



# The Anomalous Case of Axion EFTs and Massive Chiral Gauge Fields

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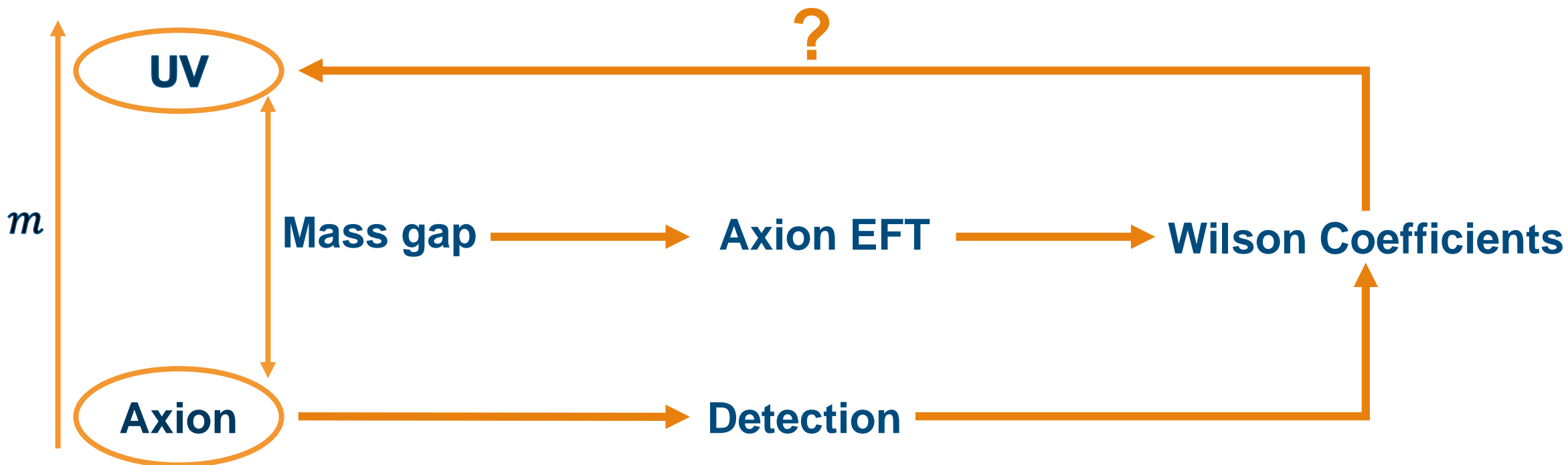
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# Axions, axions everywhere

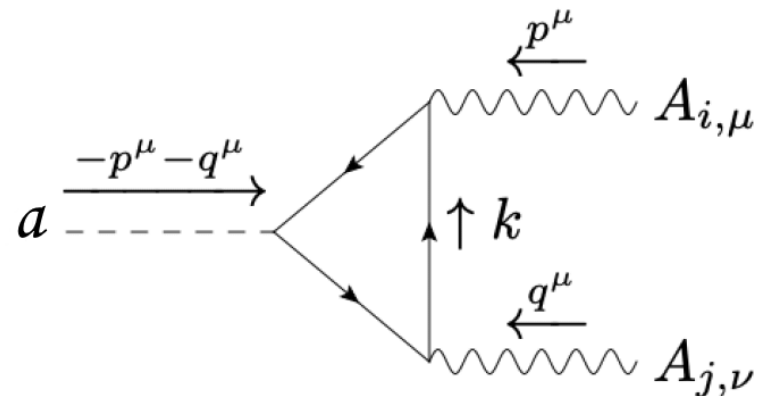
- ALPs (axions) are very appealing BSM candidates.
- There is an intense experimental program.



