An unambiguous test of positivity at lepton colliders

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[arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang

Cen Zhang (1984-2021)

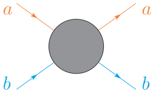
Introduction

- **Can all EFTs be UV completed?**

- Dispersion relations of forward elastic amplitudes suggest that certain operator coefficients can only be positive.
  - Assuming the UV physics is consistent with the fundamental principles of QFT (analyticity, locality, unitarity, Lorentz invariance).
  - [hep-th/0602178] Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi,
    ... many papers...
    [1902.08977, 2005.03047, ...] Zhang, Zhou et al.,
    [1908.09845, 2004.02885] Remmen, Rodd, ...

- **These positivity bounds only exist for certain Dimension-8 (or higher) operators!**

$$\frac{d^2}{ds^2}\mathcal{A}(ab \to ab)_{t\to 0}\big|_{s=0} \geq 0\,.$$



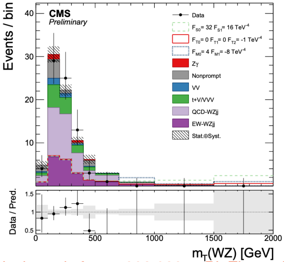
- By measuring these dim-8 operator coefficients, we can test whether the underlying new physics is consistent with the fundamental principles of QFT.

- **Can we do it?**

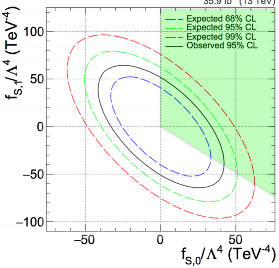
Probing positivity bounds on dimension-8 operators

- The dimension-8 contribution has a large energy enhancement $(\sim E^4/\Lambda^4)$!
- It is difficult for LHC to probe these bounds.
    - Low statistics in the high energy bins.
    - Example: Vector boson scattering.
    - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!

CMS-PAS-SMP-18-001



positivity bounds from 1902.08977 Bi, Zhang, Zhou



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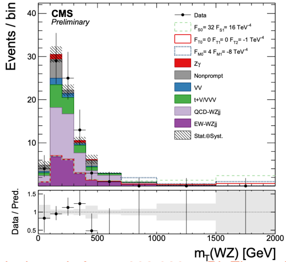
  - Example: Vector boson scattering.
  - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- Can we separate the dim-8 and dim-6 effects?
  - Precision measurements at several different $\sqrt{s}$?

    (A **very** high energy lepton collider?)
  - Or find some special process where dim-8 gives the leading new physics contribution?

CMS-PAS-SMP-18-001



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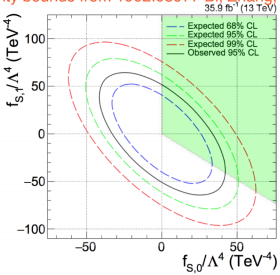
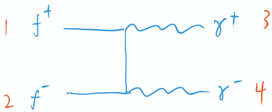
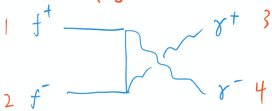


The diphoton channel [arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang

(signs denote helicities)





- $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$), SM, non-resonant.
  - Tree level SM: the only helicity configuration is $\mathcal{A}(f^+f^-\gamma^+\gamma^-)$.
- Leading order contribution: **dimension-8 contact interaction**.
  ($f^+f^- \to \bar{e}_L e_L$ or $e_R\bar{e}_R$)

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\mathrm{SM+d8}} = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 .$$

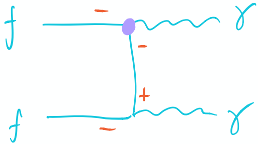
- Operators: Also contributes to $ZZ/Z\gamma$ final states with opposite helicities.
  [1806.09640] Bellazzini, Riva, see also d8 basis in
  [2005.00008] Shu et al., [2005.00059] Murphy

$$a_L = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{\ell B} - \cos\theta_W\sin\theta_W\, c^{(8)}_{\ell BW} + \sin^2\theta_W\, c^{(8)}_{\ell W}\right),$$
$$a_R = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{eB} + \sin^2\theta_W\, c^{(8)}_{eW}\right),$$

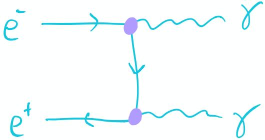
$$\mathcal{O}^{(8)}_{\ell B} = -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{eB} = -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \mathrm{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{\ell W} = -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})W^a_{\mu\nu}W^{a\,\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{eW} = -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \mathrm{h.c.})W^a_{\mu\nu}W^{a\,\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{\ell BW} = -\frac{1}{4}(i\bar{\ell}_L\sigma^a\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})B_{\mu\nu}W^{a\,\mu}{}_{\rho},$$

All other contributions are either vanishing or suppressed!

