Groomed Jet Mass in lepton collisions

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based on arXiv:1807.11472 (PLB), 2002.00942 (PLB), 2002.05730 (PRD)

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Groomed event shapes

- designed to reduce contamination from non-perturbative effects
- have complex definition
- simplest one: modified mass drop tagger:

essentially removes the soft particles from the event

mMDT grooming algorithm

1. Divide the final state of an e+e− → hadrons event into two hemispheres in any infrared and collinear safe way.
2. In each hemisphere, run the Cambridge/Aachen jet algorithm to produce an angular-ordered pairwise clustering history of particles.
3. Undo the last step of the clustering for the one hemisphere, and split it into two particles; check if these particles pass the mass drop condition, which is defined for e^+e^- collisions as:

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\rm cut}$$

where E_i and E_j are the energies of the two particles

3. If the splitting fails this condition, the softer particle is dropped and the groomer continues to the next step in the clustering at smaller angle.
4. If the splitting passes this condition the procedure ends and any observable can be measured in the remaining hemispheres

Resummation formula

Factorization formula for $\tau_{\rm L}, \tau_{\rm R} \ll z_{\rm cut} \ll 1$ $\quad \tau_i = \frac{m_i^2}{E_i^2}$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau_{\rm L}\,\mathrm{d}\tau_{\rm R}} = H(Q^2)S(z_{\rm cut})\left[J(\tau_{\rm L})\otimes S_c(\tau_{\rm L}, z_{\rm cut})\right]\left[J(\tau_{\rm R})\otimes S_c(\tau_{\rm R}, z_{\rm cut})\right]$$

C. Frye et al, arXiv: 1603.09338

Convolutions —

true product for Laplace transforms:

$$\frac{\sigma(\nu_{\rm L}, \nu_{\rm R})}{\sigma_0} = H(Q^2)S(z_{\rm cut})\tilde{J}(\nu_{\rm L})\tilde{S}_c(\nu_{\rm L}, z_{\rm cut})\tilde{J}(\nu_{\rm R})\tilde{S}_c(\nu_{\rm R}, z_{\rm cut})$$

Modified mass drop tagger groomed heavy jet mass:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\rm g}}{\mathrm{d}\rho} \equiv \int \mathrm{d}\tau_{\rm L}\,\mathrm{d}\tau_{\rm R}\,\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau_{\rm L}\,\mathrm{d}\tau_{\rm R}}\left[\Theta(\tau_{\rm L}-\tau_{\rm R})\,\delta(\rho-\tau_{\rm L}) + \Theta(\tau_{\rm R}-\tau_{\rm L})\,\delta(\rho-\tau_{\rm R})\right]$$

Resummation by RGE

Resummation is made possible by the RGEs:

$$\mu\frac{\partial\tilde{F}}{\partial\mu} = \left(d_F\Gamma_{\text{cusp}}\log\frac{\mu^2}{\mu_F^2} + \gamma_F\right)\tilde{F}\,, \quad (\tilde{F} = H\,,\ S\,,\ \tilde{J}\,,\ \tilde{S}_c)$$

The factorization theorem successfully resums all logarithms of both ρ and z_{cut} simultaneously, to leading power in the limit where $\rho \ll z_{\text{cut}} \ll 1$ in the exponent of the cross section cumulative in the mass ρ

Thus N^{n}LL refers to the resummation of all terms of the form

$$\alpha_s^m \log^{m+1-n}\rho,\ \alpha_s^m \log^{m+1-n} z_{\text{cut}}\,,\ \alpha_s^m \log^{m+1-n}\frac{\rho}{z_{\text{cut}}}$$

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order of ingredients needed for N^{n}LL resummation

	$\Gamma_{\rm cusp}$	γ_F	β	c_F
LL	α_s	-	α_s	-
NLL	α_s^2	α_s	α_s^2	-
NNLL	α_s^3	α_s^2	α_s^3	α_s
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2

known | known partially

(In the table, filled boxes mark "known" entries; the outlined boxes for γ_F at NNNLL (α_s^3) and c_F at NNNLL (α_s^2) mark "known partially".)

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not yet known: $c_{S_c}^{(2)}$

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not yet known:

$c_{S_c}^{(2)}$

$\gamma_S^{(2)}\,,\ \gamma_{S_c}^{(2)}$

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Extraction of constants

with details in the appendix

Missing constants

Constraint on the non-cusp anomalous dimensions:

$$0 = \gamma_H + \gamma_S + 2\gamma_J + 2\gamma_{S_c}$$

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We use fixed-order code EVENT2 to determine

$$c_{S_c}^{(2)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left[C_F^2\,(22 \pm 4) + C_F C_A\,(41 \pm 1) + C_F T_R n_f\,(14.4 \pm 0.1)\right]$$

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and MCCSM to find

$$\gamma_S^{(2)} = \left(\frac{\alpha_s}{4\pi}\right)^3 \left[-11600 \pm 2000\right] \qquad (n_f = 5)$$

for mMDT

mMDT groomed heavy jet mass at N³LL

Resummation is made possible by the RGEs:

