Improved Transverse Momentum Dependent factorization and its recent applications

The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences

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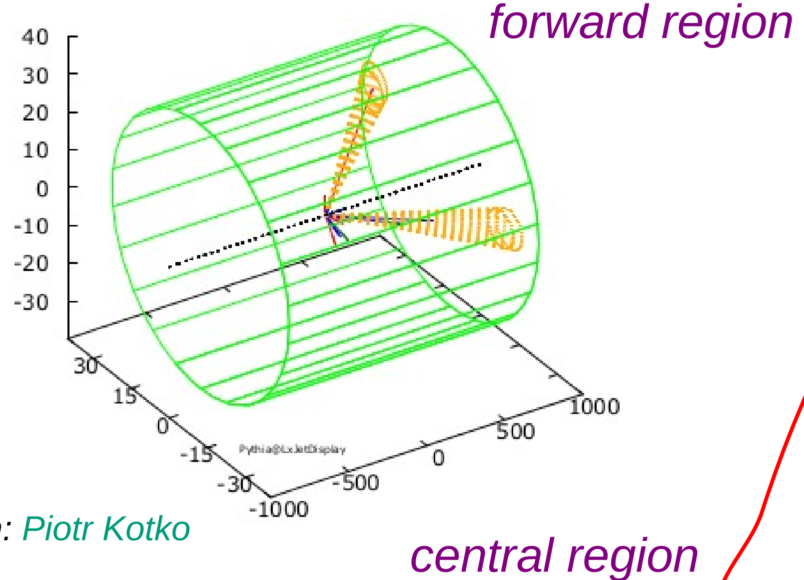
NCN

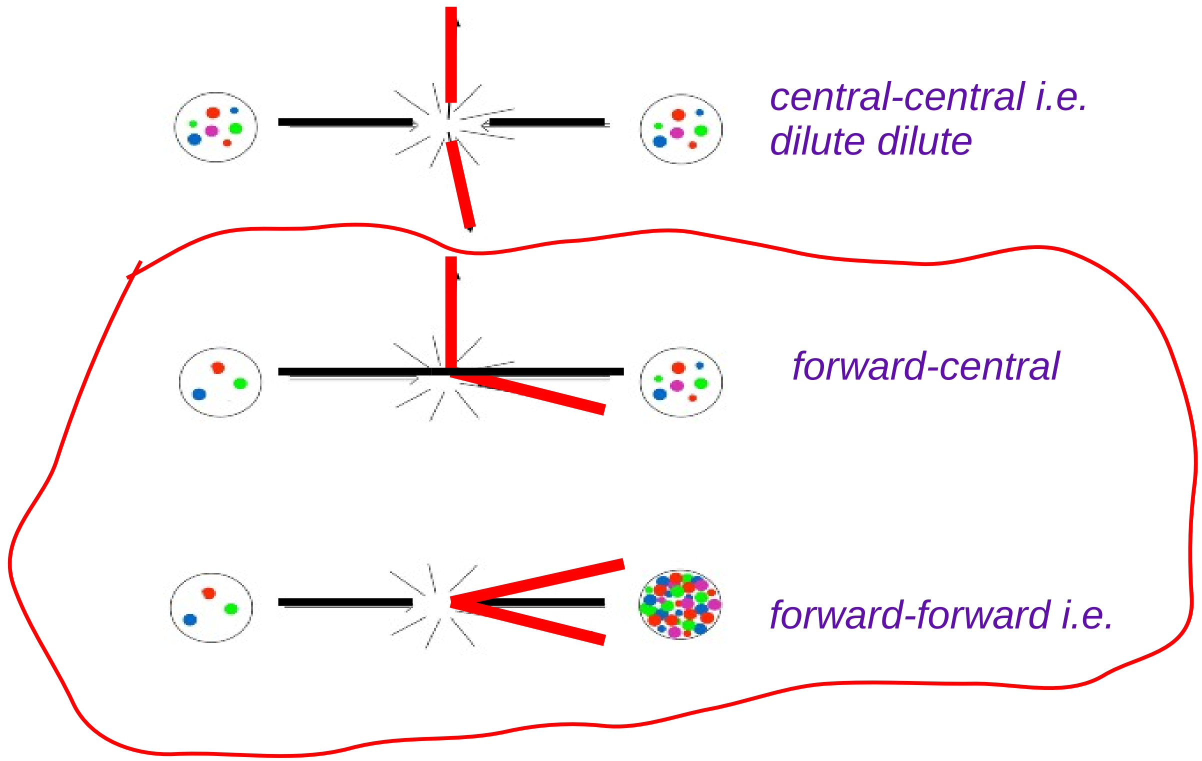
HORIZON 2020

Based on results obtained with:
M. Bury, P. Kotko, C. Marquet, E. Petreska
S. Sapeta, A. van Hameren, E. Zarow

Physics case: di-jets



From: Piotr Kotko



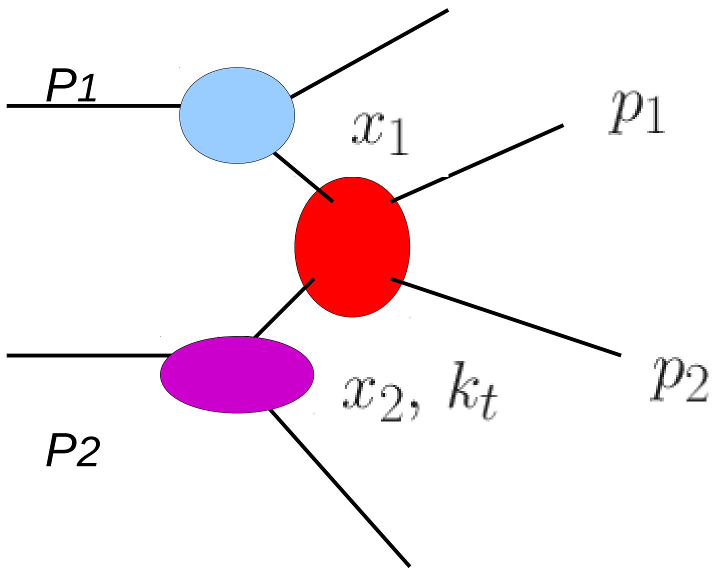
There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not

Hybrid factorization

$$\frac{d\sigma_{\rm SPS}^{P_1P_2\to {\rm dijets}+X}}{dy_1dy_2dp_{1t}dp_{2t}d\Delta\phi} = \frac{p_{1t}p_{2t}}{8\pi^2(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/P_1}(x_1,\,\mu^2)\,|\overline{\mathcal{M}_{ag^*\to cd}}|^2 \quad \mathcal{F}_{g/P_2}(x_2,k_t^2)\frac{1}{1+\delta_{cd}}$$

Can be obtained from CGC after neglecting nonlinearities
In that limit gluon density is just the dipole gluon density

Marquet '06 (coordinate space)
Deak, Jung, KK, Hautmann '09

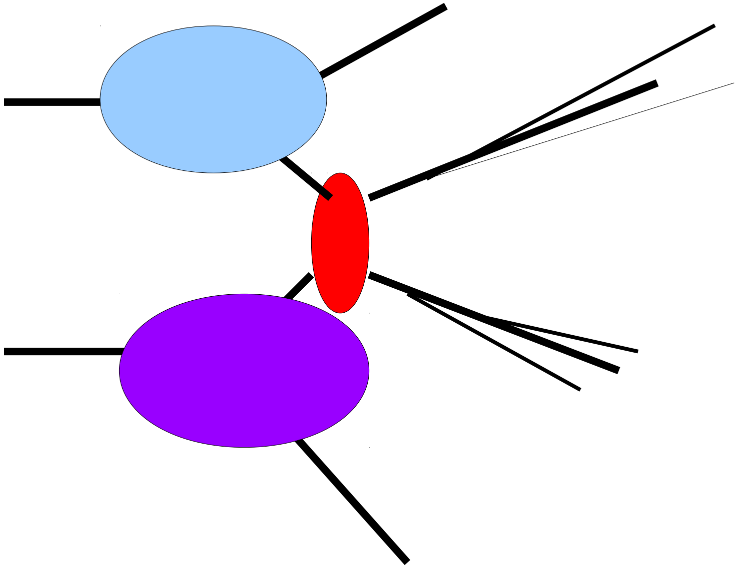


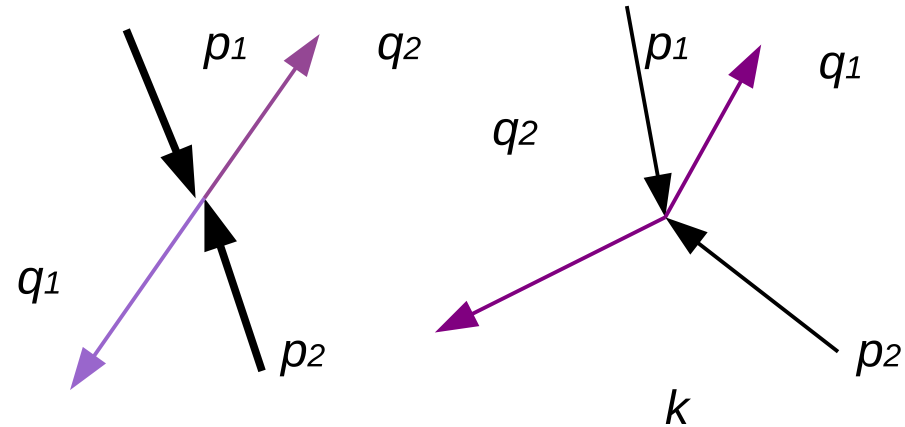
resummation of logs of x

logs of hard scale

knowing well parton densities at large x one can get information about low x physics

