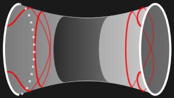
- Two-sided black hole:

$$S_{\{A_i\},\{C_i\}} = \begin{cases} \dfrac{2c}{3n}\log\left[\dfrac{\beta}{2\pi\epsilon}\cosh\left(\dfrac{4\pi^2 t}{\beta}\right)\right], & t < t_c \\ \dfrac{2c}{3n}\log\left[\dfrac{\beta}{2\pi\epsilon}\sinh\left(\dfrac{2\pi^2(L+w)}{\beta}\right)\right], & t > t_c \end{cases}$$

  geodesics stretching through the wormhole up to $t = t_c \sim w/2$



$t < t_c$



$t > t_c$

- Limit $N \to \infty$: winding number w unbounded from above
  $\Rightarrow$ can probe the BTZ geometry
  - up to the horizon in the one-sided case (extremal surface barrier)
  - in the entire space in the two-sided case
- Entanglement is enough to probe the entire BTZ black hole geometry!

Comments

String theory interpretation:

- Projection onto twisted sectors corresponds to allowing only toroidal worldsheets with particular winding numbers
- S_N orbifold dual to string theory in the tensionless limit [Gaberdiel, Gopakumar '18], [Eberhardt, Gaberdiel, Gopakumar '19], [Eberhardt '21]
- Moduli localization: only toroidal worldsheets covering the torus on the BTZ boundary an integer number of times contribute to partition function

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Moving away from the orbifold point:

- Expectation: agreement between length of winding geodesics and generalized entanglement entropy extends beyond the tensionless limit
- Known to hold for ordinary entanglement entropy
- Holds to second order in conformal perturbation theory

Summary/Outlook

Main points:

- Entanglement = geometry idea in AdS/CFT extends to (certain) measures of entanglement between different fields of the boundary CFT
  - How general is this (extension beyond D1/D5 system, general bottom up models)?
  - Proof in the spirit of [Lewkowycz, Maldacena '13]?
- Generalized entanglement measures can probe features of the bulk geometry inaccessible to RT surfaces (entanglement shadows)
  - Is this special to AdS_3/CFT_2?