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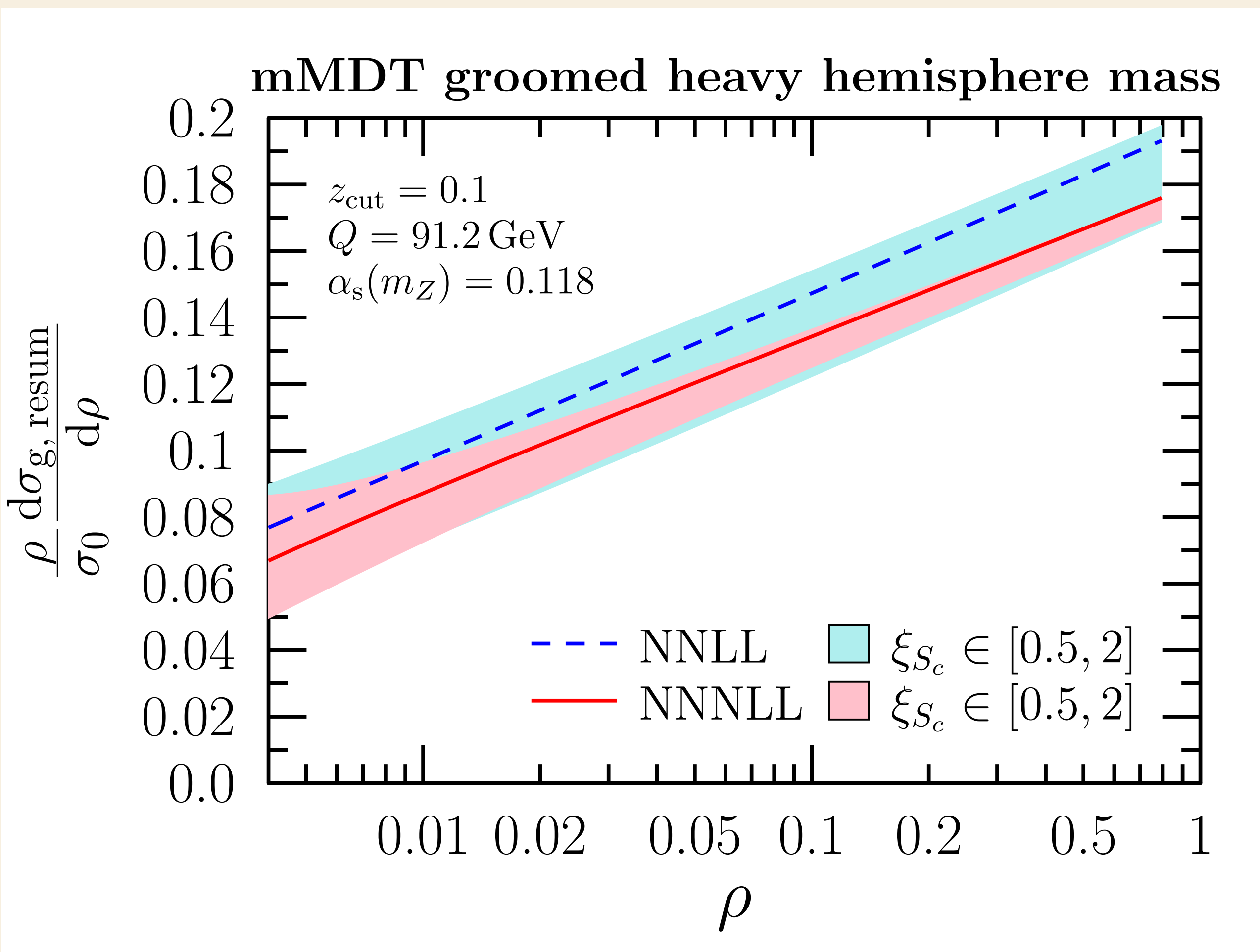

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known

new in this plot:

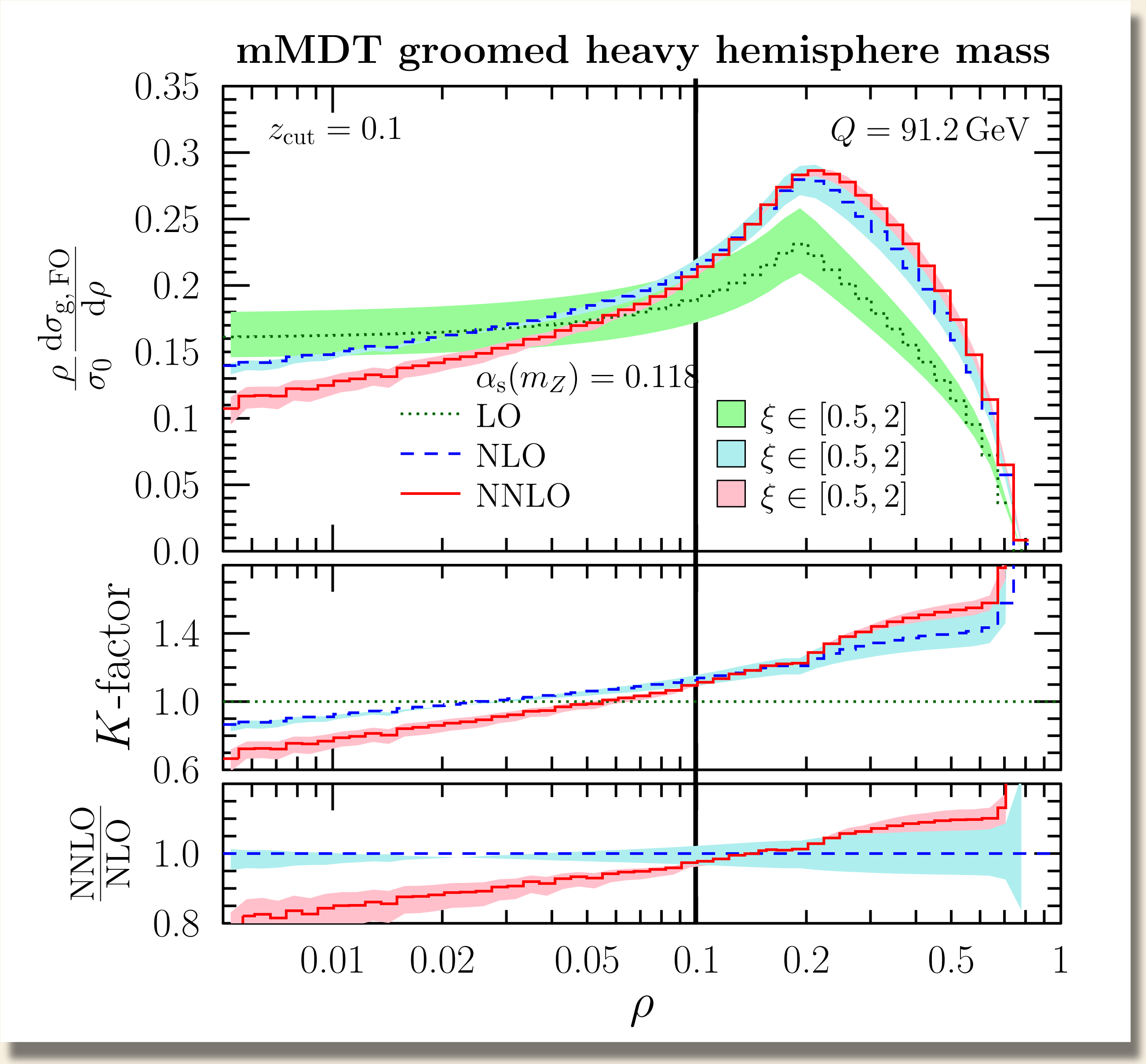
$c_{S_c}^{\rm mMDT}$ and $\gamma_S^{\rm mMDT}$

Matching to fixed order

mMDT groomed heavy jet mass

$$\rho\frac{\mathrm{d}\sigma_{\mathrm{g,NNLO}}}{\mathrm{d}\rho} = \frac{\alpha_s}{2\pi}A_{\mathrm{g}} + \left(\frac{\alpha_s}{2\pi}\right)^2\left[B_{\mathrm{g}} + A_{\mathrm{g}}\beta_0\log\frac{\mu}{Q}\right]$$

$$+\left(\frac{\alpha_s}{2\pi}\right)^3\left[C_{\mathrm{g}} + 2B_{\mathrm{g}}\beta_0\log\frac{\mu}{Q} + A_{\mathrm{g}}\left(\frac{\beta_1}{2}\log\frac{\mu}{Q} + \beta_0^2\log^2\frac{\mu}{Q}\right)\right]$$



