Recent developments on Parton Branching TMDs

EPS 2021

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Parton Branching (PB) Evolution Equations

Method to obtain transverse momentum dependent PDFs (TMDs) $\tilde{\mathcal{A}}_a(x, k_\perp, \mu^2)$ and collinear PDFs $\tilde{f}_a(x, \mu^2) = \int dk_\perp^2 \tilde{\mathcal{A}}_a(x, k_\perp, \mu^2)$:

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$$\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \times \times \int_x^{z_M} dz P_{ab}^R(z)\tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + (1-z)\mu'_\perp, \mu'^2)$$

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  b: incoming parton, a: outgoing parton, z momentum fraction of parton a to b
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- Sudakov form factor:

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  Interpretation: probability of an evolution without any resolvable branchings

Solution of PB equations

Iterative form of the PB evolution equations:

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)}\times$$
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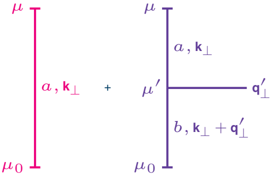

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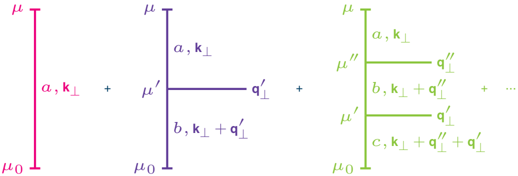

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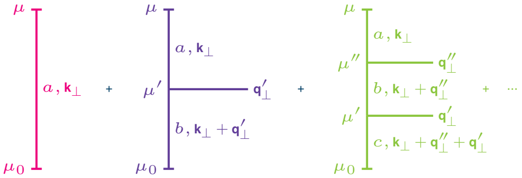

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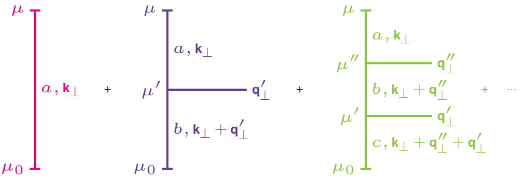

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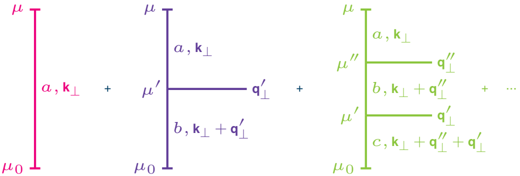

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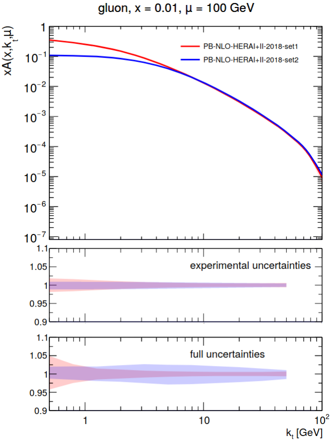
Initial distribution: $\tilde{\mathcal{A}}_a(x,k_{\perp,0},\mu_0^2) = \tilde{f}_a(x,\mu_0^2)\times\frac{1}{q_s^2}\exp\left(-\frac{k_\perp^2}{q_s^2}\right)$

Existing PDF can be chosen for $\tilde{f}_a(x,\mu_0^2)$ or can be fitted

NLO Fits

Fits with NLO splitting functions performed [Bermudez Martinez, Connor, Jung, Lelek, Zlebcik, Hautmann, Radescu, Phys. Rev. D 99 (2019) 074008, 1804.11152], available in TMDlib (new release: [arXiv:2103.09741])



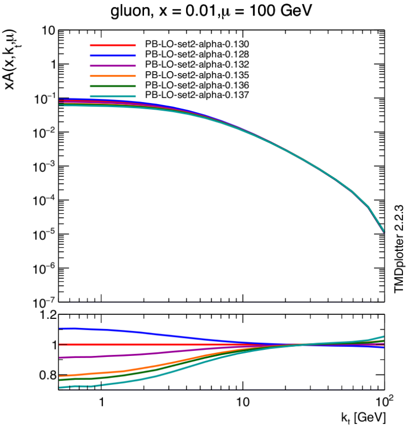
Fits performed with xFitter framework.

