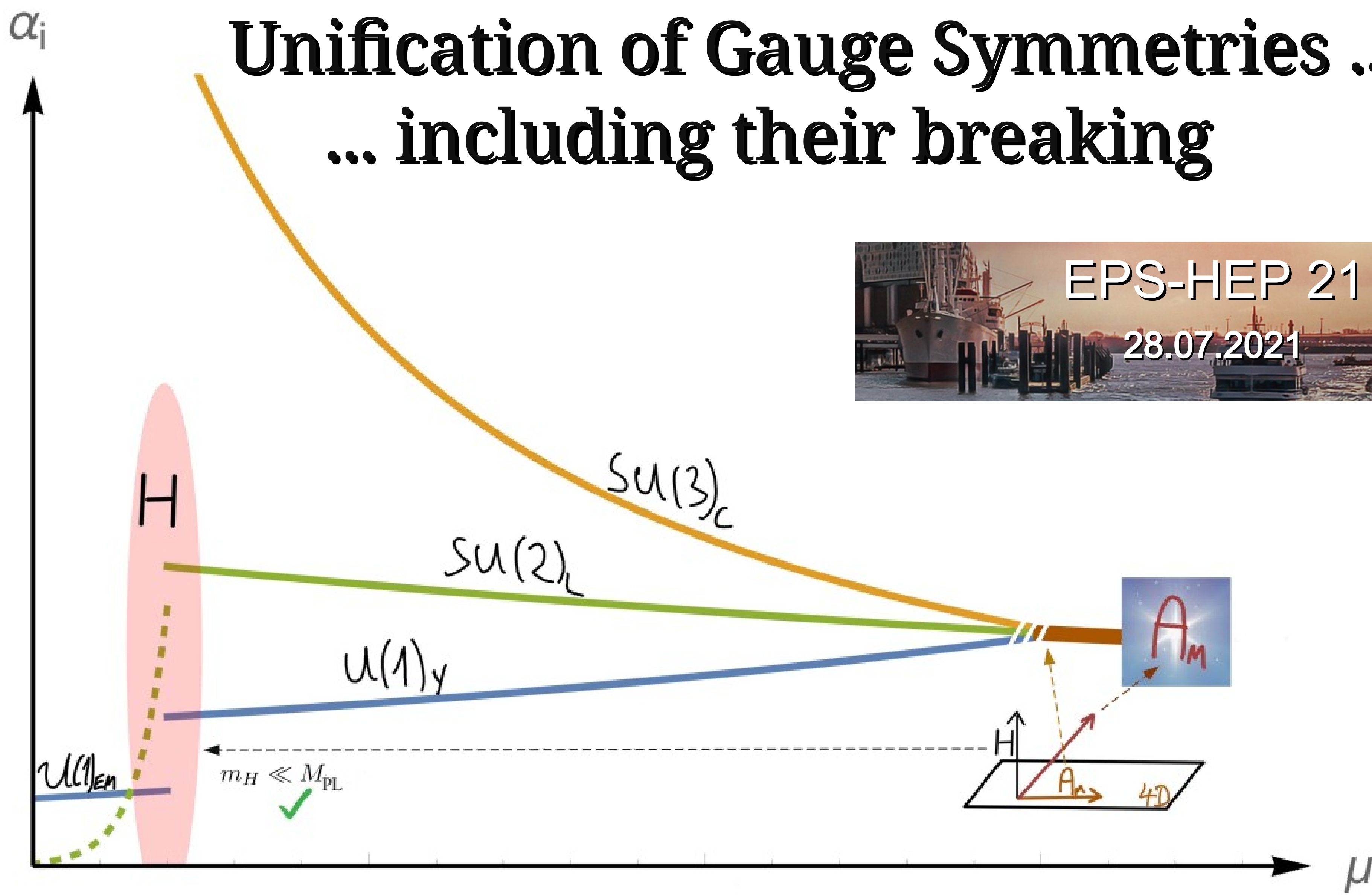
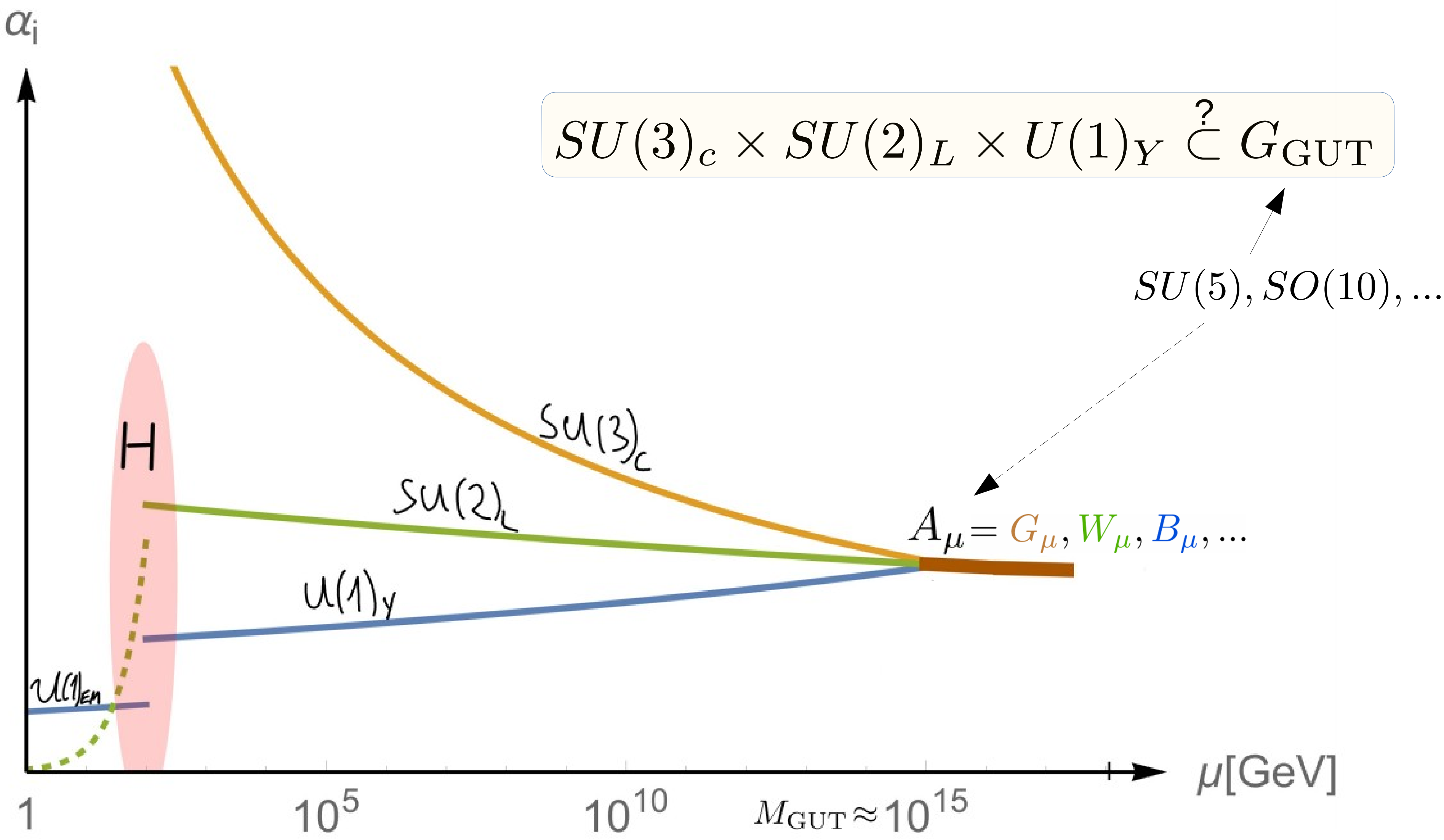
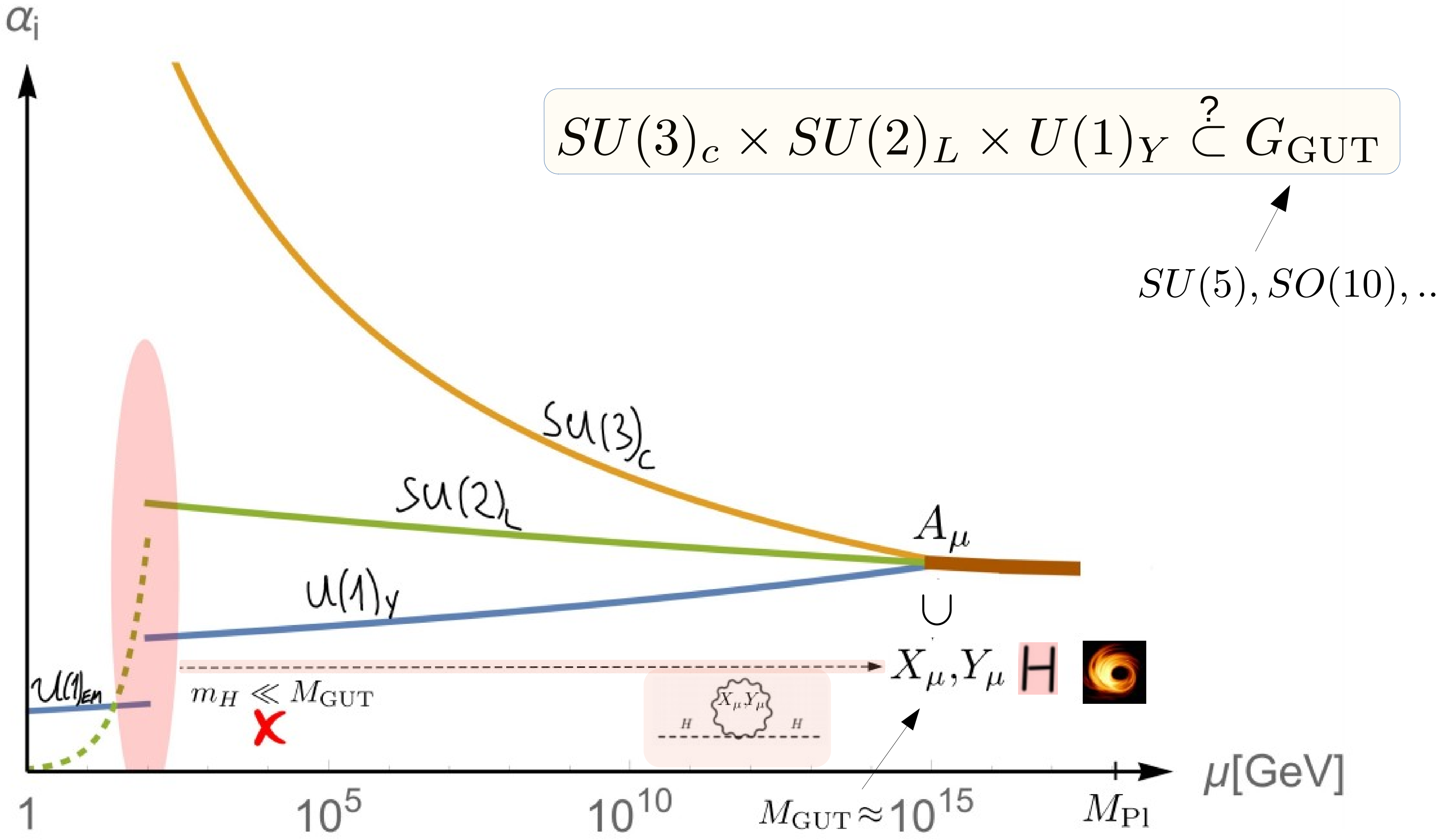
Unification of Gauge Symmetries ...
... including their breaking



Unification of Forces



The Hierarchy Problem



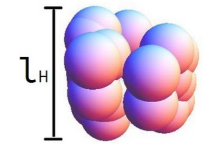
Composite Higgs Models

Kaplan, Georgi, Dimopoulos,. . .

- Higgs is composite at small distances

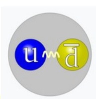
  ➡ Hierarchy Problem solved

(new strong force)



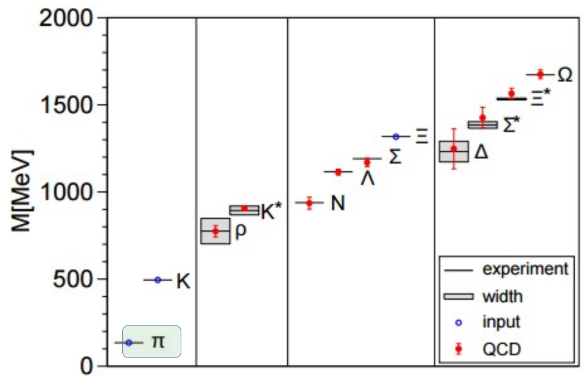
- Higgs = (pseudo) Nambu-Goldstone Boson ➡ $m_H << m_\rho$

  of spon. broken global symmetry



like pions

$$SU(2)_L \times SU(2)_R \xrightarrow{\langle \bar{q} q \rangle \neq 0} SU(2)_V$$



0906.3599

Naturally address

- Flavor Hierarchies
- Dynamical EWSB
- Tiny Neutrino Masses

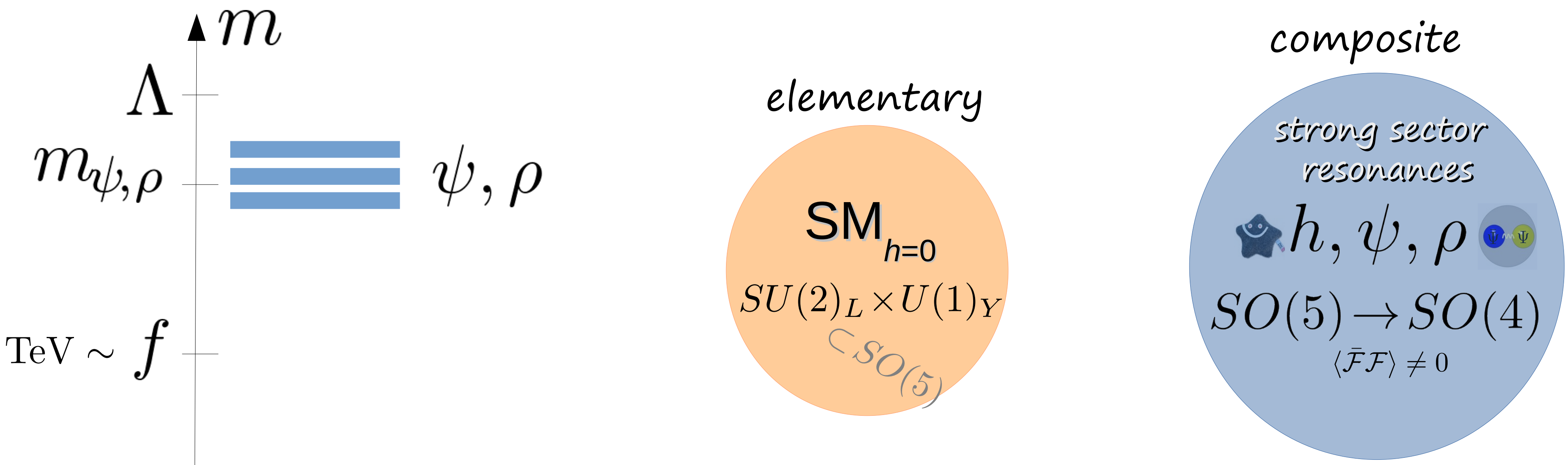
- Dark Matter
- Baryogenesis ...

- MCHM: SO(5) → SO(4)

  Contino, Nomura, Pomarol, ph/0306259
  Agashe, Contino, Pomarol, ph/0412089

  minimal: 4 Higgs dof (SO(5)/SO(4)), custodial sym.