- **The only tree-level d6 contribution are from dipole operators and have different fermion helicities.**

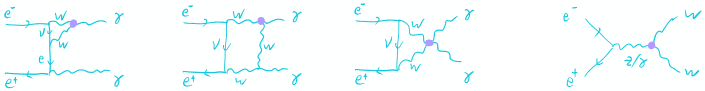


- **SM×d6 at tree level:** no interference.
- **d6^2:** Dipole operators are very well constrained by $g-2$ and EDM measurements.

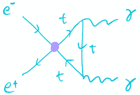


All other contributions are either vanishing or suppressed!

- **SM×d6 at 1-loop:** are either very-well constrained by other measurements with tree-level contributions, or forbidden by selection rules.
  - O_{3W} is very well constrained by $e^+e^- \to WW$ measurements.



  - Other contributions are constrained by Z-pole measurements or suppressed by the small y_e.
  - Contribution from the *eett* 4f operator is forbidden by angular momentum selection rules. ([2001.04481] Shu et al.)



$= 0$ (for e^+e^- with opposite helicities)

- **other d8:** They have different helicities and do not interfere with SM.

Positivity bounds

- Leading BSM contribution:

$$\mathcal{A}(\bar{e}_L e_L \gamma^+ \gamma^-)_{\rm d8} = \frac{a_L}{v^4}[13][23]\langle 24\rangle^2\,, \qquad \mathcal{A}(e_R \bar{e}_R \gamma^+ \gamma^-)_{\rm d8} = \frac{a_R}{v^4}[13][23]\langle 24\rangle^2\,.$$

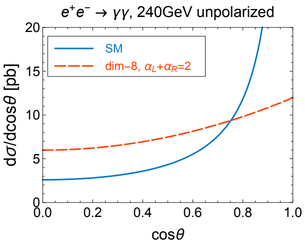
- Positivity bounds are obtained from the forward elastic amplitude $e\gamma \to e\gamma$:

$$\frac{d^2}{ds^2}\mathcal{A}(e\gamma \to e\gamma)|_{t\to 0} \geq 0\,,$$

- which implies

$$a_L \geq 0\,, \quad a_R \geq 0\,.$$

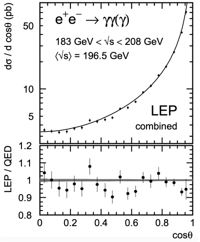
The diphoton cross section



- **Differential cross section** (The production polar angle θ is "folded" since the photon polarizations are not measured.)

$$\frac{d\sigma(e^+e^-\to\gamma\gamma)}{d\cos\theta} = \frac{(1-P_{e^-})(1+P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2}+a_L\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right) + \frac{(1+P_{e^-})(1-P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2}+a_R\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right),$$

1302.3415

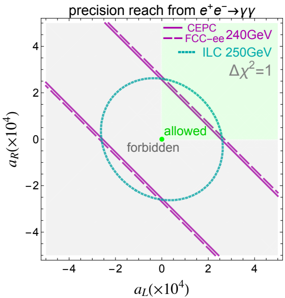


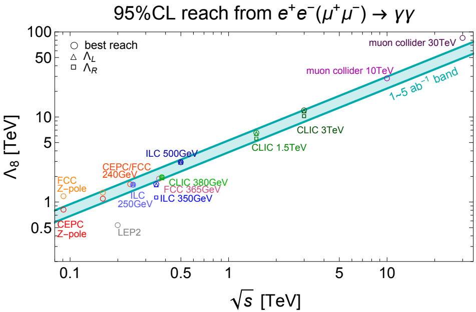
- Positivity bounds: $a_L \geq 0$, $a_R \geq 0$.
- **Positivity bound directly on the cross section!**

$$\sigma(e^+e^-\to\gamma\gamma) \geq \sigma_{\rm SM}(e^+e^-\to\gamma\gamma)\,.$$

- The LEP measurement was $\sim 1.5\sigma$ below the SM prediction.

Future projections





- χ^2 fit to the binned distribution
  - Statistics only, 19 bins in $\cos\theta\subset[0,0.95]$.
  - Agrees reasonably well with LEP result ($\lesssim 10\%$ in the reach on Λ).
- Is beam polarization useful? **Yes and no!**
  - One could measure σ_L and σ_R simultaneously.
- High energy still wins!

$$\frac{\Lambda_2}{\Lambda_1}=\left(\frac{E_2}{E_1}\right)^{\frac{3}{4}}\left(\frac{L_2}{L_1}\right)^{\frac{1}{8}}\,.$$

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy

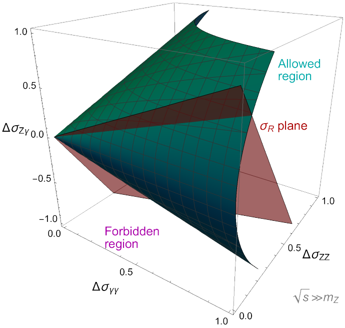
- $Z\gamma$, ZZ processes are more complicated due to the massive Z.
  - Other helicity states contribute in both SM and BSM (*e.g.* nTGCs).
  - In the high energy limit, $\mathcal{A}(f^+ f^- V^+ V^-)$ dominates in SM.

