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Probing Unified Theories with Reduced Couplings at Future Colliders

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The ad hoc Yukawa and Higgs sectors of the Standard Model induce ~ 20 free parameters. How can they be related to the gauge sector in a more *fundamental* level?

The straightforward way to induce relations among parameters is to add more symmetries.

$\rightarrow$ i.e. GUTs.

Another approach is to look for renormalization group invariant (RGI) relations among couplings at the GUT scale that hold up to the Planck scale.

$\rightarrow$ less free parameters $\rightarrow$ more predictive theories

Reduction of Couplings

About dimensionless couplings:

Any RGI expression among couplings $\mathcal{F}(g, g_1, ..., g_N) = 0$ must satisfy the pde

$$\mu \frac{d\mathcal{F}}{d\mu} = \beta_g \frac{\partial \mathcal{F}}{\partial g} + \sum_{a=1}^{N} \beta_a \frac{\partial \mathcal{F}}{\partial g_a} = 0$$

There are N independent $\mathcal{F}s$ and finding them is equivalent to solve the ode

$$\beta_g \left(\frac{dg_a}{dg} \right) = \beta_a, \qquad a = 1, ..., N$$

where g is the primary coupling $\rightarrow$ Reduction Equations (REs).

Zimmermann (1985)

Ansatz: power series solutions to the REs (preserve perturb. renormalizability):

$$g_a = \sum_n \rho_a^{(n)} \, g^{2n+1}$$

Examining in 1-loop is sufficient for uniqueness to all loops.

Oehme, Sibold, Zimmermann (1984); (1985)

Finiteness

SM $\rightarrow$ quadratic divergences

SUSY $\rightarrow$ only logarithmic divergences

Finite theories $\rightarrow$ no divergences

For a chiral, anomaly free, $N = 1$ theory the superpotential is:

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k$$

$N = 1$ non-renormalization theorem $\rightarrow$ only wave-function infinities.

The 1-loop gauge β-functions are given by

$$\beta_g^{(1)} = \frac{g^3}{16\pi^2} \left[\sum_i T(R_i) - 3C_2(G) \right]$$

The Yukawa β-functions are related to the anomalous dimensions of the matter fields:

$$\beta_{ijk}^{(1)} = C_{ijl}\gamma_k^l + C_{ikl}\gamma_j^l + C_{jkl}\gamma_i^l \qquad \gamma_j^{i(1)} = \frac{1}{32\pi^2} \left[C^{ikl} C_{jkl} - 3g^2 C_2(R) \delta_j^i \right]$$

Finiteness

1-loop β-functions vanish if the 1-loop gauge β-functions and the anomalous dimensions of all superfields vanish, imposing the conditions:

$$\sum_i T(R_i) = 3C_2(G) \quad , \qquad C_{ikl}C^{jkl} = 2\delta^i_j g^2 C_2(R_i)$$

$\rightarrow$ Gauge-Yukawa Unification (GYU)

$\rightarrow$ 1-loop finiteness is sufficient for 2-loop finiteness *Parkes, West (1984)*

- 2-loop corrections for matter fields vanish if 1-loop finite
  $\rightarrow$ sufficient for $\beta_g^{(2)} = 0 = \beta_{ijk}^{(2)}$

- $C_2[U(1)] = 0 \rightarrow$ finiteness cannot be achieved in the MSSM $\rightarrow$ GUT
- $C_2[singlet] = 0 \rightarrow$ supersymmetry can be broken only softly.

All-loop Finiteness

Theorem

Lucchesi, Piguet, Sibold (1988)

Consider an N=1 supersymmetric Yang-Mills theory with simple gauge group. If:

1. There is no gauge anomaly
2. The gauge β-function vanishes at 1-loop $\beta_g^{(1)} = 0$
3. All superfield anomalous dimensions vanish at 1-loop $\gamma_j^{i(1)} = 0$
4. The *REs* admit uniquely determined power series solution that in lowest order is a solution of the vanishing anomalous dimensions
   - $C_{ijk} = \rho_{ijk} g$
   - these solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β-functions

Then the associated Yang-Mills models depend on the single coupling constant g with a β-function which vanishes at all orders.

