Comparison of public codes for Drell-Yan processes at NNLO accuracy



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based on arXiv:2104.02400 (EPJC)



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Motto: precise measurements require precise theoretical predictions

 Experimental fiducial measurements of Drell-Yan cross-sections have now reached <0.5% accuracy apart from the luminosity uncertainty arXiv:1612.03016 (ATLAS), arXiv: 1909.0413 (CMS)

	$\sigma^{ m fid}_{W ightarrow\ell u}$ [pb]
$W^+ \to e^+ \nu$	$2939 \pm 1 \text{ (stat)} \pm 28 \text{ (syst)} \pm 53 \text{ (lumi)}$
$W^+ ightarrow \mu^+ \nu$	$2948 \pm 1 \text{ (stat)} \pm 21 \text{ (syst)} \pm 53 \text{ (lumi)}$
$W^+ \to \ell^+ \nu$	$2947 \pm 1 \text{ (stat)} \pm 15 \text{ (syst)} \pm 53 \text{ (lumi)}$
$W^- \to e^- \bar{\nu}$	$1957 \pm 1 \text{ (stat)} \pm 21 \text{ (syst)} \pm 35 \text{ (lumi)}$
$W^- ightarrow \mu^- ar{ u}$	$1964 \pm 1 \text{ (stat)} \pm 13 \text{ (syst)} \pm 35 \text{ (lumi)}$
$W^- \to \ell^- \bar{\nu}$	$1964 \pm 1 \text{ (stat)} \pm 11 \text{ (syst)} \pm 35 \text{ (lumi)}$
$W \rightarrow ev$	$4896 \pm 2 \text{ (stat)} \pm 49 \text{ (syst)} \pm 88 \text{ (lumi)}$
$W \rightarrow \mu \nu$	$4912 \pm 1 \text{ (stat)} \pm 32 \text{ (syst)} \pm 88 \text{ (lumi)}$
$W \to \ell \nu$	$4911 \pm 1 \text{ (stat)} \pm 26 \text{ (syst)} \pm 88 \text{ (lumi)}$
	$\sigma^{\mathrm{fid}}_{Z/\gamma^* ightarrow \ell\ell}$ [pb]
$Z/\gamma^* \to e^+e^-$	$502.7 \pm 0.5 \text{ (stat)} \pm 2.0 \text{ (syst)} \pm 9.0 \text{ (lumi)}$
$Z/\gamma^* \to \mu^+ \mu^-$	501.4 ± 0.4 (stat) ± 2.3 (syst) ± 9.0 (lumi)
$Z/\gamma^* \to \ell\ell$	502.2 ± 0.3 (stat) ± 1.7 (syst) ± 9.0 (lumi)

ATLAS fiducial cross sections at 7 TeV in the $66 < m_{II}/GeV < 116$ mass window

Motto: precise measurements require precise theoretical predictions

- Experimental fiducial measurements of Drell-Yan cross-sections have now reached <0.5% accuracy apart from the luminosity uncertainty (ATLAS: 1612.03016, CMS: 1909.0413)
- QCD fixed-order predictions agree in full phase space

but they differ at NNLO by as much as 1% in fiducial regions with symmetric cuts on the leptons

Data sets

Not necessary, but helpful, so chose two sets of data

- ATLAS data at E_{cm} = 7 TeV as pseudorapidity distributions for the arXiv:1612.03016
 - ${\ensuremath{\,{\circ}}}$ decay electron or muon (W+-production) and
 - decay lepton- pair (Z/γ *-production)
 - transverse momenta p_T and the pseudo-rapidities η_l of the decay leptons are subject to fiducial cuts
- $\bigcirc D \emptyset \text{ data } E_{cm} = 1.96 \text{ TeV on } W^{\pm} \text{-production} \qquad \text{arXiv:} 1412.2862$
 - measures the electron charge asymmetry distributions
 - and their dependence on the electron pseudo-rapidity
 - both symmetric as well as staggered fiducial cuts are applied on the transverse momenta and pseudo-rapidities of the electron and the neutrino

Parameters

Important for precision comparison

• G_{μ} scheme with input values G_F , M_Z , M_W (sin² θ_w , $\alpha(M_Z)$ are output), which minimizes the impact of NLO electroweak corrections

$$\begin{split} G_{\mu} &= 1.16637 \times 10^{-5} \ {\rm GeV}^{-2} \,, \\ M_Z &= 91.1876 \ {\rm GeV} \,, \qquad \Gamma_Z &= 2.4952 \ {\rm GeV} \,, \\ M_W &= 80.379 \ {\rm GeV} \,, \qquad \Gamma_W &= 2.085 \ {\rm GeV} \,, \end{split}$$

$ V_{ud} = 0.97401,$	$ V_{us} = 0.2265$,
$ V_{cd} = 0.2265$,	$ V_{cs} = 0.97320,$
$ V_{ub} = 0.00361,$	$ V_{cb} = 0.04053.$

• $\overline{\text{MS}}$ factorization scheme with $n_f = 5$ light flavors • ABMPI6 PDF with $\alpha^{(5)}(M_Z) = 0.1147$, $\mu_R = \mu_F = M_V$

Public codes

- DYNLO (version 1.5) <u>http://theory.fi.infn.it/grazzini/dy.html</u>
 uses q_T-subtraction
- FEWZ (version 3.1) <u>https://www.hep.anl.gov/fpetriello/FEWZ.html</u> uses fully local subtraction scheme
- MATRIX (version 1.0.4) <u>https://matrix.hepforge.org/</u>
 uses q_T-subtraction and scattering amps from OpenLoops
- MCFM (version 9.0)
 uses N-jettiness subtraction

https://mcfm.fnal.gov/

- Slicing parmeters:
 - $r_{\rm cut}$ for MATRIX as a cut on q_T
 - τ_{cut} for MCFM on jettiness