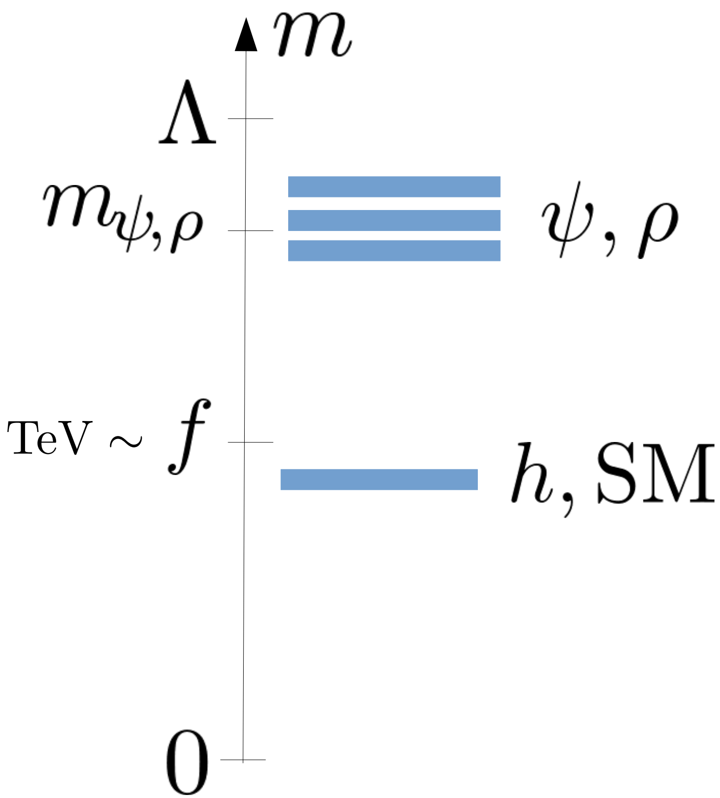
SO(5) → SO(4) Composite Higgs

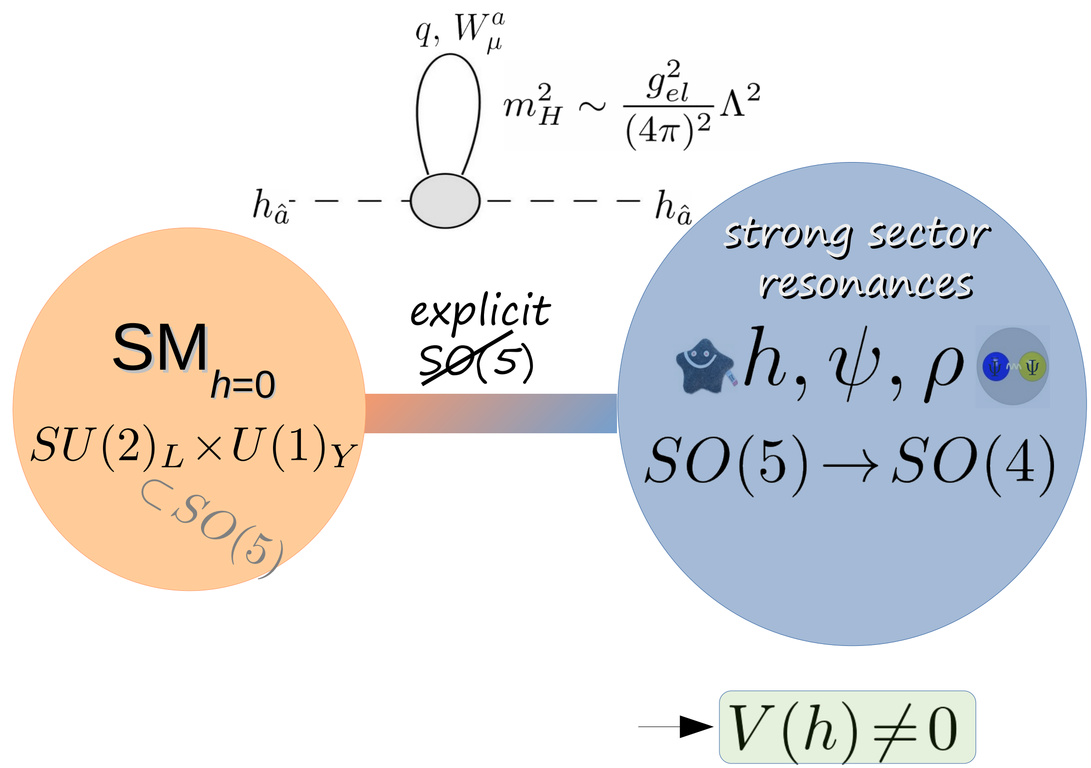


$$V(h) \propto 0$$

$$\Sigma = \Sigma_0 \, e^{-i\frac{\sqrt{2}}{f} h_{\hat{a}} T^{\hat{a}}}$$

Coupling to SM breaks SO(5)

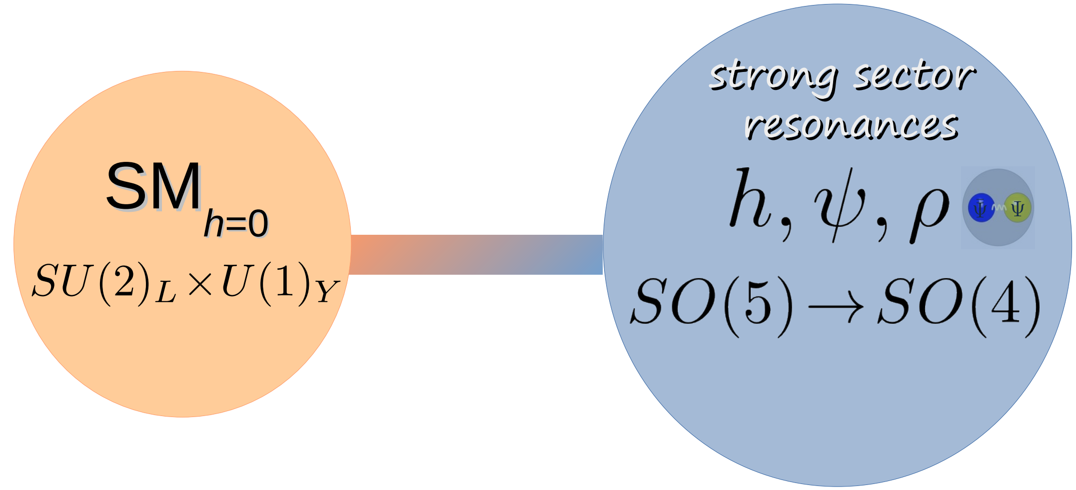


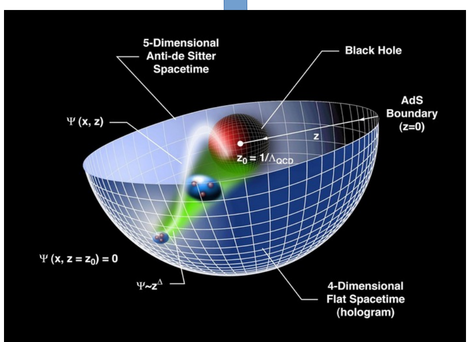


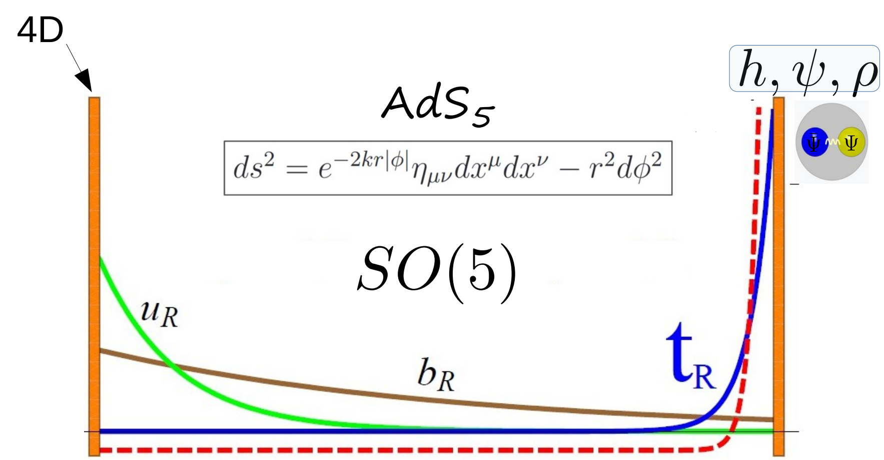
See Contino, 1005.4269, Azatov, 1212.1380, Bellazzini, Csáki, Serra 1401.2457, Panico, Wulzer, 1506.01961 & Goertz, 1812.07362 for comprehensive & condensed reviews

'Calculable' Realization of Strong Sector: Extra Dimensions

4D CH Model





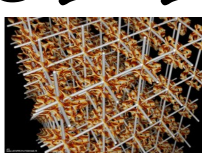


UV brane:
elementary sector

IR brane:
composite sector

5D Dual: slice of AdS$_5$

(weakly coupled)



http://www.lactamme.polytechnique.fr/

Randall, Sundrum, hep-ph/9905221

AdS/CFT correspondence

'Maldacena conjecture' & Gubser, Klebanov, Polyakov, Witten,...

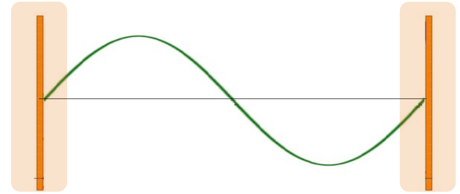
Adv. Theor. Math. Phys. 2,231 (1998)

Int. J. Theor. Phys. 38,1113 (1999)

Kaluza-Klein Decomposition and Boundary Conditions

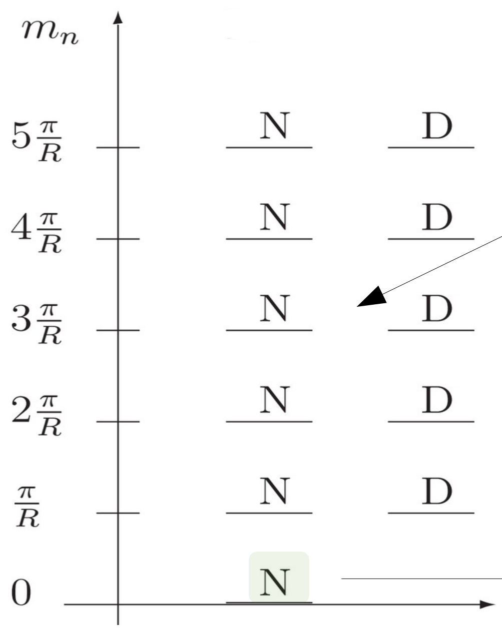
$$\Phi(x,y) = \sum_n \phi_n(x) f_n(y)$$

5D field   4D fields   'profile'

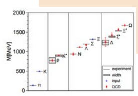


$(--)$ **Dirichlet-BCs**: $f_n(0) = f_n(R) = 0$

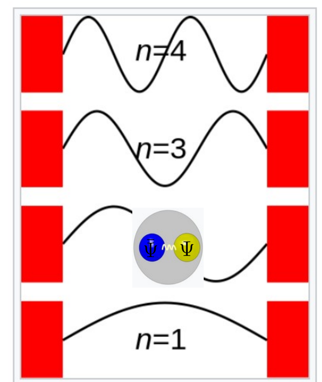
$(++)$ **Neumann BCs**: $f_n'(0) = f_n'(R) = 0$



KK resonances
= composites in dual picture



$y = 0$ $y = R$

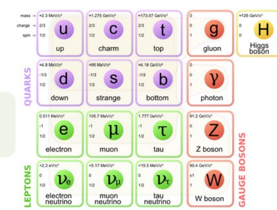


$$m_n = n\,\pi/R$$

Masses fixed by geometry, calculable!

$m_0 = 0$ (before EWSB)

light "zero mode"→ SM particles



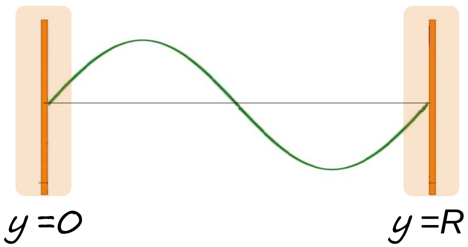
T. Kaluza, Sitzungs. Preuss. Ak. Wiss. Berlin, 966 (1921)
O. Klein, Z. Phys.37, 895 (1926)

neglect curvature of XD for simplicity

Kaluza-Klein Decomposition and Boundary Conditions

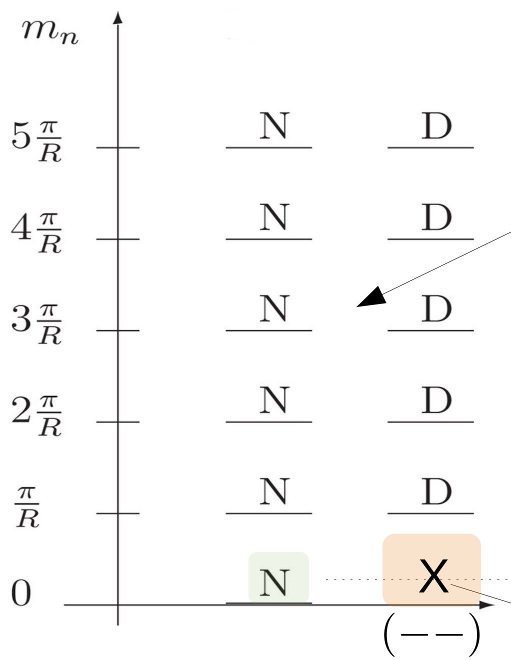
$$\Phi(x,y)=\sum_n \phi_n(x)\, f_n(y)$$

5D field 4D fields 'profile'

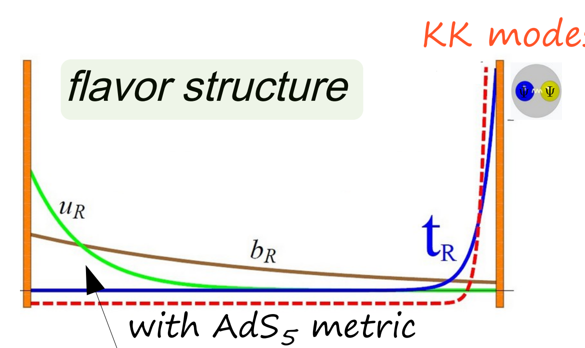


$(--)$ **Dirichlet-BCs**: $f_n(0)=f_n(R)=0$

$(++)$ **Neumann BCs**: $f_n'(0)=f_n'(R)=0$



KK resonances



$m_0 = 0$ (before EWSB)

light "zero mode" → *SM particles*

no light mode (also for $(-+)$)

T. Kaluza, Sitzungs. Preuss. Ak. Wiss. Berlin, 966 (1921)

O. Klein, Z. Phys.37, 895 (1926)

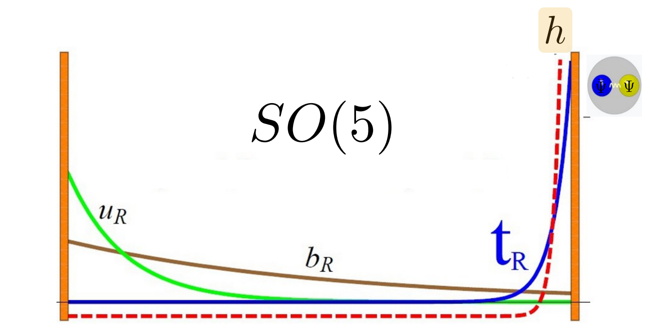
neglect curvature of XD for simplicity

Holographic pNGB Higgs

Contino, Nomura, Pomarol, ph/0306259
Agashe, Contino, Pomarol, ph/0412089

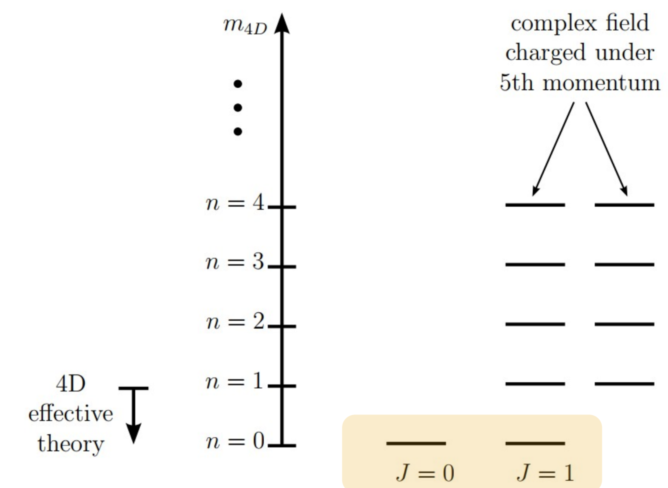
- Higgs = 5^{th} comp. of gauge field → pNGB

→ Gauge-Higgs Unification Manton, Hosotani, Fairlie, Hatanaka, Inami, Lim...



UV brane: elementary sector

IR brane: composite sector

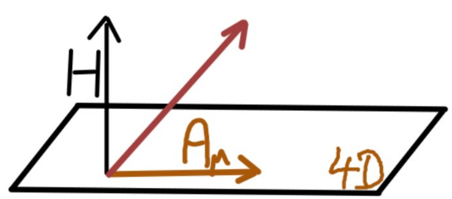


SO(5)

5D vector field $A_M^A = (A_\mu^A, \boxed{A_5^A})$

4D vector

4D scalar → Higgs



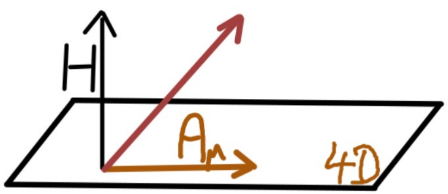
$\boxed{\text{Massless pNGB: 4D shift sym.} \leftrightarrow \text{5D gauge sym.}}$

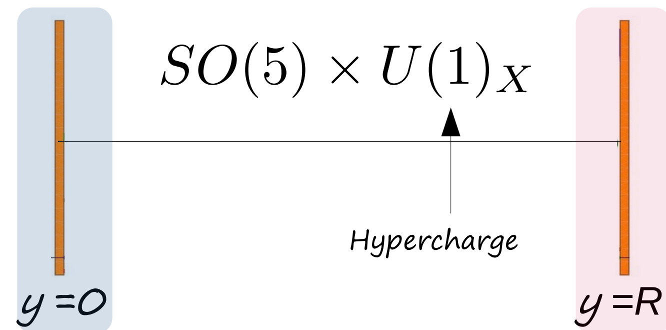
SO(5)→SO(4) Gauge-Higgs Unification

Manton, Hosotani, Fairlie, Hatanaka, Inami, Lim...

5D vector $A_M^A = (A_\mu^A, A_5^A)$

4D vector Higgs





UV brane:
elementary sector

IR brane:
composite sector

$$\mathcal{L} \supset \bar{\Psi}\left(i\not{D} + D_5\gamma^5 + \dots\right)\Psi$$

Forces

5D fermion

$$SO(5)\times U(1)_X \supset SU(2)_L \times SU(2)_R \times U(1) \quad \times \boxed{SO(5)/SO(4)}$$

$$D_M = \partial_M - ig_5 T_L^a W_M^a - ig_5 T_R^b R_M^b - ig_Y Y B_M - \frac{ig_Y}{c_\phi s_\phi} Z'_M (T_R^3 - s_\phi^2 Y) - ig_5 T^{\hat{a}} \boxed{H_M^{\hat{a}}}$$

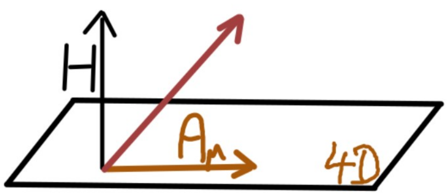
$b = 1, 2$

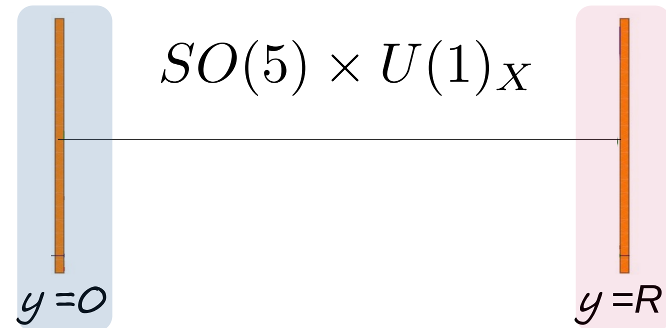
$M = \mu, 5$ and $g_Y \equiv g_5 g_X / \sqrt{g_5^2 + g_X^2}$

SO(5)→SO(4) Gauge-Higgs Unification

Manton, Hosotani, Fairlie,
Hatanaka, Inami, Lim...

