Four-dimensional treatment of positivity bounds with gravity

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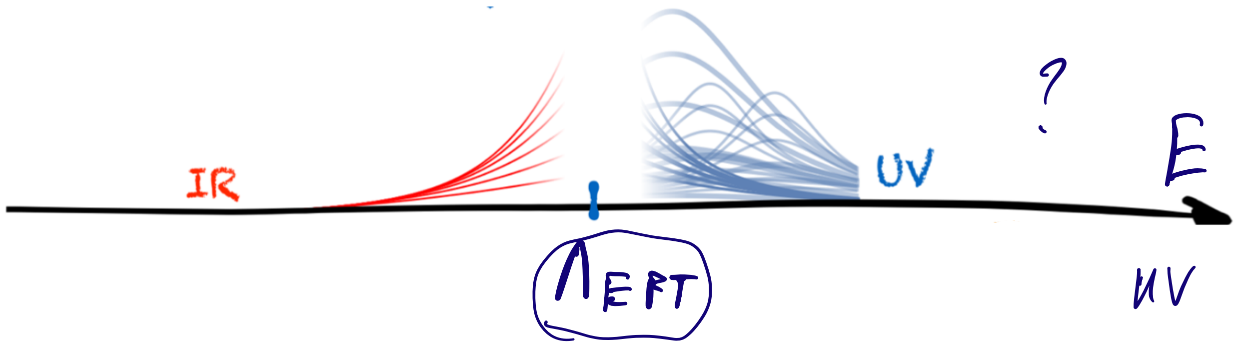
Overview

- Constraints from assuming good UV completion
- Improved positivity bounds
- Massless scalar
- Singularity in the graviton exchange
- Regge tragectory allowing to cancel IR singularities
- Examples

The talk is based on arXiv:2011.11652, 'Massless Positivity in Graviton Exchange' Mario Herrero-Valea, Raquel Santos-Garcia, Anna Tokareva

UV and IR theory

The general idea is to find out which EFT can be UV completed by a good theory and which - cannot



A 'good' UV completion

What do we mean by 'good'?

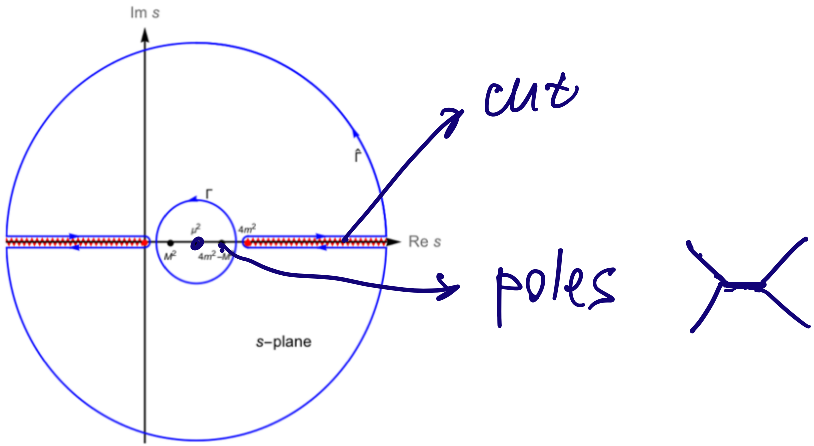
- Lorenz-invariant $\Rightarrow \mathcal{A} = \mathcal{A}(s,t,u)$
- unitary $\Rightarrow Im\,\mathcal{A} > 0$
- satisfying causality $\Rightarrow \mathcal{A}(s,t,u)$ is analytic everywhere except real axes
- local $\Rightarrow$ polynomial boundedness (Froissart-Martin bound)

$$\lim_{|s|\to\infty} \left| \frac{\mathcal{A}(s,t)}{s^2} \right| = 0, \quad t < 4m^2.$$

String theory?

What is positive?

