All-plus helicity off-shell gauge invariant multigluon amplitudes at one loop

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Motivations / Context

Formulation

$$d\sigma(ab \to \mathcal{X}) = \sum_{i,j} \int \frac{d^2\mathbf{k}_{T,i}}{\pi} \frac{d^2\mathbf{k}_{T,j}}{\pi} dx_i dx_j\, f_{i,a}(x_i, \mathbf{k}_{T,i}, \mu) f_{j,b}(x_j, \mathbf{k}_{T,j}, \mu) \times$$
$$\times d\hat{\sigma}(i^* j^* \to \mathcal{X})(x_i, \mathbf{k}_{T,i}, x_j, \mathbf{k}_{T,j}, \mu)$$

- with $k_i = x_i p_a + k_{T,i}$, $k_j = x_j p_b + k_{T,j}$
  p_{ab} : hadron momentum, $k_{T,ij}$: transverse momentum
- Suitable for high energy collisions

S. Catani, M. Ciafaloni, and F. Hautmann. HIGH-ENERGY FACTORIZATION AND SMALL X HEAVY FLAVOR PRODUCTION. ***Nucl. Phys. B*, 366:135–188, 1991**

J. C Collins and R K. Ellis. HEAVY-QUARK PRODUCTION IN VERY HIGH ENERGY HADRON COLLISIONS. ***Nuclear Physics B*, 360(1):3–30, 1991**

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$$\times d\hat{\sigma}(i^* j^* \to \mathcal{X})(x_i, \mathbf{k}_{T,i}, x_j, \mathbf{k}_{T,j}, \mu)$$

- TMDs follow BFKL equation
- Incorporate subleading power corrections
  $\to$ allow to study azimuthal correlations at lowest order

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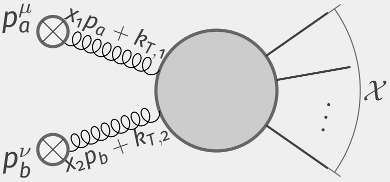
$$\times d\hat{\sigma}(i^* j \to \mathcal{X})(x_i, \mathbf{k}_{T,i}, x_j, \mu)$$

- with $k_i = x_i p + k_{T,i}$
  $p = p_a$
- Suitable for forward particle production

A. Dumitru, A. Hayashigaki, and J. Jalilian-Marian. The Color glass condensate and hadron production in the forward region. ***Nucl. Phys. A*, 765:464–482, 2006**

C. Marquet. Forward inclusive dijet production and azimuthal correlations in p(A) collisions. ***Nucl. Phys. A*, 796:41–60, 2007**

M. Deak, F. Hautmann, H. Jung, and K. Kutak. Forward Jet Production at the Large Hadron Collider. ***JHEP*, 09:121, 2009**

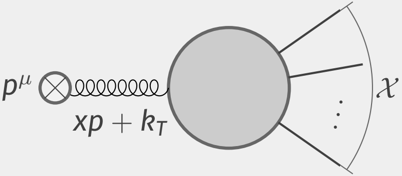


$+$ terms required for gauge invariance

Calculation

Two approaches :

- Lipatov's high energy effective action
- Explicit construction of the terms required by gauge invariance



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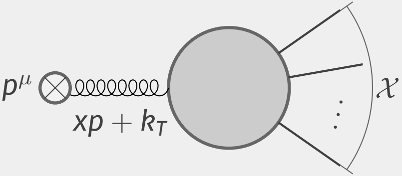


$+$ terms required for gauge invariance

2nd approach

One way : embedding the off-shell process into an on-shell one

- effective at tree-level

  A. van Hameren, P. Kotko, and K. Kutak. Helicity amplitudes for high-energy scattering. ***JHEP*, 01:078, 2013**

- implemented in KaTie (also for 0, 1 or 2 off-shell gluons)

  A. van Hameren. KaTie : For parton-level event generation with k_T-dependent initial states. ***Comput. Phys. Commun.*, 224:371–380, 2018**