• Consistency of parameters: agreement at $O(10^{-5})$ at LO

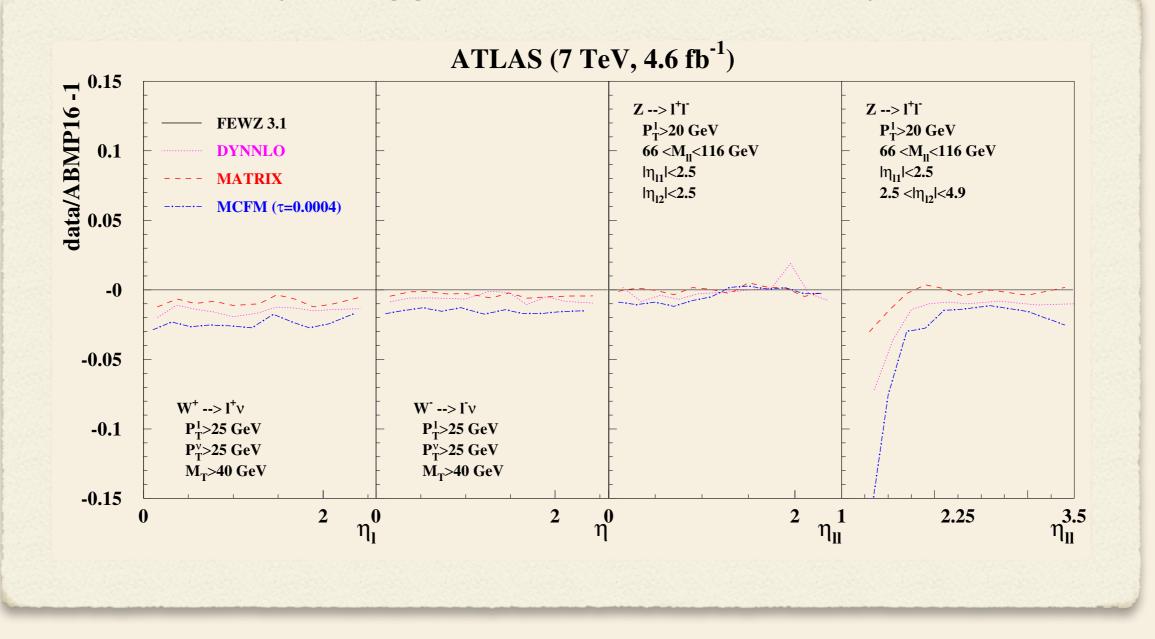
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 - accurate typically up to a few per mill and deviate in particular for distributions with challenging kinematics
 - with deviations displaying a particular pattern as a function of the (di-)lepton pseudo-rapidities

see Appendix for details

Comparison of NNLO cross sections (see Appendix for more details)



NNLO QCD cross sections for inclusive $pp \rightarrow W^{\pm} + X \rightarrow l^{\pm}v + X$ and $pp \rightarrow Z/\gamma^{*} + X \rightarrow l^{+}l^{-} + X$ as function of pseudo-rapidity, fiducial cuts are indicated in the plots $r_{\min} = 0.15(0.05)\%$ for $pp \rightarrow W^{\pm}(Z/\gamma^{*})$ (MATRIX) and $\tau_{cut} = 4 \cdot 10^{-4}$ (MCFM)

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where

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from universal soft and collinear QCD factorizationhence (schematically)

$$\sigma(\tau_{\text{cut}}) \sim 1 + \sum_{i} \ln^{i+1} \tau_{\text{cut}} + \sum_{j} \tau_{\text{cut}}^{p} \ln^{j} \tau_{\text{cut}} + O(\tau_{\text{cut}}^{p+1})$$

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• If the global subtraction term cancels only the leading soft and collinear singularities in σ then the residual power corrections in the presence of cuts on the decay leptons are enhanced to linear in q_T II

Lepton phase space and fiducial cuts

• the lepton phase space is $(q = p_1 + p_2)$

$$\Phi_L(q_T) = \left(\int \prod_{i=1}^2 \frac{d^4 p_i}{(2\pi)^3} \delta^+(p_i^2)\right) (2\pi)^4 \delta^{(4)}(q-p_1-p_2) = \frac{1}{4\pi^2} \int_0^\pi d\phi \int_{-\infty}^\infty d\Delta y \frac{p_{T1}^2}{Q^2} d\phi \int$$

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• the θ -functions break azimuthal symmetry in some parts of the phase space due to rapidity cuts — leading to linear power corrections —, whose boundary is given by q_T^* obtained as $q_T^* = |Y|$

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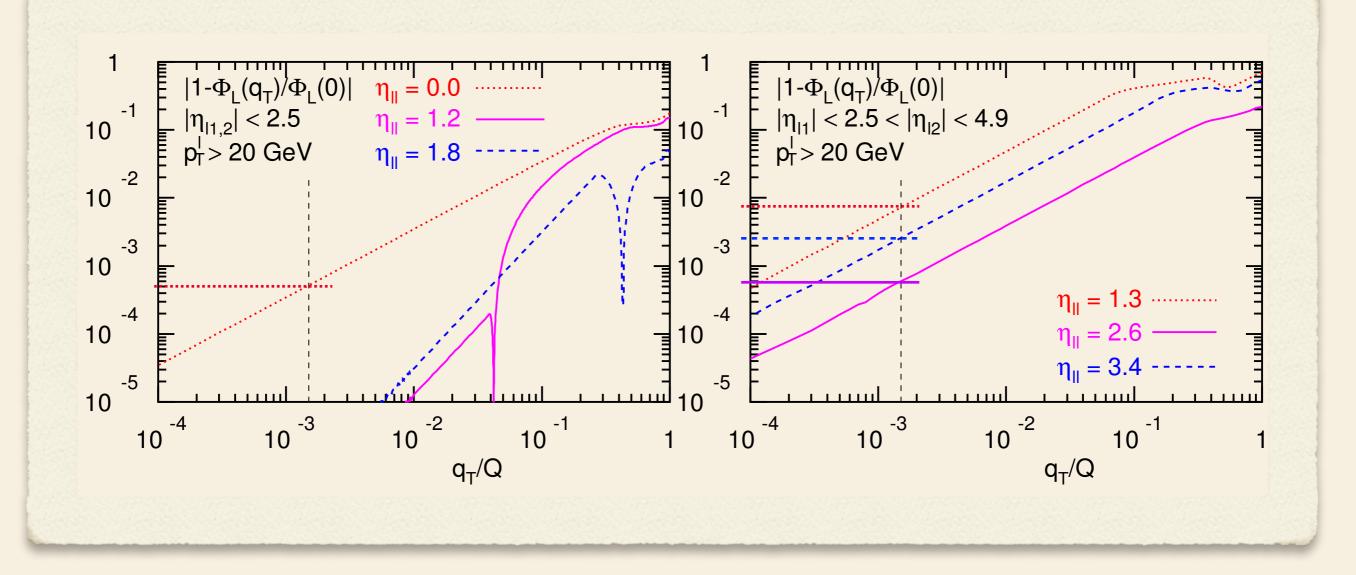
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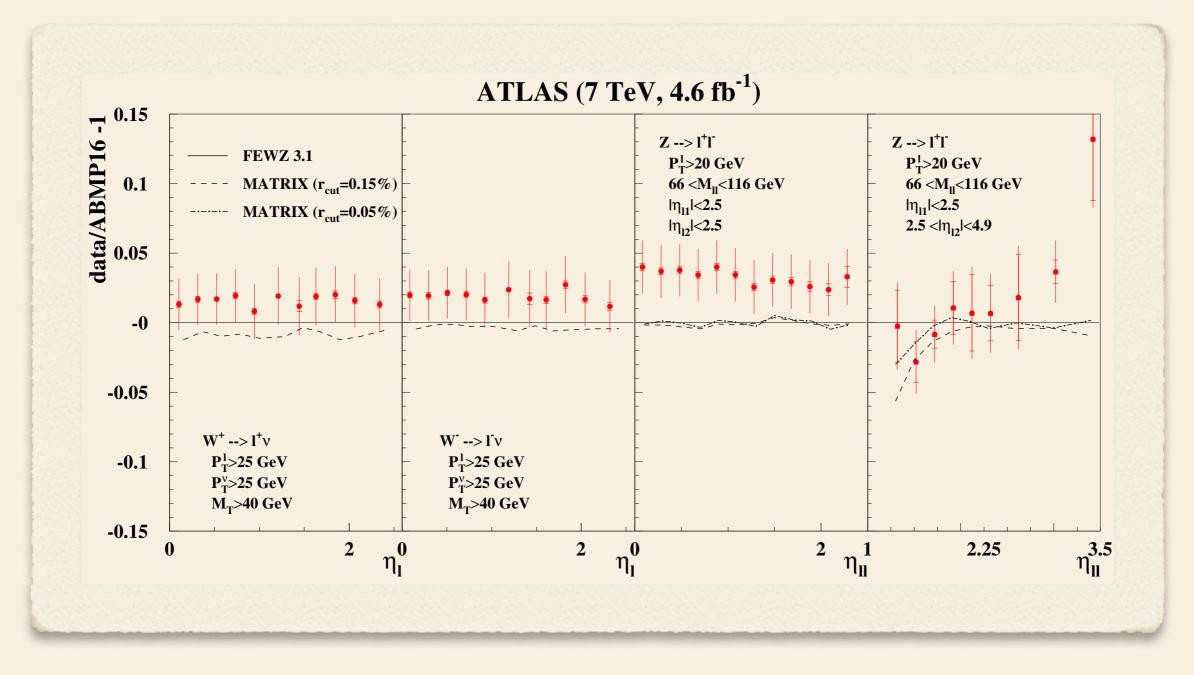
• for $q_T < q_T^*$ azimuthal symmetry is restored, and power corrections are quadratic