Soft supersymmetry breaking terms

The soft supersymmetry breaking sector introduces > 100 new free parameters.

Reduction can be extended to the dimensionful sector.

Kubo, Mondragon, Zoupanos (1996)

→ Consider a $N = 1$ supersymmetric gauge theory with soft terms:

$$-\mathcal{L}_{SSB} = \frac{1}{6} h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2} b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)^j_i \phi^{*i}\phi_j + \frac{1}{2} M\lambda\lambda + h.c.$$

1-loop finiteness can be achieved if we demand:

- $\beta_g^{(1)} = 0 = \gamma_j^{i(1)}$ — *Jones, Mezincescu, Yao (1984)*
- $h^{ijk} = -MC^{ijk}$
- $(m^2)^j_i = \frac{1}{3} MM^* \delta^j_i$ $\rightarrow (m_i^2 + m_j^2 + m_k^2) = MM^\dagger (1 + \Delta^{(2)} \frac{g^2}{16\pi^2})$
  - charge and colour breaking vacua — *Kobayashi, Kubo, Mondragon, Zoupanos (1998)*
  - incompatible with radiative EW breaking — (2-loop, can be upgraded to all-loop)

→ also sufficient for 2-loop finiteness. *Jack, Jones (1994)*

→ Possible to upgrade the soft sector to all-loop finite!

Hisano, Shifman (1997); Kazakov (1999); Jack, Jones, Pickering (1998)
Kobayashi, Kubo, Zoupanos (1998); Kobayashi, Kubo, Mondragon, Zoupanos (2000)

The All-loop Finite $N = 1$ $SU(5)$ Model

1+2-loop finite — Hamidi, Schwarz; Jones, Raby (1984); Leon, Perez-Mercader, Quiros (1985)

all-loop finite — Kapetanakis, Mondragon, Zoupanos (1993)

- $3\,(\overline{5} + 10)$ $\rightarrow$ quarks - leptons
- 24 $\rightarrow$ GUT breaking
- $4\,(5 + \overline{5}) \equiv H_i + \overline{H}_i$ $\rightarrow$ Higgs (no fast proton decay)

Heinemeyer, Mondragon, Zoupanos (2008)

In order for the model to be predictive, it should also have the properties:

- One-loop anomalous dimensions are diagonal, i.e. $\gamma_i^{(1)j} \sim \delta_i^j$
- Fermions do not couple to the adjoint rep 24
- The two Higgs doublets of the MSSM are mostly made out of a pair of Higgs $(5 + \overline{5})$ which couple to the third generation

The superpotential with an enhanced symmetry due to RoC:

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \, 10_i 10_i H_i + g_i^d \, 10_i \overline{5}_i \, \overline{H}_i \right] + g_{23}^u \, 10_2 10_3 H_4$$

$$+ g_{23}^d \, 10_2 \overline{5}_3 \, \overline{H}_4 + g_{32}^d \, 10_3 \overline{5}_2 \, \overline{H}_4 + g_2^f \, H_2 \, 24 \, \overline{H}_2 + g_3^f \, H_3 \, 24 \, \overline{H}_3 + \frac{g^\lambda}{3} \, (24)^3$$

The isolated and non-degenerate solutions to $\gamma_i^{(1)}=0$ give:

$$(g_1^u)^2=\frac{8}{5}\,g^2\,,\;(g_1^d)^2=\frac{6}{5}\,g^2\,,\;(g_2^u)^2=(g_3^u)^2=\frac{4}{5}\,g^2\,,$$
$$(g_2^d)^2=(g_3^d)^2=\frac{3}{5}\,g^2\,,\;(g_{23}^u)^2=\frac{4}{5}\,g^2\,,\;(g_{23}^d)^2=(g_{32}^d)^2=\frac{3}{5}\,g^2\,,$$
$$(g^\lambda)^2=\frac{15}{7}g^2\,,\;(g_2^f)^2=(g_3^f)^2=\frac{1}{2}\,g^2\,,\;(g_1^f)^2=0\,,\;(g_4^f)^2=0$$