Processes considered

$$g^* g^+ \cdots g^+$$

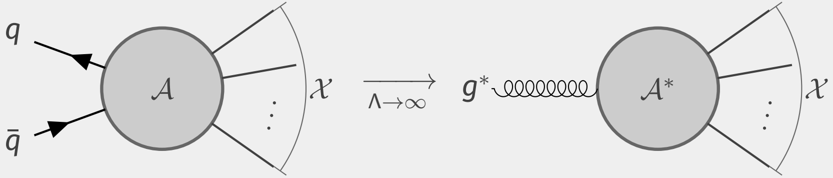
- 1st step to generalize our method at one-loop
- Null at tree level
- Important in forward particle production processes
- (Feasibility)

Color decomposition

$$\mathcal{M}^{a_1,\ldots,a_n}_{\lambda_1,\ldots,\lambda_n}(k_1,\ldots,k_n) = \sum_{\text{perm.}(2\cdots n)} \mathrm{Tr}\left(t^{a_1}t^{a_2}\ldots t^{a_n}\right) \times \\ \times \mathcal{A}\left(1^{(\lambda_1)}, 2^{(\lambda_2)}, \ldots, n^{(\lambda_n)}\right)$$

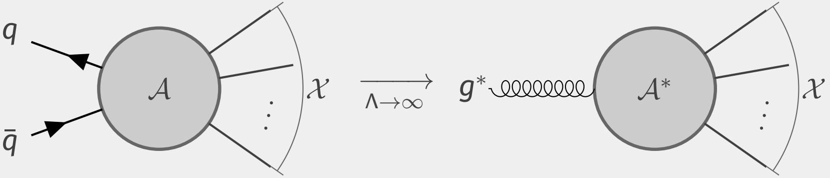
- Use of spinor-helicity methods

Method



On-shell process

- use of auxiliary quark line



Limit

- Λ parametrize the quark-antiquark pair momenta
- $k^\mu = xp^\mu + k_T^\mu$
- Limit taken such as the coupling to the quark line is eikonal

$$\lim_{\Lambda\to\infty}\left(\frac{x|k_T|}{g_s\Lambda}\mathcal{A}\left(\bar{q}(k_1)q(k_2)\mathcal{X}\right)\right) = \mathcal{A}^*\left(g^*(k)\mathcal{X}\right)$$

Auxiliary quark line kinematics

$$k_1^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \,,$$
$$k_2^\mu = (x - \Lambda) p^\mu - \alpha q^\mu + (1 - \beta) k_T^\mu \,,$$

where $\alpha = \dfrac{-\beta^2 k_T^2}{2\Lambda p \cdot q}, \quad \beta = \dfrac{1}{1 + \sqrt{1 - x/\Lambda}}$

$$p^2 = q^2 = 0, \quad p \cdot q > 0, \quad p \cdot k_T = q \cdot k_T = 0$$

High energy kinematics

$$k^\mu = k_1^\mu + k_2^\mu = x p^\mu + k_T^\mu$$

k_T decomposition

$$k_T^\mu = -\bar{\kappa} e^\mu - \bar{\kappa}^* e_*^\mu,$$

with $\quad e^\mu = \frac{1}{2}\langle p|\gamma^\mu|q], \quad e_*^\mu = \frac{1}{2}\langle q|\gamma^\mu|p]$

and $\quad \bar{\kappa} = \frac{\kappa}{[pq]} = \frac{\langle q|\not{k}|p]}{2p\cdot q},$

$$\bar{\kappa}^* = \frac{\kappa^*}{\langle qp\rangle} = \frac{\langle p|\not{k}|q]}{2p\cdot q}$$

we notice that $\quad k_T^2 = -\kappa\kappa^*$

Spinors

$$\begin{aligned}
|i^\pm\rangle &= u_\pm(k_i) \\
&= v_\mp(k_i) \\
\langle i^\pm| &= \bar{u}_\pm(k_i) \\
&= \bar{v}_\mp(k_i) \\
\langle ij\rangle &= \langle i^-|j^+\rangle \\
&= \bar{u}_-(k_i)u_+(k_j) \\
[ij] &= \langle i^+|j^-\rangle \\
&= \bar{u}_+(k_i)u_-(k_j)
\end{aligned}$$

On-shell limit

$$\lim_{|k_T|\to 0} \mathcal{A}_n^{*(0)}(g^* \mathcal{X}) = \frac{|k_T|}{\kappa^*}\mathcal{A}_n^{(0)}(g^- \mathcal{X}) + \frac{|k_T|}{\kappa}\mathcal{A}_n^{(0)}(g^+ \mathcal{X})$$