Lepton phase space and fiducial cuts



difference between the Born and real emission phase spaces $\Phi_L(0) - \Phi_L(q_T)$ of the decay leptons relative to the Born one at fiducial cuts applied to ATLAS data set for Z/γ^* -boson production ($Q = M_Z$) for different values of the gauge boson pseudo-rapidity η_{ll} , $p_{lT}^{_l} \ge 20$ GeV. Left: cuts selecting central pseudo-rapidities. Right: Cuts selecting one lepton at central pseudo-rapidity and the other at forward pseudo-rapidity. The vertical dashed line indicates the minimum value $r_{cut} = 0.15\%$ used in MATRIX as a slicing cut

Recall: MATRIX vs FEWZ



NNLO QCD cross sections for inclusive $pp \rightarrow W^{\pm} + X \rightarrow l^{\pm}v + X$ and $pp \rightarrow Z/\gamma^{*} + X \rightarrow l^{+}l^{-} + X$ as function of pseudo-rapidity, fiducial cuts as before and indicated in the plots $r_{cut} = 0.15\%$ (dashed) and $r_{cut} = 0.05\%$ (dashed-dotted)

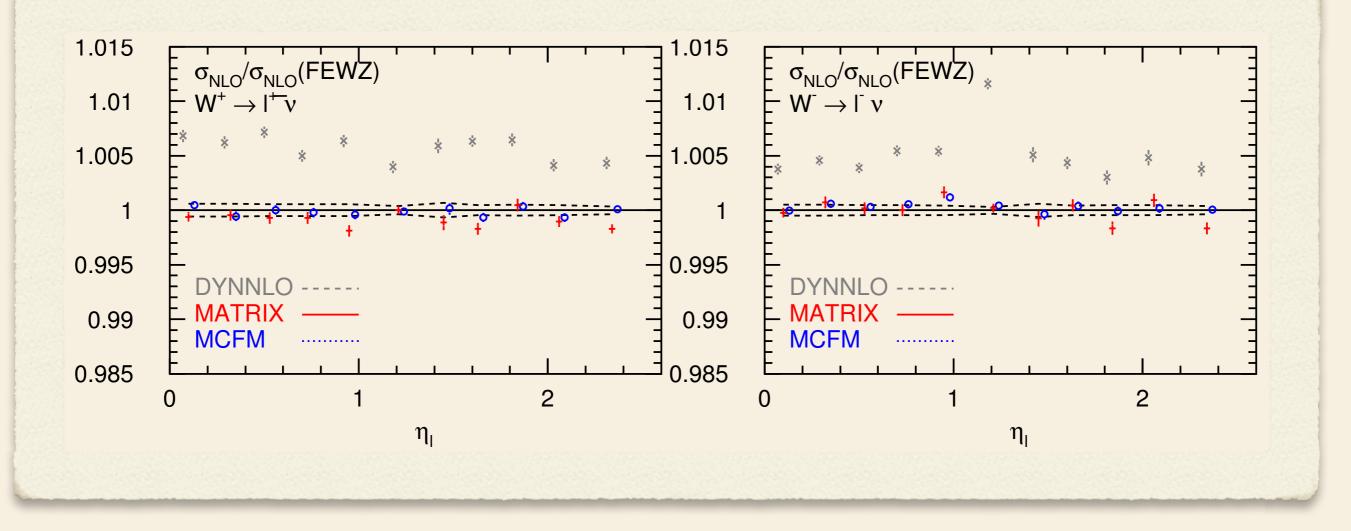
Conclusions

- at NLO MATRIX, MCFM and FEWZ are in agreement
- ✓ at NNLO accuracy we found differences among the predictions comparable in size to the NNLO correction itself — see Appendix
- ✓ fiducial cuts on the transverse momenta and pseudo-rapidities of the decay leptons lead to linear power corrections in the slicing parameter
- ✓ deviations share certain patterns across the range of pseudo-rapidities in the considered distributions, which have been correlated with the appearance of linear power corrections in the lepton decay phase space $Φ_L$ as a function of q_T
- ✓ the continuous increase in the precision of the experimental measurements, the theory predictions are pressed to provide cross sections at NNLO (or beyond) where the systematic uncertainties due to choices of particular schemes or algorithms for the computation can be safely neglected in comparison to the experimental uncertainties