From the dimensionful sector we obtain:

$$\begin{aligned} h &= -MC \\ m_{H_u}^2+2m_{10}^2 &= M^2 \\ m_{H_d}^2-2m_{10}^2 &= -\frac{M^2}{3} \\ m_{\bar{5}}^2+3m_{10}^2 &= \frac{4M^2}{3} \end{aligned}$$

Independent parameters:

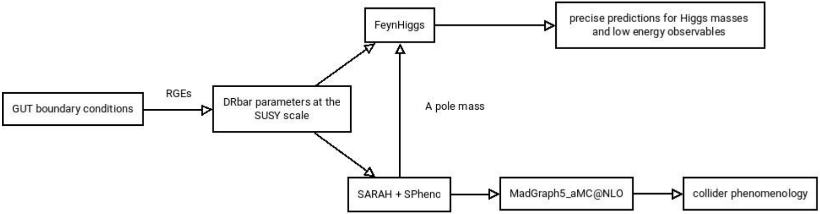
— Dimensionless: g

— Dimensionful: M, m_{10}

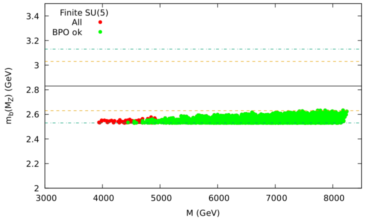
Computational Setup

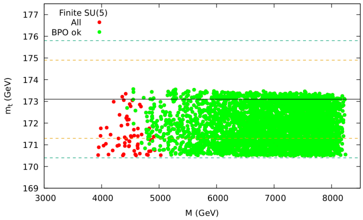
Heinemeyer, Mondragon, GP, Tracas, Zoupanos (2018), (2020);
Heinemeyer, Kalinowski, Kotlarski, Mondragon, GP, Tracas, Zoupanos (2020)

- Broken GUT $\rightarrow$ MSSM $\rightarrow$ REs' solutions act as boundary conditions at M_{GUT}
- 1-loop β-functions for the soft sector, everything else in 2 loops
- $\mu < 0$ only acceptable choice because of quark masses
- m_τ is used as input

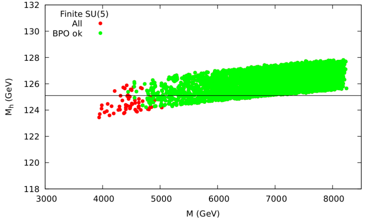


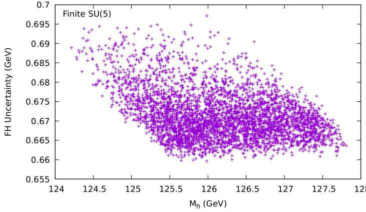
3^{rd} generation analysis:





Points in green satisfy $b \to s\gamma$, $B_u \to \tau\nu$, $B_s \to \mu^+\mu^-$ and $\Delta M_{B_s}^{\rm exp}$