A. Kardos et al, arXiv: 1807.11472

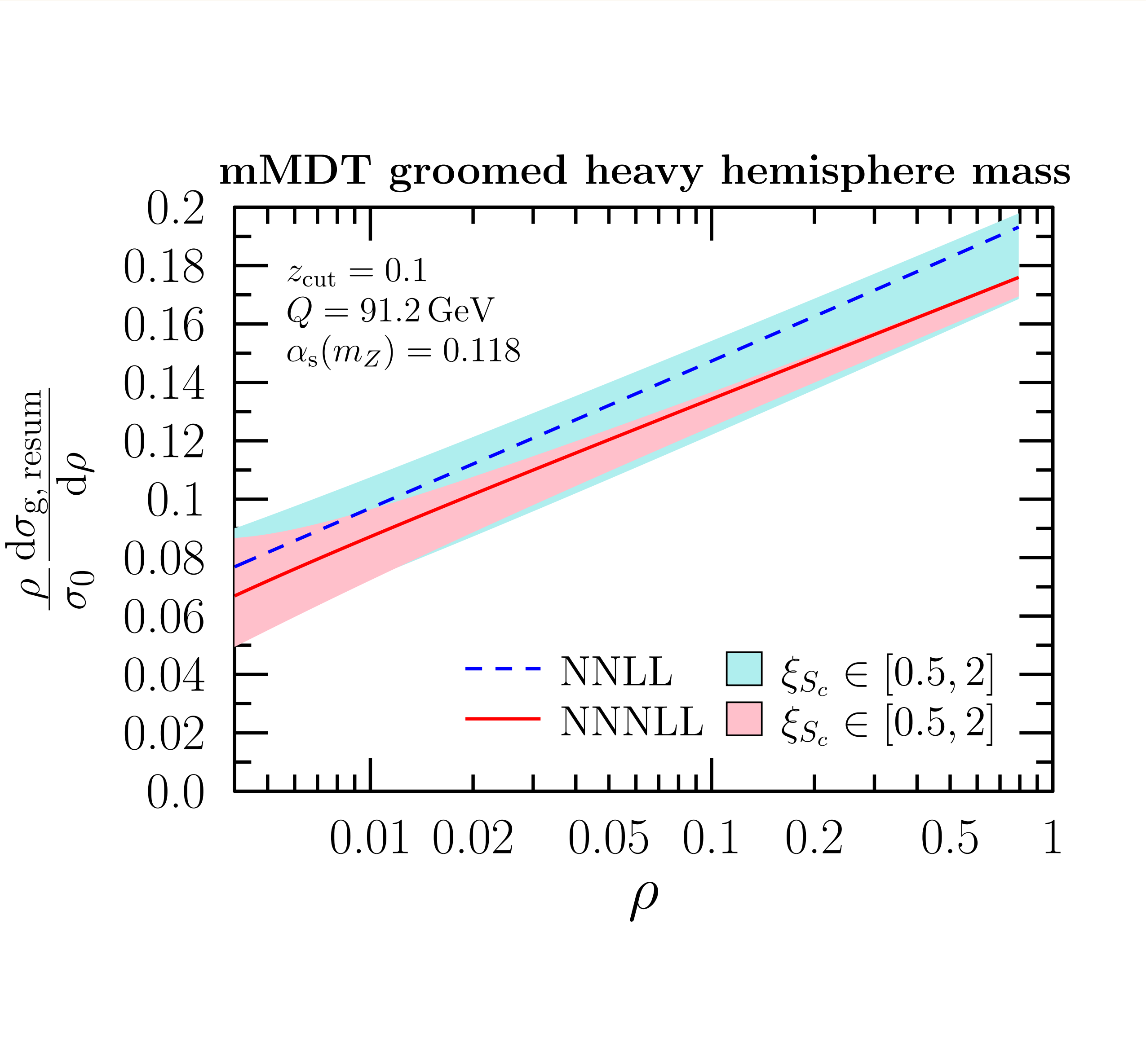
A, B and C are computed with MCCSM (=Monte Carlo for the CoLoRFulNNLO Subtraction Method)

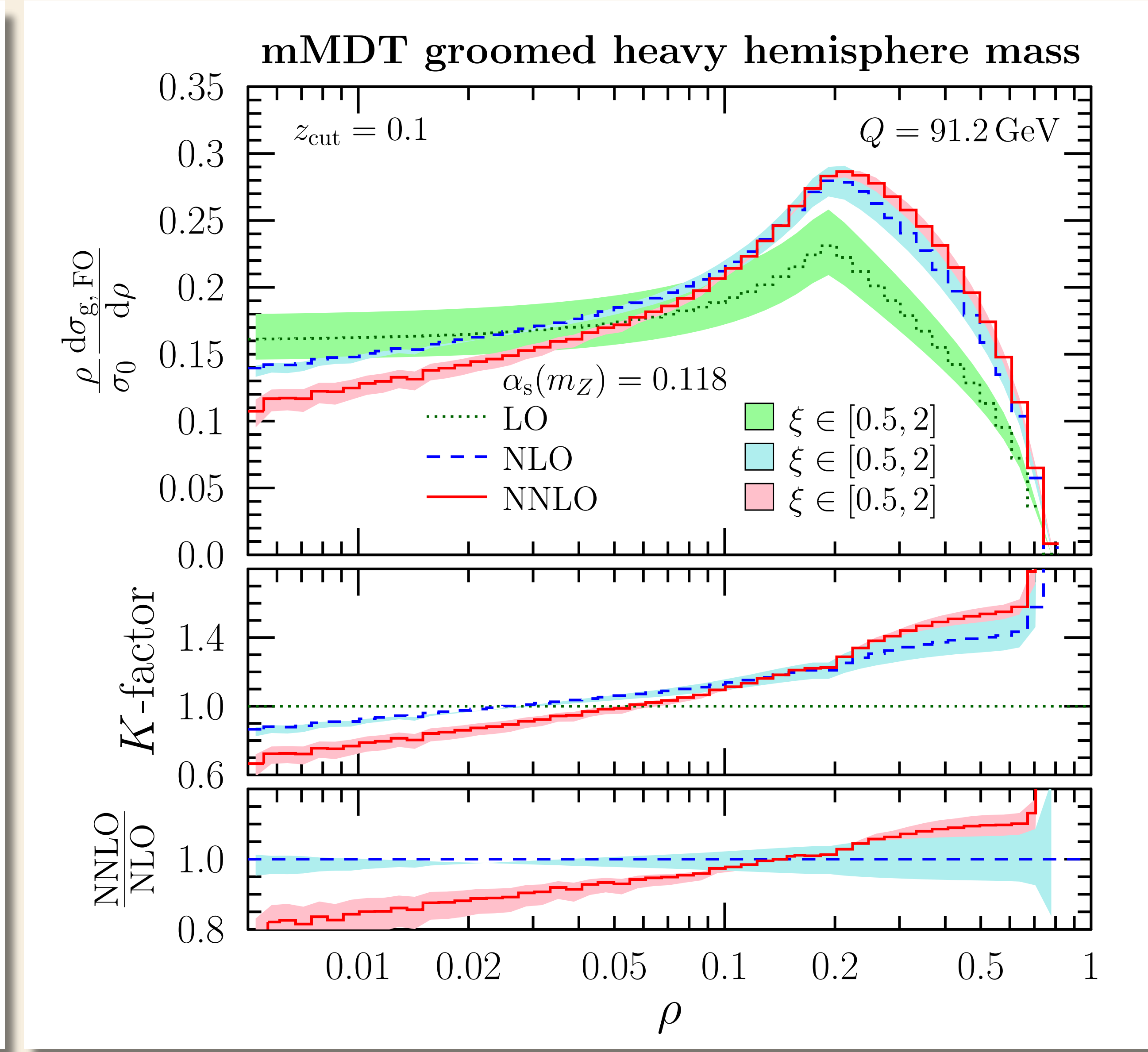
Converges for $\rho > 0.1$,
cannot be trusted for $\rho < 0.1$