Hybrid factorization





$p_1+p_2=q_1+q_2$ $\quad$ $p_1+p_2=q_1+q_2+k$

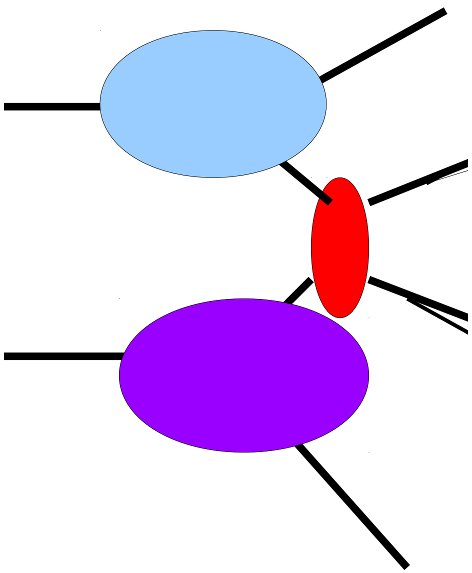
Schematic formula for cross section

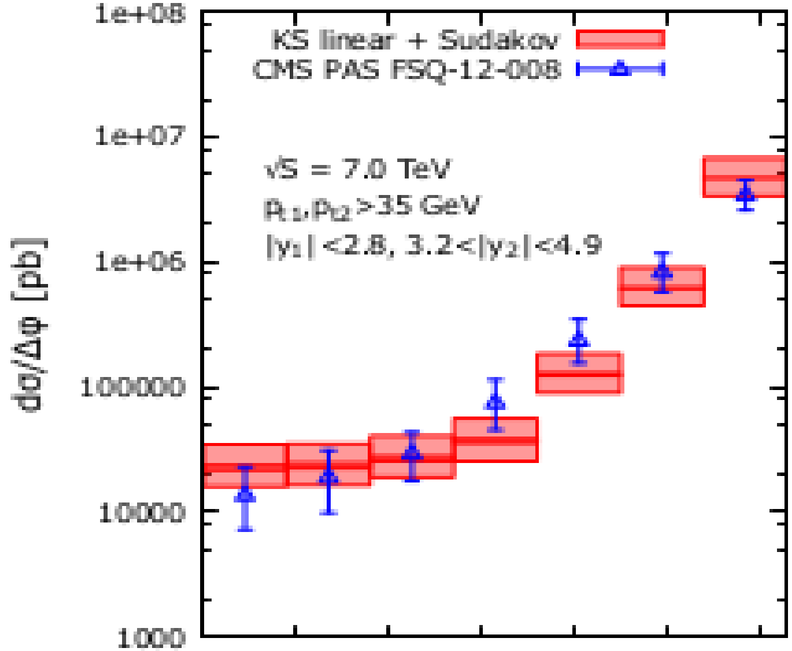
$$\frac{d\sigma}{dPS} \propto xf(x_1,\mu) \otimes \hat{\sigma}(x_1,x_2,k_T) \otimes \mathcal{F}(x_2,k_T)$$

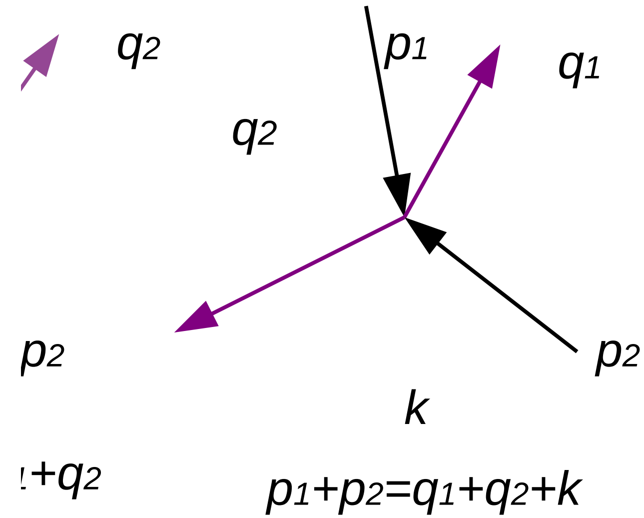
Ciafaloni, Catani, Hautman '93
Collins, Ellis '93

New helicity based methods for ME
van Hameren, Kotko, K.Kutak, JHEP 1301 (2013) 078

Hybrid factorization







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New helicity based methods for ME
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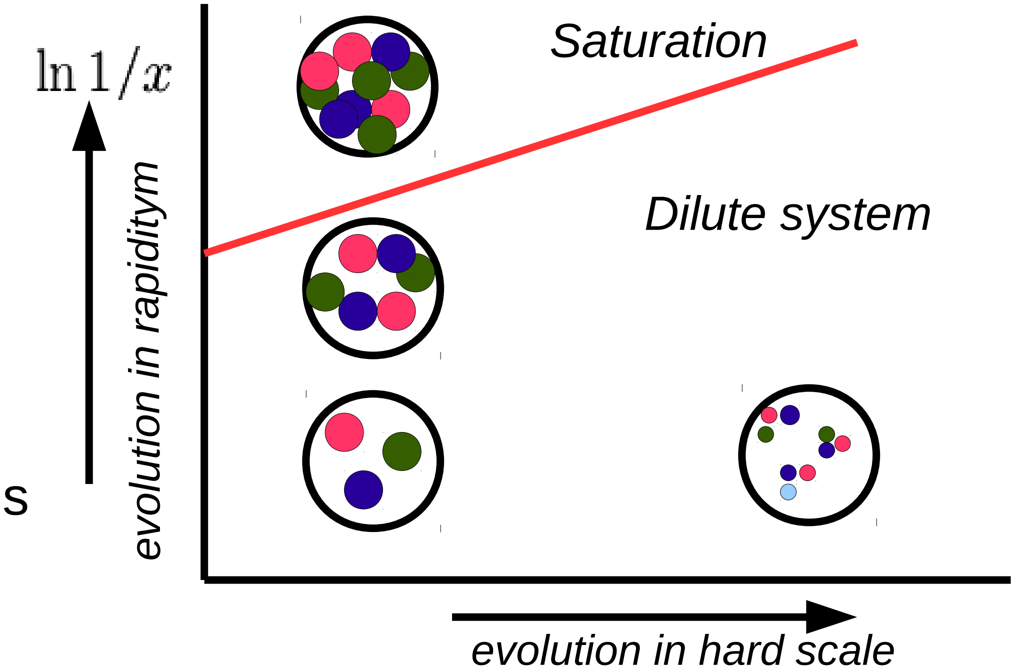
The saturation problem: sensitivity to gluons at small k_t

Saturation – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

Color Glass Condensate

effective field theory describing interactions and evolution of dense system of partons

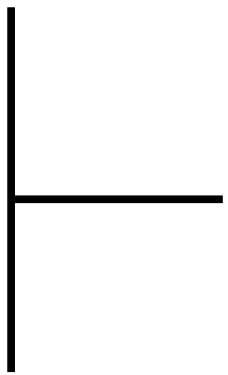
Venugopalan, McLerran, Balitsky, Kovchegov, Iancu, Mueller ….



On microscopic level it means that gluon apart splitting recombine

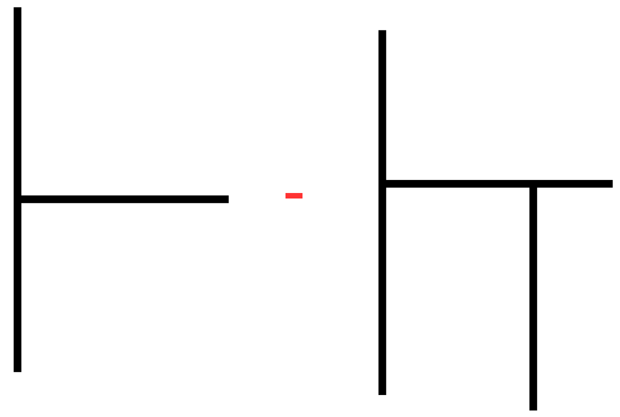
splitting

Linear evolution equation



Nonlinear evolution equations

splitting



recombination

The saturation problem: sensitivity to gluons at small k_t

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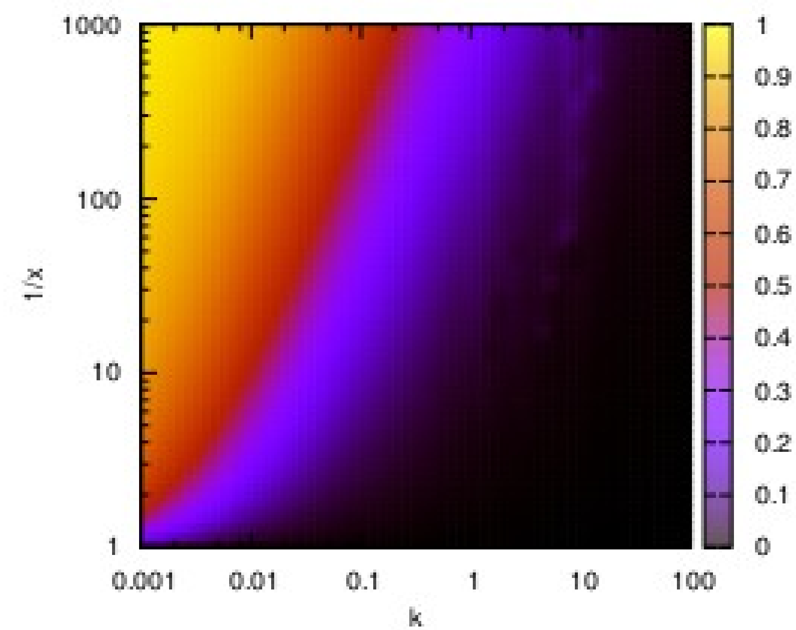