Appendix

Validation at NLO

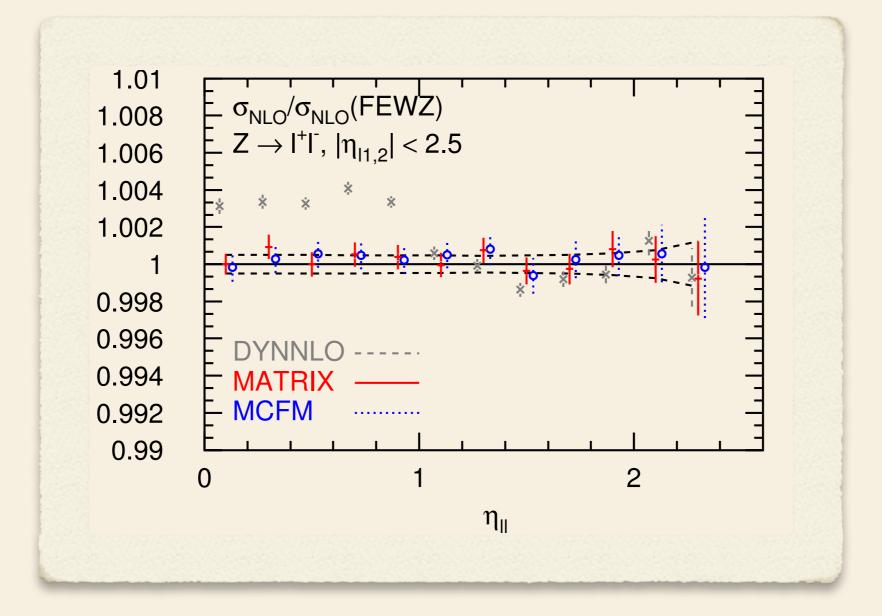
- Consistency of parameters: agreement at $O(10^{-5})$ at LO
- At NLO all but DYNNLO employ local subtraction



NLO QCD cross sections for inclusive $pp \rightarrow W^{\pm} + X \rightarrow l^{\pm}v + X$ as function of pseudorapidity, $p^{l}_{T}, p^{v}_{T} \ge 25$ GeV and $M_{T} \ge 40$ GeV

Validation at LO and NLO

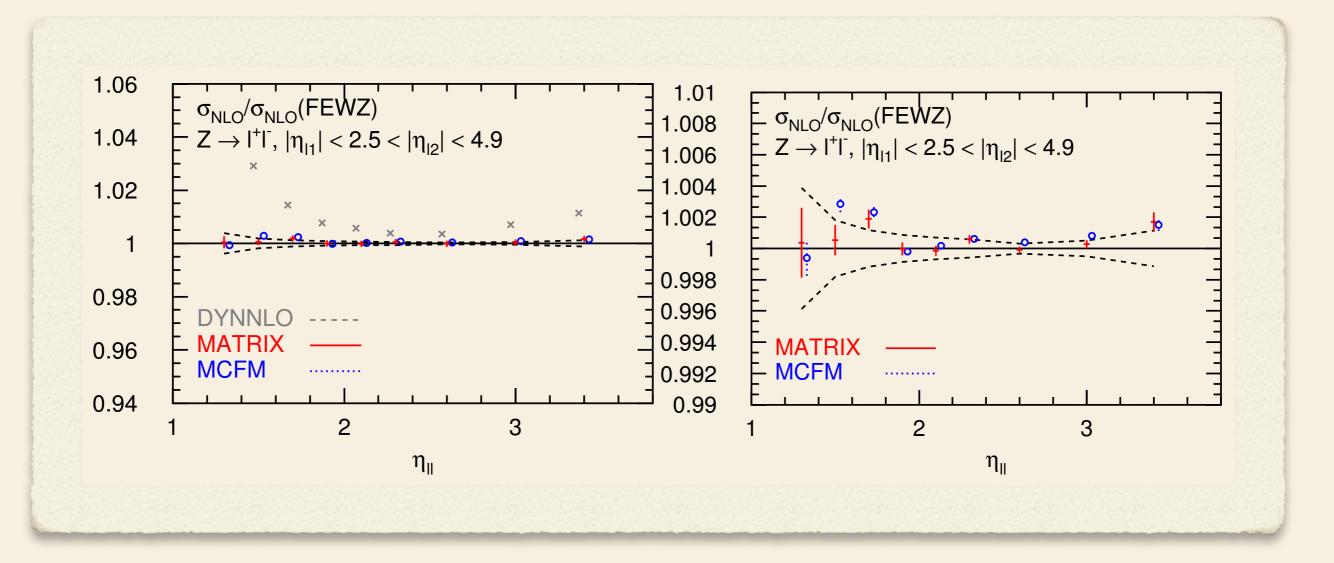
At NLO all but DYNNLO employ local subtraction



As previous for $pp \rightarrow Z/\gamma^* + X \rightarrow l^+ l^- + X$, $p^i_T \ge 25$ GeV, $116 \ge M_{ll}/\text{GeV} \ge 66$, $|\eta_{l_i}| \le 2.5$, i = 1, 2

Validation at LO and NLO

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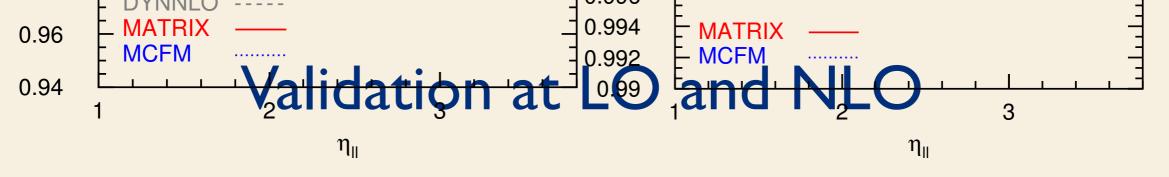
As previous but with $|\eta_{l_1}| \le 2.5, 2.5 \le |\eta_{l_2}| \le 4.9$

1.08 1.06

 A^{e}_{NLO}/A^{e}_{NLO} (FEWZ)