A. van Hameren. BCFW recursion for off-shell gluons. ***JHEP*, 07:138, 2014**

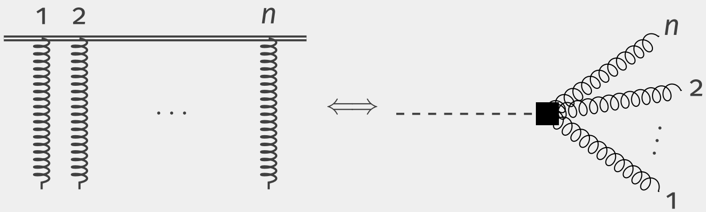
Equivalence between auxiliary partons

Using auxiliary quarks or auxiliary gluons has been shown to be equivalent at tree level.

A. van Hameren. Calculating off-shell one-loop amplitudes for k_T-dependent factorization: a proof of concept. **2017**

Equivalence

- Equivalent to Lipatov's effective action
- Equivalent to using eikonal Feynman rules for the auxiliary quark line



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Eikonal Feynman rules

- Vertex : $-ig_s T^a_{ij}\gamma^\mu \to ig_s T^a_{ij} p^\mu$
- Propagators : $i\frac{\delta_{ij}}{\not{k}} \to \frac{\delta_{ij}}{p\cdot k}$

A. van Hameren, P. Kotko, and K. Kutak. Helicity amplitudes for high-energy scattering. *Journal of High Energy Physics*, 2013(1):1–26, 2013

Results

Amplitudes calculated

1. $g^* g^+ g^+$
2. $g^* g^+ g^+ g^+$
3. $g^* g^+ g^+ g^+ g^+$
4. $g^* g^+ \cdots g^+$

$\bar{q}^- q^+ g^+ \cdots g^+$ amplitude

$$\mathcal{A}^{(1)}_{n+1}(1^-_{\bar{q}}, 2^+_q, 3^+, \cdots, (n+1)^+) = \frac{i g_s^{n+1}}{32\pi^2}\left(1 + \frac{1}{N_c^2}\right) \frac{\sum_{l=3}^{n} \langle 1|\not{K}_{2\cdots l}\not{k}_l|1\rangle}{\langle 23\rangle \cdots \langle (n+1)1\rangle} + \frac{i g_s^{n+1}}{48\pi^2}\left(1 + \frac{n_s - n_f}{N_c}\right) \frac{S_1 + S_2}{\langle 12\rangle\langle 23\rangle \cdots \langle (n+1)1\rangle},$$

with

$$S_1 = \sum_{j=3}^{n} \frac{\langle 2j\rangle \langle 1(j+1)\rangle \langle 1|\not{K}_{j,j+1}\not{K}_{(j+1)\cdots(n+1)}|1\rangle}{\langle j(j+1)\rangle},$$

$$S_2 = \sum_{j=3}^{n-1} \sum_{l=j+1}^{n} \frac{\langle 1|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|1\rangle^2 \langle 2|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|1\rangle}{\langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|(j-1)\rangle \langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|j\rangle} \times \frac{\langle (j-1)j\rangle \langle l(l+1)\rangle \langle 1|\not{K}_{2\cdots(j-1)}[\mathcal{F}(j,l)]^2 \not{K}_{(l+1)\cdots(n+1)}|1\rangle}{\langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|l\rangle \langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|(l+1)\rangle s_{j\cdots l}},$$

where $\mathcal{F}(j,l) = \sum_{i=j}^{l-1} \sum_{m=i+1}^{l} \not{k}_i \not{k}_m$.

Z. Bern, L. J Dixon, and D. A Kosower. LAST OF THE FINITE LOOP AMPLITUDES IN QCD. ***Physical Review D*, 72(12):125003, 2005**