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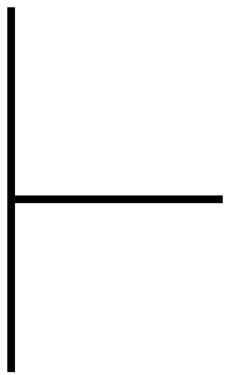

Nonlinear evolution equations

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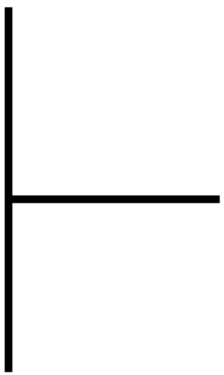

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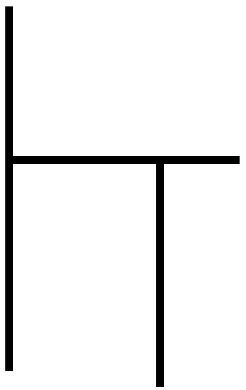
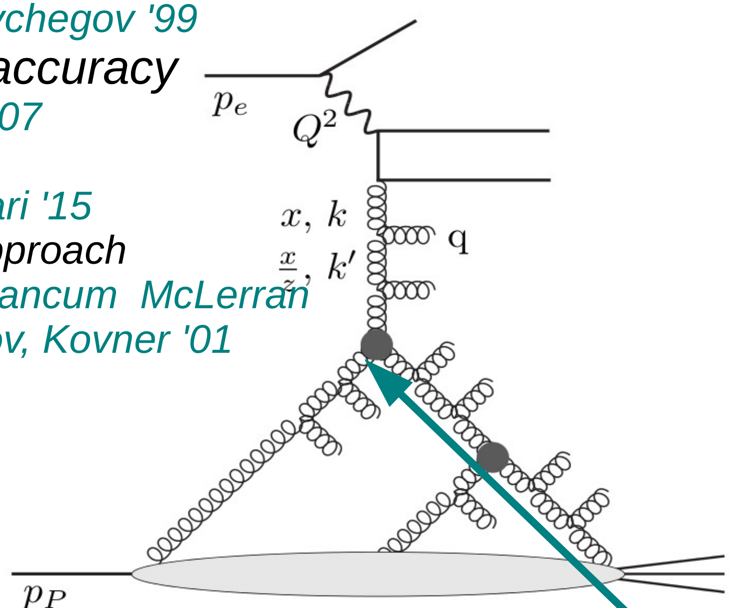
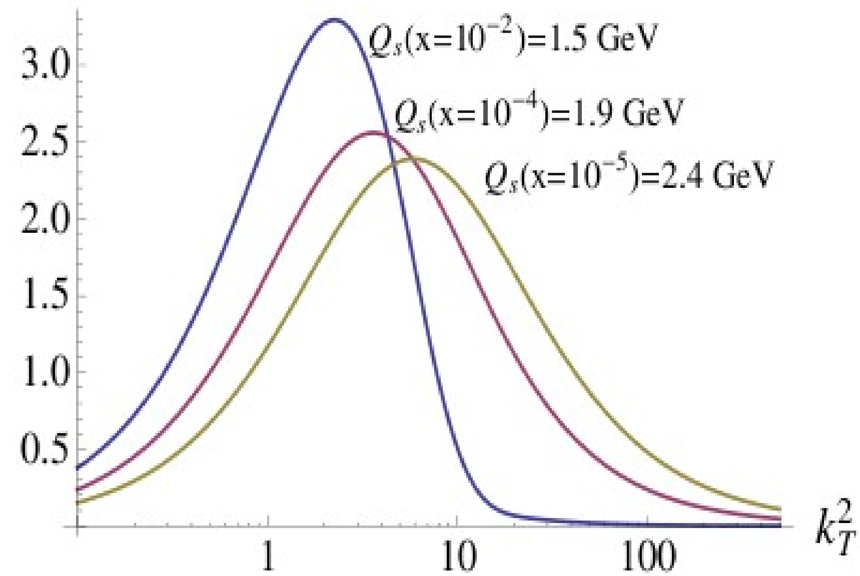
recombination



The saturation problem: supressing gluons at small k_t

Originally formulated in coordinate space
Balitsky '96, Kovchegov '99
Now at NLO accuracy
Balitsky, Chirilli '07
and solved
Lappi, Mantysaari '15
More general approach
Jalian-MarianI, Iancum McLerran Weigert Leonidov, Kovner '01





Solution of the BK equation

The BK equation for dipole gluon density

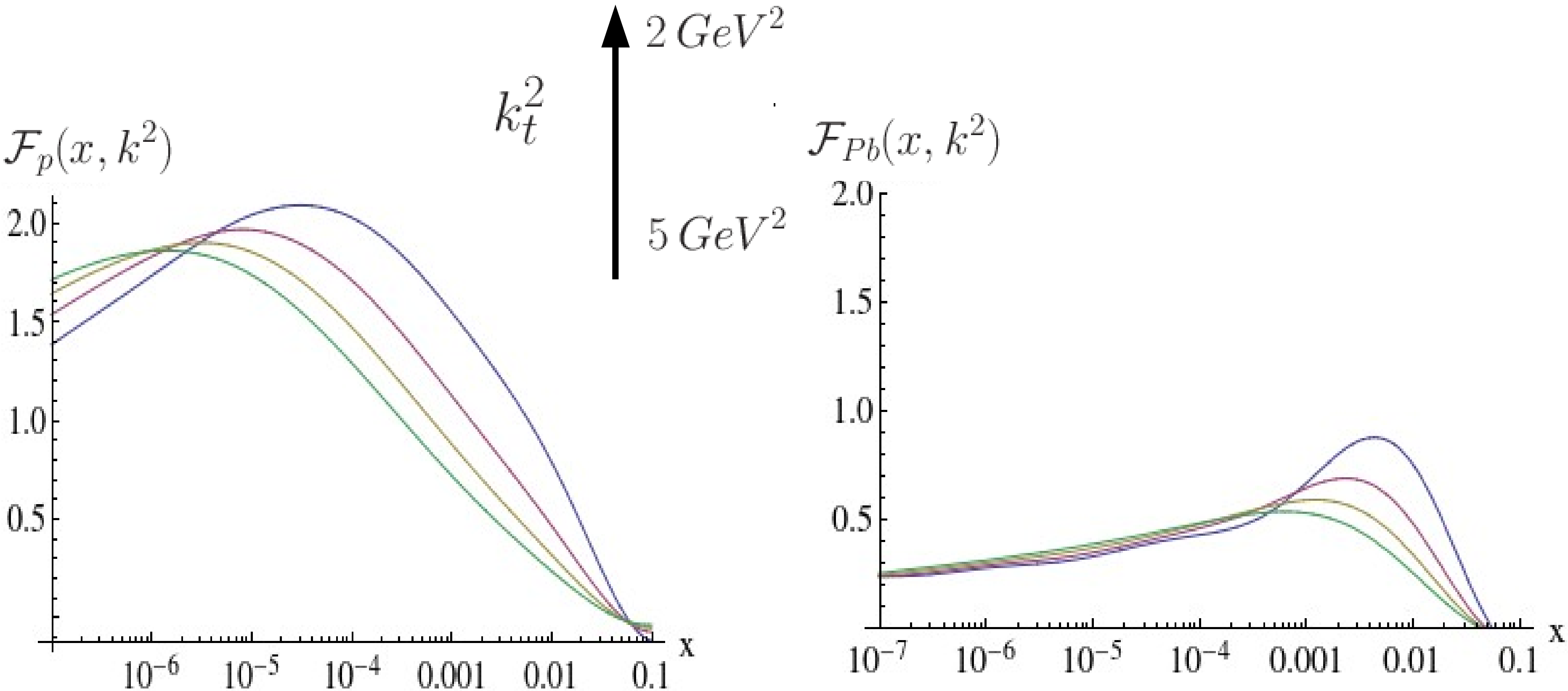
$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F} - \frac{1}{R^2} V \otimes \mathcal{F}^2$$

hadron's radius

Momentum space

Kwiecinski, Kutak '02
Nikolaev, Schafer '06

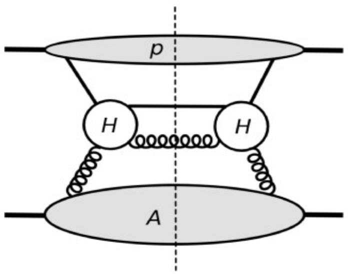
Glue in p vs. glue in Pb



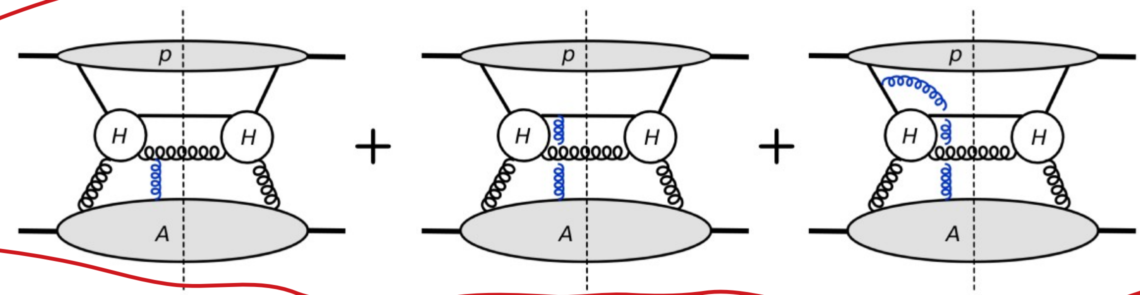
Maximum signalize emergence of saturation scale