mMDT groomed heavy jet mass

N³LL can be matched to N²LO additively by subtracting the expansion of N³LL through O(α_s^3)

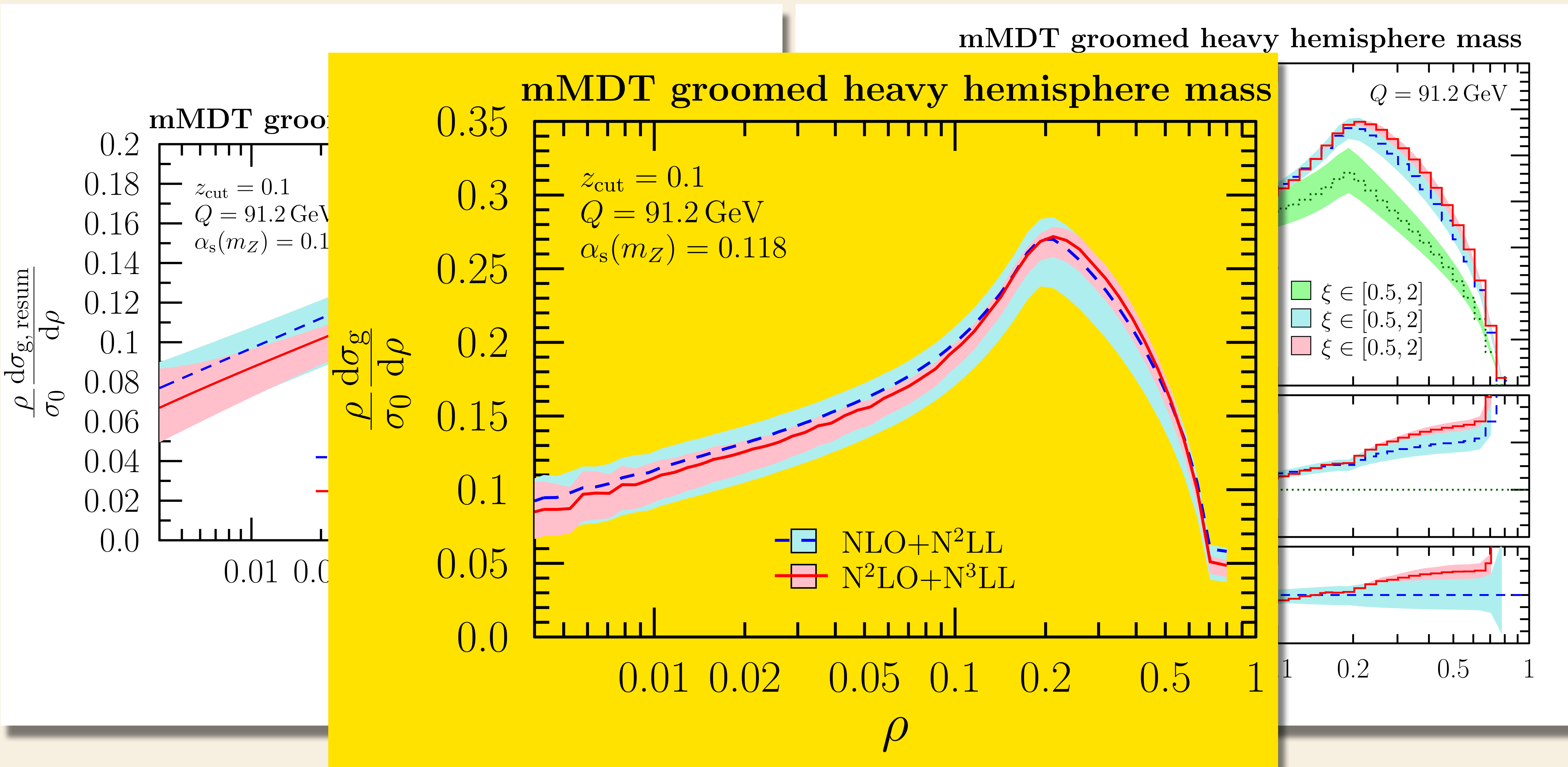
$$\frac{\rho}{\sigma_0}\frac{d\sigma_{g,\text{FO+res}}}{d\rho} = \frac{\rho}{\sigma_0}\left(\frac{d\sigma_{g,\text{N}^3\text{LL}}}{d\rho} + \frac{d\sigma_{g,\text{N}^2\text{LO}}}{d\rho} - \frac{d\sigma_{g,\text{LP}}}{d\rho}\right)$$





$$\frac{d\sigma_{g,\text{LP}}}{d\rho} = \delta(\rho)D_{\delta,g} + \frac{\alpha_s}{2\pi}(D_{A,g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^2(D_{B,g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^3(D_{C,g}(\rho))_+$$

mMDT groomed heavy jet mass



Conclusions

- ✓ Precise determination of the strong coupling using hadronic final states in electron-positron annihilation requires

  careful selection of observables with small perturbative and non-perturbative corrections (and data — not discussed here)

- ✓ MCCSM was used to compute differential distributions for groomed event shapes — mMDT groomed heavy jet mass among others