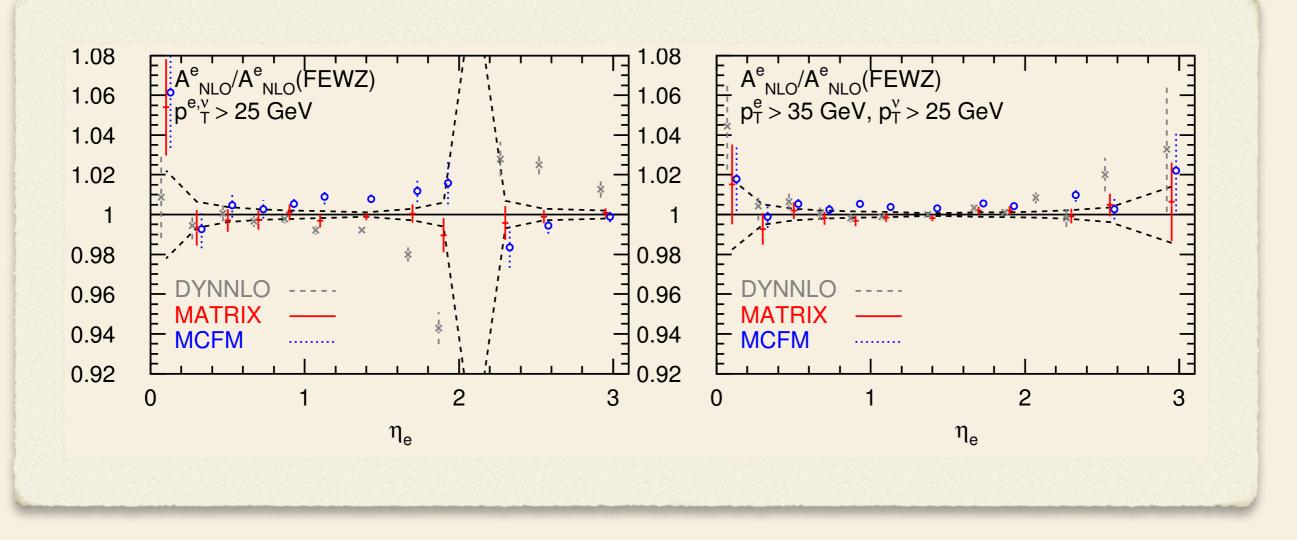
1.08 20 1.06

 A^{e}_{NLO}/A^{e}_{NLO} (FEWZ) $p^{e} > 35 \text{ GeV}$ $p^{\vee} > 25 \text{ GeV}$



Consistency of parameters: agreement at O(10⁻⁵) at LO

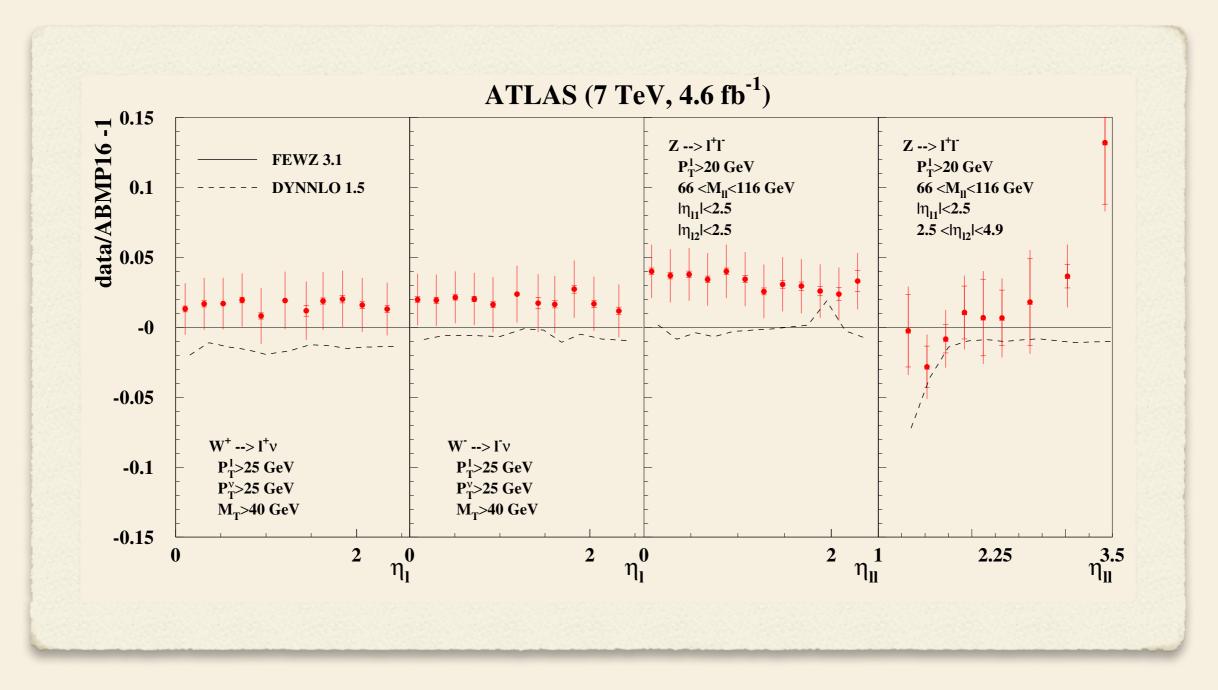
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electron charge asymmetry distribution A_e in W^{\pm} boson production Left: symmetric cuts, p^{l_T} , $p^{v_T} \ge 25$ GeV Right: staggered cuts $p^{l_T} \ge 35$ GeV, $p^{v_T} \ge 25$ GeV