Formula for TMD gluons and gauge links

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}|P\rangle$$



Not gauge invariant



From S. Sapeta

similar diagrams with 2,3,….gluon exchanges. All this need to be resummed

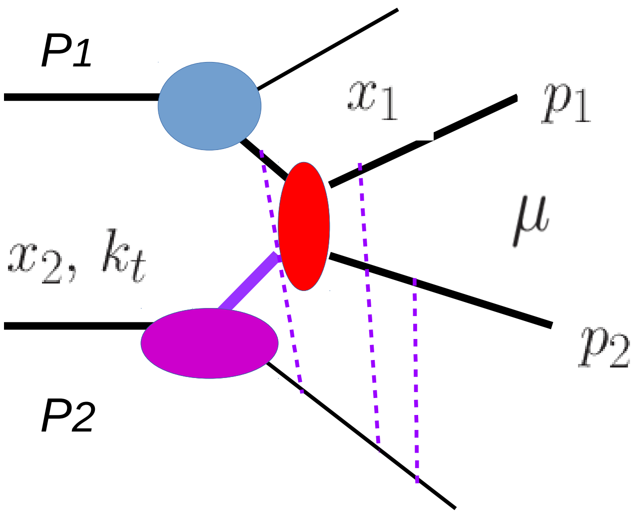
C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\mathcal{U}_{C_1}\hat{F}^{i+}\left(\xi\right)\mathcal{U}_{C_2}\right\}|P\rangle$$

Hard part defines the path of the gauge link

$$\mathcal{U}^{[C]}(\eta;\xi)=\mathcal{P}\exp\left[-ig\int_C \mathrm{d}z\cdot A(z)\right]$$

The ITMD factorization for di-jets



- The color structure is separated from kinematic part of the amplitude by means of the color decomposition.
- The TMD gluon distributions are derived for the color structures following

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska JHEP 12 (2016) 034

The same gauge link and as in TMD 's

Fabio Dominguez, Bo-Wen Xiao, Feng Yuan Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan Phys.Rev. D83 (2011) 105005

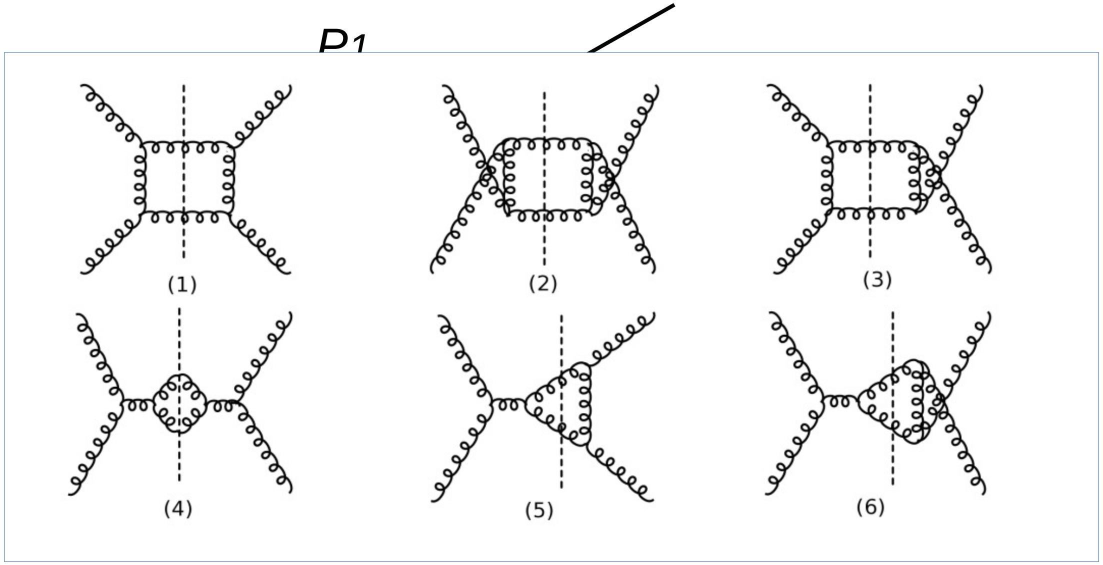
Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

gauge invariant amplitudes with k_t and TMDs

Example for g g → g g*

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2}\, x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

Improved Transverse Momentum Dependent Factorization



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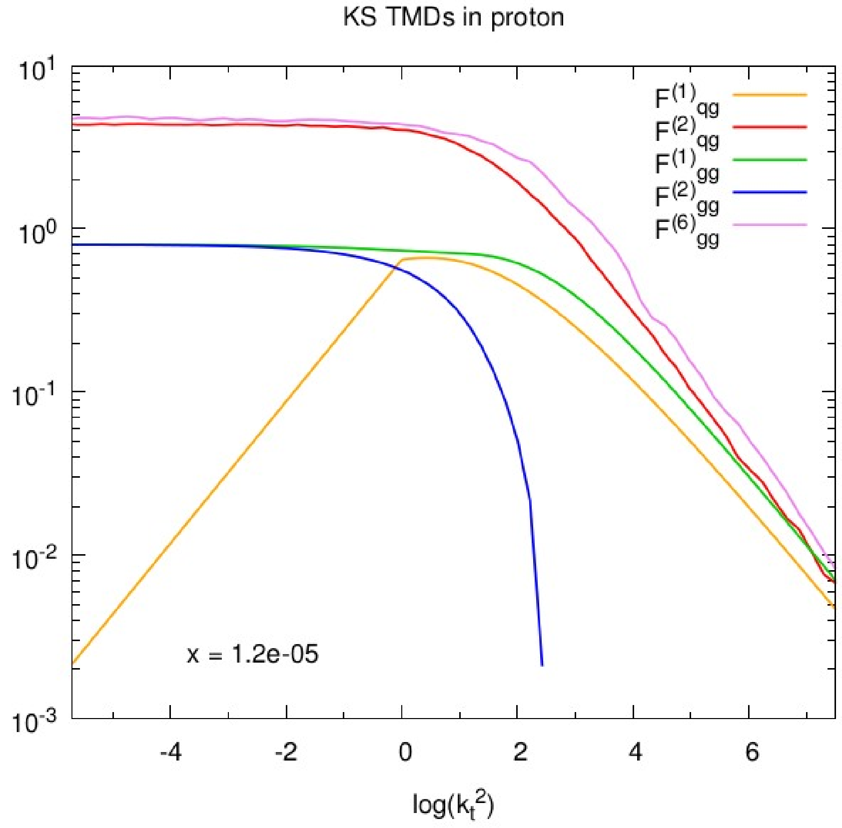
Plots of ITMD gluons



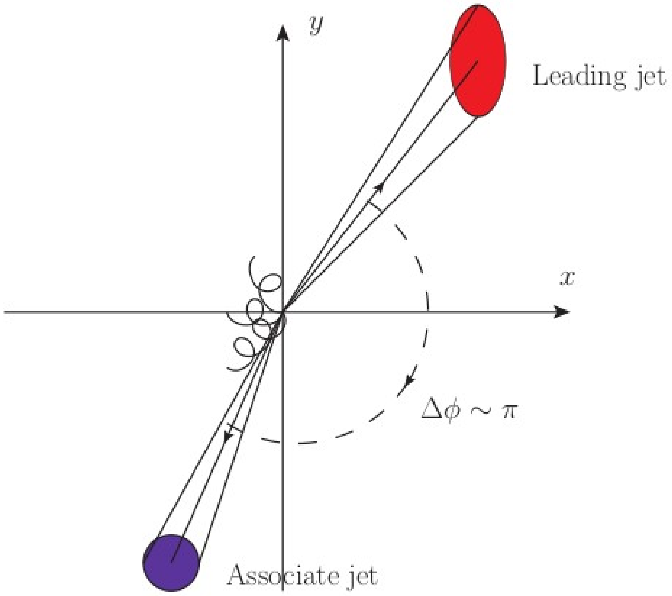
Calculation – in large Nc approximation all gluons can be calculated from the dipole one.