5D vector $A_M^A = (A_\mu^A, A_5^A)$





UV brane:
elementary sector

IR brane:
composite sector

Yukawa

$$\mathcal{L} \supset \bar{\Psi}\,(i\not{D} + \boxed{D_5}\gamma^5 + \dots)\Psi$$

$$D_M = \partial_M - ig_5 T_L^a W_M^a - ig_5 T_R^b R_M^b - ig_Y Y B_M - \frac{ig_Y}{c_\phi s_\phi} Z'_M (T_R^3 - s_\phi^2 Y) - ig_5 T^{\hat{a}} \boxed{H_M^{\hat{a}}}$$

$SO(5)/SO(4)$

$$\boxed{SO(5)\times U(1)_X \supset SU(2)_L \times SU(2)_R \times U(1)}$$

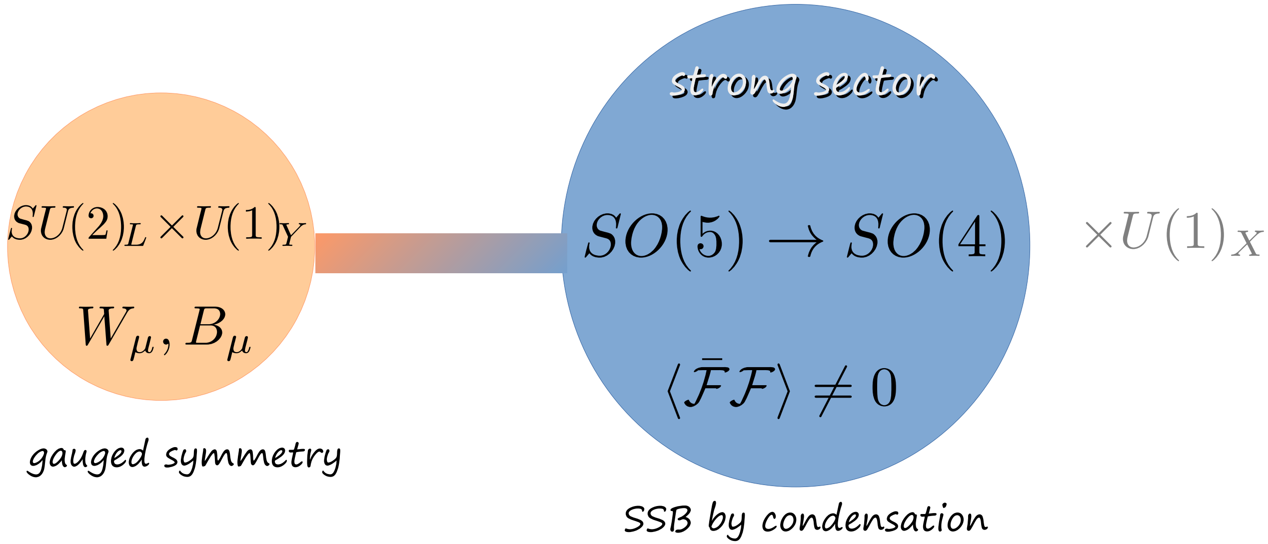
$$\downarrow$$

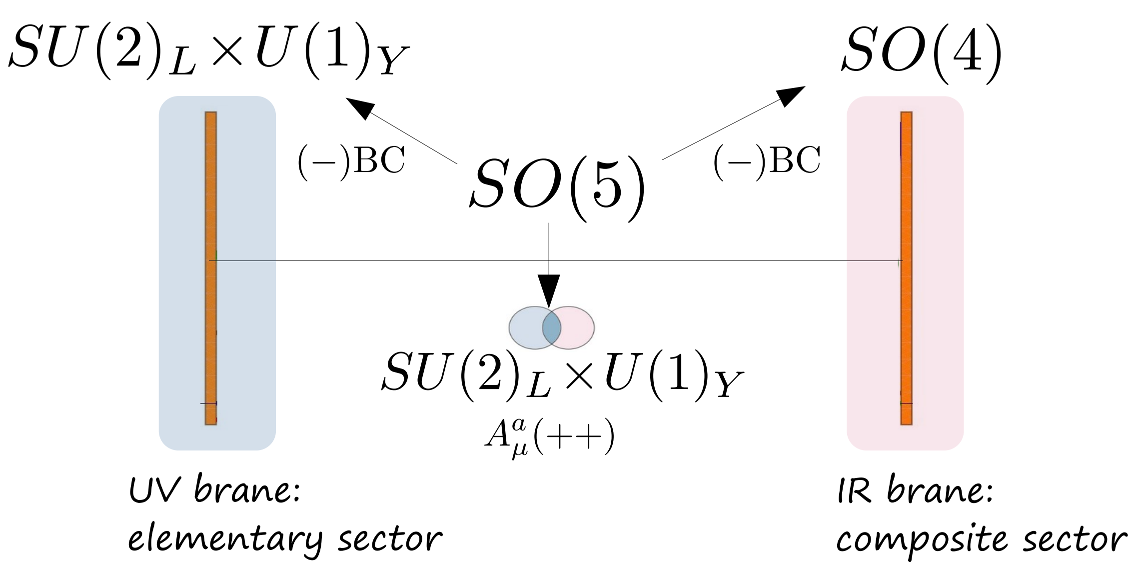
$$\underrightarrow{E \ll \mathrm{TeV}}: G_{\mathrm{EW}} = SU(2)_L \times U(1)_Y$$

$W_\mu^a(+,+)$ $W_5^a(-,-)$
$\cancel{R}_\mu^b(-,+)$ $R_5^b(+,-)$
$B_\mu(+,+)$ $B_5(-,-)$
$\cancel{Z}'_\mu(-,+)$ $Z'_5(+,-)$
$\cancel{H}_\mu^{\hat{a}}(-,-)$ $\boxed{H_5^{\hat{a}}(+,+)}$

Higgs

Symmetry Breaking by Boundary Conditions





$W^a_\mu(+,+)$ $W^a_5(-,-)$

$\cancel{R^b_\mu}(-,+)$ $R^b_5(+,-)$

$B_\mu(+,+)$ $B_5(-,-)$

$\cancel{Z'_\mu}(-,+)$ $Z'_5(+,-)$

$\cancel{H^{\hat{a}}_\mu}(-,-)$ $\boxed{H^{\hat{a}}_5}(+,+)$

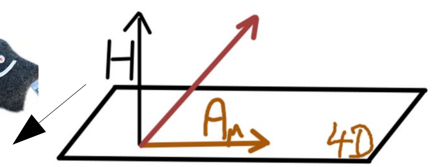
$SO(5)/SO(4)$ GB

One Step Further:

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!

Gauge-Higgs Unification

$$SO(5): \; A_M^A = (A_\mu^A, \boxed{A_5^A})$$



$$U(1)_X: \; X_M = (X_\mu, X_5)$$

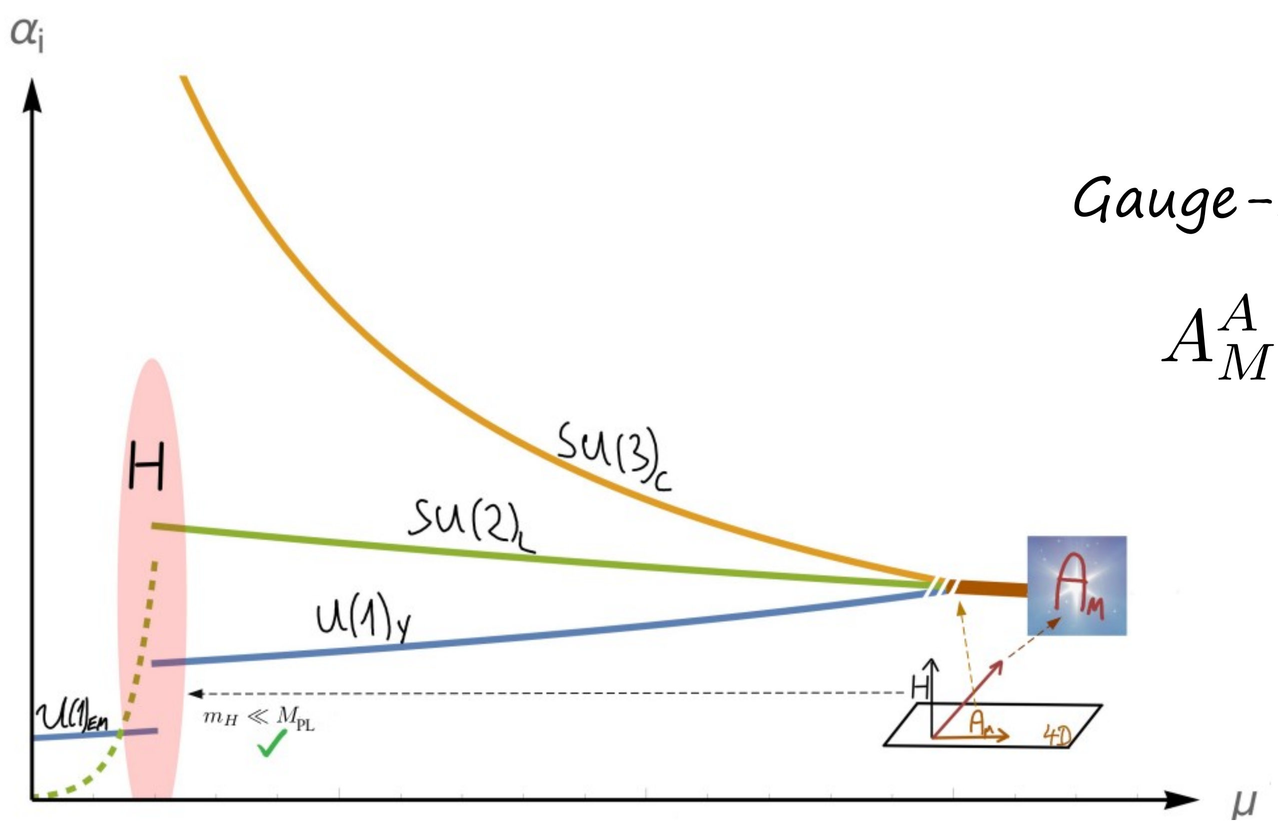
$$SU(3)_c: \; G_M^A = (G_\mu^A, G_5^A)$$

Gauge-Higgs Grand Unification

$$A_M^A = (A_\mu^A, \boxed{A_5^A}) \quad G_{\text{GUT}}$$

Gauge-Higgs Grand Unification

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!



Gauge-Higgs ~~Grand~~ Unification

$$A_M^A = (\boxed{A_\mu^A}, \boxed{A_5^A}) \quad G_{\mathrm{GUT}}$$

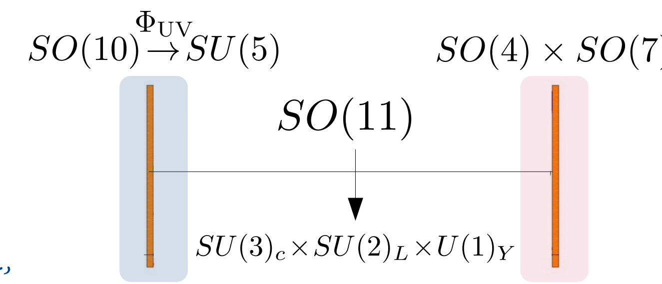
Gauge-Higgs Grand Unification

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!

- Two considered setups:

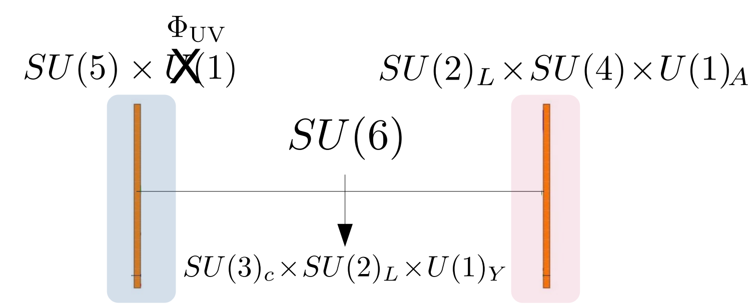
$SO(11)$

Hosotani, Yamatsu, 1504.03817
Furui, Hosotani, Yamatsu, 1606.07222
Hosotani, 1606.08108
(see also Agashe, Contino, Sundrum, hep-ph/0502222,
Frigerio, Serra, Varagnolo, 1103.2997,...)



$SU(6)$

Burdman, Nomura, hep-ph/0210257
Lim, Maru, 0706.1397
...



Too Good To be True?

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!

$SO(11)$

$SU(6)$

Severe phenomenological challenges:

- (too) light exotic states due to large irreps of bigger symmetry
- Difficult to obtain correct EWSB/m_H
- Degenerate/massless quarks&leptons

...

Too Good To be True?

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!

Severe phenomenological challenges:

$SO(11)$

$SU(6)$

- go to 6D Hosotani, Yamatsu, 1706.03503, 1710.04811
- abandon bulk SM & introduce new BSM 5D multiplets + addtl. mirror fermions
  Maru, Yatagai, 1903.08359, 1911.03465 ...

Too Good To be True?

- Embed GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!

$SO(11)$

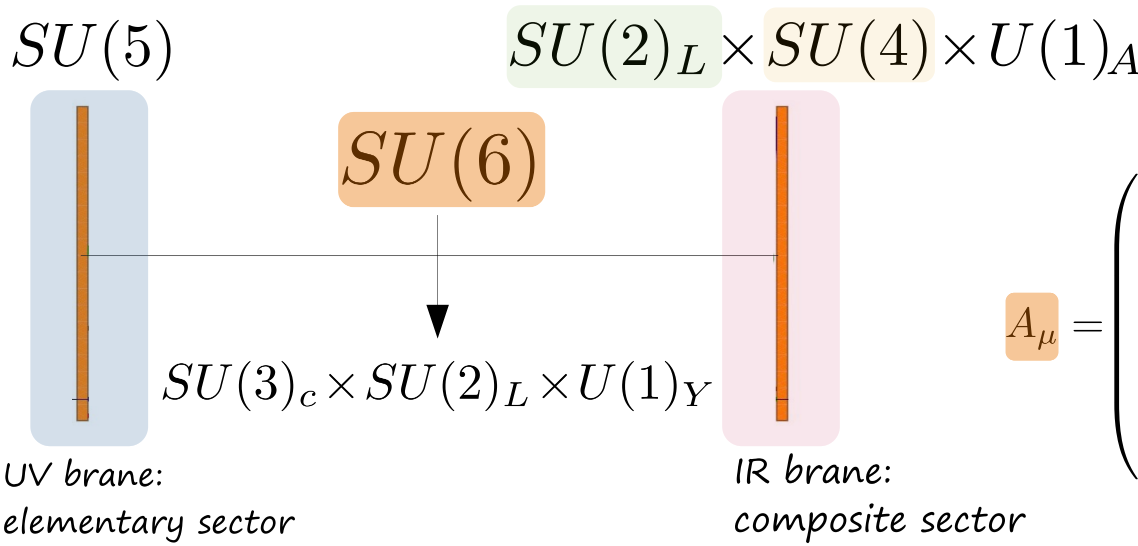
$SU(6)$

Minimal Alternative

SU(6) In warped space with different breaking pattern and brane masses

Original SU(6) Breaking

but warped...



$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (-+) \\ (+-) & (+-) & (++) & (++) & (++) & (-+) \\ (+-) & (+-) & (++) & (++) & (++) & (-+) \\ \hline (--) & (--) & (-+) & (-+) & (-+) & (-+) \end{array}\right)$$

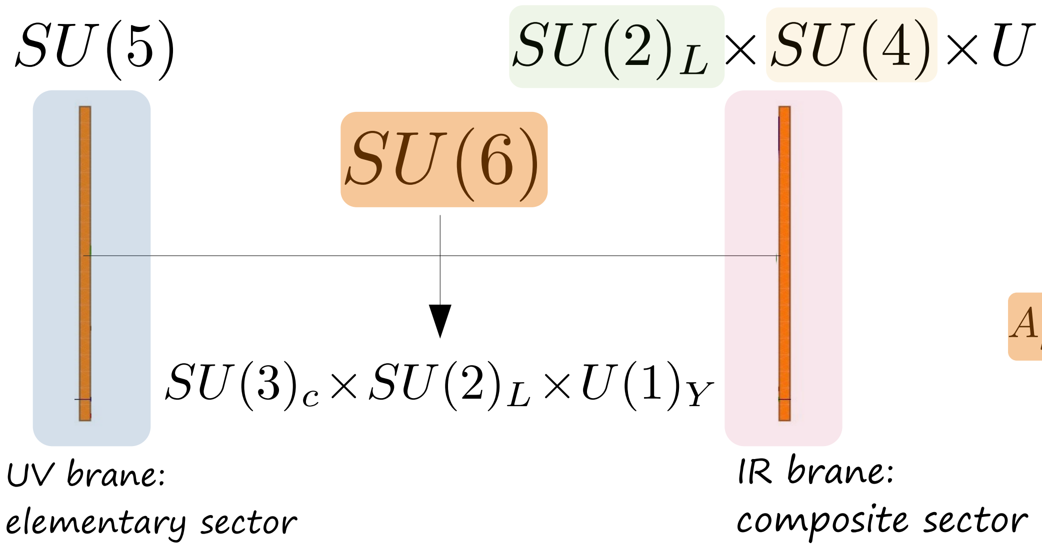
$A_5 : + \leftrightarrow -$

$(16 - 12) = 4 \text{ GBs} \to \text{Higgs}$ 

Original SU(6) Breaking

but warped...