Comparison at NNLO

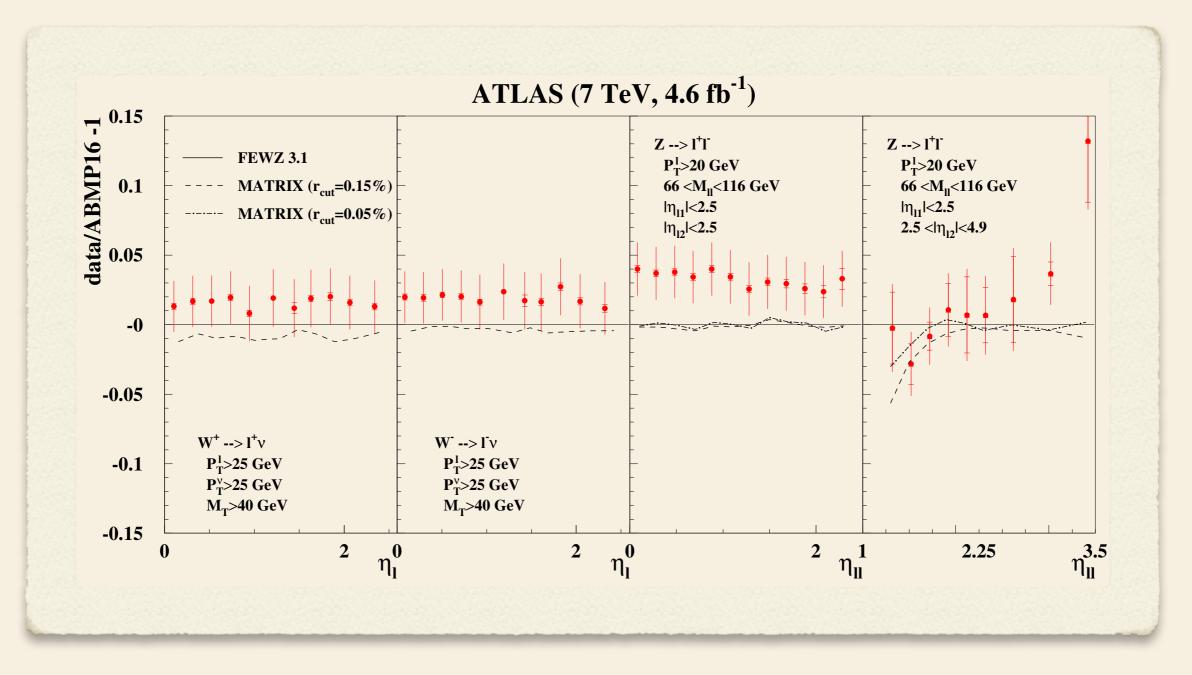
DYNNLO vs FEWZ



NNLO QCD cross sections for inclusive $pp \rightarrow W^{\pm} + X \rightarrow l^{\pm}v + X$ and $pp \rightarrow Z/\gamma^{*} + X \rightarrow l^{+}l^{-} + X$ as function of pseudo-rapidity, fiducial cuts as before and indicated in the plots

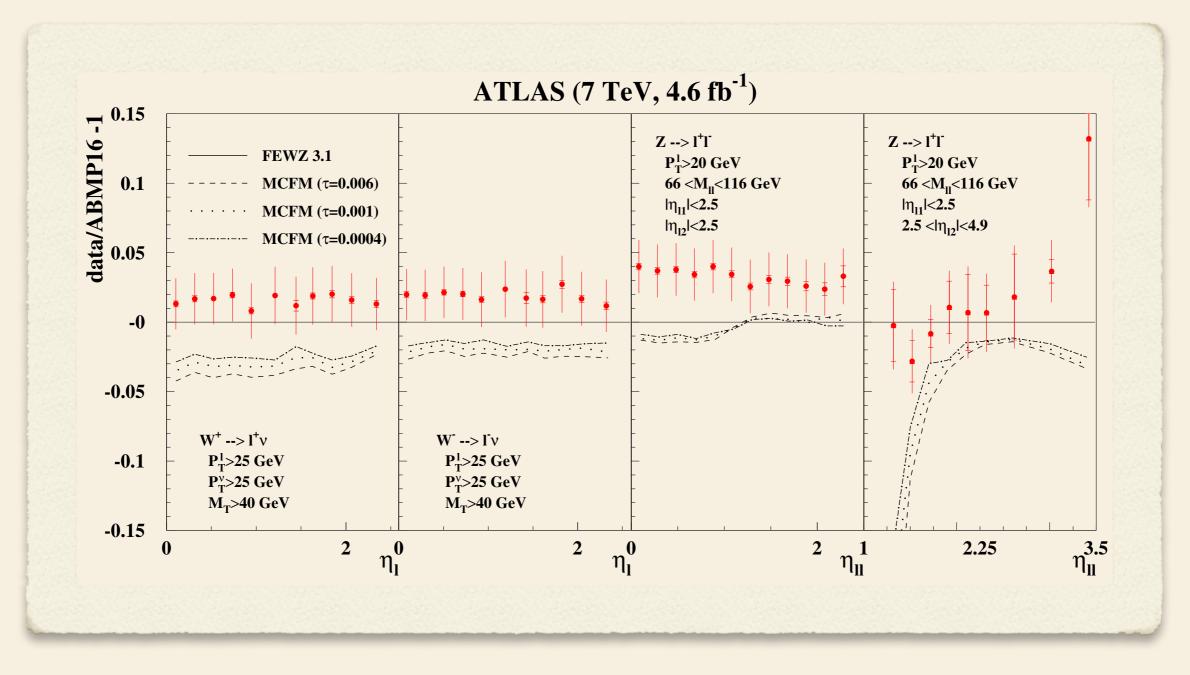
> 23 ATLAS (7 TeV, 4.6 fb⁻¹)

MATRIX vs FEWZ



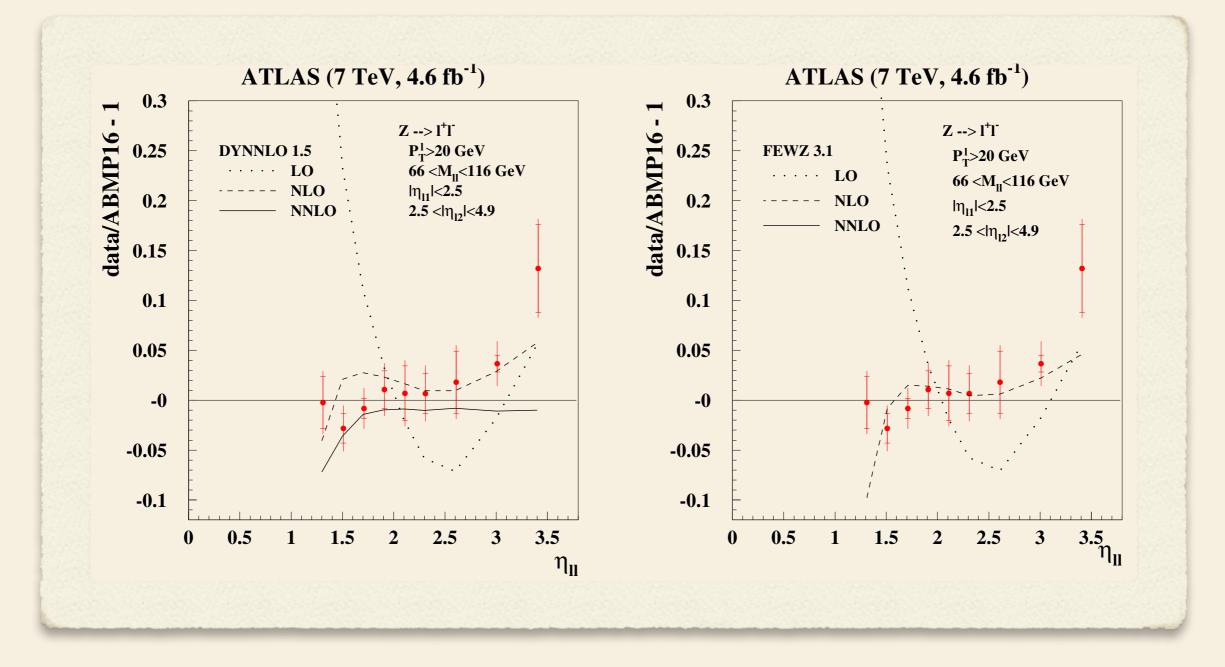
As previous with different values for the q_T -slicing cut: $r_{\text{cut}} = 0.15\%$ (dashed) and $r_{\text{cut}} = 0.05\%$ (dashed-dotted)