Example: forward limit $t = 0$

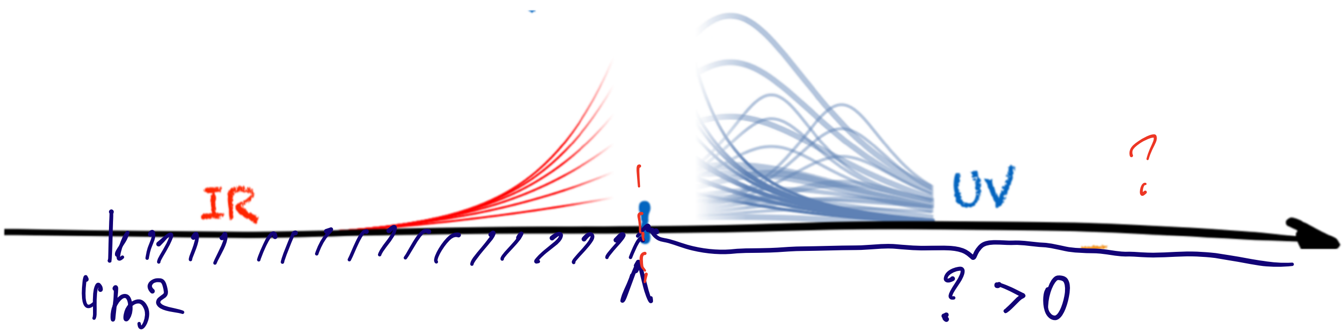


$$\Sigma_{IR} = \frac{1}{2\pi i}\int_\Gamma ds \frac{\mathcal{A}(s)}{(s-\mu^2)^3} = \int_{4m^2}^{\infty} \frac{ds}{\pi}\left(\frac{Im\mathcal{A}(s)}{(s-\mu^2)^3} + \frac{Im\mathcal{A}^+(s)}{(s-4m^2+\mu^2)^3}\right)$$

$$\Sigma_{IR} = \frac{1}{2}\mathcal{A}''(s) > 0$$

Improved positivity bounds

Part of the rhs integrals can be computed in the effective theory



$$\Sigma_{IR} = \frac{1}{2}\mathcal{A}''(s) > \int_{4m^2}^{\Lambda^2} \frac{ds}{\pi} \left(\frac{Im\mathcal{A}(s)}{(s-\mu^2)^3} + \frac{Im\mathcal{A}^+(s)}{(s-4m^2+\mu^2)^3} \right)$$

How powerful are the bounds?

- Galileon model is (almost) closed!

$$L = -\frac{1}{2}(\partial\phi)^2 + \frac{a}{4\Lambda^4}(\partial\phi)^4 + \frac{c}{\Lambda^3}\Box\phi(\partial\phi)^2 - \frac{m^2}{2}\phi^2$$

not allowed by symmetry

$$a > \frac{3}{640}\frac{c^4}{16\pi^2}\left(\frac{E}{\Lambda}\right)^8$$

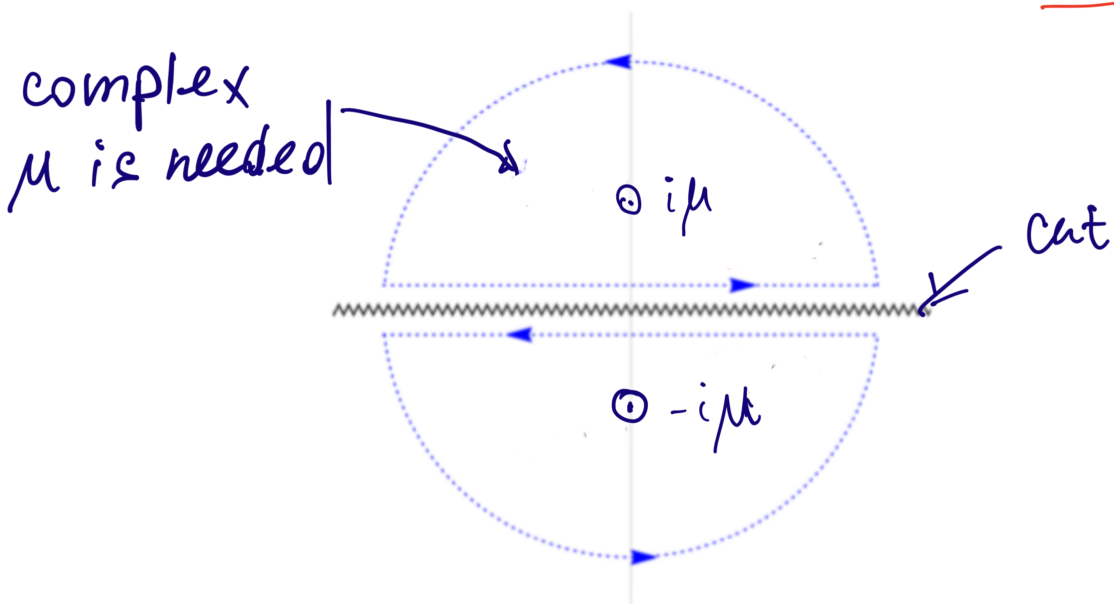
- massive gravity has low cutoff $\Lambda < (10^5 \text{ km})^{-1}$ Bellazzini et al., 1710.02539
- model with two scalars (Higgs-dilaton inflation) got meaningful constraints:

For a certain range of parameters higher derivative operators are not suppressed Herrero-Valea, Timiryasov, AT, 1905.08816

There are also recent works providing systematic approach to constraining generic EFTs: Bellazzini et al 2011.00037, Armani-Hamed et al 2012.15849, Li et al 2101.01191

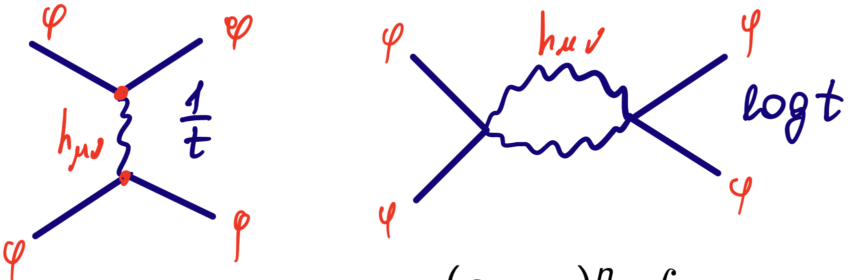
Issues with massless particles

- Branch cuts divide the complex plane $\Rightarrow$ contours should be chosen in a different way
- Froissart-Martin bound can be no longer satisfied
- IR singularities
- the function in the RHS is positive definite only for $\mu < 4m^2$



Issues with gravitons

Where does the singularity come from?

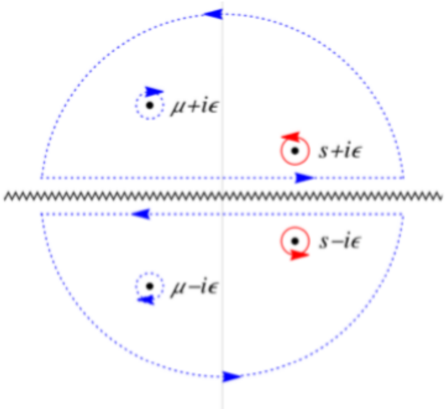


$$\mathcal{A}(s,t) \equiv \mathcal{A}^{(n)}(s,t) = \frac{(s-\mu)^n}{2\pi i} \oint_{\gamma_s} dz \, \frac{\mathcal{A}(z,t)}{(z-s)(z-\mu)^n}$$

$$\mathcal{A}(s,0^-) \equiv \mathcal{A}(s,t)|_{t\to 0} = \frac{f(s)}{t} + g(s)\log(t) + \mathcal{A}_\circ(s) + \mathcal{O}(t).$$

The validity of positivity bounds with gravitons were studied in D>4 (Caron-Huot, Mazac, Rastelli, Simmons-Duffin, arXiv:2102.08951) and D<4 (Bellazzini, Lewandowski, Serra, arXiv:1902.03250)