$SU(2)_L \times SU(4) \times U(1)_A$



$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (-+) \\ (+-) & (+-) & (++) & (++) & (++) & (-+) \\ (+-) & (+-) & (++) & (++) & (++) & (-+) \\ \hline (--) & (--) & (-+) & (-+) & (-+) & (-+) \end{array}\right)$$

$A_5 : + \leftrightarrow -$

Fermion irreps (min. attempt):

RH 0-mode

$$\mathbf{20_L} \to (\mathbf{3},\mathbf{2})^{-,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus \tilde{e}_R(\mathbf{1},\mathbf{1})^{-,-}_{1}$$
$$(\mathbf{3}^*,\mathbf{2})^{-,+}_{-1/6} \oplus u_R(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{-1}$$

$$\mathbf{15_L} \to q_L(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e^c_R(\mathbf{1},\mathbf{1})^{+,+}_{1}$$
$$\oplus\, d_R(\mathbf{3},\mathbf{1})^{-,-}_{-1/3} \oplus (\mathbf{1},\mathbf{2})^{-,+}_{1/2}$$

$$\mathbf{6_L} \to (\mathbf{3},\mathbf{1})^{-,+}_{-1/3} \oplus l^c_L(\mathbf{1},\mathbf{2})^{-,-}_{1/2} \oplus \nu^c_R(\mathbf{1},\mathbf{1})^{+,+}_{0}$$

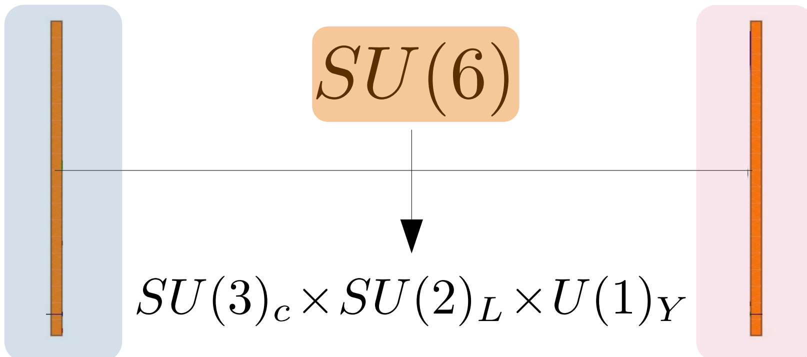
$$\mathbf{1_L} \to (\mathbf{1},\mathbf{1})^{+,-}_{0}$$

$$m_e (= m_u) = 0$$

Novel Breaking Pattern

but warped...

$SU(5)$ $\qquad$ $SU(2)_L \times SU(3)_c \times U(1)_Y = G_{\text{SM}}$



$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)$$

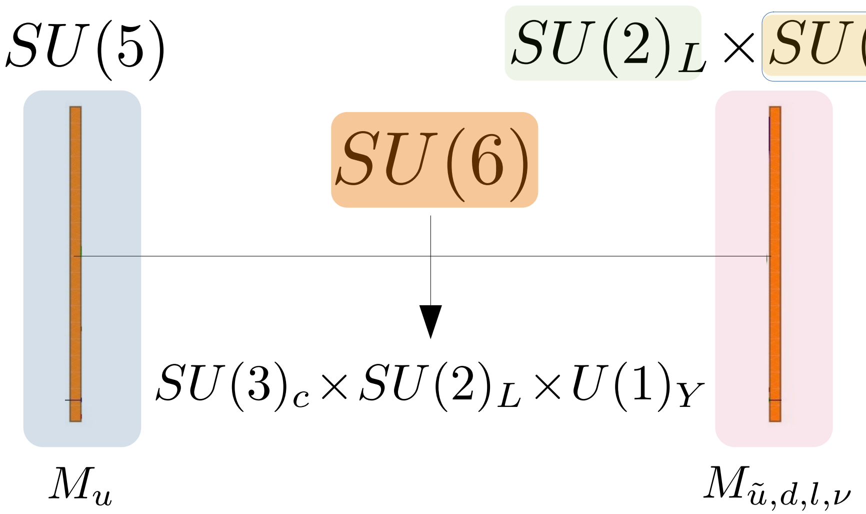
$A_5 : + \leftrightarrow -$

$$(23 - 12) = 11 \text{ GBs} \to \text{Higgs} \boxed{+ \text{ singlet } + (\mathbf{3}, \mathbf{1})_{-1/3} \text{ LQ}}$$

- Scalars want to be heavy ... :)
- Could use them ...

Novel Breaking Pattern

but warped...



$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)$$

$A_5 : + \leftrightarrow -$

Fermion irreps:

$$\mathbf{20_L} \to (\mathbf{3},\mathbf{2})^{-,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{1}$$
$$(\mathbf{3}^*,\mathbf{2})^{-,+}_{-1/6} \oplus u_R(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{-1}$$

$$\mathbf{15_L} \to q_L(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e^c_R(\mathbf{1},\mathbf{1})^{+,+}_{1}$$
$$\oplus (\mathbf{3},\mathbf{1})^{-,+}_{-1/3} \oplus (\mathbf{1},\mathbf{2})^{-,+}_{1/2}$$

$$\mathbf{6_L} \to d_R(\mathbf{3},\mathbf{1})^{-,-}_{-1/3} \oplus l^c_L(\mathbf{1},\mathbf{2})^{-,-}_{1/2} \oplus \nu^c_R(\mathbf{1},\mathbf{1})^{+,+}_{0}$$

$$\mathbf{1_L} \to (\mathbf{1},\mathbf{1})^{+,-}_{0}$$

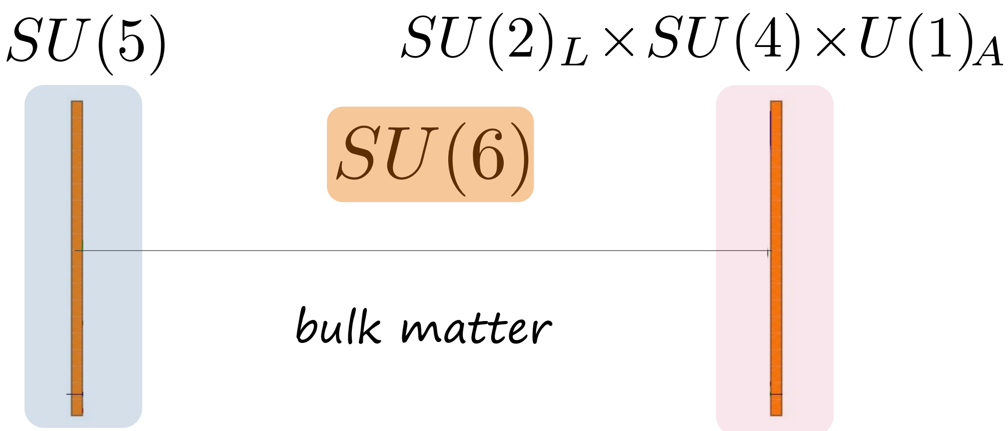
$M_u \to m_u$
$M_{\tilde{u}} \to V(H)$
$M_{d,l} \to m_e \neq m_d$
$M_\nu \to$ light ν

viable spectrum ✓

+EWSB!

SU(6) GHGUT

<u>Original Model</u>



SM at low energies? ✗

<u>New Pattern</u>

Reproduces SM at low energies! ✓
(incl. EWSB, m_H, ...)

Phenomenology

Reproduces SM at low energies!

- Extended Scalar Sector (incl. Higgs Mass)
- New X,Y Gauge Bosons
- (Fermionic) resonances
- …

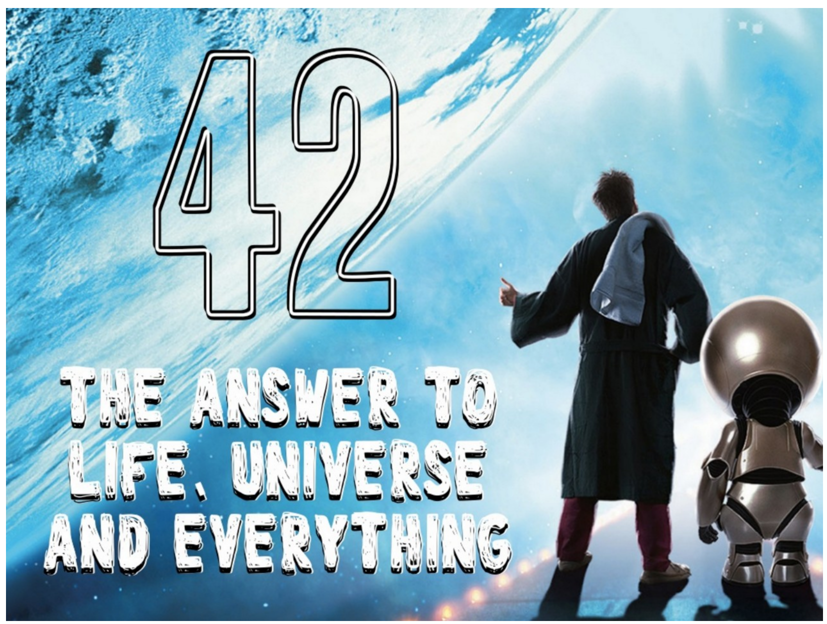
The Model is True!

$\boxed{\text{Higgs} + \text{singlet} + (\mathbf{3},\mathbf{1})_{-1/3}\ \text{LQ}}$

- The mode is true….

  … well, because 20+15+6+1 =

:)



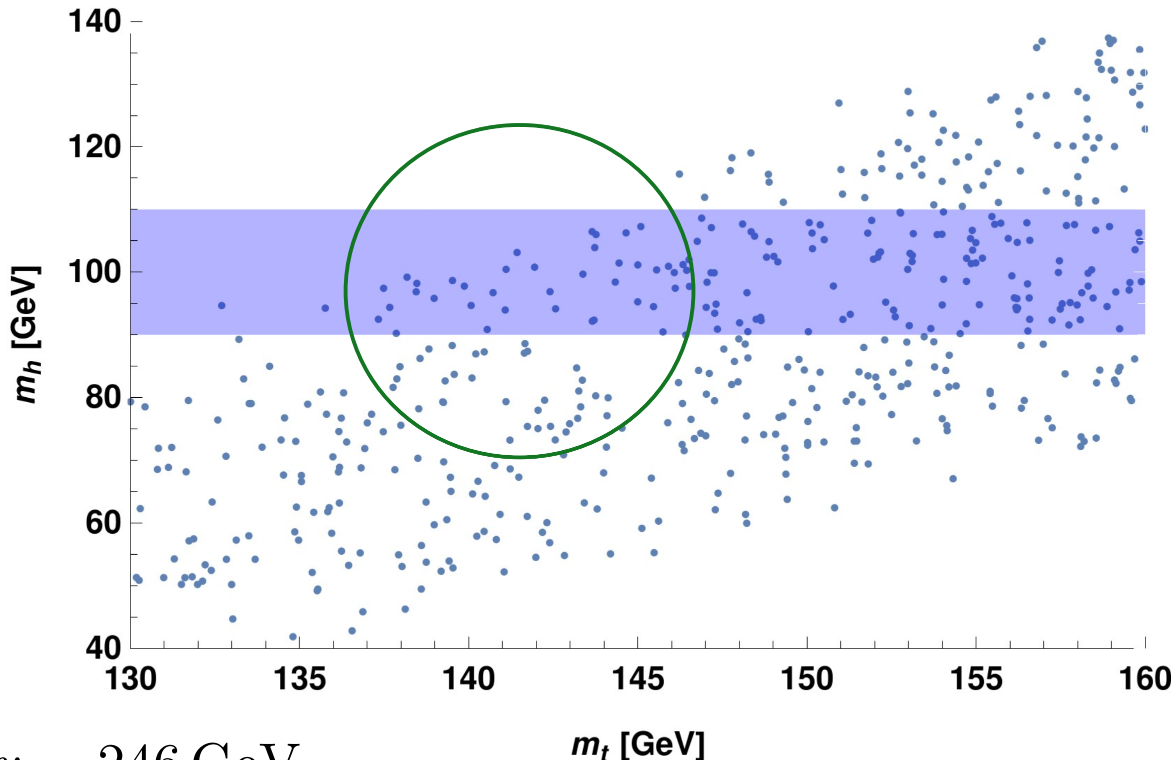
Scalar Potential

$$V_r(v,c,s) = \frac{N_r}{(4\pi)^2}\int_0^\infty dp\, p^3 \log(\rho_r(-p^2, v, c, s))$$

Higgs LQ singlet

$\boxed{\text{Higgs} + \text{singlet} + (\mathbf{3},\mathbf{1})_{-1/3}\,\text{LQ}}$

Spectral function:
$\rho_r(m^2_{n;r}, v, c, s) = 0$



$v = 246\,\text{GeV}$

→ Correct quartic predicted!

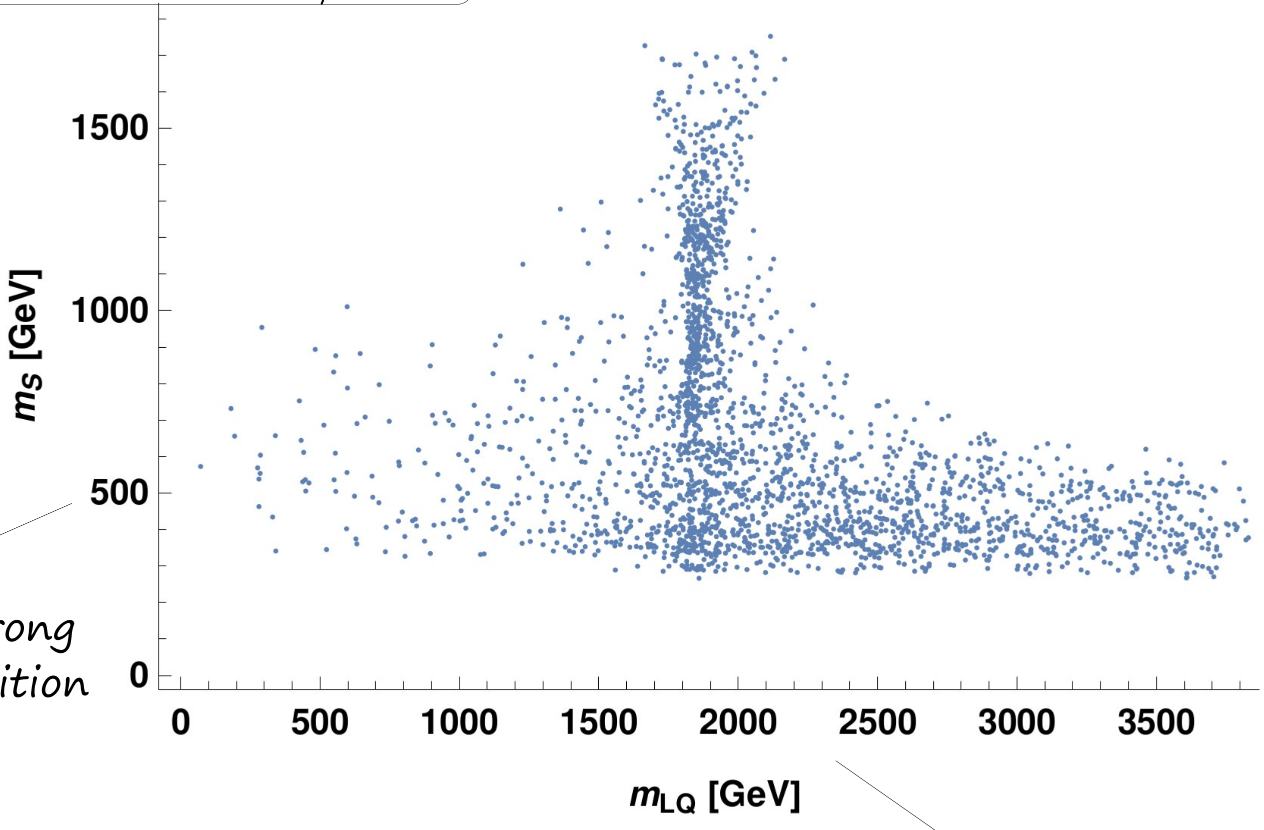
$m_h(\mu), m_t(\mu)$, with $\mu \sim f$

$M_{\tilde{u}} \neq 0$ *crucial*

$R'^{-1} = 10\,\text{TeV} \approx m_\rho/2$

Scalar Potential

Higgs + singlet + $(\mathbf{3},\mathbf{1})_{-1/3}$ LQ



Potentially induce strong
1[st] order Phase transition
→ Baryogenesis

address (CC) B anomalies

Vector Leptoquarks

$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & \boxed{(+-)} & \boxed{(+-)} & \boxed{(+-)} & (--) \\ (++) & (++) & \boxed{(+-)} & \boxed{(+-)} & \boxed{(+-)} & (--) \\ \hline \boxed{(+-)} & \boxed{(+-)} & (++) & (++) & (++) & (--) \\ \boxed{(+-)} & \boxed{(+-)} & (++) & (++) & (++) & (--) \\ \boxed{(+-)} & \boxed{(+-)} & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)$$

$$m_{(+,-)} = \frac{2}{R'\sqrt{2\log(\frac{R'}{R}) - 1}} \sim \boxed{0.25}/R' \sim m_\rho/10 \ll m_\rho$$

$$(m_{(-,+)} \sim 2.5/R' \sim m_\rho)$$

$$R'^{-1} \approx m_\rho/2 \sim 10\,\mathrm{TeV} \implies \boxed{m_{X,Y} \sim 2.5\,\mathrm{TeV}}$$

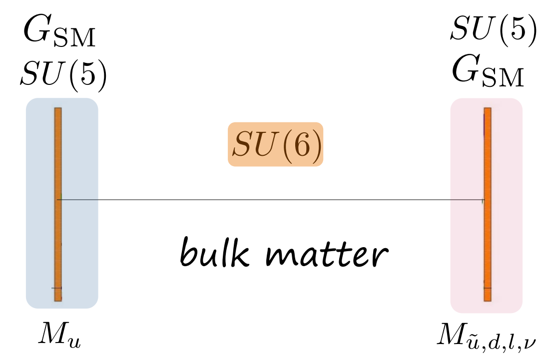
ok with di-lepton searches...

e.g. Crivellin, Müller, Schnell, 2101.07811

Same ballpark as EWPT and Flavor in GHU!