MCFM vs FEWZ



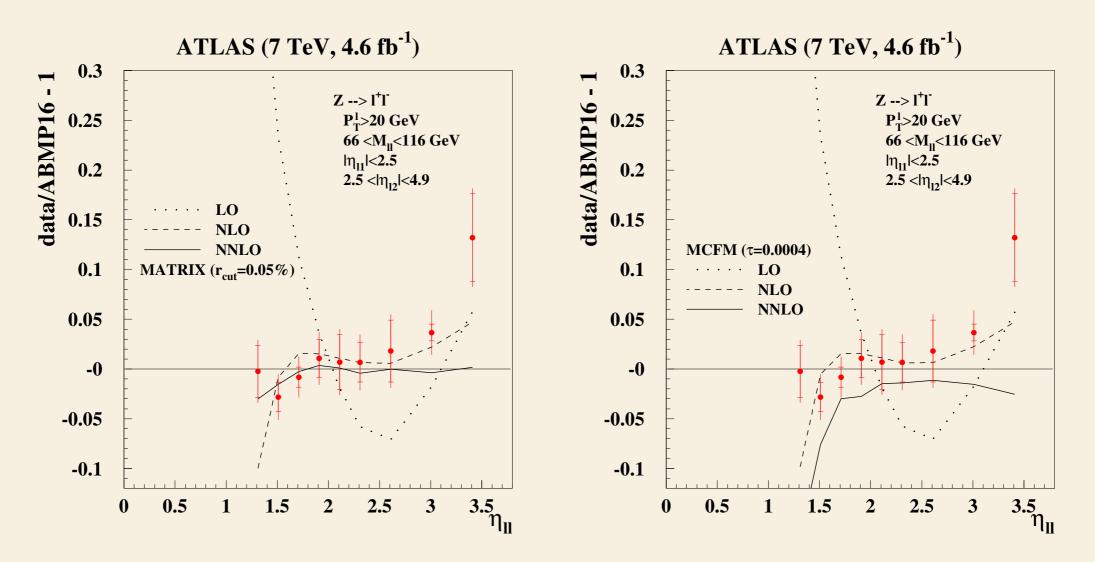
As previous with different values for the jettiness slicing cut: $\tau_{cut} = 6 \cdot 10^{-3}$ (dashed), $\tau_{cut} = 10^{-3}$ (dotted), $\tau_{cut} = 4 \cdot 10^{-4}$ (dashed-dotted)

DYNNLO vs FEWZ



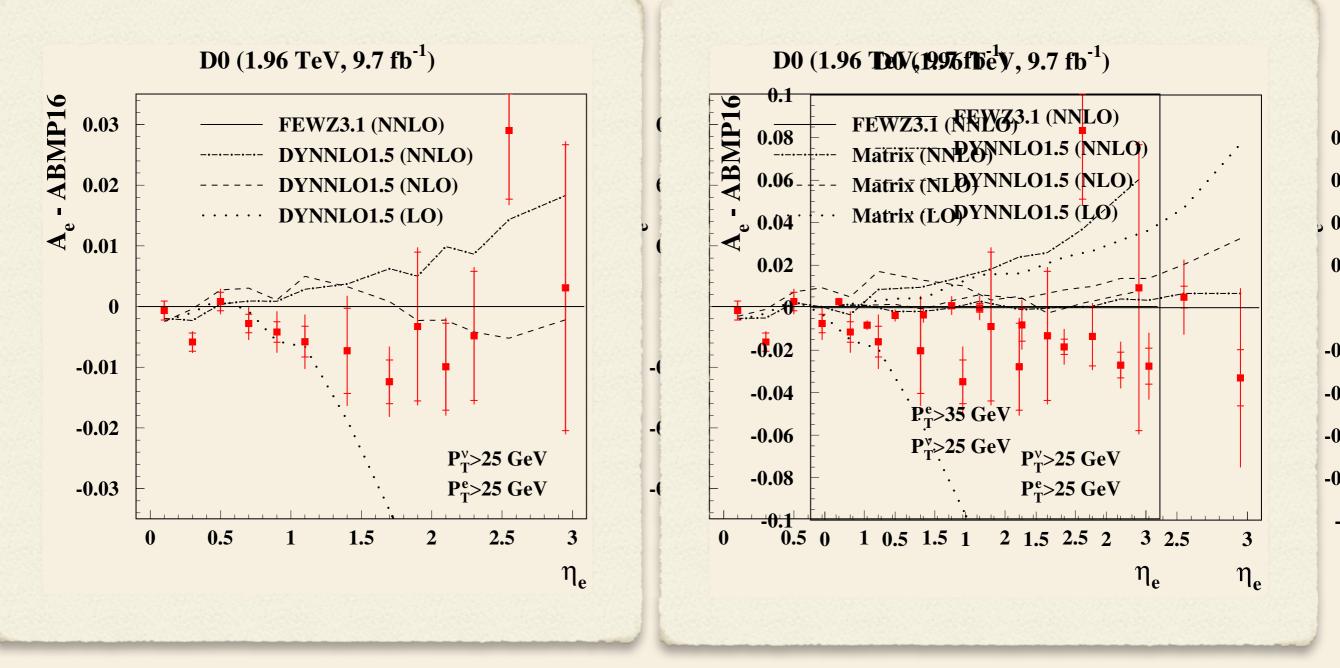
LO, NLO and NNLO QCD cross sections normalized to **FEWZ** at NNLO for inclusive $pp \rightarrow Z/\gamma^* + X \rightarrow l^+l^- + X$ as function of pseudo-rapidity of the lepton pair with staggered cuts indicated in the plots