- Full coupled evolution with all flavours
- HERAPDF parametrization form for all flavours
- Using full HERAI+II inclusive DIS data (3.5$< Q^2 <$50000 GeV^2 & $4{\cdot}10^{-5} < x < 0.65$)
- Fixed z_M
- set1 $\alpha_s(\mu)$, set2 $\alpha_s(q_\perp)$

New - work in progress: LO Fits

- TMDs are now being used for jet production at the LHC
- "TMD jet merging" methods developed at LO [arXiv:2107. 01224]
  $\rightarrow$parton showers + multi-jet ME
- Consistent treatment: LO fits

- Good fits with different values of α_s
- Influence low-$p_\perp$ range of TMDs
- Impact on low-$p_\perp$ range of DY



$\alpha_s(M_Z^2)$	0.128	0.13	0.132	0.135	0.136	0.137
χ^2/dof	1.385	1.392	1.455	1.537	1.569	1.579

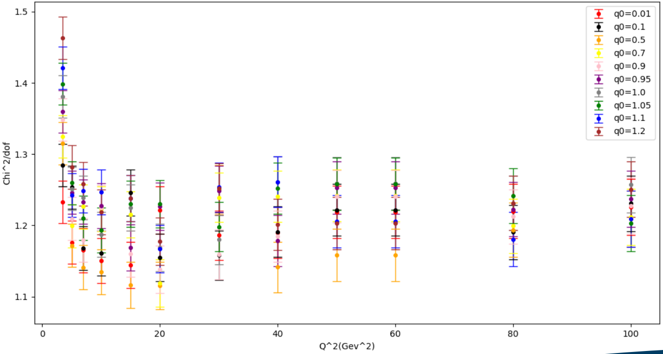
New - work in progress: Fits with dynamical resolution scale

First time full effects of AO!

$z_M = 1 - \frac{q_0}{\mu'}$ with q_0 minimal transverse momentum of emitted parton

χ^2/dof in NLO fits vs minimal Q^2 in fits

$\rightarrow$ Works well down to low Q^2_{min}, even with $q_0 \approx 1$GeV

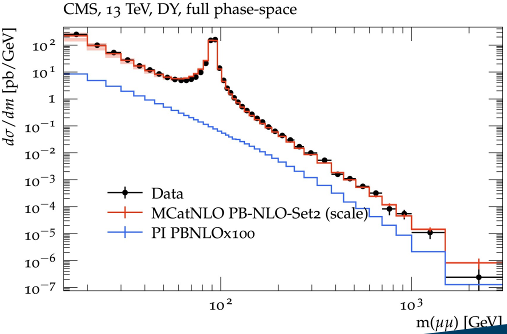


Photon TMD

[H. Jung, S. Taheri Monfared, T. Wening,Physics Letters B 817 (2021) 136299, arXiv:2102.01494]

- Include QED corrections, $\alpha \sim \alpha_s^2$: LO QED and NLO QCD splitting functions
- QCD partons fitted, photons generated by perturbation
- $\chi^2/dof = 1.21 \rightarrow$ Same as only QCD

Differential Drell-Yan cross section in pp-collisions at 13 TeV (CMS-2018-I1711625) [JHEP 12 (2019), 059]:



Calculation with CASCADE3

[Eur. Phys. J. C 81 (2021) no.5, 425],

ME: MC@NLO

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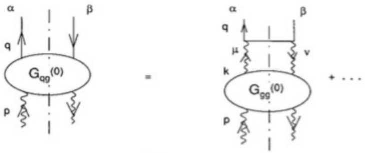

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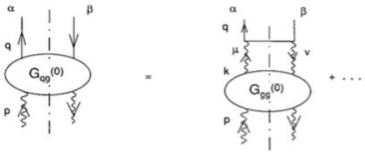

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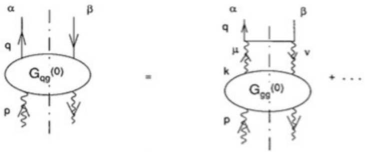

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- $\lim_{\mathbf{k}_\perp \to 0} \tilde{P}_{ab}(z, \mathbf{k}_\perp, \mathbf{q}_\perp) = P_{ab}^{\text{DGLAP}}(z)$

$k_\perp$-dependent Sudakov form factor
New - work in Progress

Resolvable branchings:
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- Definition of $\mathbf{k}_\perp$-dependent Sudakov form factor
- Should sum over all possible splittings $\to$ Angular averaged splitting functions

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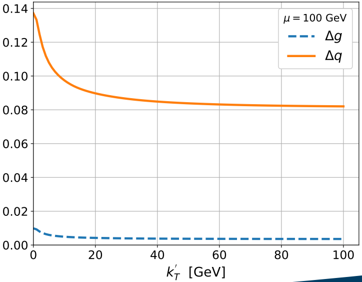

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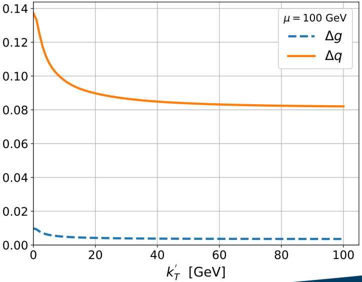

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Studies with:

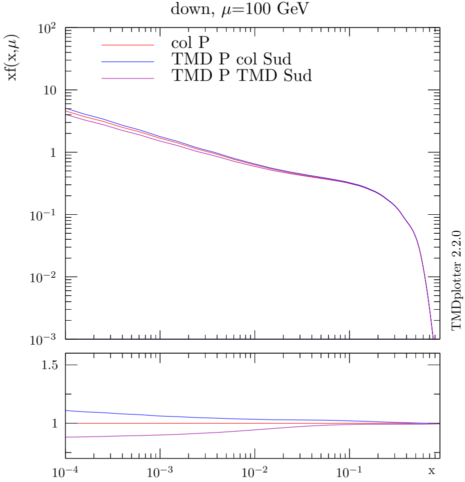
1. col. P and col. Sudakov form factor (normal PB)
2. TMD P and col. Sudakov form factor
3. TMD P and TMD Sudakov form factor

Model 1 and 3 conserve momentum of proton, 2 not.

Numerical Results with TMD P
New - work in Progress

We studied effects of TMD Splitting functions on the evolution. No fits has been done yet.
Implementation with:

- PB method (LO)
- PB with TMD Splitting functions in resolvable branchings, but collinear Sudakov
- PB with TMD Splitting functions and TMD Sudakov



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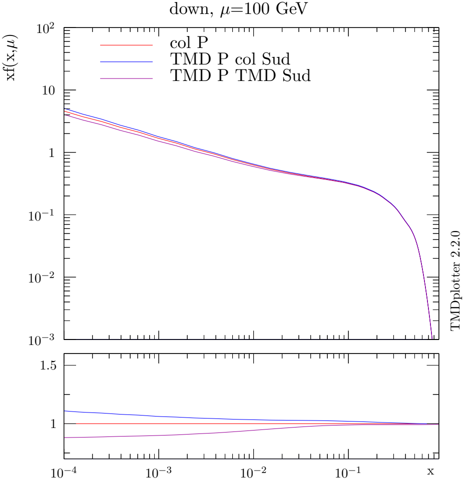
- Effects are small for large x $\longrightarrow$ reasonable since $\tilde{P}_{ab}(z, \mathbf{k}_\perp, \mathbf{q}_\perp) \xrightarrow[k_\perp \to 0]{} P_{ab}(z)$
- TMD Sudakov suppression compared to collinear Sudakov



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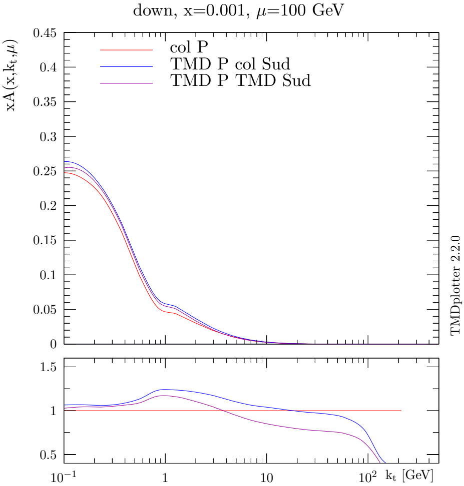
- Whole $\mathbf{k}_\perp$-region is affected



Numerical Results with TMD P
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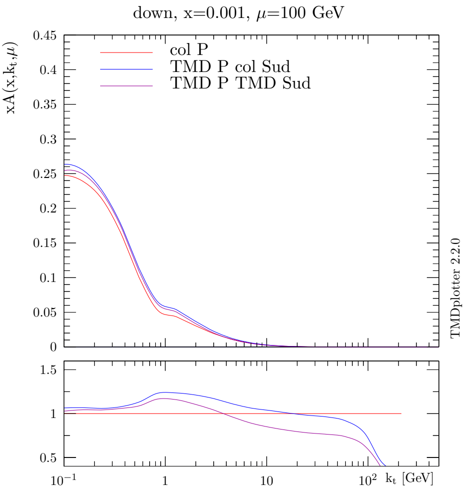
- TMD Sudakov suppression compared to collinear Sudakov
- Whole $\mathbf{k}_\perp$-region is affected
- PB is capable of handling TMD P and TMD sudakov



Summary and outlook

Fits obtained with PB method:

- NLO fits, set1 and set2, available in TMDlib

New fits:

- QCD+QED fit (including first photon TMD in PB)
- **Flavor-variable-number schemes 4FLVN/5FLVN** [H. Jung, S. Taheri Monfared,arXiv:2106.09791]

Fits in progress:

- LO fits
- Fits with dynamical resolution scale

Inclusion of TMD splitting functions in PB:

- $k_\perp$-dependent splittings affect both real emissions and Sudakov form factors
- Working code to produce TMD and collinear PDFs
- Prospects to do phenomenology: fits will be performed and the fitted parton densities will be used for predictions of LHC and EIC processes

Thank you!