Issues with gravitons

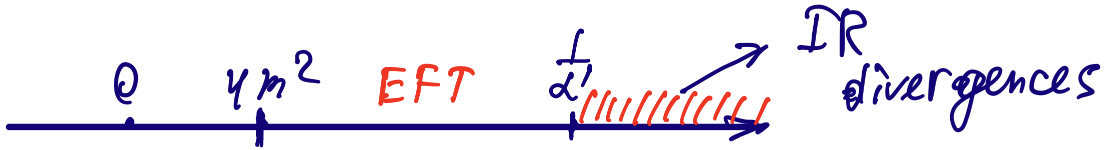


$$\Sigma^{(j)} = \int_{4m^2}^{\infty} dz \left(\frac{z^3 \mathrm{Im}\mathcal{A}(z+i\epsilon, 0^-)}{2\pi(z^2+\delta^2)^{2j+1}} + \frac{(z-4m^2)^3 \mathrm{Im}\mathcal{A}^{\times}(z+i\epsilon, 0^-)}{2\pi((z-4m^2)^2+\delta^2)^{2j+1}} \right)$$

$$\Sigma^{(j)} > 0$$

$$\Sigma^{(j)} = \sum Res \frac{s^3 \mathcal{A}(s)}{(s^2+\delta^2)^{2j+1}}$$

Regge trajectory



$$\Sigma^{(j)} = \int_{4m^2}^{\infty} dz \left(\frac{z^3 \mathrm{Im}\mathcal{A}(z+i\epsilon,0^-)}{2\pi(z^2+\delta^2)^{2j+1}} + \frac{(z-4m^2)^3 \mathrm{Im}\mathcal{A}^{\times}(z+i\epsilon,0^-)}{2\pi((z-4m^2)^2+\delta^2)^{2j+1}} \right)$$

LHS is divergent at $t \to 0$. LHS also must have divergence. The only source is the part at ∞.

$$s \to \infty,\ \ t \to 0 \Rightarrow \boxed{\mathrm{Im}\mathcal{A}(s,t) = r(t)\left(\alpha' s\right)^{2+l(t)}}$$

$$l(t) < 0,\ \ l(t) = l'(0)t + l''(0)t^2 + \dots$$

$$\Sigma^{(1)} \sim \int_{M^2}^{\infty} ds \frac{r(t) z^{l'(0)t}}{z} \sim \frac{1}{t}$$

Tokuda, Aoki, Hirano 2007.15009

Regge trajectory

$$\int_{M_*^2}^{\infty} dz\, \frac{\operatorname{Im} A(z,t)}{z^3} = \frac{a}{t}$$

$$\operatorname{Im} A(z,t) = \tau(z,t)\, z^{2+\ell(t)}$$ – regular in t, $t \to 0$ $z > M_*^2$

$$\int_{M_*^2}^{\infty} \tau(z,0)\, z^{\ell(t)}\, d\ln z = \frac{a}{t}, \quad z = e^{\sigma}$$

$$\int_{\log M_*^2}^{\infty} \tau(\sigma,0)\, e^{\sigma \ell'(0) t}\, d\sigma = \frac{a}{t}, \quad \ell'(0) < 0$$ $a < 0$

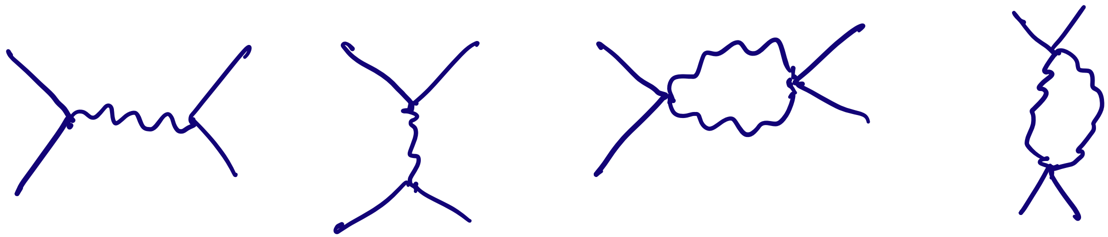
$$\int_0^{\infty} \tau(\sigma,0)\, e^{\sigma \ell'(0) t}\, d\sigma = \frac{a}{t} + (\text{regular in } t \to 0)$$

$\Downarrow$

$\tau(\sigma)$ is an inverse Laplace transform of $\frac{a}{t}$

$\tau(\sigma)$ is uniquely determined by IR divergences!