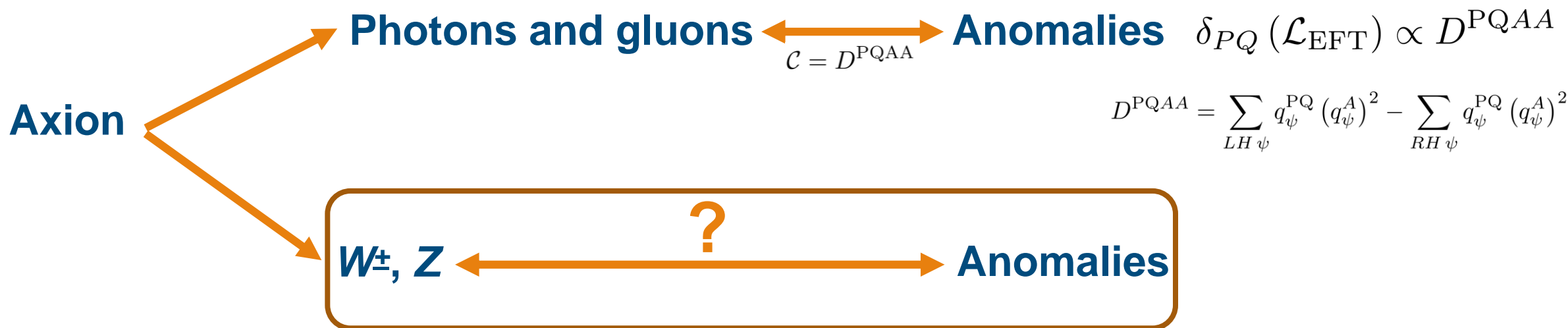
What does an IR measurement say about the UV?

# Axion phenomenology

Effective axion-gauge boson couplings play the main role.



$$\mathcal{A}(a \rightarrow AA) \propto \mathcal{C}$$



$$\delta_{PQ}(\mathcal{L}_{\text{EFT}}) \propto D^{\text{PQAA}}$$

$$D^{\text{PQAA}} = \sum_{LH\psi} q_{\psi}^{\text{PQ}} (q_{\psi}^A)^2 - \sum_{RH\psi} q_{\psi}^{\text{PQ}} (q_{\psi}^A)^2$$

This relation is not so clear...

E.g. it is violated in DFSZ [Q&S, 1903.12559]

# Axion EFT with a massive abelian gauge field

Non-anomalous coupling linked to longitudinal degrees of freedom.

$$\mathcal{L} \supset -g^2 \frac{D^{\text{PQAA}}}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu} - g \frac{\mathcal{C} - D^{\text{PQAA}}}{8\pi^2 f} \partial_\mu a \left( \frac{\partial_\nu \theta_A}{m_A} - g A_\nu \right) \tilde{F}^{\mu\nu}$$

(1): anomalous

(2): non-anomalous

$$\delta_{PQ}(\mathcal{L}_{\text{EFT}}) \propto D^{\text{PQAA}}$$

$$\mathcal{A}(a \rightarrow AA) \propto \mathcal{C}$$

## When does this happen?

Massive chiral  $\longrightarrow \mathcal{C} \neq D^{\text{PQAA}} \longrightarrow$  (2) Non-zero

# Non-abelian: EFT terms for the SM

## Anomalous

$$-\frac{g^2 C_{WW}}{16\pi^2} \frac{a}{f} W^a \tilde{W}^a$$

$$-\frac{g'^2 C_{BB}}{16\pi^2} \frac{a}{f} B \tilde{B}$$

## Non-anomalous

$$\partial_\mu a \operatorname{Tr}(TV_\nu) \tilde{B}^{\mu\nu}$$

$$\partial_\mu a \operatorname{Tr}(V_\nu \tilde{W}^{\mu\nu})$$

$$\partial_\mu a \operatorname{Tr}(TV_\nu) \operatorname{Tr}(T\tilde{W}^{\mu\nu})$$

$$U = e^{i\frac{\pi^a}{v}\sigma^a}, \quad D_\mu U = \partial_\mu U - igW_\mu U + ig'B_\mu U \frac{\sigma_3}{2} \quad V_\mu = D_\mu U U^\dagger, \quad T = U \sigma_3 U^\dagger$$

**Non-linear EW is a consequence of chiral matter in the UV.**

[Cohen et al, 2008.08597]

# Sum rules for the axion EFT

## Sum rules from anomalous-only operators

$$C_{\gamma\gamma} + s_W^{-2}(1 - t_W^2)C_{Z\gamma} - \frac{1}{s_W^2 c_W^2}C_{ZZ} = 0, \quad C_{\gamma\gamma} + s_W^{-2}C_{Z\gamma} - (1 + t_W^{-2})C_{WW} = 0$$

**In the mass basis:**  $-\frac{16\pi^2}{e^2}\mathcal{L} \supset C_{\gamma\gamma}\frac{a}{f}F\tilde{F} + 2\frac{C_{Z\gamma}}{c_W s_W}\frac{a}{f}F\tilde{Z} + \frac{C_{ZZ}}{c_W^2 s_W^2}\frac{a}{f}Z\tilde{Z} + \frac{2C_{WW}}{s_W^2}W^+\tilde{W}^-$