FeynHiggs 2.16.0 $\to$ uncertainty lowered by more than $\sim 1\ GeV$

	$\tan\beta$	$M_{A,H}$	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}^0_1}$	$M_{\tilde{\chi}^0_2}$	$M_{\tilde{\chi}^0_3}$	$M_{\tilde{\chi}^0_4}$	$M_{\tilde{\chi}^\pm_1}$	$M_{\tilde{\chi}^\pm_2}$
FUTSU5-1	49.9	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	50.1	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	49.9	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV	scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV
$\tilde{\chi}^0_2\tilde{\chi}^+_1$	0.17	0.08	0.03	$\tilde{u}_i\tilde{\chi}^-_1$, $\tilde{d}_i\tilde{\chi}^+_1 + h.c.$	0.15	0.06	0.02
$\tilde{g}\tilde{g}$	0.20	0.05	0.01	$Hb\bar{b}$	2.76	0.85	
$\tilde{q}_i\tilde{q}_j$, $\tilde{q}_i\tilde{q}^*_j$	3.70	1.51	0.53	$Ab\bar{b}$	2.73	0.84	
$\tilde{\chi}^+_1\tilde{\chi}^-_1$	0.10	0.05	0.02	$H^+b\bar{t} + h.c.$	1.32	0.42	
$\tilde{e}_i\tilde{e}^*_j$	0.23	0.13	0.05	H^+W^-	0.38	0.12	
$\tilde{q}_i\tilde{g}$, $\tilde{q}^*_i\tilde{g}$	2.26	0.75	0.20				

- No discovery at the HL-LHC
- Heavy Higgs masses (FUTSU5-1 and FUTSU5-2) accessible at the FCC-hh
- Lighter stop mass of FUTSU5-1 accessible at the FCC-hh (no 5σ)

2-loop Finite $N = 1\ SU(3)^3$

Ma, Mondragon, Zoupanos (2004)

1-loop : $\beta_i = (16\pi^2)^{-1}\alpha_i g_i^3$, $\alpha_i = 3n_G - 9$, n_G : #of fermion gens

$$q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, \quad L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}$$

Two trilinear invariants for the superpotential (only third gen considered):

$$f\ Tr(Lq^cQ) + \frac{1}{6}f'\ \epsilon_{ijk}\epsilon_{abc}(L_{ia}L_{jb}L_{kc} + q^c_{ia}q^c_{jb}q^c_{kc} + Q_{ia}Q_{jb}Q_{kc})$$

After the vanishing of anomalous dimensions is imposed:

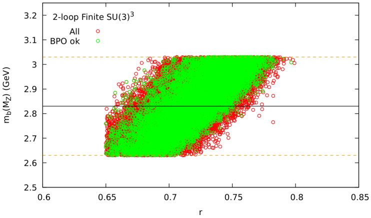
$$|f|^2 + \frac{2}{3}|f'|^2 = \frac{16}{9}g^2$$

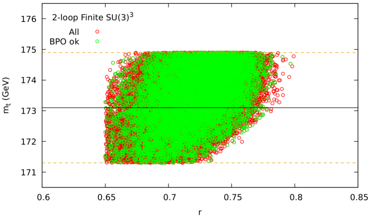
— Isolated and non-degenerate solution $\rightarrow |f|^2 = \frac{16}{9}g^2$ — All-loop finite; No lepton masses

— Unique (but no isolated) solution $\rightarrow |f|^2 = r\frac{16}{9}g^2$, $|f'|^2 = (1-r)\frac{8}{3}g^2$ — 2-loop finite

Dimensionful: $h = -Mf \qquad m^2_{H_u} + m^2_{\tilde{t}^c} + m^2_{\tilde{q}} = M^2 = m^2_{H_d} + m^2_{\tilde{b}^c} + m^2_{\tilde{q}}$

Intermediate scales: 10^{13} & 10^{14} GeV $\rightarrow$ contributions from exotic particles





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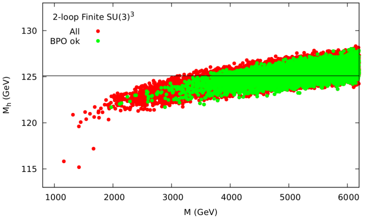