- ✓ Our predictions
  - show good perturbative stability for $\rho > 10^{-1}$ (smaller scale dependence than un-groomed event shapes)
  - are stable numerically to $\rho \sim 10^{-4}$
  - were used to extract unknown constants needed for NNNLL resummation and matching

- ✓ NNLO+NNNLL additive matching is made possible the first time

The end

Appendix

Kinematics

For $z_{\rm cut} \ll 1$ only soft particles are groomed away,

hence the mMDT constraint is (E_H: hemisphere energy)

$$E_s > E_H z_{\rm cut} = \frac{Q}{2} z_{\rm cut}$$

Contribution to ρ of a soft particle, just passing mMDT, is

$$\rho = \frac{m_H^2}{E_H^2} = \frac{2E_H E_s(1-\cos\theta_s)}{E_H^2} \leq 2\frac{E_s}{E_H}$$

Taking the upper bound, we get parametric scaling as

$$E_s \sim \frac{\rho E_H}{2} \quad \text{and then} \quad \rho \gtrsim 2 z_{\rm cut}$$

mMDT grooming acts where $\rho < 2z_{\rm cut}$

Extraction of two-loop constants

The leading-power (LP) differential cross section for $\rho \to 0$

$$\frac{\mathrm{d}\sigma_{\mathrm{g,LP}}}{\mathrm{d}\rho} = D_{\delta,\mathrm{g}}\,\delta(\rho) + \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}$$

σ^{sing} is defined to integrate to 0 on $[0,1]$, hence the total sec is

$$\sigma_{\mathrm{tot}} = D_{\delta,\mathrm{g}} + \int_0^1 \mathrm{d}\rho\left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right)$$

known contains $c_{S_c}^{(2)}$ difficult to integrate numerically

need a better strategy: will be achieved through steps of identities

Extraction of two-loop constants

We want numerical integrals in the region where grooming acts

$$\int_0^1 \mathrm{d}\rho\left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho}-\frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right)=\int_0^{2z_{\mathrm{cut}}}\mathrm{d}\rho\left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho}-\frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right)+\int_{2z_{\mathrm{cut}}}^1\mathrm{d}\rho\,\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho}-\int_{2z_{\mathrm{cut}}}^1\mathrm{d}\rho\,\frac{d\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}$$

the integral of $\mathrm{d}\sigma_{\mathrm{g}}/\mathrm{d}\rho$ can be rewritten into

$$\int_{2z_{\mathrm{cut}}}^1\mathrm{d}\rho\,\frac{d\sigma_{\mathrm{g}}}{\mathrm{d}\rho}=\int_{2z_{\mathrm{cut}}}^4\mathrm{d}\rho\left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho}\Theta(1-\rho)-\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\right)+\int_{2z_{\mathrm{cut}}}^4\mathrm{d}\rho\,\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}$$

mMDT has effect near z_{cut}, so can drop the Θ function

upper limit is 4 in the ungroomed xsec because there ρ is normalized to the cm instead of the hemisphere energy,

(yet the integrand of the 1st integral vanishes below 1)

Extraction of two-loop constants

Resume:

$$\int_0^1 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right) = \int_0^{2z_{\mathrm{cut}}} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right) + \int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \frac{d\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}$$

$$\int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} = \int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\right) + \int_{2z_{\mathrm{cut}}}^4 \mathrm{d}\rho \frac{\mathrm{d}\sigma}{\mathrm{d}\rho}$$

Up to power corrections, the ungroomed xsec is

$$\int_{2z_{\mathrm{cut}}}^4 \mathrm{d}\rho \frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \int_0^4 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\rho}\right) + \mathcal{O}(z_{\mathrm{cut}}) + \int_{2z_{\mathrm{cut}}}^4 \mathrm{d}\rho \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\rho}$$

Collecting all steps we find

$$\sigma_{\mathrm{tot}} = D_{\delta,\mathrm{g}} + \int_0^{2z_{\mathrm{cut}}} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right) + \int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\right)$$
$$+ \int_0^4 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\rho}\right) + \int_{2z_{\mathrm{cut}}}^1 \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g}}^{\mathrm{sing}}}{\mathrm{d}\rho}\right) + \int_1^4 \mathrm{d}\rho \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\rho} + \mathcal{O}(z_{\mathrm{cut}})$$

Extraction of two-loop constants

For σ_{tot} we have from previous page:

$$\sigma_{\text{tot}} = D_{\delta,\text{g}} + \int_0^{2z_{\text{cut}}} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\text{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\text{g}}^{\text{sing}}}{\mathrm{d}\rho} \right) + \int_{2z_{\text{cut}}}^{1} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\text{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma}{\mathrm{d}\rho} \right)$$
$$+ \int_0^{4} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} \right) + \int_{2z_{\text{cut}}}^{1} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\text{g}}^{\text{sing}}}{\mathrm{d}\rho} \right) + \int_1^{4} \mathrm{d}\rho \, \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} + \mathcal{O}\left(z_{\text{cut}}\right)$$

but it can also be expressed with the ungroomed distribution:

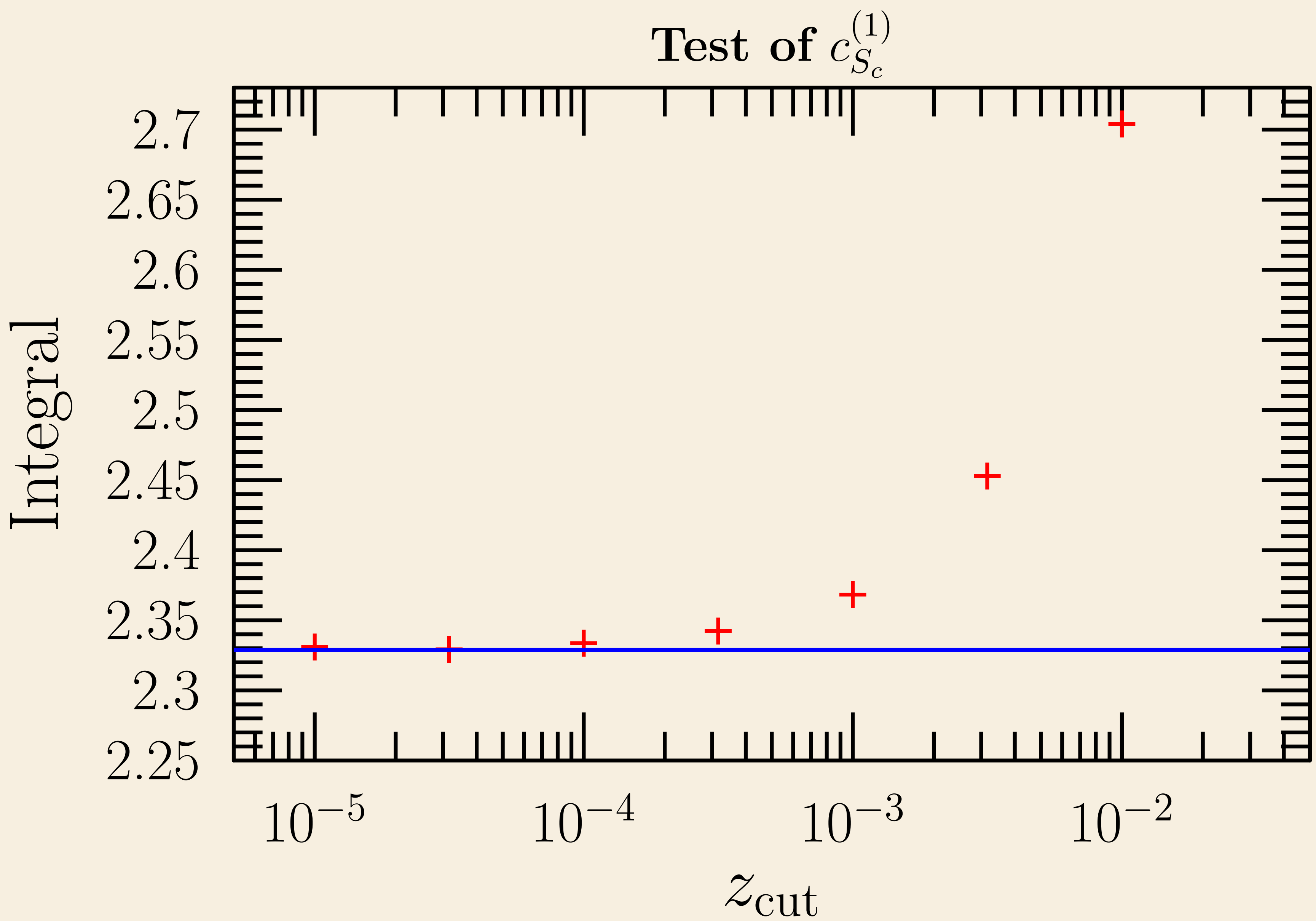
$$\sigma_{\text{tot}} = D_{\delta} + \int_0^{4} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} \right)$$

so finally

$$D_{\delta} = D_{\delta,\text{g}} + \int_0^{2z_{\text{cut}}} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\text{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\text{g}}^{\text{sing}}}{\mathrm{d}\rho} \right) + \int_{2z_{\text{cut}}}^{1} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma_{\text{g}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma}{\mathrm{d}\rho} \right)$$
$$+ \int_{2z_{\text{cut}}}^{1} \mathrm{d}\rho \left(\frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\text{g}}^{\text{sing}}}{\mathrm{d}\rho} \right) + \int_1^{4} \mathrm{d}\rho \, \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}\rho} .$$