## Sum-rules violation by non-anomalous effects

$$C_{\gamma\gamma} + s_W^{-2}(1 - t_W^2)C_{Z\gamma} - \frac{1}{s_W^2 c_W^2}C_{ZZ} = \frac{c_1 - c_2 - c_3}{2c_W^2 s_W^2}, \quad C_{\gamma\gamma} + s_W^{-2}C_{Z\gamma} - (1 + t_W^{-2})C_{WW} = \frac{c_1 - c_2 + c_3}{2s_W^2}$$

$$i\frac{c_1 g'}{8\pi^2 f}\partial_\mu a \text{Tr}(TV_\nu)\tilde{B}^{\mu\nu} + i\frac{c_2 g}{4\pi^2 f}\partial_\mu a \text{Tr}(V_\nu\tilde{W}^{\mu\nu}) + i\frac{c_3 g}{4\pi^2 f}\partial_\mu a \text{Tr}(TV_\nu)\text{Tr}(T\tilde{W}^{\mu\nu})$$

... which only appear after integrating out chiral matter



**The violation of these sum rules is the hallmark of chiral matter!**

# A minimal model with axions and chiral matter

## DFSZ-like extended scalar sector

$(SU(3)_c, SU(2)_L, U(1)_Y)_{U(1)_{PQ}}$

$$H_{1,2} \sim (1, 2, \pm \frac{1}{2})_{X_{1,2}} \quad a = \frac{1}{f} (V_\Phi a_\Phi + X_1 v_1 a_1 + X_2 v_2 a_2), \quad f^2 = V_\Phi^2 + X_1^2 v_1^2 + X_2^2 v_2^2$$

$$\Phi \sim (1, 1, 0)_1 \quad H_1 \supset \frac{v_1}{\sqrt{2}} e^{i \frac{a_1}{v_1}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad H_2 \supset \frac{v_2}{\sqrt{2}} e^{i \frac{a_2}{v_2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \Phi \supset \frac{V_\Phi}{\sqrt{2}} e^{i \frac{a_\Phi}{V_\Phi}}$$

## New heavy fermions

$$L_1 \sim (1, 2, +Y)_{X_{L_1}} \quad E_1 \sim (1, 1, +Y - \frac{1}{2})_{X_{L_1} - X_1} \quad N_1 \sim (1, 1, +Y + \frac{1}{2})_{X_{L_1} + X_2}$$

$$L_2 \sim (1, 2, -Y)_{X_{L_2}} \quad E_2 \sim (1, 1, -Y + \frac{1}{2})_{X_{L_2} + X_2} \quad N_2 \sim (1, 1, -Y - \frac{1}{2})_{X_{L_2} - X_1}$$

**Left Handed**

**Right Handed**

$$- \mathcal{L}_Y = y_{E_1} \bar{L}_1 E_1 H_1 + y_{E_2} \bar{L}_2 E_2 \tilde{H}_2 + y_{N_1} \bar{L}_1 N_1 \tilde{H}_2 + y_{N_2} \bar{L}_2 N_2 H_1 + \text{h.c.}$$

**Vector-like UV charges, but chiral spectrum**

# Axion-gauge bosons EFTs and sum rules

$$-g'^2 \frac{(1 + 12Y^2)(X_1 - X_2)}{96\pi^2} \frac{a}{f} B\tilde{B} - g^2 \frac{X_1 - X_2}{96\pi^2} \frac{a}{f} W^a \tilde{W}^a \Big|_{\text{lin.}} - gg' \underbrace{\frac{X_1 - X_2}{96\pi^2} \frac{a}{f} B\tilde{W}^3 \Big|_{\text{lin.}}}_{\text{Non-anomalous}}$$

**Broken  
Sum rules**

**Non-anomalous**

$$C_{\gamma\gamma} + s_W^{-2}(1 - t_W^2)C_{Z\gamma} - \frac{1}{s_W^2 c_W^2} C_{ZZ} = \frac{X_1 - X_2}{12c_W^2 s_W^2} \quad C_{\gamma\gamma} + s_W^{-2}C_{Z\gamma} - (1 + t_W^{-2})C_{WW} = \frac{X_2 - X_1}{12s_W^2}$$