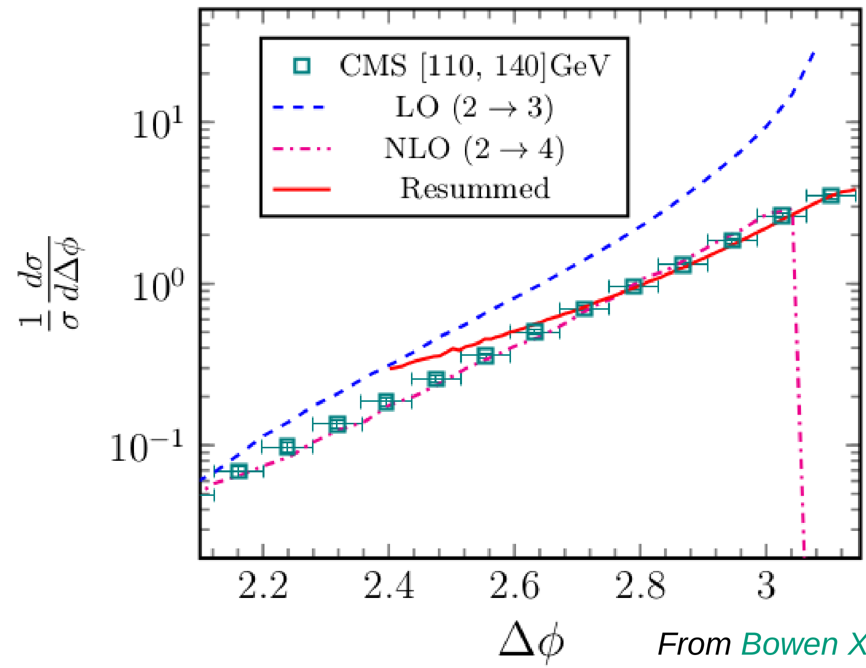
Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren JHEP 1612 (2016) 034

The other densities are flat at low k_t → less saturation

Not negligible differences at large k_t → differences at small angles

Sudakov, back-to-back jets and collinear physics





From Bowen Xiao lecture at QCD 2019 master class

In collinear physics at LO for 2 → 2 we get delta function since the colliding partons do not carry transverse momentum. Adding more jet we get some improvement 2 → 3, 2 → 4. The unobserved partons can be soft and can introduce large logs. Note: k_t factorization also smears the delta function but takes into account also low x effects

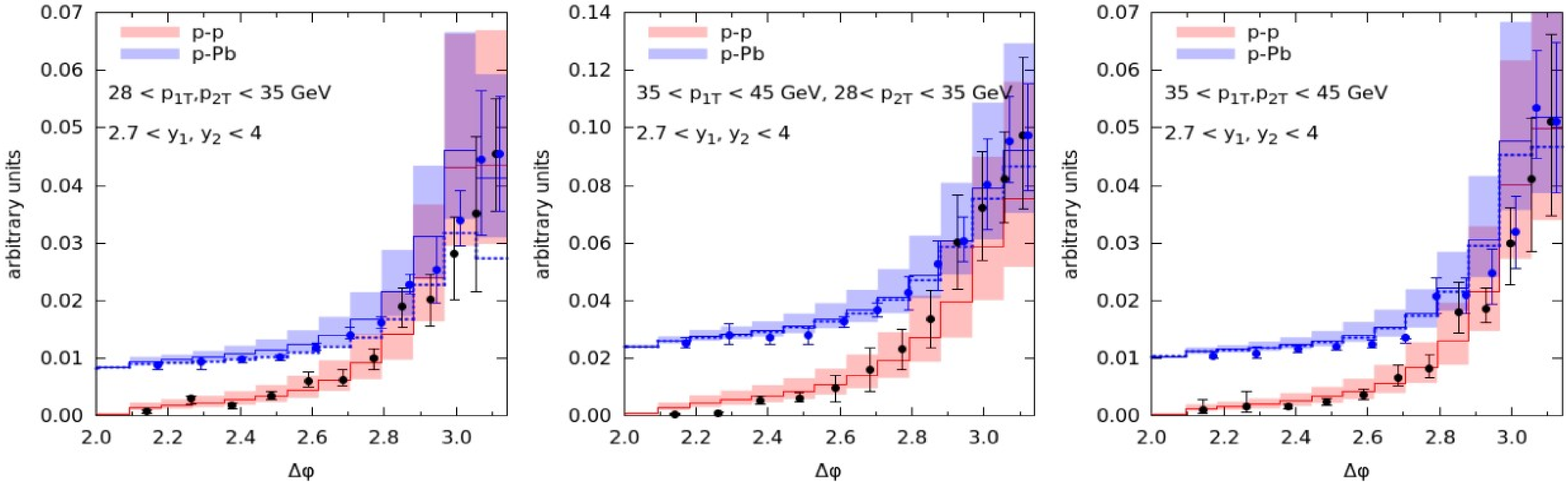
$$p_t >> k_t$$

divergence $L \sim \ln^2 \frac{p_t^2}{k_t^2}$ *needs to be resummed*

leading jet

imbalance between leading jet and associated jet – in forward jet scenario this can be linked to k_t of incoming parton

The ITMD factorization for di-jets in p-p and p-Pb



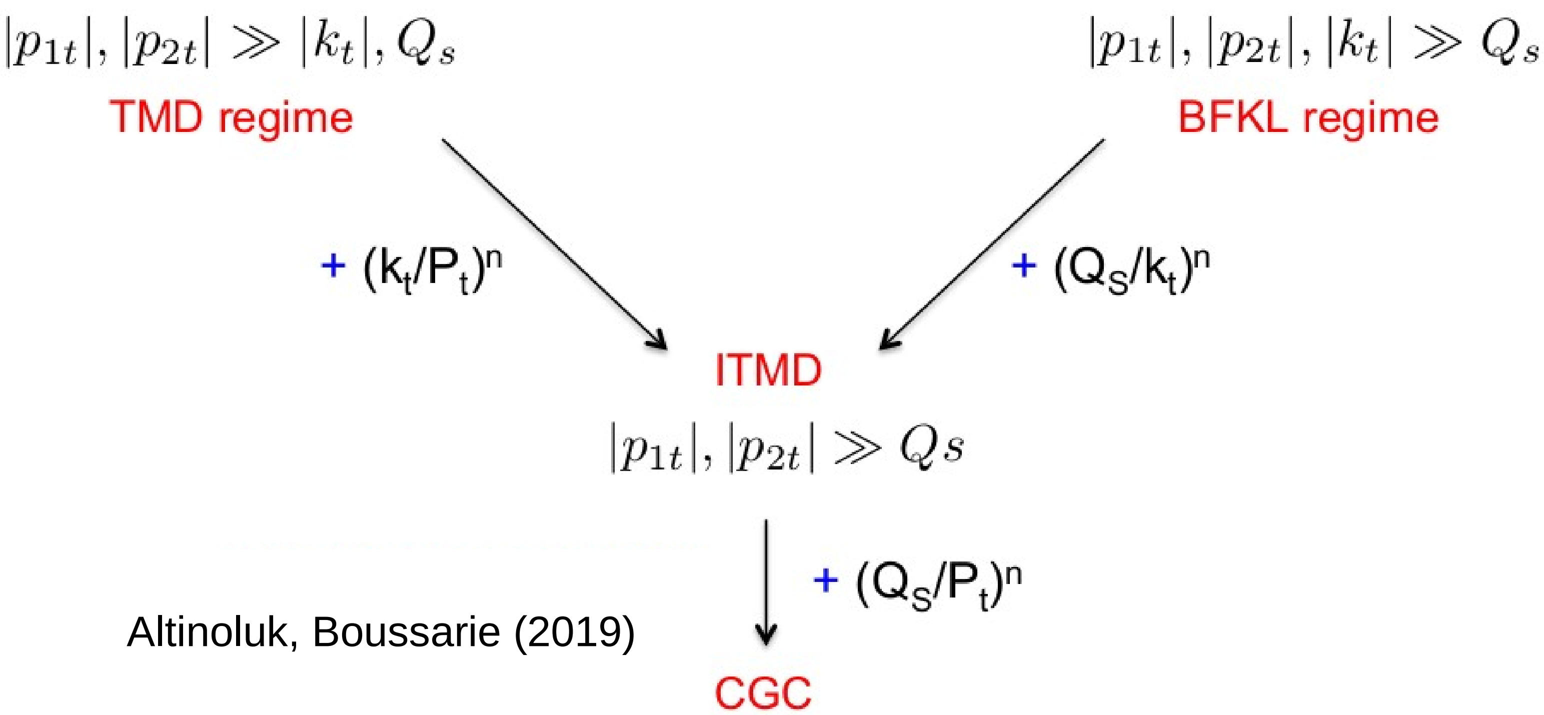
A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

To describe data we had to account both for Sudakov effects and for saturation.

Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

ITMD from Color Glass Condensate

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156
T. Altinoluk, R. Boussarie, JHEP10(2019)208



Altinoluk, Boussarie (2019)

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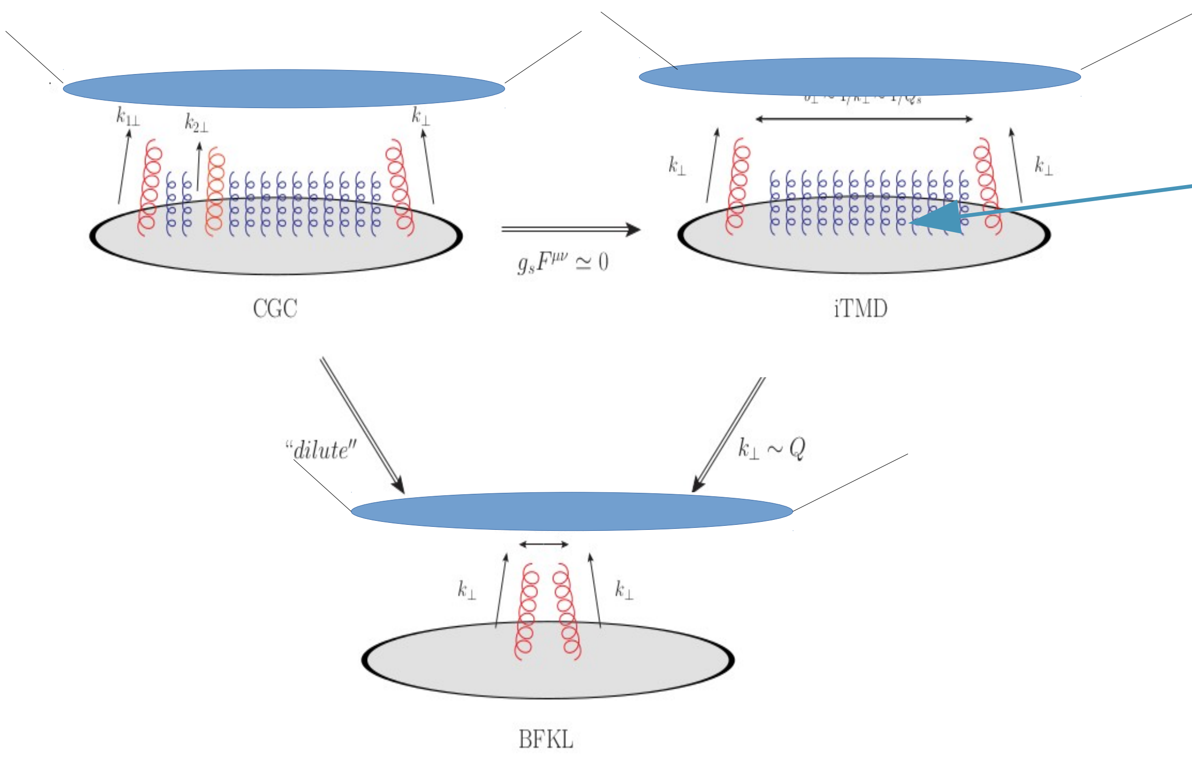
T. Altinoluk, R. Boussarie, JHEP10(2019)208



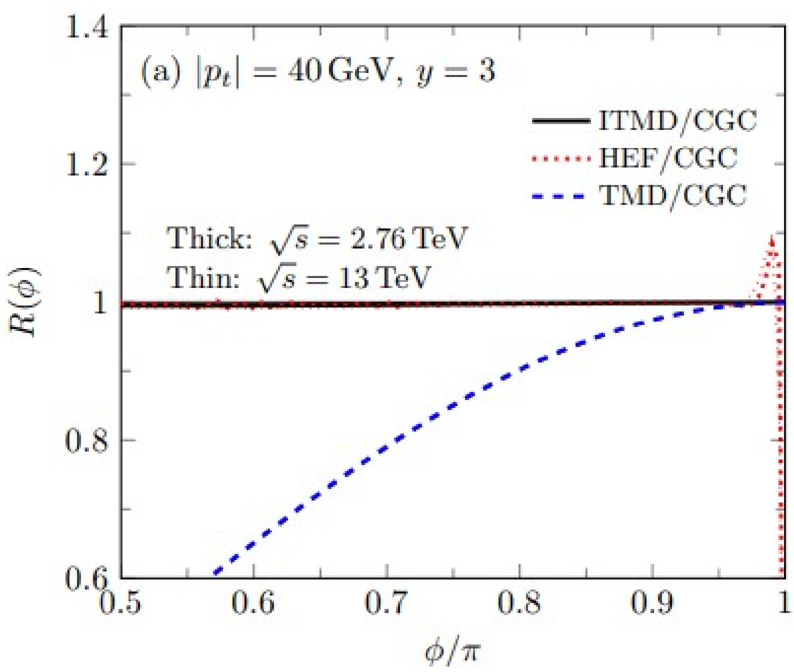
Multiple soft emissions

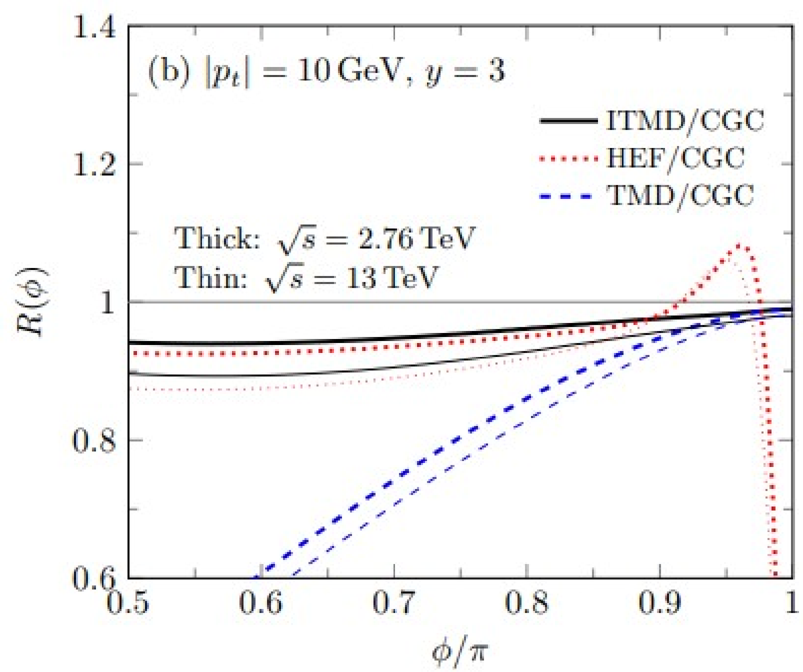
from R. Boussarie
Initial Stages 2019

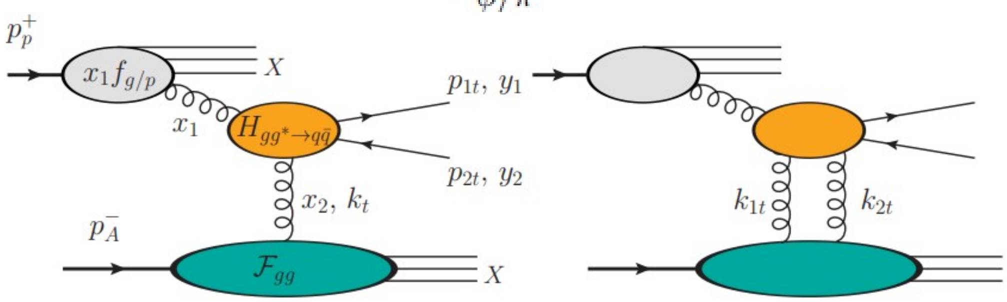
Wandzura-Wilczek approximmation ITMD neglects higher genuine twist contributions i.e. hard gluon exchanges between the target and the amplitude, while it resums all kinematic twist

$$\mathrm{d}\sigma_{\mathrm{CGC}} = \underbrace{\mathrm{d}\sigma_{\mathrm{TMD}} + \overbrace{\mathcal{O}\left(\frac{k_\perp}{Q_\perp}\right)}^{\text{kinematic}}}_{\mathrm{d}\sigma_{\mathrm{ITMD}}} + \overbrace{\mathcal{O}\left(\frac{Q_s}{Q_\perp}\right)}^{\text{genuine}}$$

ITMD vs CGC







H. Fujii, C. Marquet, K. Watanabe, JHEP12(2020)181

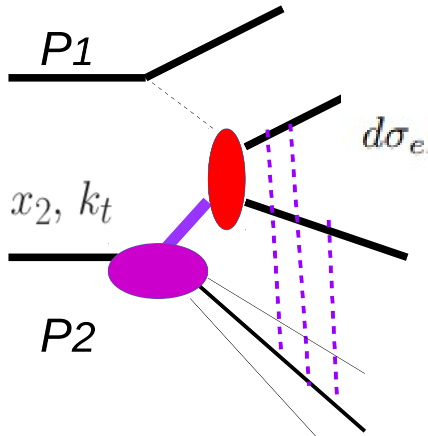
The ITMD formula is a good approximation to the CGC formula in a wide range of azimuthal angle.