Λ prescription

$$\mathcal{A}_n^{*(1)}(g^*, 3^+, \cdots, (n+1)^+) = \frac{i g_s^n x |k_T|}{48\pi^2} \left(1 + \frac{n_s - n_f}{N_c}\right) \frac{U_1^* + U_2^* + U_3^*}{\kappa^* \langle p3\rangle\langle 34\rangle \cdots \langle np\rangle} \,,$$

$$\text{with } U_1^* = \sum_{j=3}^{n} \frac{\langle pj\rangle\langle p(j+1)\rangle\langle p|\not{K}_{j,j+1}\not{K}_{(j+1)\cdots(n+1)}|p\rangle}{\langle j(j+1)\rangle} \,,$$

$$U_2^* = \sum_{j=4}^{n-1}\sum_{l=j+1}^{n} \frac{\langle p|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|p\rangle^3}{\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|(j-1)\rangle\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|j\rangle}$$

$$\times \frac{\langle (j-1)j\rangle\langle l(l+1)\rangle\langle p|\not{K}'_{3\cdots(j-1)}[\mathcal{F}(j,l)]^2\not{K}_{(l+1)\cdots(n+1)}|p\rangle}{\langle p|\not{K}_{3\cdots(j-1)}\not{K}_{j\cdots l}|l\rangle\langle p|\not{K}_{3\cdots(j-1)}\not{K}_{j\cdots l}|(l+1)\rangle s_{j\cdots l}} \,,$$

$$U_3^* = \sum_{l=4}^{n} \frac{\langle p|\not{K}_{3\cdots l}\not{K}_{(l+1)\cdots(n+1)}|p\rangle^3}{\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{3\cdots l}|p\rangle\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{3\cdots l}|3\rangle}$$

$$\times \frac{\langle p3\rangle\langle l(l+1)\rangle[p|[\mathcal{F}(3,l)]^2\not{K}_{(l+1)\cdots(n+1)}|p\rangle}{\kappa^*[p|\not{K}_{3\cdots l}|l\rangle[p|\not{K}_{3\cdots l}|(l+1)\rangle s_{3\cdots l}} \,.$$

Λ prescription

$$\mathcal{A}_n^{*(1)}(g^*, 3^+, \cdots, (n+1)^+) = \frac{i g_s^n x |k_T|}{48\pi^2}\left(1 + \frac{n_s - n_f}{N_c}\right)\frac{U_1^* + U_2^* + U_3^*}{\kappa^* \langle p3\rangle\langle 34\rangle \cdots \langle np\rangle},$$

$$\text{with } U_1^* = \sum_{j=3}^{n} \frac{\langle pj\rangle\langle p(j+1)\rangle\langle p|\slashed{K}_{j,j+1}\slashed{K}_{(j+1)\cdots(n+1)}|p\rangle}{\langle j(j+1)\rangle},$$

- Same result with auxiliary gluon line
- Right on-shell limit
- 3-point amplitude agrees with Lipatov's effective action results
  M. Nefedov. One-loop corrections to multiscale effective vertices in the EFT for multi-Regge processes in QCD. **2019**

$$\times \frac{\cdots}{\kappa^*[p|\slashed{K}_{3\cdots l}|l\rangle[p|\slashed{K}_{3\cdots l}|(l+1)\rangle s_{3\cdots l}}.$$

Loop integrals

- Λ prescription works at loop-level for all $g^* g^+ \cdots g^+$ amplitudes !

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- Here, the Λ prescription was applied on finite amplitudes
- No regularization scheme needed

Issues to solve for an arbitrary amplitude

- Λ prescription and loop-integration do NOT commute

work in progress

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- Λ prescription and loop-integration do NOT commute
- Emergence of diagrams non equivalent to Lipatov's effective action ones

work in progress

Issues to solve for an arbitrary amplitude

- Λ prescription and loop-integration do NOT commute
- Emergence of diagrams non equivalent to Lipatov's effective action ones
- Results are different using auxiliary gluons or auxiliary quarks, but the difference seems to be universal

work in progress

Conclusion

- We calculated expression for the following amplitudes at loop-level in high energy factorization :
    - $g^*g^+g^+$ (in agreement with Lipatov's effective action result)
    - $g^*g^+g^+g^+$
    - $g^*g^+g^+g^+g^+$
    - $g^*g^+\cdots g^+$
- The obtained amplitudes have the expected properties (same as tree-level ones) :
    - On-shell limit
    - Equivalence in the use of auxiliary quarks or auxiliary gluons

Outlook

- Expanding this embedding method to non-finite amplitudes
- Addressing real correction
- Automatize NLO calculations in k_T-factorization / hybrid factorization

Thanks for your attention!

Find more at arXiv:2008.07916

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