**The sum rules can be written with observables**

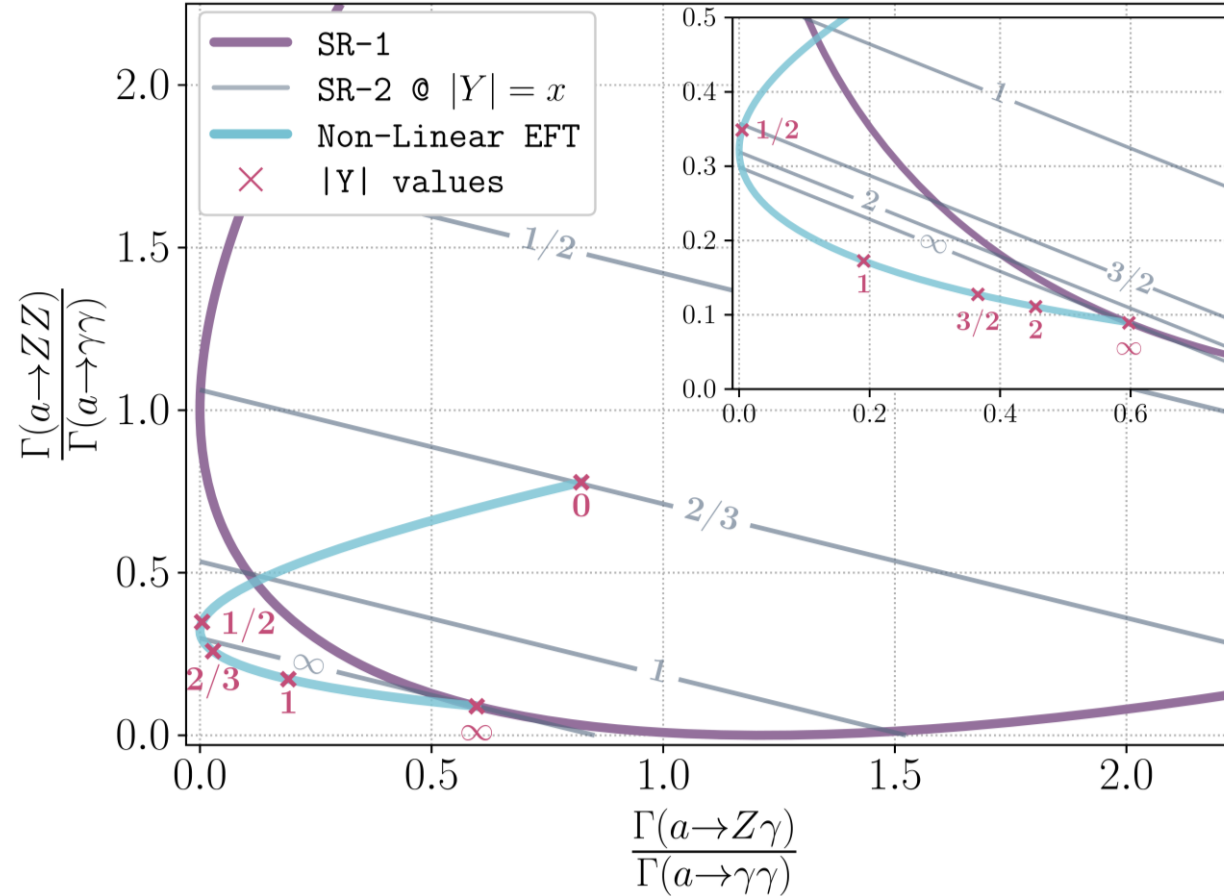
$$m_a > 2m_Z$$

$$\text{SR-1: } \left[ \frac{\Gamma(a \rightarrow ZZ)}{\Gamma(a \rightarrow \gamma\gamma)} - 1 - \frac{(t_W^2 - 1)^2}{2t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} \right]^2 - \frac{2(t_W^2 - 1)^2}{t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} = 0$$

$$\text{SR-2: } \Gamma(a \rightarrow \gamma\gamma) + \frac{1}{2}(t_W^{-2} - 1)\Gamma(a \rightarrow WW) - t_W^{-2}\Gamma(a \rightarrow ZZ) + \frac{1}{2}(1 - t_W^{-2})\Gamma(a \rightarrow Z\gamma) = 0$$