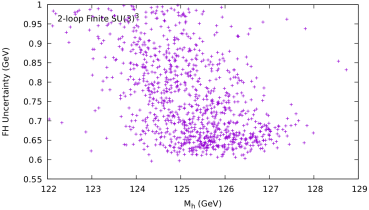

FeynHiggs 2.16.0 $\rightarrow$ uncertainty lowered by more than ~ 1 *GeV*

	$\tan\beta$	$M_{A,H}$	$M_{H^{\pm}}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}^0_1}$	$M_{\tilde{\chi}^0_2}$	$M_{\tilde{\chi}^0_3}$	$M_{\tilde{\chi}^0_4}$	$M_{\tilde{\chi}^{\pm}_1}$	$M_{\tilde{\chi}^{\pm}_2}$
FSU33-1	46.2	7.029	7.028	6.526	1.506	2.840	6.108	6.109	2.839	6.109
FSU33-2	45.5	6.484	6.431	8.561	2.041	3.817	7.092	7.093	3.817	7.093
FSU33-3	49.7	6.539	6.590	10.159	2.473	4.598	6.780	6.781	4.598	6.781
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_{\tau}}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FSU33-1	2.416	2.415	1.578	2.414	5.375	5.411	4.913	5.375	4.912	5.411
FSU33-2	3.188	3.187	2.269	3.186	7.026	7.029	6.006	7.026	6.005	7.029
FSU33-3	3.883	3.882	2.540	3.882	8.334	8.397	7.227	8.334	7.214	7.409

scenarios $\sqrt{s}$	FSU33-1 100 TeV	FSU33-2 100 TeV	FSU33-3 100 TeV	scenarios $\sqrt{s}$	FSU33-1 100 TeV	FSU33-2 100 TeV	FSU33-3 100 TeV
$\tilde{\chi}^0_2\tilde{\chi}^+_1$	0.58	0.16	0.07	$\tilde{\nu}_i\tilde{\nu}^*_j$	0.10	0.03	0.01
$\tilde{g}\tilde{g}$	2.61	0.30	0.07	$\tilde{u}_i\tilde{\chi}^-_1$, $\tilde{d}_i\tilde{\chi}^+_1 + h.c.$	1.22	0.25	0.08
$\tilde{g}\tilde{\chi}^0_1$	0.20	0.05	0.02	$\tilde{q}_i\tilde{\chi}^0_1$, $\tilde{q}^*_i\,\tilde{\chi}^0_1$	0.55	0.13	0.05
$\tilde{g}\tilde{\chi}^0_2$	0.20	0.04	0.01	$\tilde{q}_i\tilde{\chi}^0_2$, $\tilde{q}^*_i\,\tilde{\chi}^0_2$	0.60	0.13	0.04
$\tilde{g}\tilde{\chi}^+_1$	0.42	0.09	0.03	$\tilde{\nu}_i\tilde{e}^*_j$, $\tilde{\nu}^*_i\tilde{e}_j$	0.36	0.12	0.04
$\tilde{q}_i\tilde{q}_j$, $\tilde{q}_i\tilde{q}^*_j$	25.09	6.09	2.25	$Hb\bar{b}$	0.71	1.23	1.19
$\tilde{\chi}^+_1\tilde{\chi}^-_1$	0.37	0.10	0.04	$Ab\bar{b}$	0.72	1.23	1.18
$\tilde{e}_i\tilde{e}^*_j$	0.39	0.12	0.06	$H^+b\bar{t} + h.c.$	0.37	0.75	0.58
$\tilde{q}_i\tilde{g}$, $\tilde{q}^*_i\,\tilde{g}$	22.12	3.71	1.05	H^+W^-	0.10	0.25	0.19

- No discovery at the HL-LHC
- Heavy Higgs masses of all benchmarks accessible at the FCC-hh
- Stop masses of all benchmarks accessible at the FCC-hh (no 5σ)

Summary

- Reduction of Couplings: powerful tool that implies Gauge-Yukawa Unification
- Finiteness: old dream of HEP, very predictive models
- completely finite theories $\rightarrow$ both in dimensionless and dimensionful sector

- Reduced models re-examined in 2-loop (1-loop for the SSB sector)
- $\mu < 0$ survives phenomenological constraints
- Both models agree with all LHC measurements
- The models elude discovery at HL-LHC — part of their spectra could be observed at the FCC-hh

Thank you for your attention!