See also

Boussarie, Mäntysaari, Salazar, Schenke
2106.11301
Marquet, Altinoluk, Taels, 2021

ITMD with Sudakov for dijets in DIS

P. Kotko, KK. S. Sapeta, A. van Hameren, E. Zarow



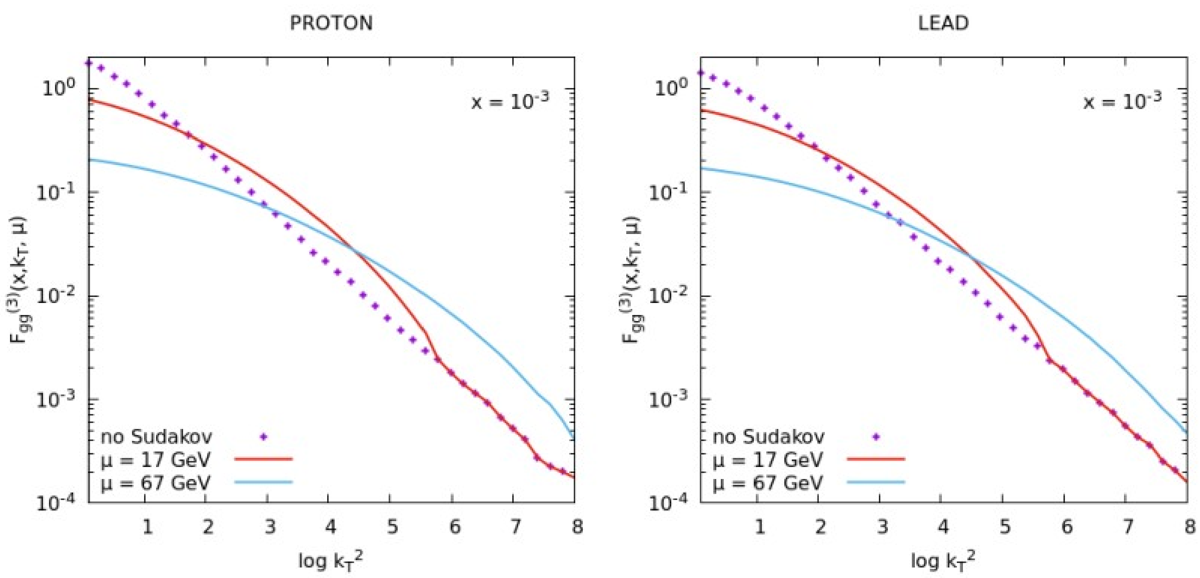
$$d\sigma_{eh\to e'+2j+X} = \int \frac{dx}{x}\frac{d^2k_T}{\pi}\mathcal{F}^{(3)}_{gg}(x,k_T,\mu)\,\frac{1}{4xP_e\cdot P_h}\,d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|$$

$$S^{g\to q\bar{q}}_{\mathrm{Sud}}(\mu, b_T) = \frac{\alpha_s N_c}{4\pi}\ln^2\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

A. Mueller, B-W. Xiao, F. Yuan, 2013

This process allows to probe the Weizsacker-Williams TMD

The Weizsacker-Williams TMD with Sudakov resummation to account for soft emissions



Related studies for dijet/dihadron at EIC

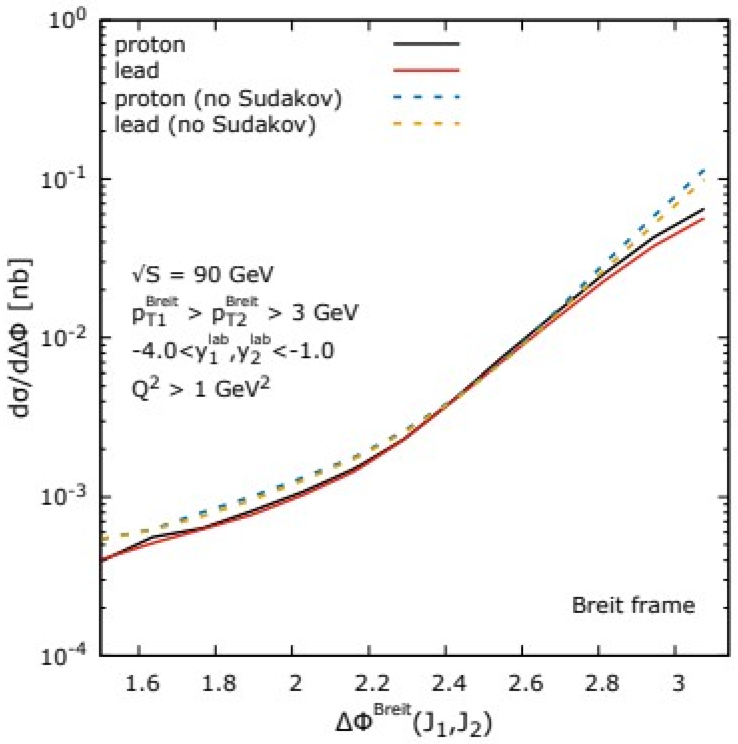
Back-to-back regime using MV model + Sudakov
L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014

Full CGC calculations (no Sudakov)
A. Dumitru, V. Skokov, 2018

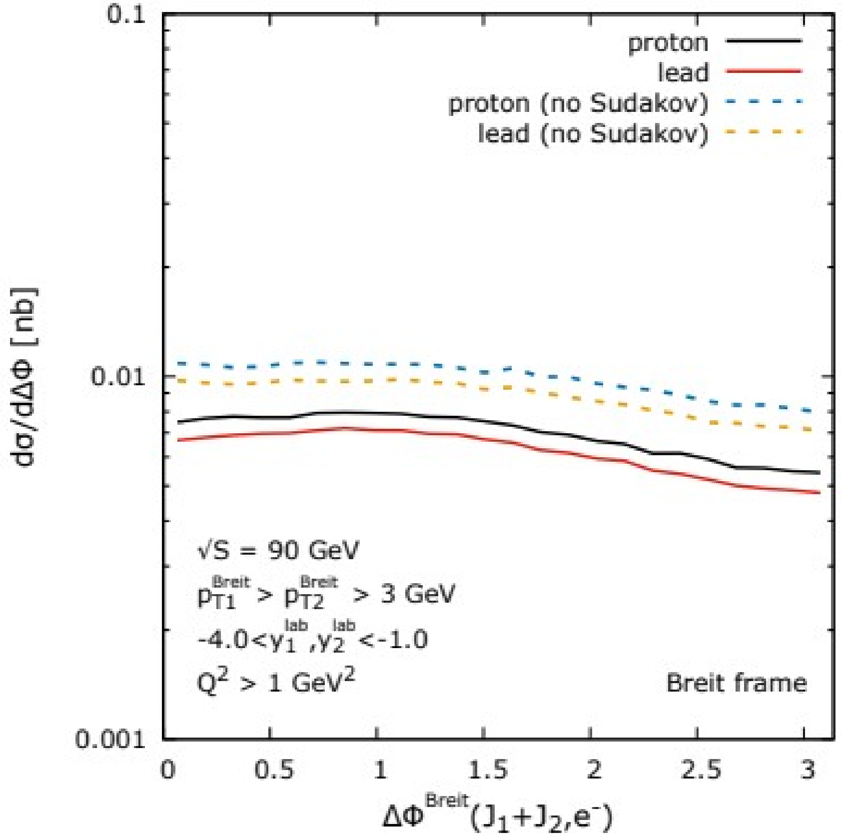
H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019
F. Salazar, B. Schenke, 2020

Azimuthal correlations EIC kinematics

Angle between dijets



New observable – angle between di-jet and electron



Large Sudakov effects.
Rather small saturation effects.

Summary

- *ITMD is a new framework for calculations at forward rapidities*
- *The framework is a very good approximation of CGC when the momenta of jets are larger than Qs*
- *Necessity to have both Sudakov resummation*

I did not have time to discuss formulation of ITMD for:

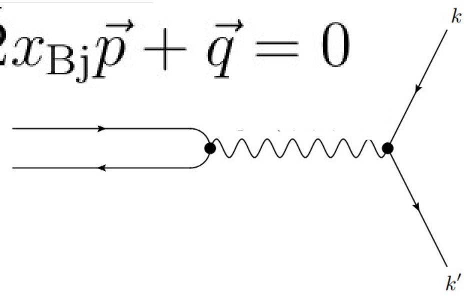
- *Hadrons*
- *Tri-jets ITMD*.*
  In this cases new elements enter like so called linearly polarized gluon distribution