# Sum rules for partial widths, graphically

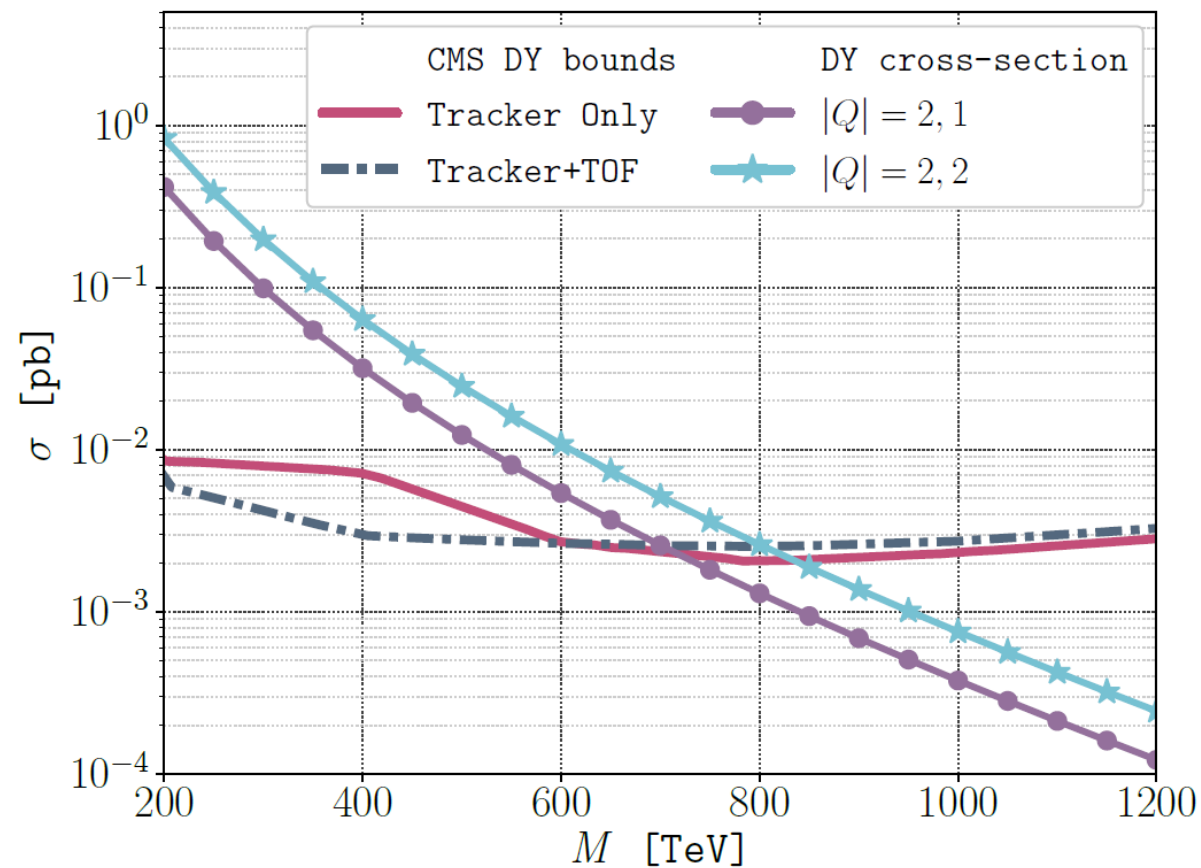
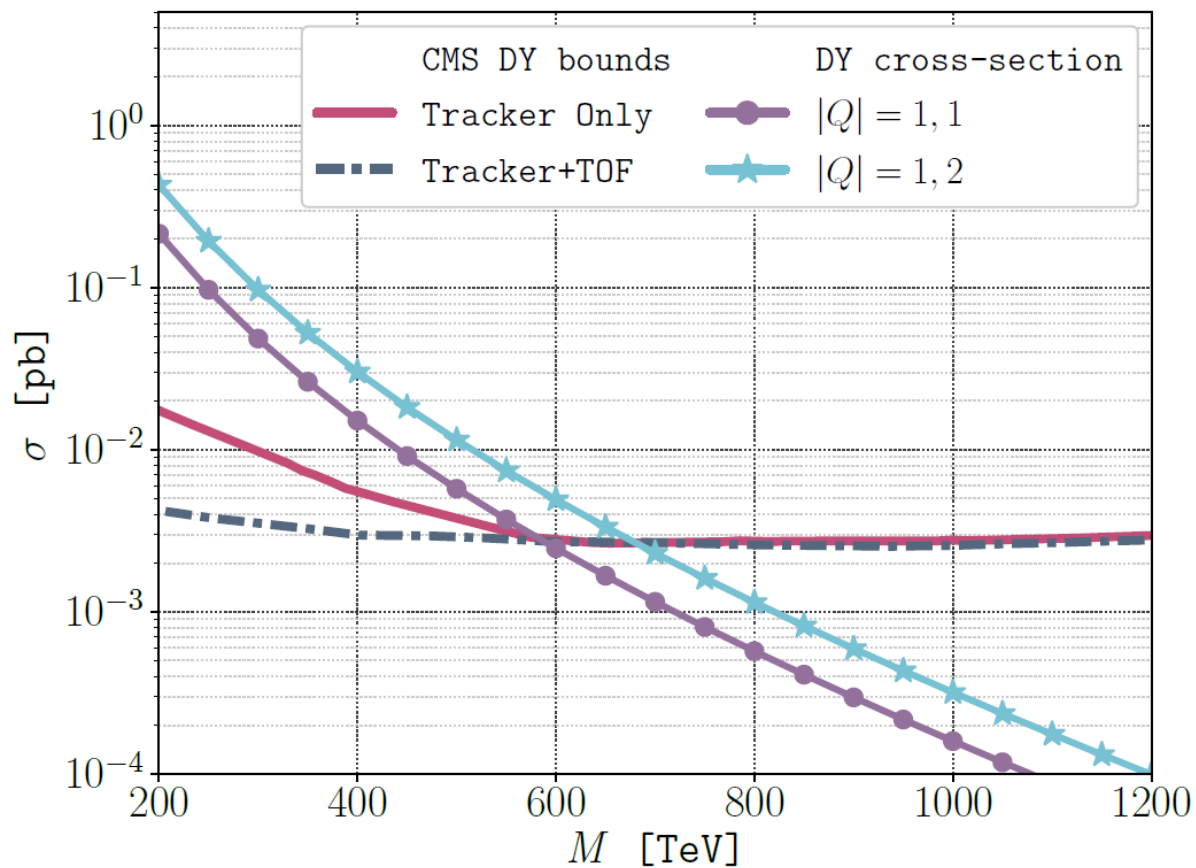