MATRIX and MCFM vs FEWZ



As previous for MATRIX (left) and MCFM (right)

DYNNLO vs FEWZ

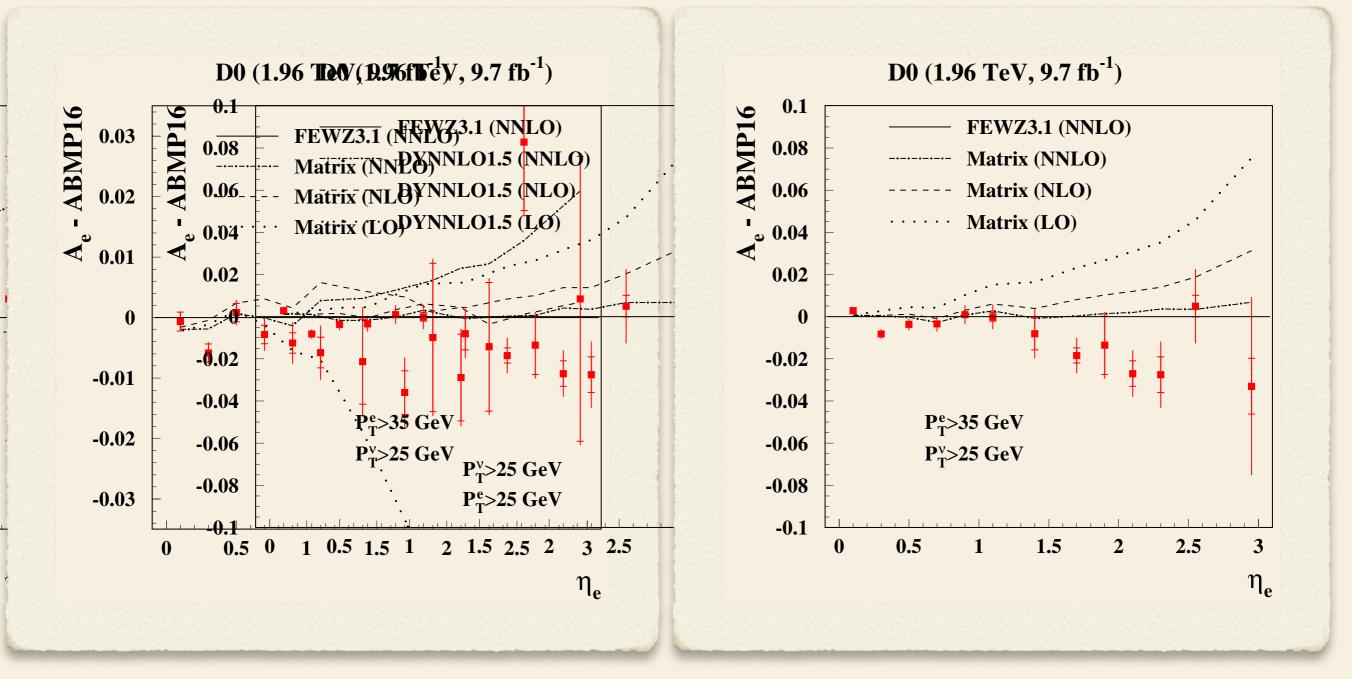


electron charge asymmetry distribution A_e in W^{\pm} boson production at LO, NLO and NNLO normalized to FEWZ at NNLO

Left: symmetric cuts Right: staggered cuts as indicated in the plots D0 (1.96 TeV, 9.7



MATRIX vs FEWZ

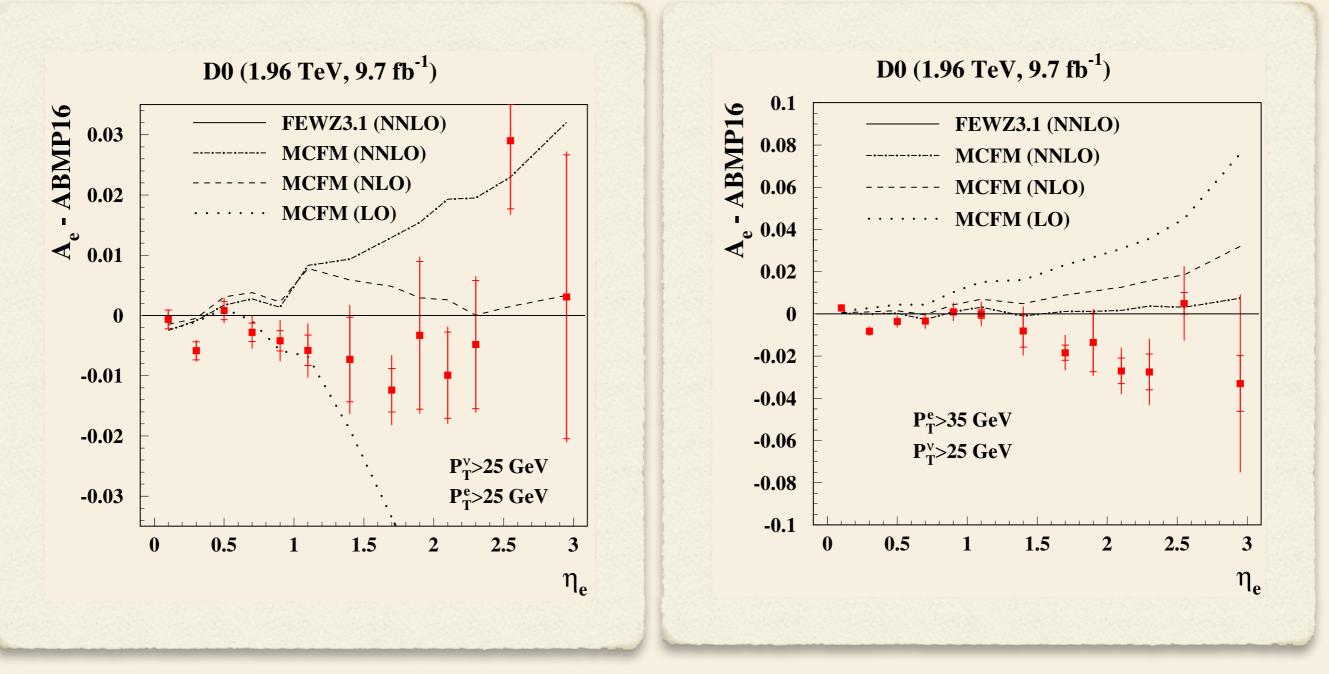


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