Trace anomaly for Weyl fermions using the Breitenlohner-Maison scheme for γ^*

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1. THE MODEL

In a curved *n*-dimensional space, the action for Weyl fermions reads:

$$S = -\int \bar{\psi}\,\mathcal{P}_- \not{\nabla} (\mathcal{P}_+ \psi) \sqrt{-g}\, \mathrm{d}^n y, \tag{1}$$

where $\mathcal{P}_-$ and $\mathcal{P}_+$ are the chiral projectors used to obtain Weyl fermions from Dirac ones.

The associated symmetric stress-energy tensor reads:

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu}\nabla^{\nu)}\psi - \frac{1}{2}\gamma^{(\mu}\nabla^{\nu)}\bar{\psi}\psi + \frac{1}{2} g^{\mu\nu}(\not{\nabla}\bar{\psi}\mathcal{P}_+\psi - \bar{\psi}\mathcal{P}_-\not{\nabla}\psi). \tag{2}$$

Classically, the theory is conformal invariant, which means that $T^\mu_\mu \overset{\text{class.}}{=} 0$. At the quantum level it could become nontrivial, developing a linear dependence on certain geometrical quantities:

$$\langle T^\mu_\mu \rangle = -\left(w\, C^2 + b\, \mathcal{E}_4 + c\, \nabla^2 R + f \tilde{R} R\right). \tag{3}$$

2. OVERVIEW

In a series of recent articles [1-3], it has been claimed that $f \propto \mathrm{i}$. Given that:

$$H = \int T^{00}(x)\, \mathrm{d}^3 x, \tag{4}$$

having an imaginary term in the stress-energy tensor could result in a non-Hermitean Hamiltonian. This could have serious consequences on the standard model.

3. THE METHOD

We use Feynman diagrams in conjunction with dimensional regularization to compute $\langle T^\mu_\mu \rangle$ [4]. The use of γ^* is subtle in arbitrary dimensions: we employ the Breitenlohner-Maison scheme, which separates space-time into a strictly four-dimensional piece and another having (n – 4) dimensions.

$$\eta_{\mu\nu} = \bar{\eta}_{\mu\nu} + \hat{\eta}_{\mu\nu}\,, \quad \gamma_\mu = \bar{\gamma}_\mu + \hat{\gamma}_\mu\,, \quad p_\mu = \bar{p}_\mu + \hat{p}_\mu\,, \ \ldots \tag{5}$$

4. THE VEV $\langle T^{\mu\nu}(x) \rangle$

We consider fluctuations around Minkowski space:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}\,, \tag{6}$$

then expand the stress tensor and compute its vacum expectation value using the Gell-Mann-Low theorem. At first order we obtain:

$$\begin{aligned} \langle T^{\mu\nu}(x)\rangle^{(1)}_{\text{ren}} = & -\frac{1}{16\,\pi^2}\iint \frac{1}{120}\left[\frac{12}{5} - \ln\left(\frac{p^2 - \mathrm{i}\,0}{\mu^2}\right)\right] e^{\mathrm{i}p(x-y)} \\ & \times \Bigg[\left(\Pi^{\mu\nu}(p)\,\Pi^{\rho\sigma}(p) - 3\,\Pi^{\rho(\mu}(p)\,\Pi^{\nu)\sigma}(p)\right) h_{\rho\sigma}(y) \\ & - \frac{1}{180}\Pi^{\mu\nu}(p)\,\Pi^{\rho\sigma}(p)\Bigg] \frac{\mathrm{d}^4 p}{(2\,\pi)^4}\, \mathrm{d}^4 y, \end{aligned} \tag{7}$$

where:

$$\Pi^{\mu\nu}(p) \equiv p^\mu p^\nu - \eta^{\mu\nu} p^2. \tag{8}$$

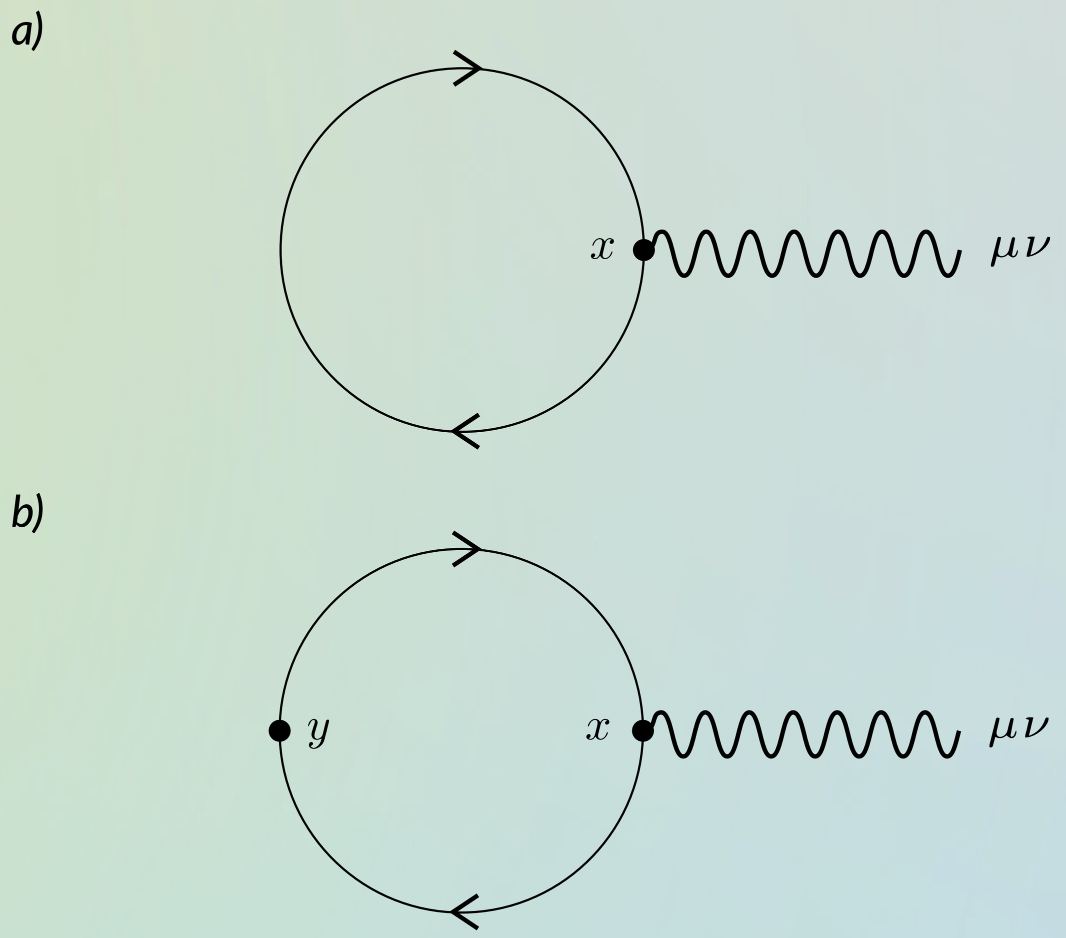

Figure 1. Examples of graviton-fermion Feynman diagrams that appear at first order in ***h****. Here (external) gravitons are represented by wavy lines while fermions are represented by solid lines.*

We have carried out the computation up to second order in ***h***, since this is enough to corroborate the anomaly.