$$\text{SR-1: } \left[ \frac{\Gamma(a \rightarrow ZZ)}{\Gamma(a \rightarrow \gamma\gamma)} - 1 - \frac{(t_W^2 - 1)^2}{2t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} \right]^2 - \frac{2(t_W^2 - 1)^2}{t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} = 0$$

$$\text{SR-2: } \Gamma(a \rightarrow \gamma\gamma) + \frac{1}{2} (t_W^{-2} - 1) \Gamma(a \rightarrow WW) - t_W^{-2} \Gamma(a \rightarrow ZZ) + \frac{1}{2} (1 - t_W^{-2}) \Gamma(a \rightarrow Z\gamma) = 0$$

# Chiral extensions: heavy fermions direct searches

$$Y = 3/2$$



**Custodial  
limit**



$$M \gtrsim 720 \text{ GeV}$$

**Fully  
degenerate**



$$M \gtrsim 840 \text{ GeV}$$

# Conclusions

- **The longitudinal component of massive gauge fields allows us to write non-anomalous axion-gauge field operators.**
- **They break the correlation between couplings and anomalies.**
- **Integrated-out chiral matter generates those operators.**
- **In the non-abelian case, sum rules distinguish between couplings fully related or not to anomalies.**
- **This is relevant for axion couplings to EW gauge bosons.**
- **It links axions with chiral extensions of the SM.**
- **We provided an explicit chiral extension of the SM that showcases this phenomenon and is still allowed by experiments.**

# Thank you for your attention



Contact

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# Appendix

# UV and IR chirality

Vector-like gauge charges can yield a *chiral* mass spectrum.

Charge under	$\psi_L$	$\psi'_L$	$\psi_R$	$\psi'_R$
U(1)	+1	0	0	+1

$$\mathcal{L} \supset -y\overline{\psi}_L\psi_R\phi - y'\overline{\psi}'_L\psi'_R\phi^\dagger + h.c. \supset -m\overline{\psi}\psi - m'\overline{\psi}'\psi'$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \psi' = \begin{pmatrix} \psi'_L \\ \psi'_R \end{pmatrix}$$