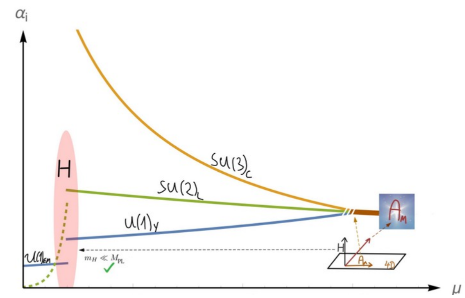
Conclusions

- Embedded GUT group in enhanced global symmetry of CH:
  Unification of all forces & EWSB (Higgs)!



- Modified symmetry breaking solves all problems of earlier models

- Minimal fermion realization (cf. MCHM$_5$) leads to fully viable spectrum



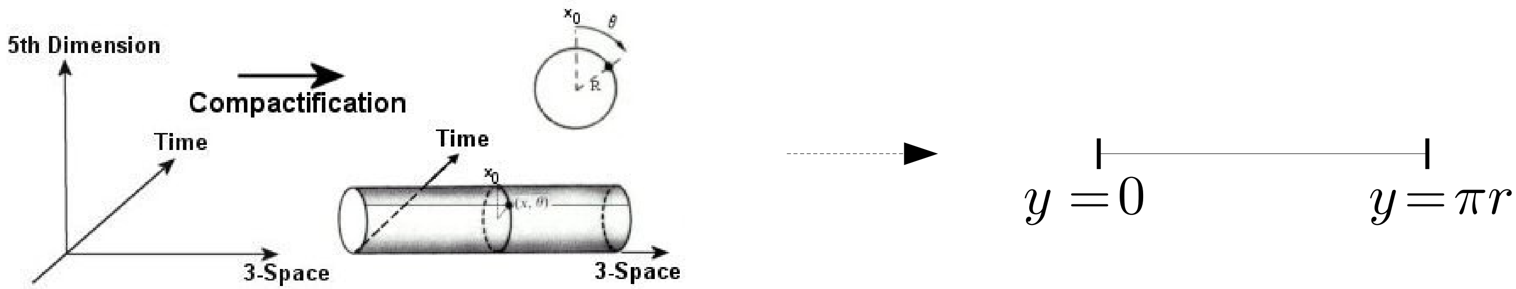
- KK modes of GUT gauge bosons at TeV scale

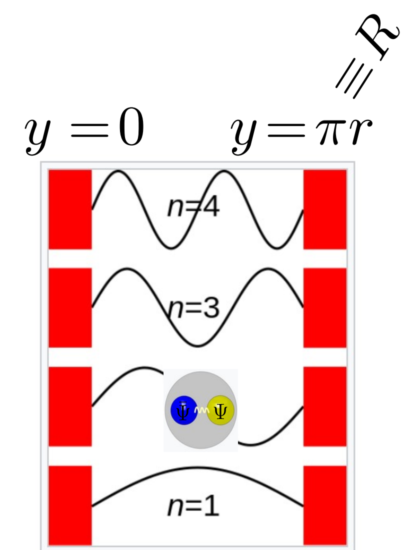
- Lots of directions to explore
  Top partners & tuning, Flavor anomalies, Unification, Baryogenesis, …

Backup

Kaluza-Klein Decomposition

- Additional Dimensions compactified to escape detection





- Particle in extra dimension
  → tower of (4D) Kaluza-Klein excitations

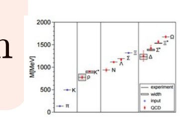
$$\Phi(x,y)=\sum_n \phi_n(x) f_n(y)$$

$m_n = n\,\pi/R$

Masses fixed by geometry, calculable!