- In the $\sqrt{s} \gg m_Z$ limit,

$$\sigma(e^+e^- \to ZZ) \geq \sigma_{\rm SM}(e^+e^- \to ZZ)\,.$$

- Consider the elastic amplitude of $eV \to eV$,
  - V is an arbitrary mixing state of γ and Z,
  - scan over the mixing angle to obtain the strongest bound ($\Delta\sigma \equiv \sigma - \sigma_{\rm SM}$),

$$(\Delta\sigma_{Z\gamma})^2 \leq 4\Delta\sigma_{\gamma\gamma}\Delta\sigma_{ZZ}\,.$$



- σ_Rs only occupy a plane in the 3d parameter space.
  - 3 operators with ℓ_L, 2 operators with e_R.

Conclusion

- Measurements of $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$) offer a unique opportunity to directly probe dimension-8 operators and their positivity bounds.
- We can do it at a lepton collider with $\sqrt{s} \sim 240\,\text{GeV}$ (but higher energy is always better).
  - **Build a Higgs factory, get a positivity test for free!**
- If there is a deviation from SM, we can measure $e^+e^- \to \gamma\gamma$ at several energies (*e.g.* Z-pole and 240 GeV) to check whether it comes from d8 operators ($\sim s^2$).

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- If there is a deviation from SM, we can measure $e^+e^- \to \gamma\gamma$ at several energies (*e.g.* Z-pole and 240 GeV) to check whether it comes from d8 operators ($\sim s^2$).
- **Can QFT really break down at the TeV scale?**
  - Example from history: Nobody expected classical physics to break down, nobody expected parity to be violated, ...

- **It's important to do the experiment!**

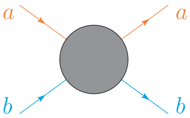
backup slides

Dispersion relations

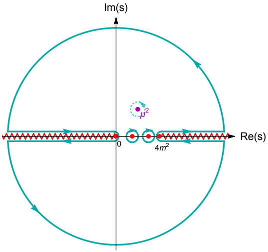
- Consider a forward ($t \to 0$) elastic amplitude ($s + t + u = 4m^2$)

$$\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s - \mu^2)^n ,$$

$$c_n = \frac{1}{2\pi i} \oint_{s=\mu^2} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s-\mu^2)^{n+1}} ,$$



- Applying the fundamental principles of QFT
  - Analyticity (Cauchy's theorem applies)
  - Locality (poles from tree-level factorization, branch cuts from loops, Froissart Bound)
  - Unitarity (Optical theorem, $\mathrm{Im}\mathcal{A} \sim \sigma_{\mathrm{tot}}$)
  - Lorentz invariance (Crossing symmetry)



- Dispersion relation tells us that

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma_{\mathrm{tot}}^{ab}}{(s-\mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\mathrm{tot}}^{a\bar{b}}}{(s-4m^2+\mu^2)^{n+1}} \right) + c_n^{\infty} ,$$

Sum rules and positivity bounds

- Sum rule:

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma_{\rm tot}^{ab}}{(s-\mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\rm tot}^{a\bar{b}}}{(s-4m^2+\mu^2)^{n+1}} \right) + c_n^{\infty} \,,$$

  - Froissart bound: $\mathcal{A} < \text{const} \cdot s \log^2 s \quad \Rightarrow \quad c_n^{\infty} = 0$ for $n > 1$.
  - For even n, the two terms with cross sections are both positive, so $c_n > 0$.

- Consider the limit $m^2 \ll \mu^2 \ll \Lambda^2$ (massless SMEFT).

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \cdots \,.$$

$$\mathcal{A}_4 = g_{[0]} \mathcal{A}_4^{[0]} + g_{[-2]} \mathcal{A}_4^{[2]} + g_{[-4]} \mathcal{A}_4^{[4]} + \dots.$$

- $c_{n=1} \;\Leftrightarrow$ dimension-6 (no positivity bounds, boundary can be nonzero),
  $c_{n=2} \;\Leftrightarrow$ dimension-8 (or d6^2) (has positivity bounds),
  ...

- See *e.g.* [2011.00037] Bellazzini, Miró, Rattazzi, Riembau, Riva, [2012.15849] Arkani-Hamed, Huang, Huang for more general positivity bounds also for non-forward amplitudes.

Collider scenarios

$\int \mathcal{L}dt$ [ab^{-1}]				
unpolarized	91 GeV	161 GeV	240 GeV	365 GeV
CEPC	8	2.6	5.6	
FCC-ee	150	10	5	1.5
ILC	250 GeV	350 GeV	500 GeV	
$(-0.8, +0.3)$	0.9	0.135	1.6	
$(+0.8, -0.3)$	0.9	0.045	1.6	
$(\pm 0.8, \pm 0.3)$	0.1	0.01	0.4	
CLIC	380 GeV	1.5 TeV	3 TeV	
$(-0.8, 0)$	0.5	2	4	
$(+0.8, 0)$	0.5	0.5	1	
muon collider	10 TeV	30 TeV		
unpolarized	10	90		