$$\overline{\psi}_L\psi'_R + h.c.$$

$$\overline{\psi}'_L\psi_R + h.c.$$



**Can be forbidden  
by symmetries**

# Axion EFT in the abelian case

Consider a pseudoscalar axion,  $a$ , and abelian gauge field  $A_\mu$

Lowest dim. CP-inv. Operator with those fields

$$\begin{array}{l}
 \mathcal{L}_{\text{EFT}} \supset -g^2 \frac{\mathcal{C}}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu} \xrightarrow{\delta_{PQ} a = \epsilon_{PQ} f} \delta_{PQ} (\cdot) \propto \mathcal{C} \\
 \text{Sum and subtract } D^{\text{PQAA}} \text{ and integrate by parts} \downarrow \\
 -g^2 \frac{D^{\text{PQAA}}}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu} + g^2 \frac{\mathcal{C} - D^{\text{PQAA}}}{8\pi^2 f} \partial_\mu a A_\nu \tilde{F}^{\mu\nu} \xrightarrow{\mathcal{A} (a \rightarrow AA) \propto \mathcal{C}} \delta_{PQ} (\cdot) \propto D^{\text{PQAA}}
 \end{array}$$

**Anomalies may not capture all of the axion-gauge bosons couplings**

# EFT with a fake PQ-gauge symmetry

Couple your theory to an auxiliary gauge field  $A_\mu^{\text{PQ}}$   $\partial_\mu \epsilon_{\text{PQ}} \neq 0$   
 And consider “local” PQ transformations

Anomaly matching must hold for consistency of the fake gauge theory

$D^{\text{PQAA}}$  is the same for global and local PQ transformations



$$\delta_{\text{PQ}}(\mathcal{L}_{\text{EFT}}) = -g^2 \epsilon_{\text{PQ}} \frac{D^{\text{PQAA}}}{16\pi^2 f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

**But:**

$$\delta_{\text{PQ}} \left( -g^2 \frac{D^{\text{PQAA}}}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \underbrace{g^2 \frac{\mathcal{C} - D^{\text{PQAA}}}{8\pi^2 f} \partial_\mu a A_\nu \tilde{F}^{\mu\nu}}_{\neq D^{\text{PQAA}}} \right) = -g^2 \epsilon_{\text{PQ}} \frac{\mathcal{C}}{16\pi^2 f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

**We'd like this term to not contribute to the anomaly**



# Introducing the GCS term

**Generalized  
Chern-Simons  
(GCS) term**

$$\frac{\partial_\mu a}{f} A_\nu \tilde{F}^{\mu\nu} \quad \longrightarrow \quad \left( \frac{\partial_\mu a}{f} - g_{\text{PQ}} A_\mu^{\text{PQ}} \right) A_\nu \tilde{F}^{\mu\nu}$$

**Non PQ invariant**      **PQ invariant**      **Non gauge invariant**

**Gauge invariant**

**If  $A_\nu^{\text{PQ}}$  is massive**

$$A_\mu \rightarrow A_\mu - \frac{\partial_\mu \theta_A}{g m_A}$$

$$-\frac{1}{g} \left( \frac{\partial_\mu a}{f} - g_{\text{PQ}} A_\mu^{\text{PQ}} \right) \left( \frac{\partial_\nu \theta_A}{m_A} - g A_\nu \right) \tilde{F}^{\mu\nu}$$

**At the end, set  $A_\nu^{\text{PQ}} = 0$**

# Minimal chiral axion model: Yukawa sector

$$-\mathcal{L}_Y = y_{E_1} \bar{L}_1 E_1 H_1 + y_{E_2} \bar{L}_2 E_2 \tilde{H}_2 + y_{N_1} \bar{L}_1 N_1 \tilde{H}_2 + y_{N_2} \bar{L}_2 N_2 H_1 + \text{h.c.}$$

$$\begin{pmatrix} L_1 = (N_{L_1}, E_{L_1})^T \\ L_2 = (E_{L_2}, N_{L_2})^T \end{pmatrix} \quad \downarrow \quad \text{Axion couplings}$$

$$-\mathcal{L}_Y \supset m_{E_1} \left( e^{iX_1 \frac{a}{f}} \right) \bar{E}_{L_1} E_1 + m_{E_2} \left( e^{-iX_2 \frac{a}{f}} \right) \bar{E}_{L_2} E_2 \\ + m_{N_1} \left( e^{-iX_2 \frac{a}{f}} \right) \bar{N}_{L_1} N_1 + m_{N_2} \left( e^{iX_1 \frac{a}{f}} \right) \bar{N}_{L_2} N_2 + \text{h.c.}$$