4D fields ‘profile’ along extra dimension

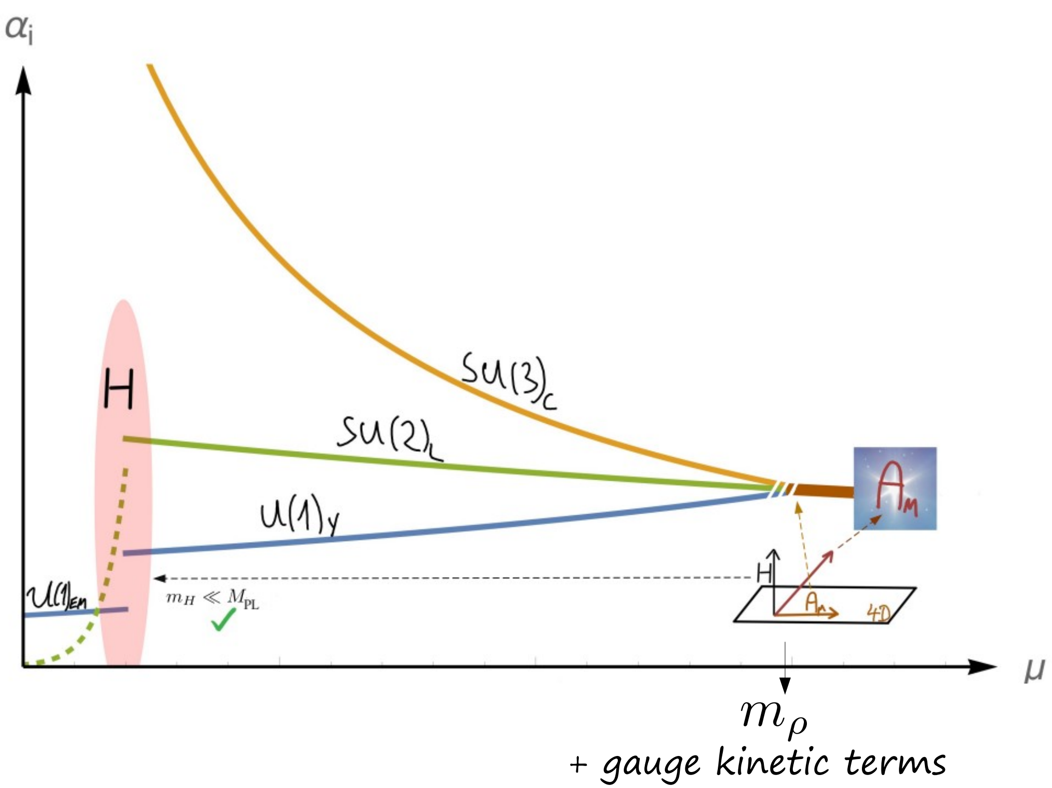
Composite resonances in dual picture

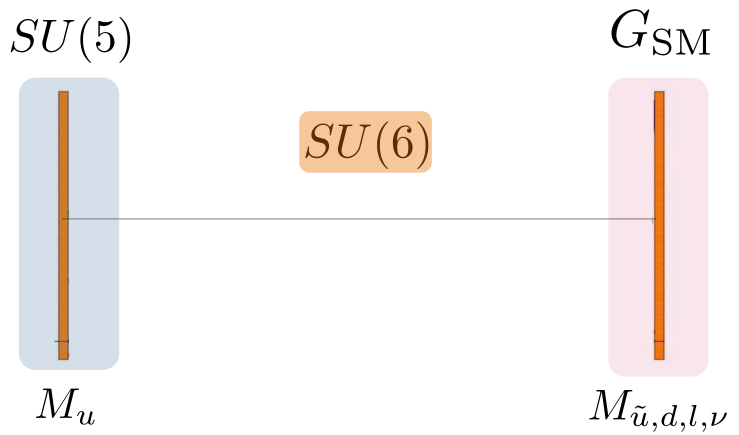


T. Kaluza, Sitzungs. Preuss. Ak. Wiss. Berlin, 966 (1921)
O. Klein, Z. Phys.37, 895 (1926)

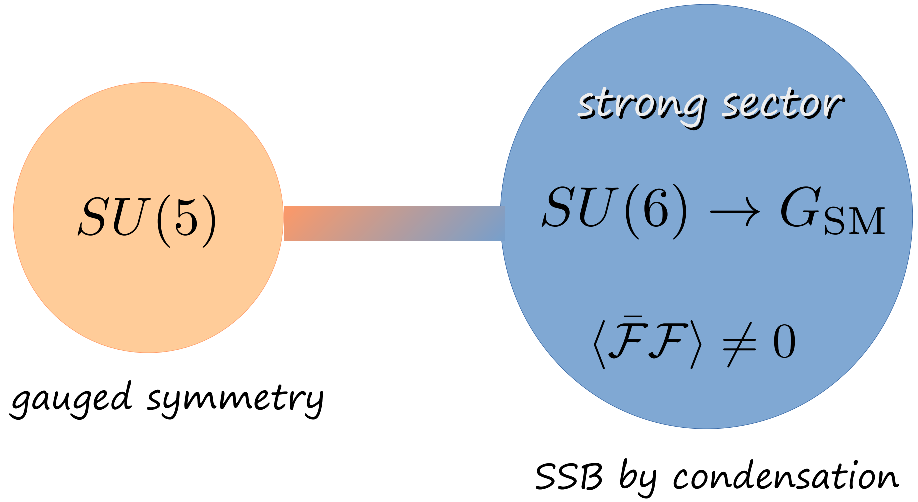
neglect curvature of XD for simplicity

Unification: different options

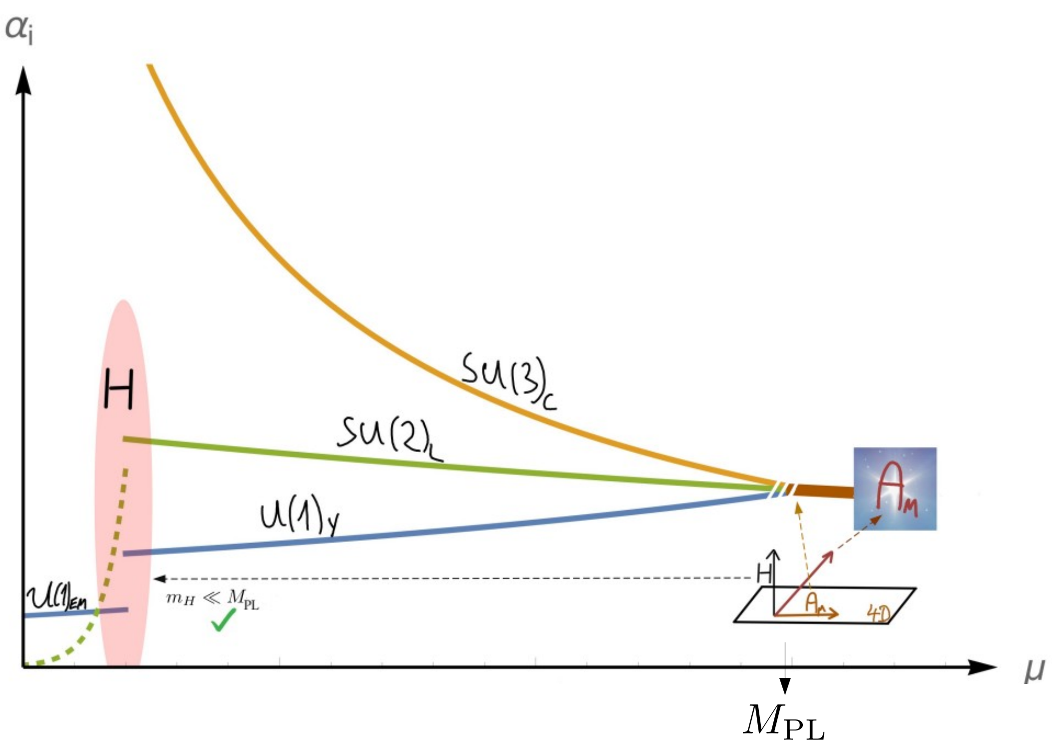




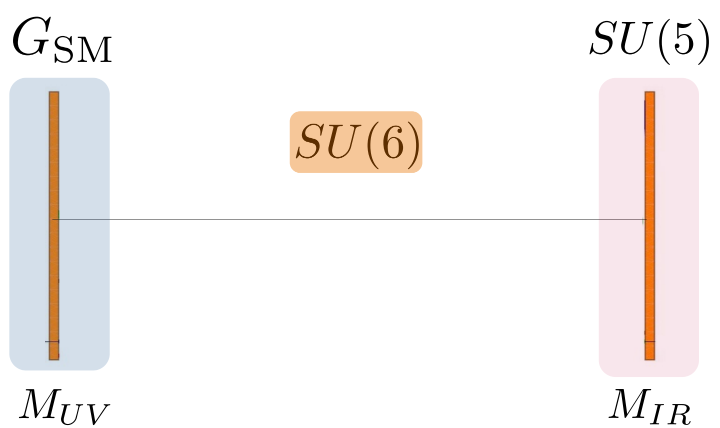
$m_\rho \sim 10\,\mathrm{TeV}$

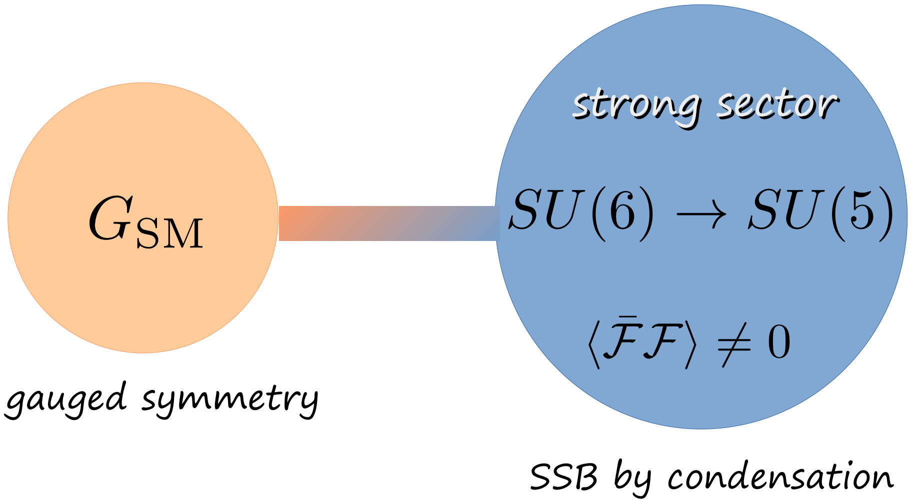


Unification: different options

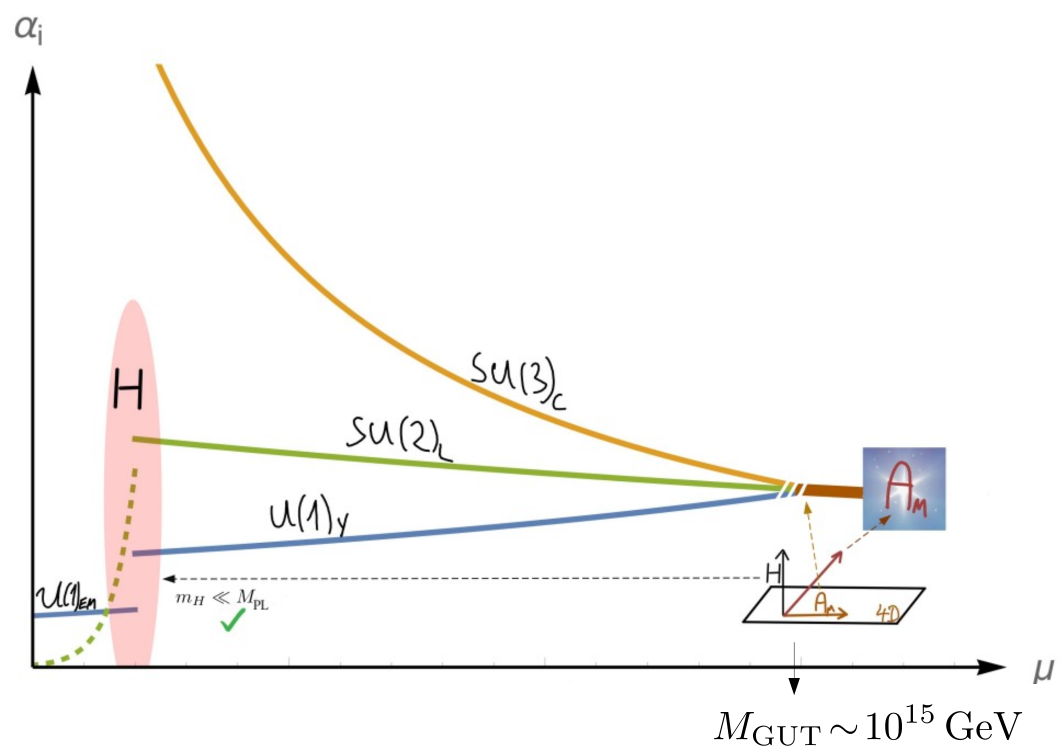


- KK modes of X,Y gauge bosons at TeV scale

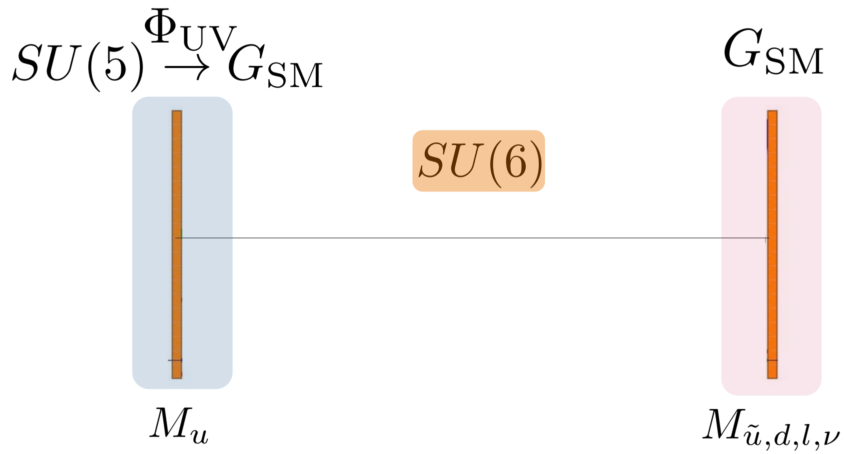


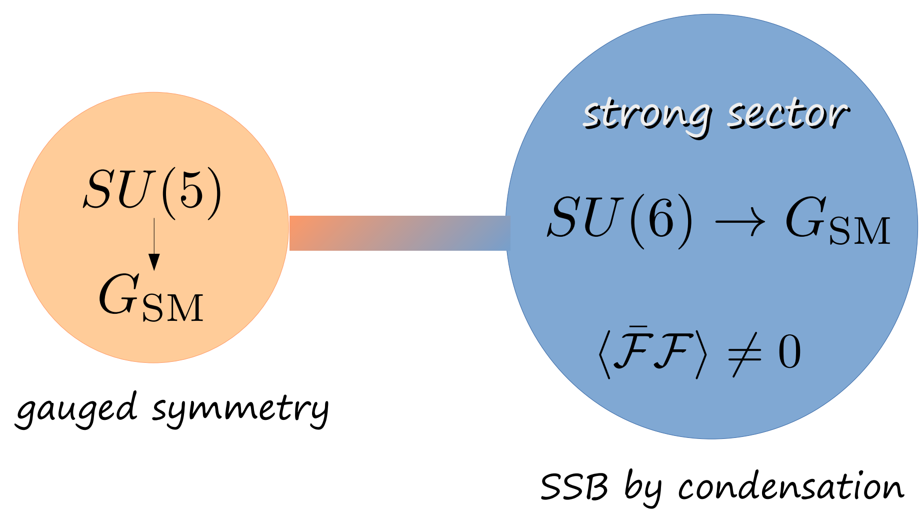


Unification: different options



- KK modes of X,Y gauge bosons at TeV scale





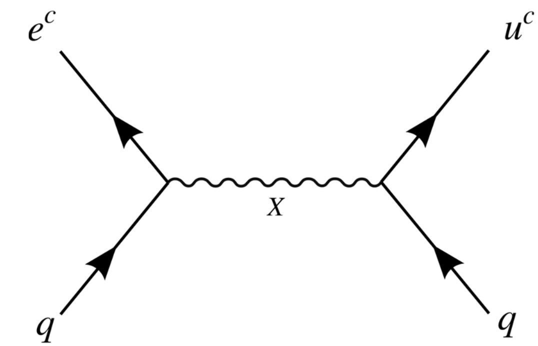
Proton Decay?

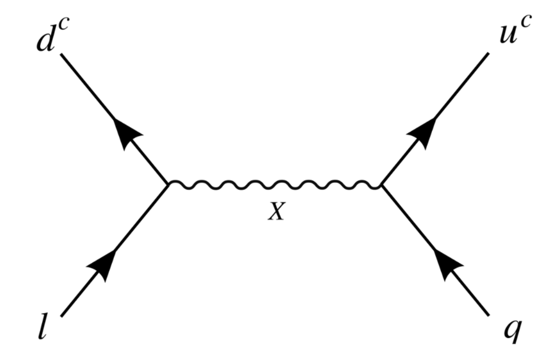
Fermion embeddings help: no $p \to \pi_0 + e^+$

$$\mathbf{20_L} \to (\mathbf{3},\mathbf{2})^{-,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{1}$$
$$(\mathbf{3}^*,\mathbf{2})^{-,+}_{-1/6} \oplus u_R(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{-1}$$

$$\mathbf{15_L} \to q_L(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e^c_R(\mathbf{1},\mathbf{1})^{+,+}_{1}$$
$$\oplus (\mathbf{3},\mathbf{1})^{-,+}_{-1/3} \oplus (\mathbf{1},\mathbf{2})^{-,+}_{1/2}$$

$$\mathbf{6_L} \to d_R(\mathbf{3},\mathbf{1})^{-,-}_{-1/3} \oplus l^c_L(\mathbf{1},\mathbf{2})^{-,-}_{1/2} \oplus \nu^c_R(\mathbf{1},\mathbf{1})^{+,+}_{0}$$

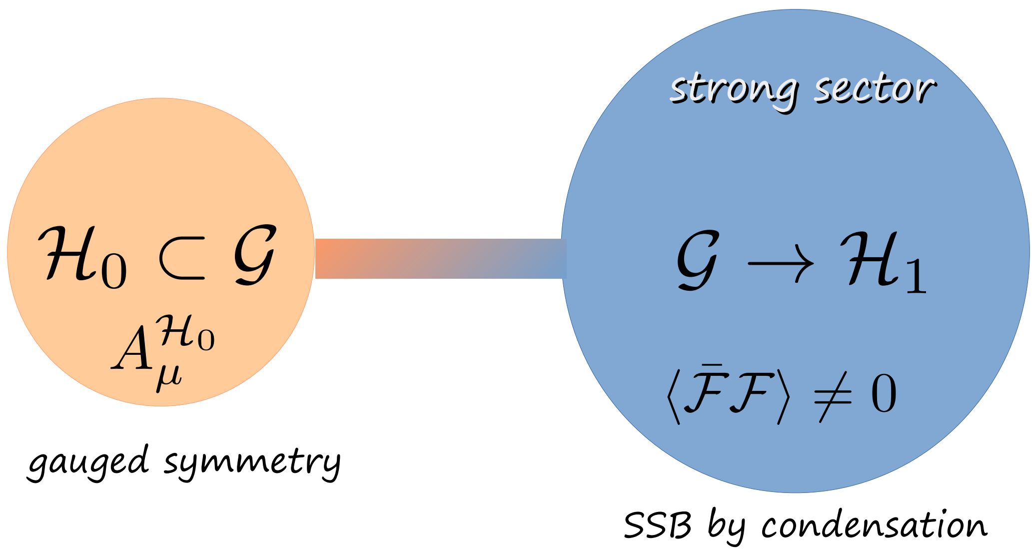


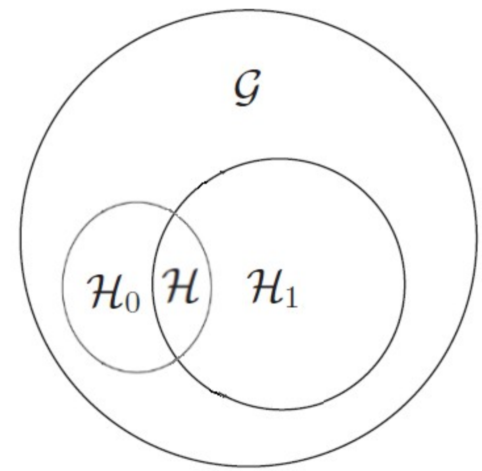


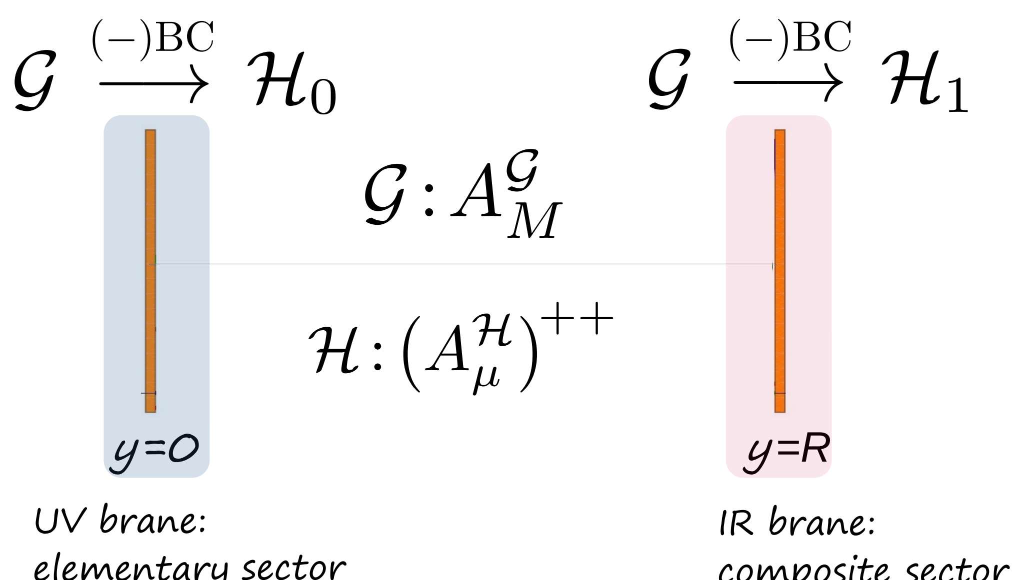
& secret baryon symmetry:

SM-fields with usual charges + charge new fermions and vector&scalar LQs

Symmetry Breaking by Boundary Conditions







$n = \dim(\mathcal{G}) - \dim(\mathcal{H}_1)$ goldstone bosons

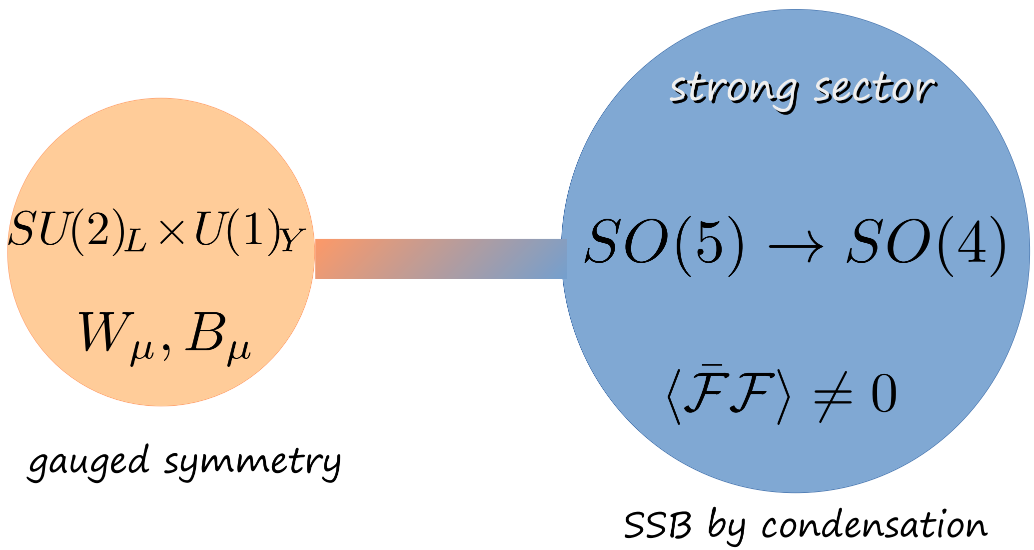
$n_0 = \dim(\mathcal{H}_0) - \dim(\mathcal{H})$ absorbed by gauging

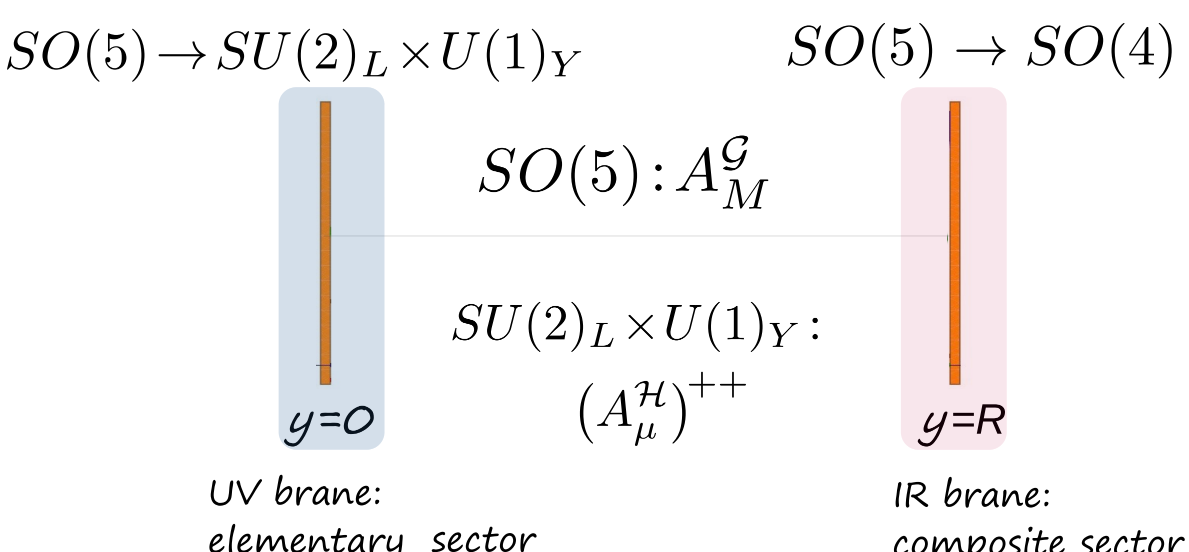
where $\mathcal{H} = \mathcal{H}_0 \cap \mathcal{H}_1$ unbroken gauge group

$\Rightarrow (n - n_0)$ pNGBs

$$C_5^{\hat{a}}(+,+)\,, \quad \hat{a} = 1, .., 4$$

Symmetry Breaking by Boundary Conditions





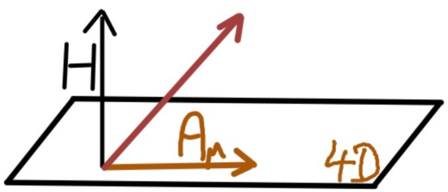
$$
\begin{array}{ll}
L_\mu^a(+,+) & L_5^a(-,-) \\
\cancel{R_\mu^b}(-,+) & R_5^b(+,-) \\
B_\mu(+,+) & B_5(-,-) \\
\cancel{Z'_\mu}(-,+) & Z'_5(+,-) \\
\cancel{C_\mu^{\hat{a}}}(-,-) & \boxed{C_5^{\hat{a}}(+,+)}
\end{array}
$$

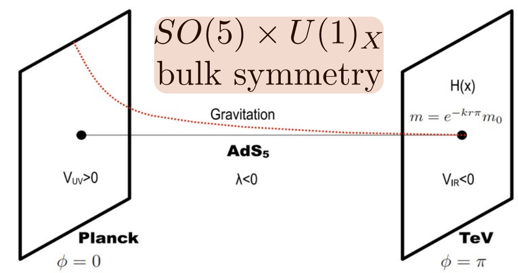
SO(5) Gauge-Higgs Unification

Manton, Hosotani, Fairlie,
Hatanaka, Inami, Lim...

5D vector $A_M^A = (A_\mu^A, A_5^A)$

$$ds^2 = a^2(z)\left(\eta_{\mu\nu}dx^\mu x^\nu - dz^2\right) \qquad a(z) = \frac{R}{z}$$





$$\mathcal{S} \supset \sum_{k=1,2}\int d^4x\int_R^{R'} dz\ a^4\left\{\bar{\zeta}_k\left[i\not{D} + \left(D_5 + 2\frac{a'}{a}\right)\gamma^5 - aM_k\right]\zeta_k\right\}$$

$$D_M = \partial_M - ig_5T_L^aL_M^a - ig_5T_R^bR_M^b - ig_YYB_M - i\frac{g_Y}{c_\phi s_\phi}Z'_M\left(T_R^3 - s_\phi^2Y\right)$$
$$-ig_5T^{\hat{a}}C_M^{\hat{a}}$$

→ Yukawa $\mathcal{S} \supset -\sum_{k=1,2} ig_5\int d^4x\int_R^{R'} dz\ a^4\bar{\zeta}_k(x,z)\gamma^5T^4\zeta_k(x,z)C_5^4(x,z)$

Novel Breaking Pattern

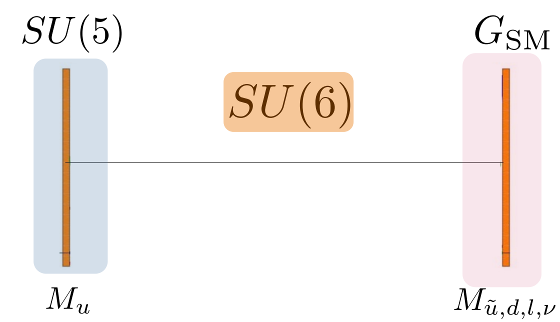
Fermion irreps:

$$\mathbf{20_L} \to (\mathbf{3},\mathbf{2})^{-,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{1}$$
$$(\mathbf{3}^*,\mathbf{2})^{-,+}_{-1/6} \oplus u_R(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus (\mathbf{1},\mathbf{1})^{-,+}_{-1}$$

$$\mathbf{15_L} \to q_L(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e^c_R(\mathbf{1},\mathbf{1})^{+,+}_{1}$$
$$\oplus (\mathbf{3},\mathbf{1})^{-,+}_{-1/3} \oplus (\mathbf{1},\mathbf{2})^{-,+}_{1/2}$$

$$\mathbf{6_L} \to d_R(\mathbf{3},\mathbf{1})^{-,-}_{-1/3} \oplus l^c_L(\mathbf{1},\mathbf{2})^{-,-}_{1/2} \oplus \nu^c_R(\mathbf{1},\mathbf{1})^{+,+}_{0}$$

$$\mathbf{1_L} \to (\mathbf{1},\mathbf{1})^{+,-}_{0}$$



+ boundary mass terms!

$$S_{UV} = \int \mathrm{d}^4x \left(M_u \psi_{\mathbf{20},10}\chi_{\mathbf{15},10} + \mathrm{h.c.}\right)$$

$$S_{IR} = \int \mathrm{d}^4x \left(\frac{R}{R'}\right)^4 \Big(M_{\tilde u}\psi_{\mathbf{15},(3^*,1)}\chi_{\mathbf{20},(3^*,1)} + M_d \chi_{\mathbf{15},(3,1)}\psi_{\mathbf{6},(3,1)}$$
$$+ M_l \chi_{\mathbf{15},(1,2)}\psi_{\mathbf{6},(1,2)} + M_\nu \chi_{\mathbf{6},1}\psi_{\mathbf{1}} + \mathrm{h.c.}\Big)$$

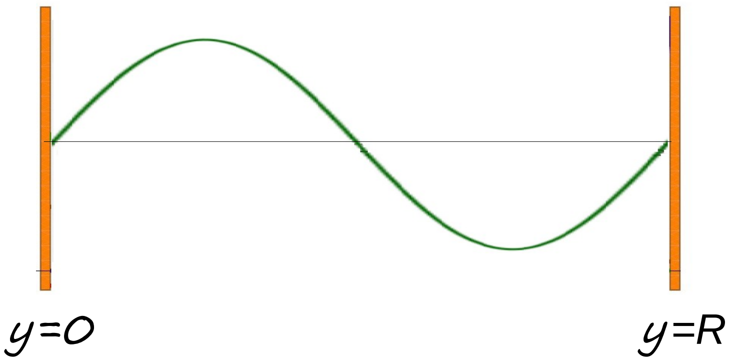
Breaks SU(4), but crucial to get spectrum (and correct EWSB)!