Graviton loops



$$\mathcal{A} \propto \frac{s^2}{t} + s^2 \log t, \quad t \to 0$$

To obtain this divergense we need add a correction

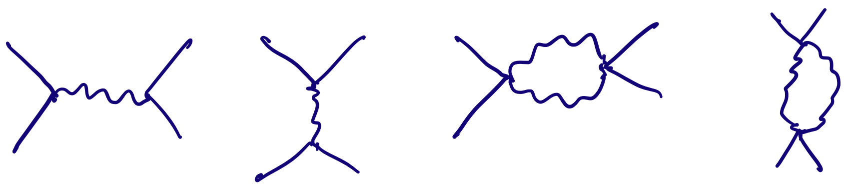
determined by $s^2 \log t$

$$\mathrm{Im}\mathcal{A}(s,t) = r(t)\,(\alpha' s)^{2+l(t)}\left(1 + \frac{\zeta}{\log(\alpha' s)}\right)$$

IR pole and log divergences are cancelled by assuming this form of the amplitude in Regge limit. $l'(0)$ and ζ can be obtained from the IR form of the amplitude.

The simplest example: scalar with gravity

$$S = \int d^4x \sqrt{|g|} \left(-\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right)$$



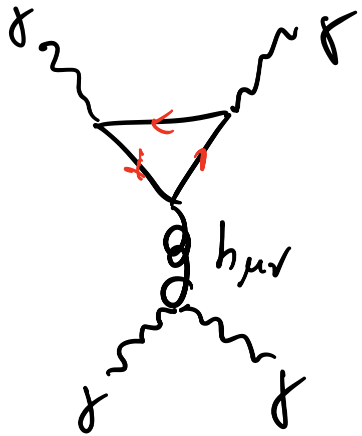
$$\mathcal{A}(s,0^-) = -\frac{\kappa^2 s^2}{t} - \frac{33\kappa^4 s^2}{24\pi^2}(\log(s) + \log(-s)) - \frac{33\kappa^4 s^2}{24\pi^2}\log(t)$$

$$\Sigma^{(1)} = -\frac{\kappa^2}{t} - \frac{33\kappa^4}{24\pi^2}\left(\frac{3}{2} + \log(t) + \log(\delta^2)\right), \quad \Sigma^{(j>1)} = \frac{y(j)\kappa^2}{\pi^2\delta^{4j-4}}$$

Cancelling the divergences determines $r(0)\alpha'^2 \sim -I'(0)\kappa^2$ and $r(0)\alpha'^2\zeta \sim \kappa^4$, which fixes $\zeta > 0$

QED with gravity

$$S = \int d^4x \sqrt{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi - m\bar{\psi}\psi\right)$$



$$\mathcal{M} = -\frac{11 e^2 s^2}{360\pi^2 m^2 M_{Pl}^2} + \dots, \quad \mathcal{M}_{ss} < 0$$

Needs a positive contribution

$$\mathcal{L} = \frac{1}{\Lambda^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

$$\Lambda \sim \sqrt{m M_{Pl}} \ll M_{Pl}, \quad \Lambda \sim 10^8\,\text{GeV for QED}$$

interpretation is still an issue

Alberte, de Rham, Jaitly, Tolley '20

Conclusions

- Low energy theories can be constrained from the requirement to have good UV completion (Lorentz invariance, unitarity, causality, locality)
- In the massless limit, extra assumptions about UV physics are needed to cancel IR divergencies - Regge form of the amplitude
- This allows to justify the bounds obtained without gravity
- Inclusion of graviton scatterings typically make the positivity bounds weaker, due to terms with unknown signs left after cancellation of the poles
- Renormalizable theory with gravity can have lower cutoff than expected (Planck mass)

Thank you for your attention!