**Mass eigenstates:**

$$N_i = N_i + N_{L_i}$$
$$E_i = E_i + E_{L_i}$$

**Custodial limit**

$$m_{N_i} = m_{E_i}$$

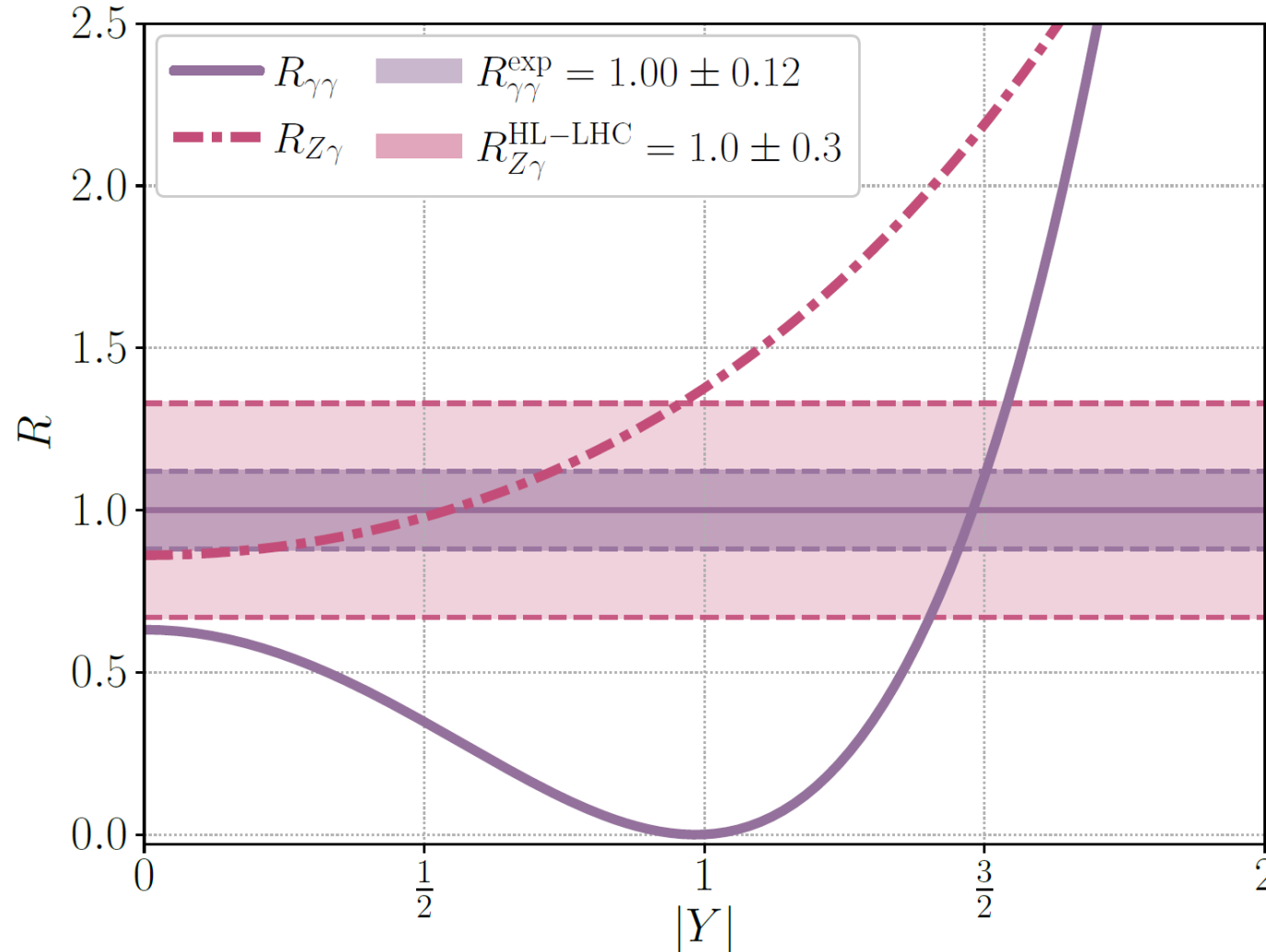
# Chiral extensions + 2HDM: Higgs couplings

**Alignment limit**



$$\beta - \alpha = \pi/2$$

$$R_{(Z,\gamma)\gamma} = \frac{|\mathcal{A}_{(Z,\gamma)\gamma}^{\text{SM}} + \mathcal{A}_{(Z,\gamma)\gamma}^{\text{new}}|^2}{|\mathcal{A}_{(Z,\gamma)\gamma}^{\text{SM}}|^2}$$



**Custodial limit**

$$m_{N_i} = m_{E_i}$$



**Constraints on S and T are satisfied**