$M_u \to m_u$
$M_{\tilde u} \to V(H)$
$M_{d,l} \to m_e \neq m_d$
$M_\nu \to$ *light* ν

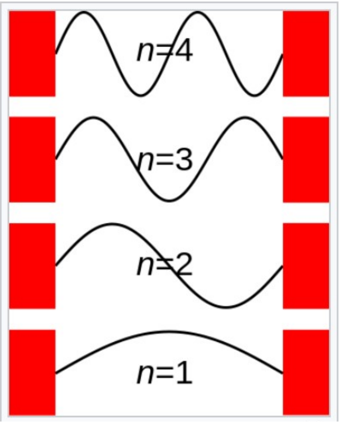
viable spectrum

Kaluza-Klein Decomposition

T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, 966 (1921)
O. Klein, Z. Phys.37, 895 (1926)



Original Idea:
unify gravity and electromagnetism, by merging the photon vector field together with the 4D Minkowski metric into a 5×5 metric



Simple Example: Scalar field in 5D

$$S_5 = \frac{1}{2}\int d^4x \int_0^R dy\ \partial_M \Phi(x,y)\partial^M \Phi(x,y) - m_5^2 \Phi^2(x,y)$$

Kaluza-Klein Decomposition

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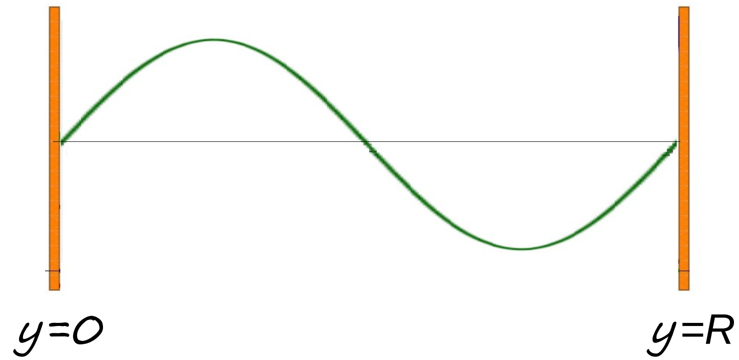

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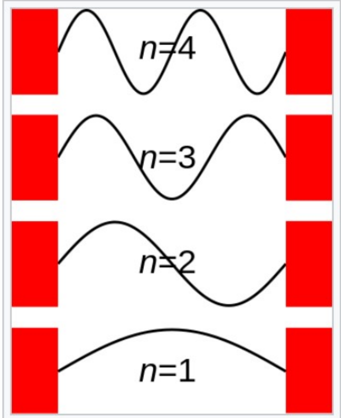

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$$S_5 = \frac{1}{2}\int d^4x \int_0^R dy\ \partial_M \Phi(x,y)\partial^M \Phi(x,y) - m_5^2 \Phi^2(x,y)$$

KK decomposition into complete set of orthonormal functions $f_n(y)$ on interval

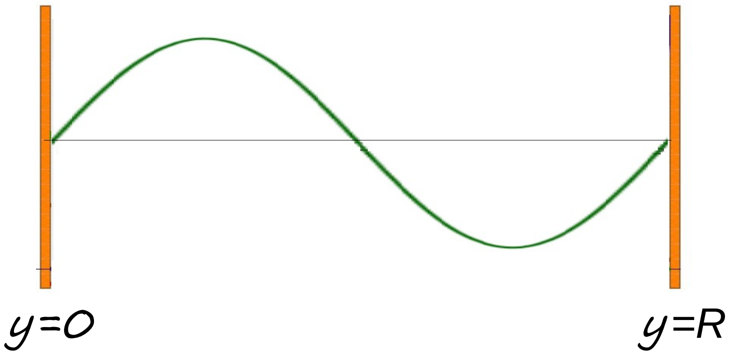
$$\Phi(x,y) = \sum_n \phi_n(x) f_n(y)\,, \qquad \int_0^R dy\ f_n(y) f_m(y) = \delta_{nm}$$

4D scalars

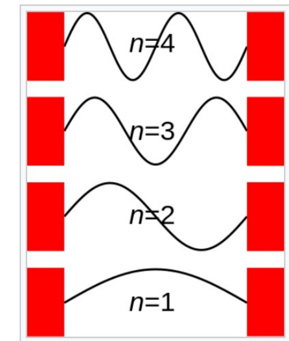
Kaluza-Klein Decomposition

T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, 966 (1921)

O. Klein, Z. Phys.37, 895 (1926)



$$M = \overbrace{0, 1, 2, 3,}^{x^\mu} \overset{y}{\mathbf{4}}$$



$$\boxed{S_5 = \frac{1}{2}\int d^4x \int_0^R dy \ \partial_M \Phi(x,y)\partial^M \Phi(x,y) - m_5^2 \Phi^2(x,y)}$$

$$\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$$

$$\Phi(x,y) = \textstyle\sum_n \phi_n(x) f_n(y)$$

$$\textstyle\int_0^R dy \ f_n(y) f_m(y) = \delta_{nm}$$

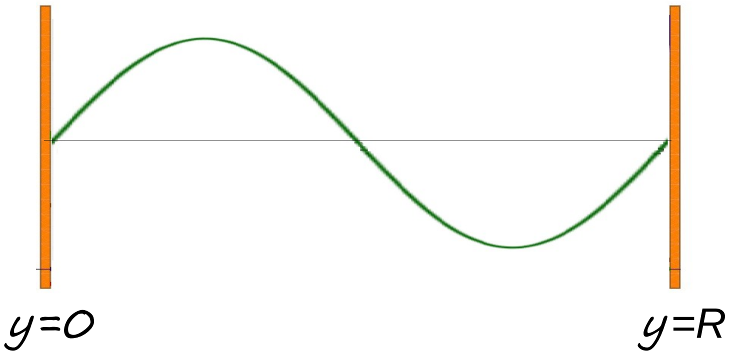
$$S_5 = \frac{1}{2}\int d^4x \sum_n \{\partial_\mu \phi_n(x)\partial^\mu \phi_n(x) - m_5^2 \phi_n^2(x)\}$$

$$+\frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n} \phi_m(x)\phi_n(x) \ f_m(y)\partial_y^2 f_n(y)$$

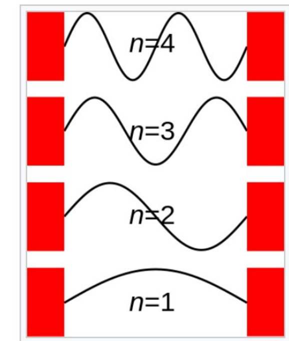
$$-\frac{1}{2}\int d^4x \sum_{m,n} \phi_m(x)\phi_n(x) \ \left(f_m(R)f_n'(R) - f_m(0)f_n'(0)\right)$$

Kaluza-Klein Decomposition

T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, 966 (1921)
O. Klein, Z. Phys.37, 895 (1926)



$$M = \overbrace{0,1,2,3,}^{x^\mu}\ \overset{y}{\mathbf{4}}$$



$$\boxed{S_5 = \frac{1}{2}\int d^4x \int_0^R dy\ \partial_M\Phi(x,y)\partial^M\Phi(x,y) - m_5^2\Phi^2(x,y)}$$

$$\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$$

$$\Phi(x,y) = \textstyle\sum_n \phi_n(x) f_n(y)$$

$$\int_0^R dy\ f_n(y) f_m(y) = \delta_{nm}$$

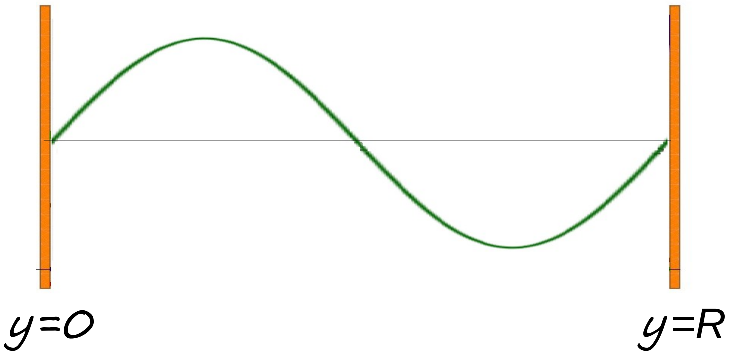
$$S_5 = \frac{1}{2}\int d^4x \sum_n \{\partial_\mu\phi_n(x)\partial^\mu\phi_n(x) - m_5^2\phi_n^2(x)\}$$

$$+\frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n}\phi_m(x)\phi_n(x)\ f_m(y)\partial_y^2 f_n(y)$$

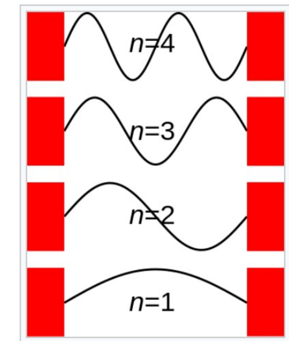
$$-\frac{1}{2}\int d^4x \sum_{m,n}\phi_m(x)\phi_n(x)\ \left(f_m(R)f_n'(R) - f_m(0)f_n'(0)\right)$$

Kaluza-Klein Decomposition

T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, 966 (1921)
O. Klein, Z. Phys.37, 895 (1926)



$$M = \overbrace{0,1,2,3,}^{x^\mu} \overset{y}{4}$$



$$\boxed{S_5 = \frac{1}{2}\int d^4x \int_0^R dy\ \partial_M \Phi(x,y)\partial^M \Phi(x,y) - m_5^2 \Phi^2(x,y)}$$

$$\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$$

$$\Phi(x,y) = \sum_n \phi_n(x) f_n(y)$$

$$\int_0^R dy\ f_n(y) f_m(y) = \delta_{nm}$$

$$S_5 = \frac{1}{2}\int d^4x \sum_n \{\partial_\mu \phi_n(x)\partial^\mu \phi_n(x) - m_5^2 \phi_n^2(x)\}$$

$$+\frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n} \phi_m(x)\phi_n(x)\ f_m(y)\partial_y^2 f_n(y)$$

IBP

$$-\frac{1}{2}\int d^4x \sum_{m,n} \phi_m(x)\phi_n(x)\ \left(f_m(R)f_n'(R) - f_m(0)f_n'(0)\right)$$

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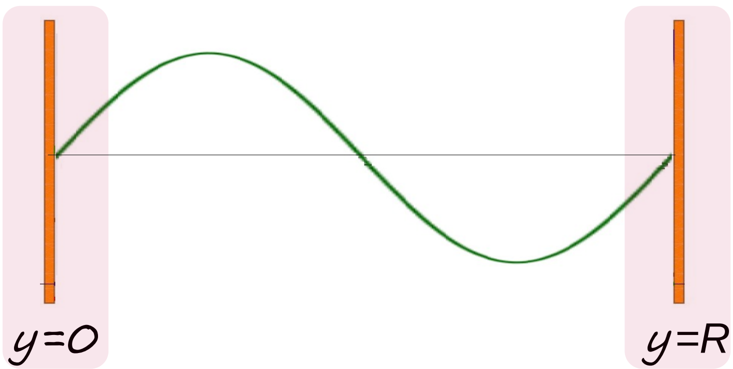

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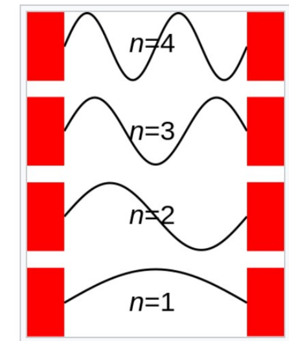

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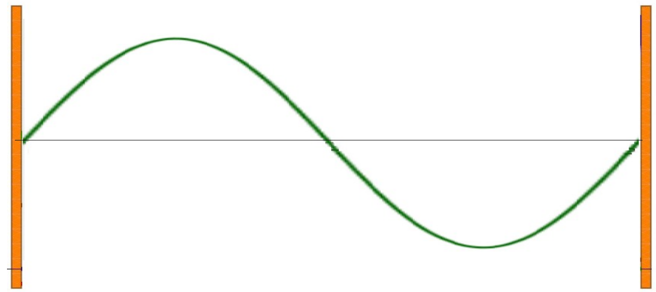

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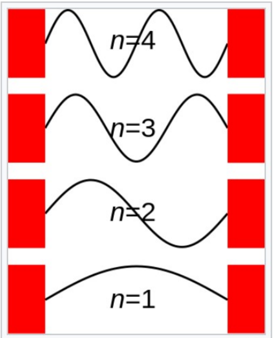
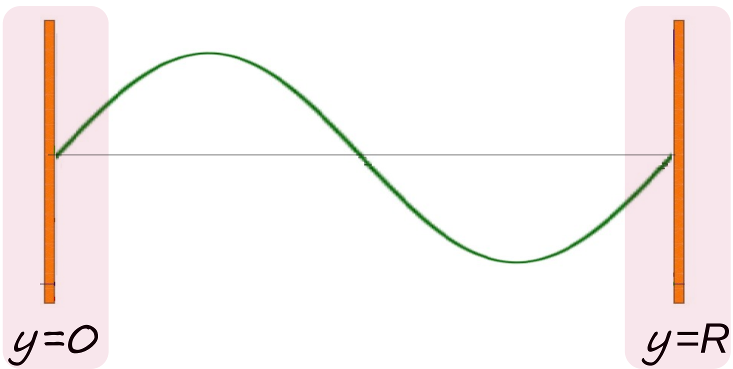


'Matching' or variation of S

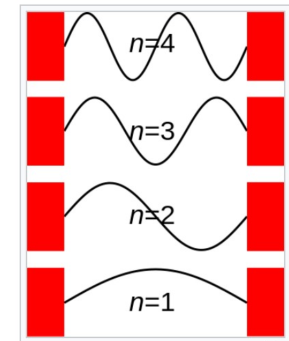
$$\begin{aligned} S_5 &= \frac{1}{2}\int d^4x \sum_n \{\partial_\mu \phi_n(x) \partial^\mu \phi_n(x) - m_5^2 \phi_n^2(x)\} \\ &+ \frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n} \phi_m(x)\phi_n(x)\ f_m(y)\partial_y^2 f_n(y) \\ &- \frac{1}{2}\int d^4x \sum_{m,n} \phi_m(x)\phi_n(x)\left(f_m(R)f_n'(R) - f_m(0)f_n'(0)\right) \end{aligned}$$

$$\Rightarrow \quad \partial_y^2 f_n(y) = -a_n^2 f_n(y)$$

Boundary Conditions



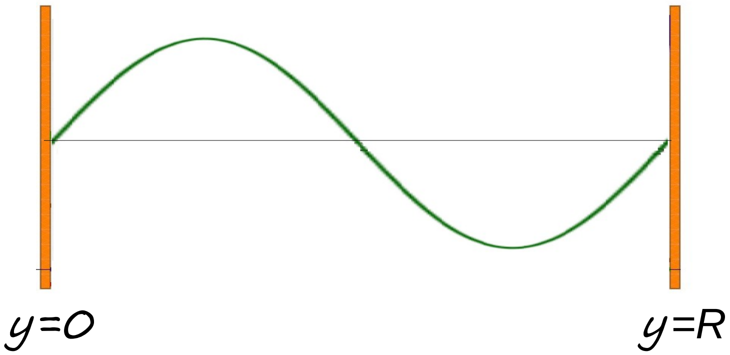
$$\Phi(x,y) = \sum_n \phi_n(x) f_n(y)$$
$$\int_0^R dy\ f_n(y) f_m(y) = \delta_{nm}$$



'Matching' or variation of S

$$S_5 = \frac{1}{2}\int d^4x \sum_n \{\partial_\mu \phi_n(x)\partial^\mu \phi_n(x) - m_5^2 \phi_n^2(x)\}$$
$$+\frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n} \phi_m(x)\phi_n(x)\ f_m(y)\partial_y^2 f_n(y) \quad \Rightarrow \quad \partial_y^2 f_n(y) = -a_n^2 f_n(y)$$
$$-\frac{1}{2}\int d^4x \sum_{m,n} \phi_m(x)\phi_n(x)\left(f_m(R) f_n'(R) - f_m(0) f_n'(0)\right) \quad \Rightarrow \quad f_n(y) = 0 \vee f_n'(y) = 0$$
$$y = 0, R$$