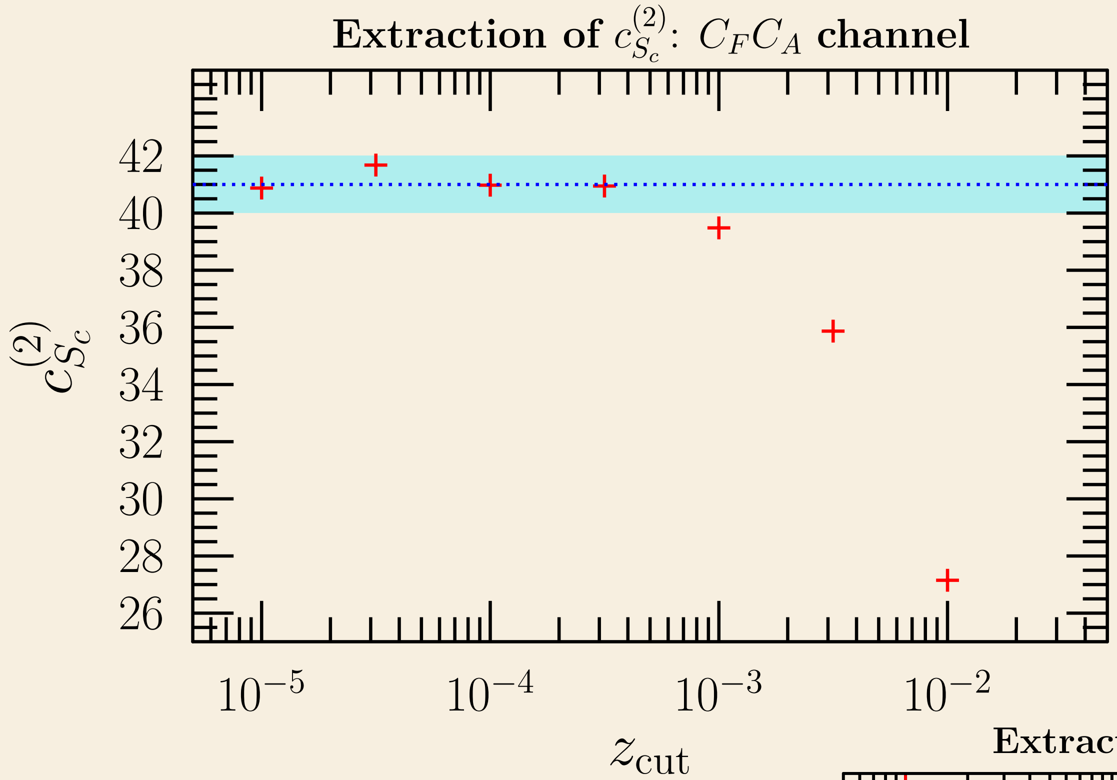
Validation at one loop

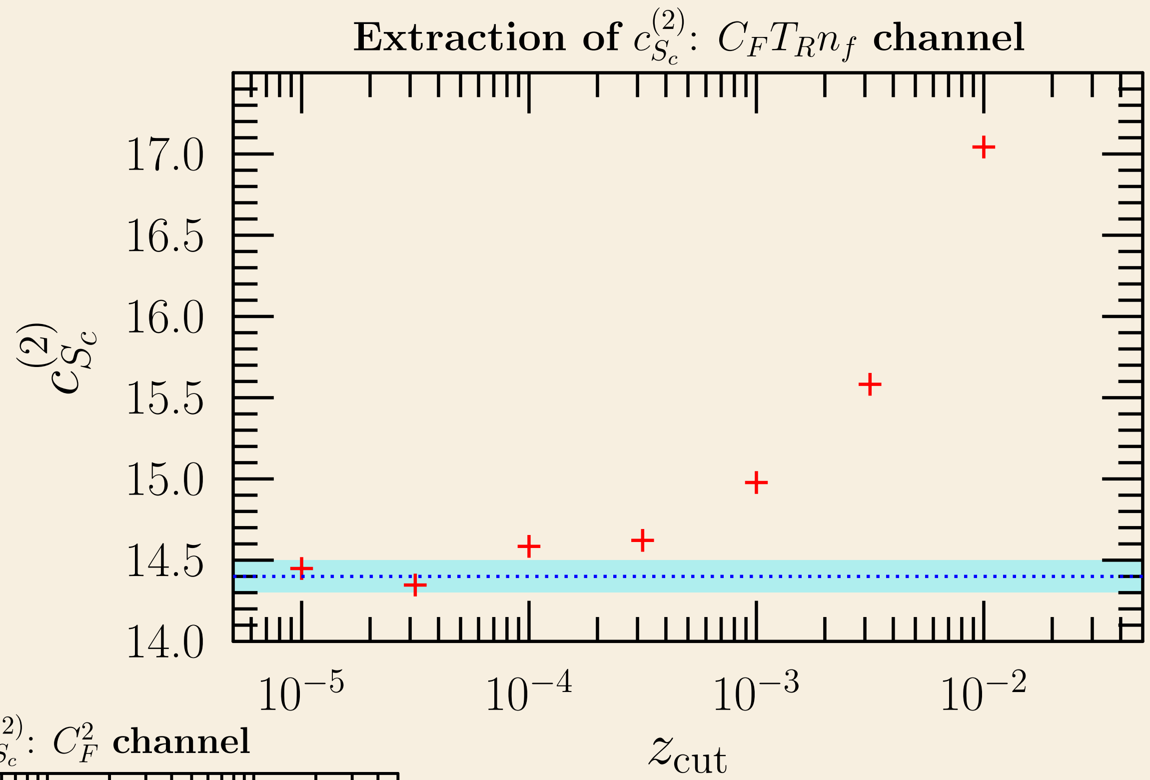
At one loop the integral can be computed numerically, but also known analytically:

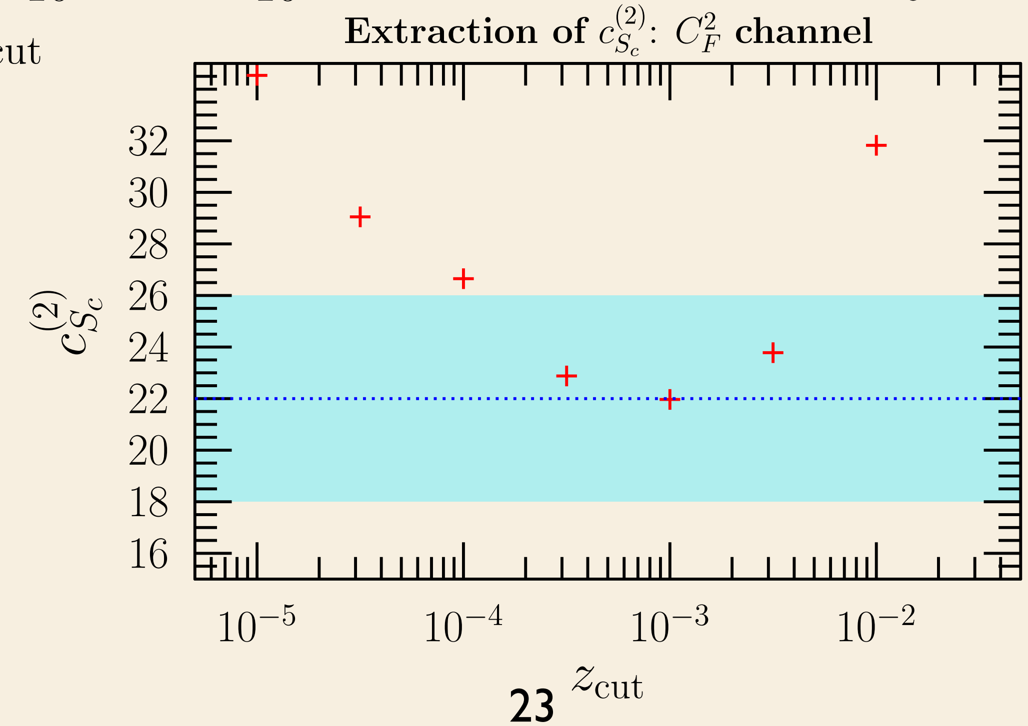


Fits at two loops

At two loops the integral computed numerically with EVENT2, can be fitted (separately for each color channel)







Extraction of three-loop non-cusp anomalous dimension

the formal expansion of the mMDT groomed distribution for $\rho \ll z_{\rm cut} \ll 1$:

$$\frac{d\sigma_{\rm g,LP}}{d\rho} = \delta(\rho) D_{\delta,\rm g} + \frac{\alpha_s}{2\pi}(D_{A,\rm g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^2 (D_{B,\rm g}(\rho))_+ + \left(\frac{\alpha_s}{2\pi}\right)^3 (D_{C,\rm g}(\rho))_+$$

mMDT grooming removes double logarithms in ρ to all orders:

$$D_{\delta,g} = c_\delta(z_{\rm cut})$$

$$\rho D_{A,g} = c_A(z_{\rm cut})$$

$$\rho D_{B,g} = b_B(z_{\rm cut}) \log\rho + c_B(z_{\rm cut})$$

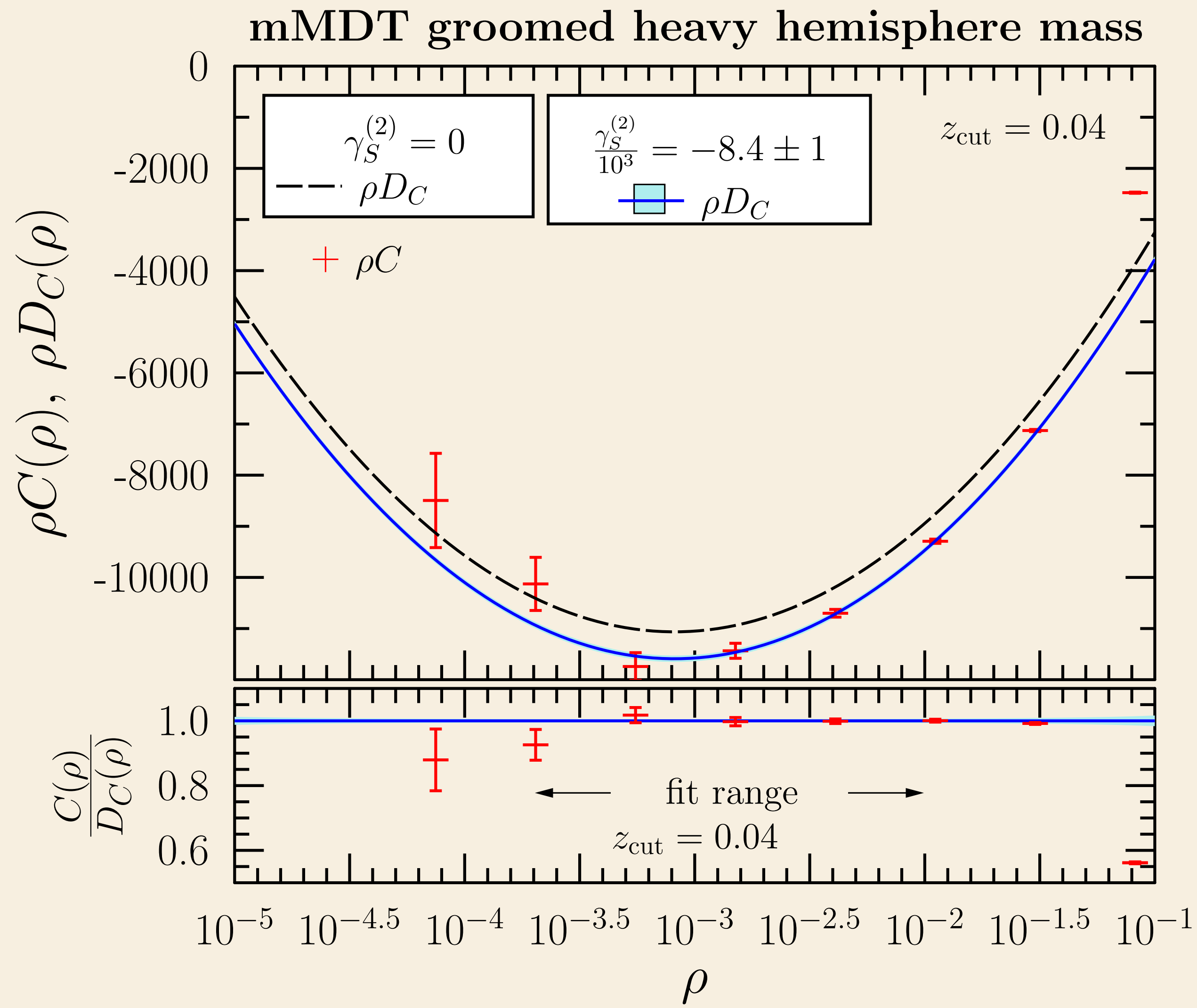
$$\rho D_{C,g} = a_C(z_{\rm cut}) \log^2\rho + b_C(z_{\rm cut}) \log\rho + c_C(z_{\rm cut})$$

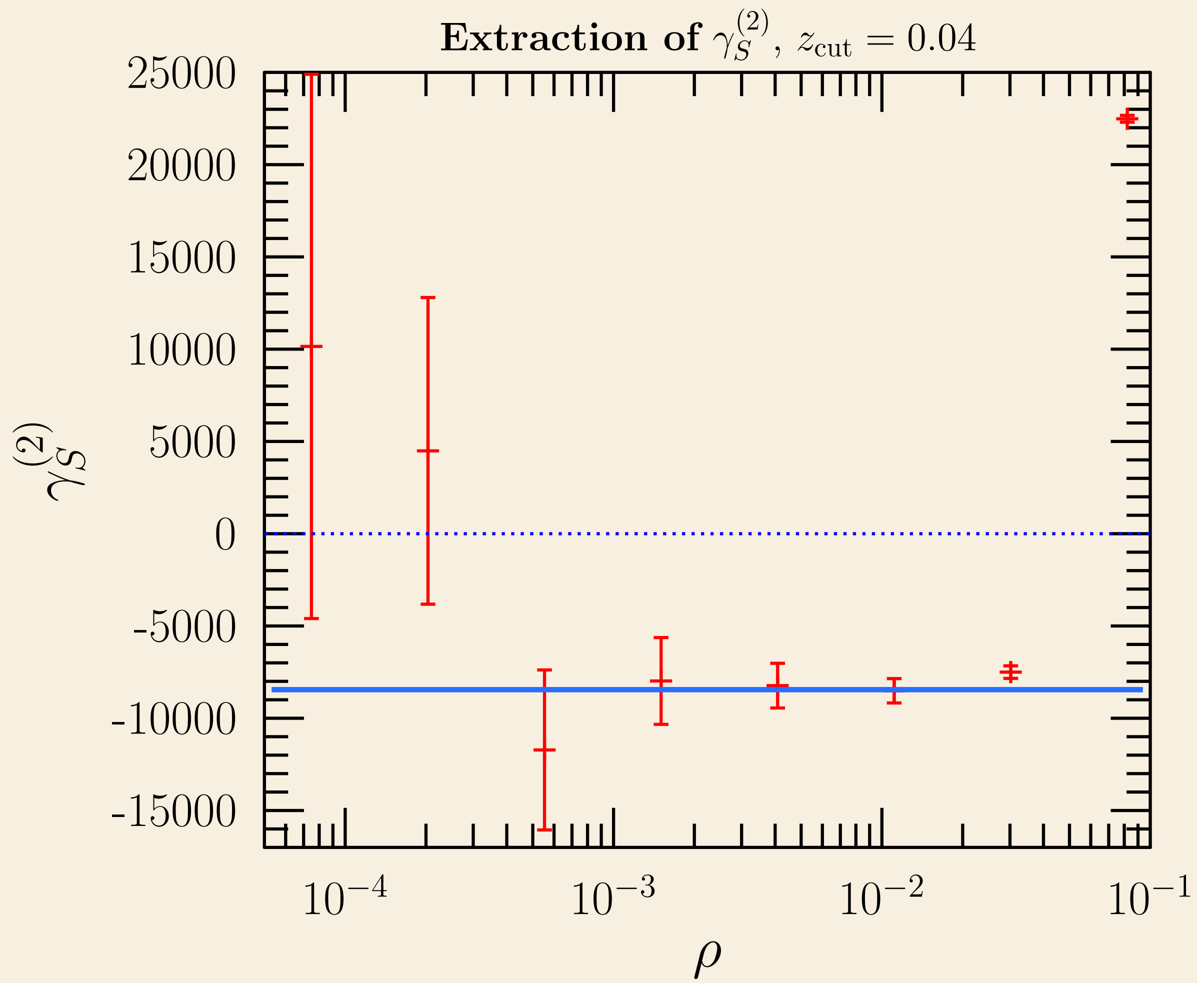
$\uparrow$ can be computed by MCCSM

$z_{\rm cut} \to 0$ (applied to $c_C(z_{\rm cut})$):

$$\gamma_S^{(2)}/16 - 1944.97$$

Fit a parabola in log ρ for fixed z_{cut}





Extrapolation of constant term to $z_{\text{cut}}=0$