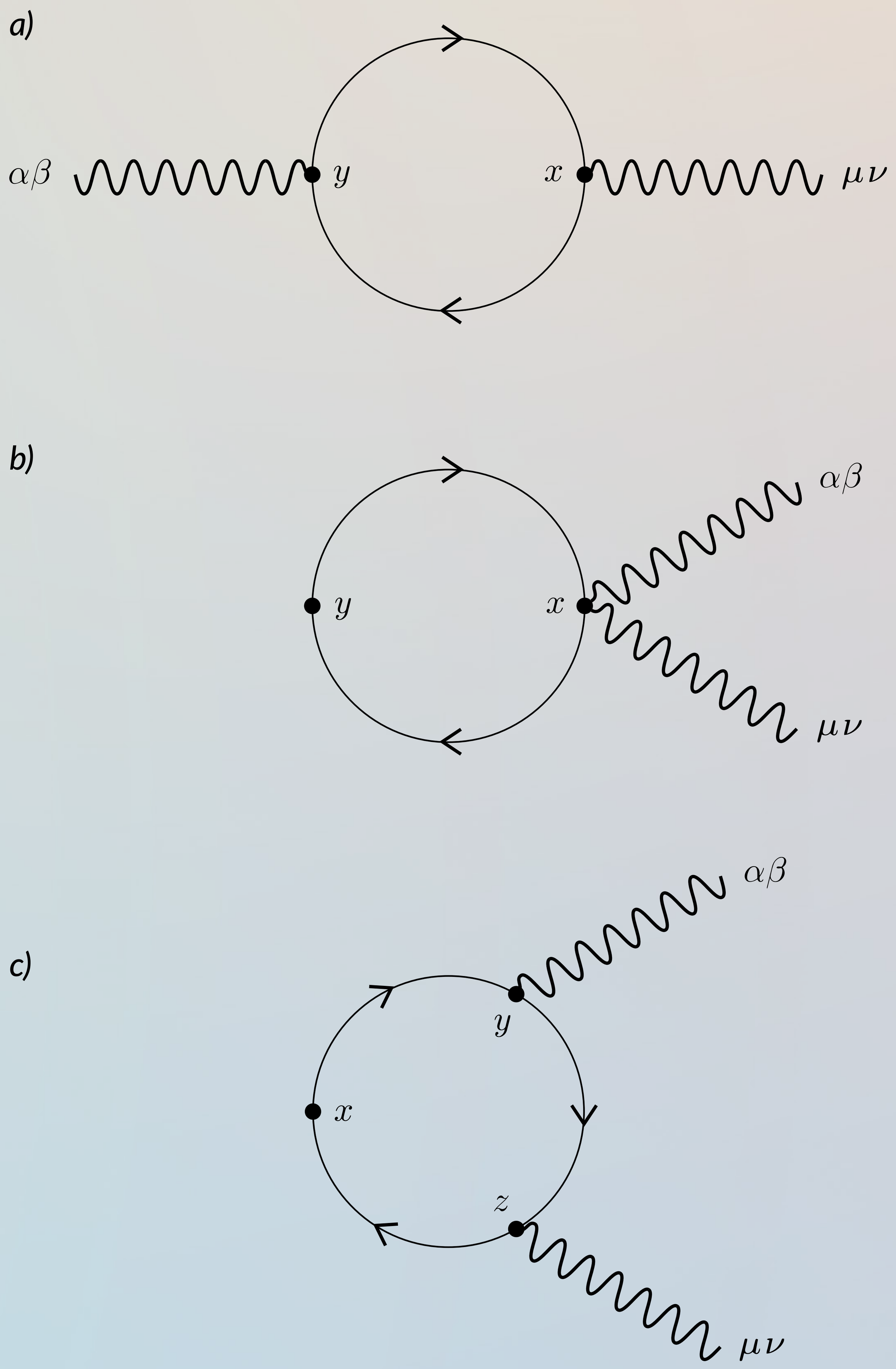

Figure 2. Examples of Feynman diagrams which appear at second order in ***h****. Gravitons are represented by wavy lines while fermions are represented by solid lines.*

5. ANOMALY

Our computation up to second order concludes that the coefficients of the anomaly are:

$$w = -\frac{18}{720\,(4\,\pi)^2}\,,\ c = \frac{11}{720\,(4\,\pi)^2}\,,\ b = -\frac{12}{720\,(4\,\pi)^2}\,,\ f = 0. \tag{9}$$

6. CONCLUSION

Our computation:

- demonstrates that the calculated trace anomaly does not contain any parity-odd term proportional to the Pontryagin density;
- shows that the result for the trace anomaly for a Weyl fermions is half the one of a Dirac fermion, as also confirmed using other methods [5-7]; and
- reaffirms the validity of the equivalence principle for the coupling of matter to gravity in the present case, since there is no difference between left- or right-handed fermions.

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