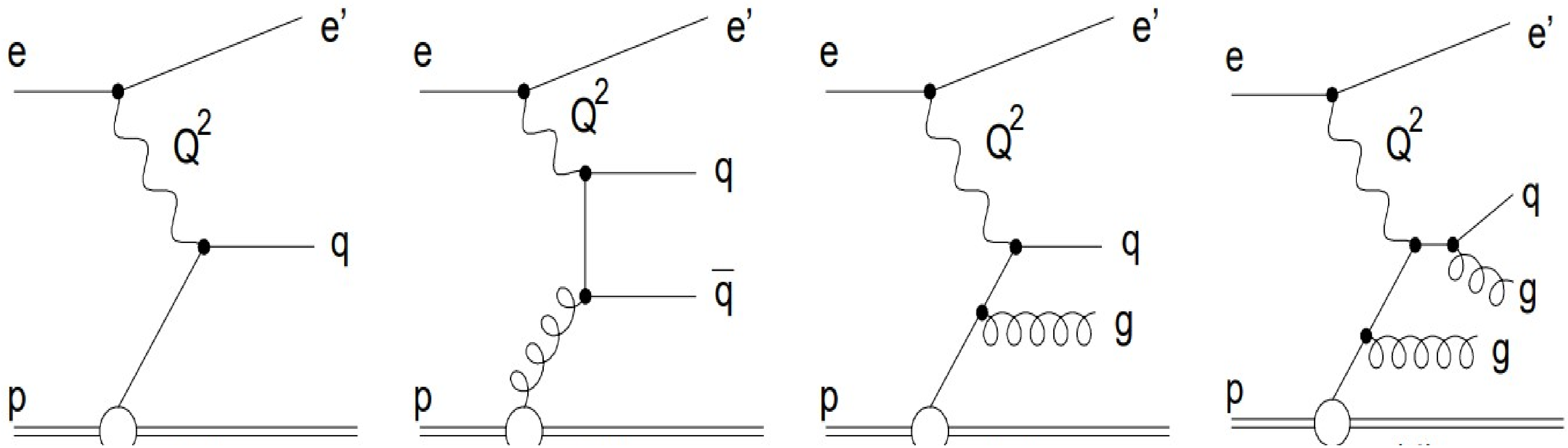
Backup

Background

~ 0 p_T in Breit frame

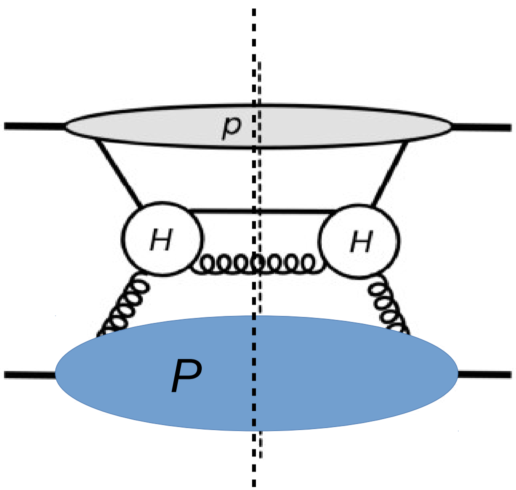
$$2x_{\mathrm{Bj}}\vec{p} + \vec{q} = 0$$

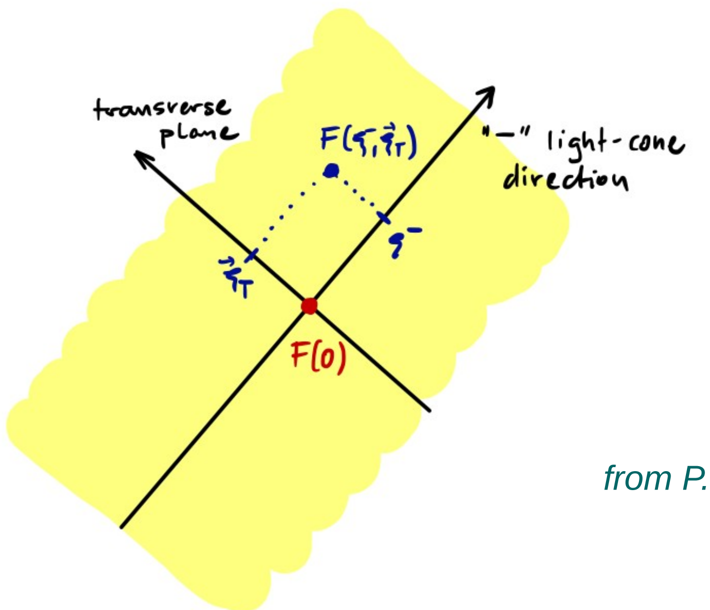




Definition of TMD – gauge links

The formula for HEF is strictly valid for large transversal momentum and was obtained in a specific gauge. Ultimately we want to go beyond this.





from P. Kotko, Bialasówka 2019

Naive definition of gluon distribution

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}|P\rangle$$

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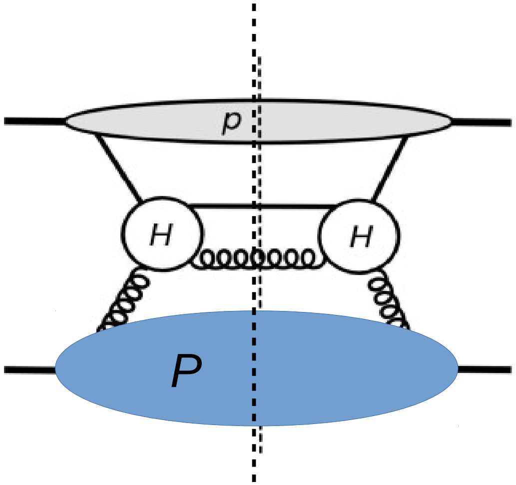
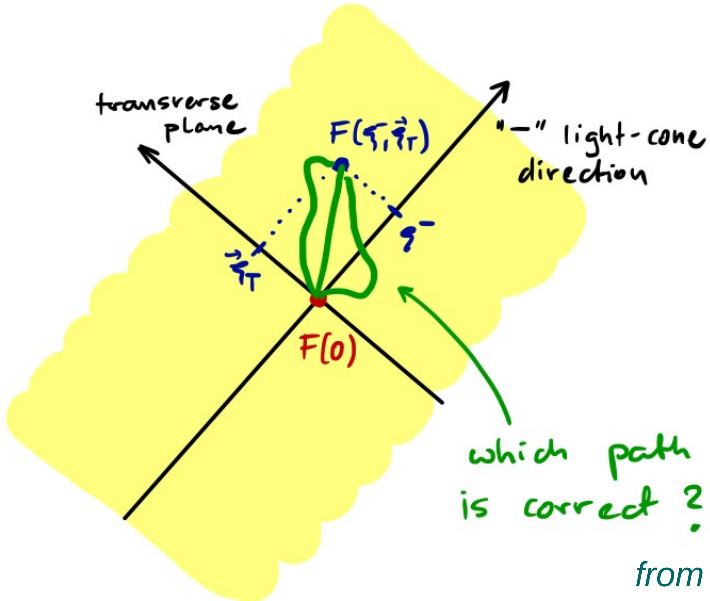




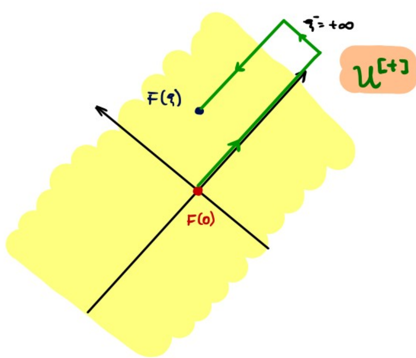
from P. Kotko, Bialasówka 2019

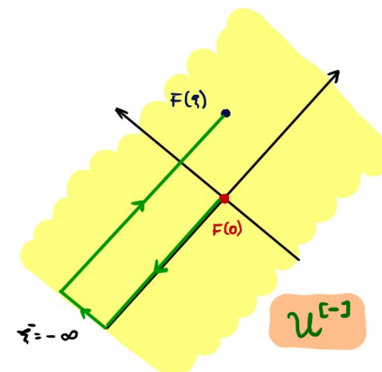
Naive definition

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- -i\vec{k}_T\cdot\vec{\xi}_T}\,\left\langle P\right|\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}\left|P\right\rangle$$

Definition of TMD – gauge links

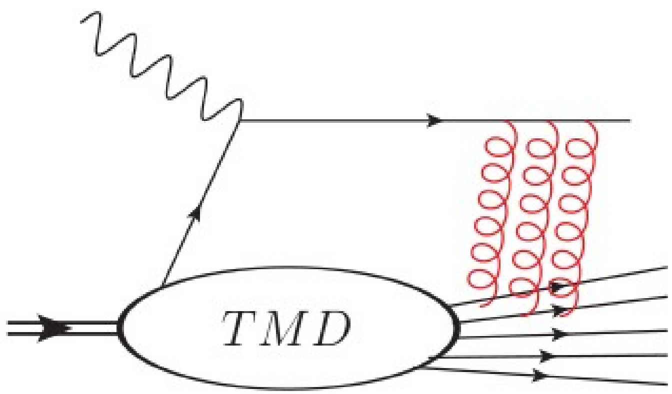
Two basic structures arise:





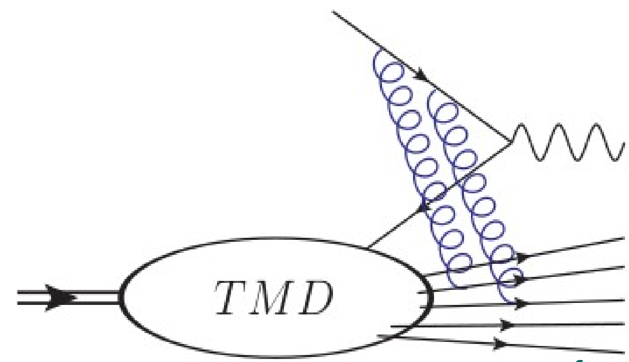
from P. Kotko, Bialasówka 2019

Semi Inclusive DIS



final state interactions

Drell-Yan



initial state interactions

from R. Boussarie Initial Stages 2019

$$\Phi_q^{[+]}(x,p_T) = \int \frac{\mathrm{d}(\xi\cdot P)\,\mathrm{d}^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle H|\,\overline{\psi}(0)\,\mathcal{U}^{[+]}\,\psi(\xi)\,|H\rangle$$

C.J. Bomhof, P.J. Mulders, F. Pijlman Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{U}^{[\pm]} = U^n_{[(0^-,\mathbf{0}_T);(\pm\infty^-,\mathbf{0}_T)]}\,U^T_{[(\pm\infty^-,\mathbf{0}_T);(\pm\infty^-,\boldsymbol{\infty}_T)]}\,U^T_{[(\pm\infty^-,\boldsymbol{\infty}_T);(\pm\infty^-,\boldsymbol{\xi}_T)]}\,U^n_{[(\pm\infty^-,\boldsymbol{\xi}_T);(\xi^-,\boldsymbol{\xi}_T)]}$$