Kaluza-Klein Decomposition



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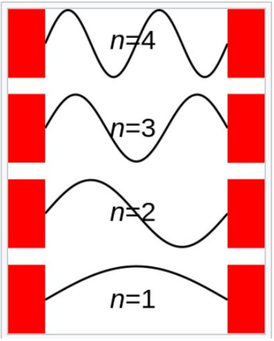

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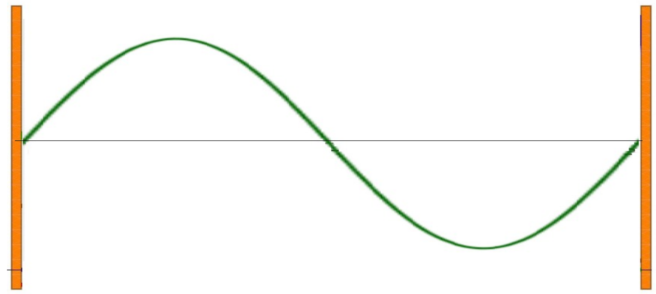
$$S_5 = \frac{1}{2}\int d^4x \sum_n \{\partial_\mu \phi_n(x) \partial^\mu \phi_n(x) - m_5^2 \phi_n^2(x)\}$$
$$+\frac{1}{2}\int d^4x \int_0^R dy \sum_{m,n} (-a_n^2 \phi_m(x)\phi_n(x))\ f_m(y) f_n(y)$$

Kaluza-Klein Decomposition



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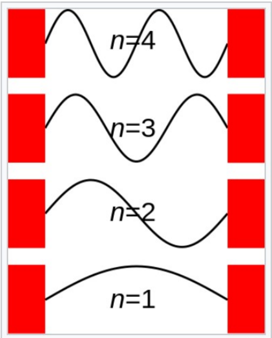

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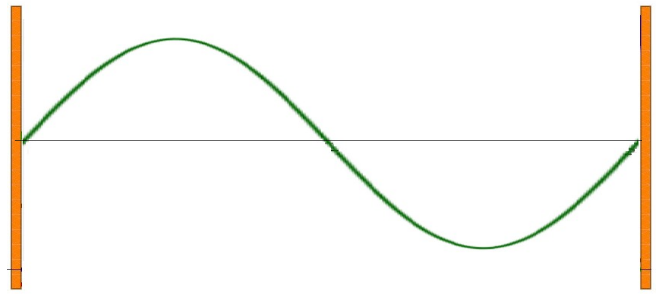
$$m_n^2 = m_5^2 + a_n^2$$

Kaluza-Klein Decomposition



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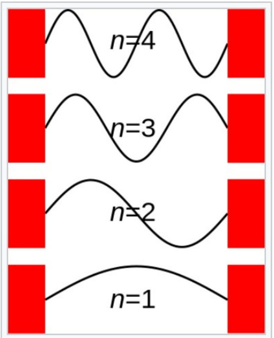

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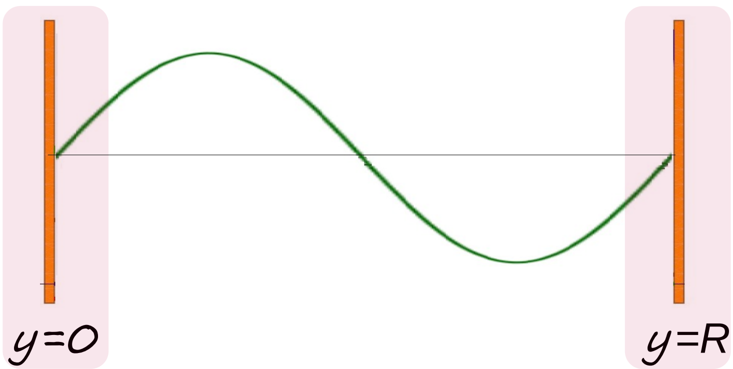
$$m_n^2 = m_5^2 + a_n^2$$

Scalar in compact. XD = infinite tower of 4D scalars with different masses

Kaluza-Klein Profiles



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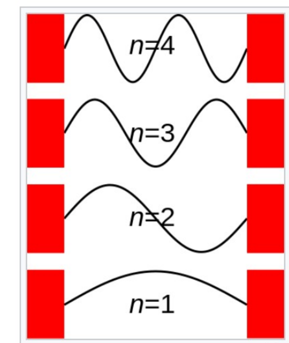

$$S = \frac{1}{2} \int d^4x \sum_n \{\partial_\mu \phi_n(x) \partial^\mu \phi_n(x) - m_n^2 \phi_n^2(x)\}$$

$$m_n^2 = m_5^2 + a_n^2$$

$$\partial_y^2 f_n(y) = -a_n^2 f_n(y)$$

$$f_n(y) = c_n \cos(a_n y) + d_n \sin(a_n y)$$

$$f_n(y) = 0 \vee f_n'(y) = 0$$
$$y = 0, R$$

Mass spectrum from boundary conditions (BCs)

Solution → Randall-Sundrum Scenario

Randall, Sundrum, hep-ph/9905221

- Non-trivial metric such that (4D) length scales get contracted differently along fifth dimension → '**warp factor**'

$$\boxed{ds^2 = e^{-2kr|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r^2 d\phi^2}$$

$$S_{\rm IR} \supset \int d^4x\, r \int_{-\pi}^{\pi} d\phi\, \frac{\sqrt{G}}{r^2}\left[G^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) - V(\Phi)\right]\delta(|\phi|-\pi)$$

$$V(\Phi) = \frac{\lambda_5}{2}\left(\Phi^\dagger\Phi - \frac{v_5^2}{2}\right)^2$$

$$v_5 = \sqrt{2\mu_5^2/\lambda_5} \sim M_{\rm Pl}$$

$$= \int d^4x\sqrt{-\bar{g}}\, e^{-4kr\pi}\left\{e^{2kr\pi}\bar{g}^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) - \frac{\lambda_5}{2}\left(\Phi^\dagger\Phi - \frac{v_5^2}{2}\right)^2\right\}$$

$$\bar{g}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x)$$

$$\Phi \to e^{kr\pi}\Phi$$

$$= \int d^4x\sqrt{-\bar{g}}\left\{\bar{g}^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) - \frac{\lambda_5}{2}\left(\Phi^\dagger\Phi - e^{-2kr\pi}\frac{v_5^2}{2}\right)^2\right\}$$

$$v = e^{-kr\pi}v_5 \sim \text{TeV}$$

→ **HP solved** ✓

(gravitational redshifting)

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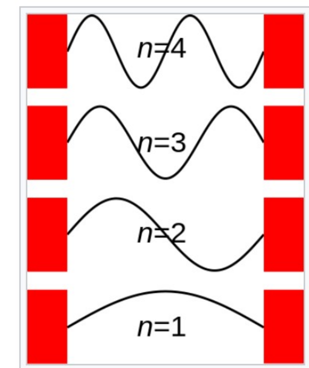


- In complete analogy to flat case → 5D SM (just different metric)
- Light Fermion fields can be localized differently along XD

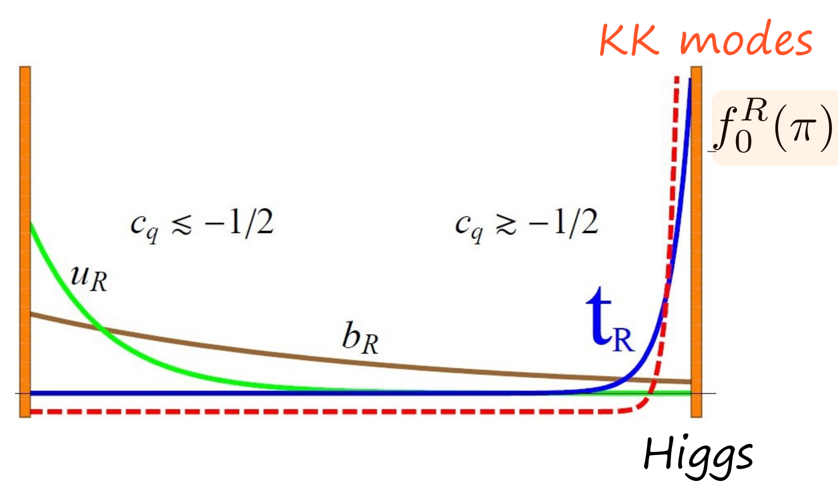
$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi \, \sqrt{G} \left\{ E_a^A \left[\frac{i}{2} \, \bar{\Psi}\gamma^a(\partial_A - \overleftarrow{\partial_A})\Psi + \frac{\omega_{bcA}}{8} \, \bar{\Psi}\{\gamma^a, \sigma^{bc}\}\Psi \right] - m \, \mathrm{sgn}(\phi) \, \bar{\Psi}\Psi \right\}$$

- Profiles $f_0^R(\phi) \propto e^{ckr|\phi|}$ (RH)

$c \equiv -m/k$

$$f_0^R(\pi) \propto \sqrt{\frac{1+2c}{1-\epsilon^{1+2c}}}$$

$$\epsilon \equiv e^{-kr\pi} \approx 10^{-16}$$



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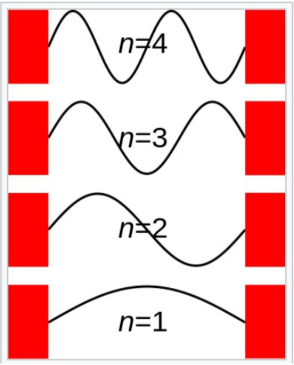

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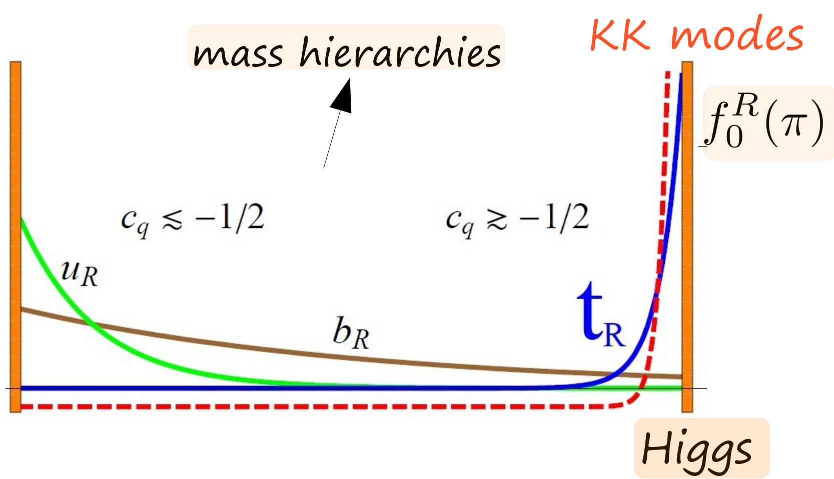
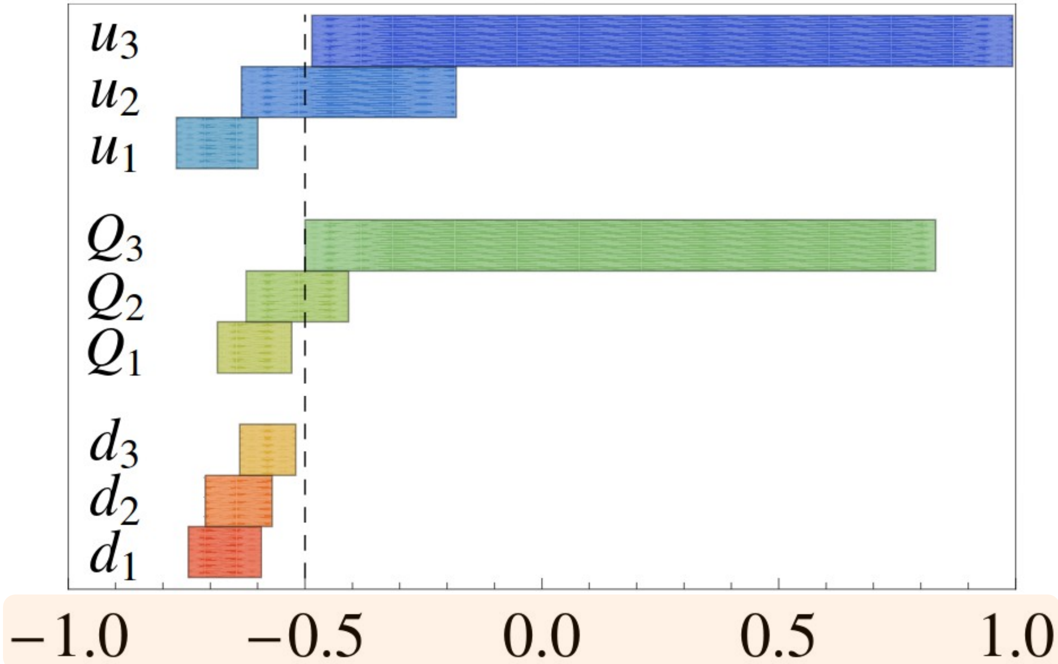


Solution to Flavor Puzzle



Goertz, 1112.6387

$c_{u_i,Q_i,d_i} \sim \mathcal{O}(1)$

- Profiles $f_0^R(\phi) \propto e^{ckr|\phi|}$
  (RH)

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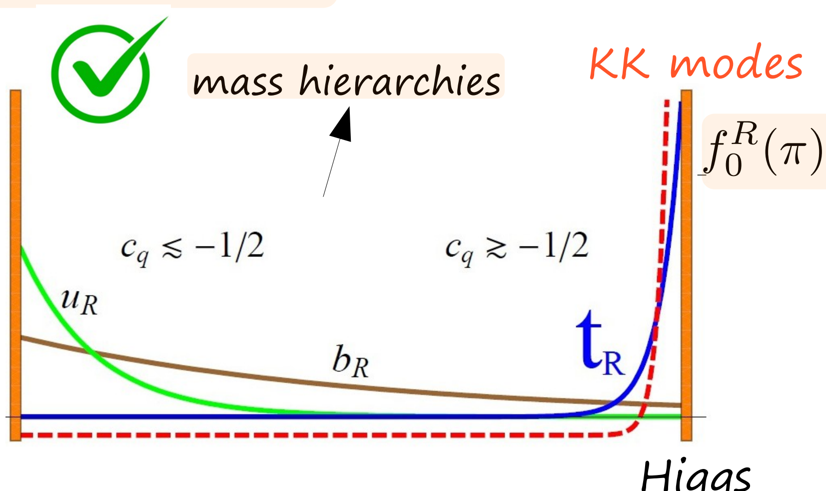
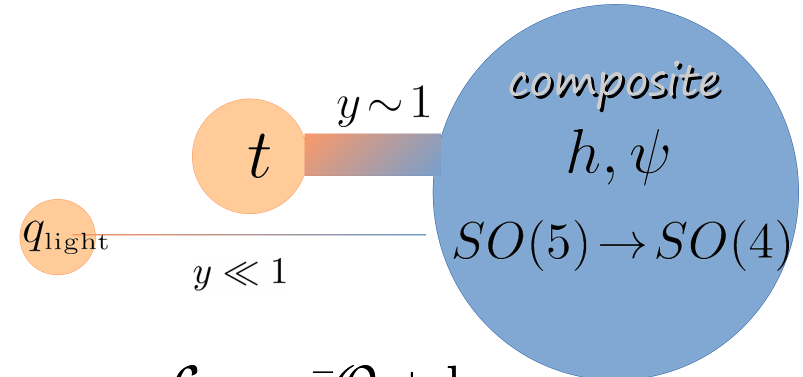


RS - Partial Compositeness

- Addresses the flavor puzzle:

$$y^q_{L,R} \sim (\Lambda/\Lambda_{UV})^{\gamma_{L,R}}\,, \qquad \gamma_{L,R} = [\mathcal{O}_{L,R}] - 5/2$$

→ Hierarchies generated naturally



$$\mathcal{L} = y\bar{q}\mathcal{O} + \text{h.c.}$$

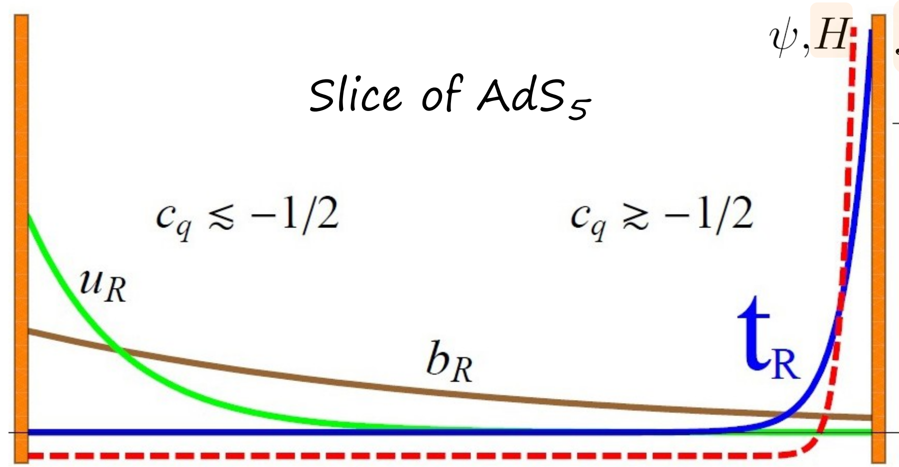
$$\gamma_R = |c_R - 1/2| - 1$$

Overlap with Higgs → mass

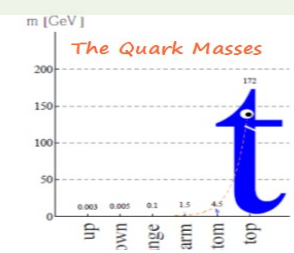
$$f_0^R(\pi) \sim (1 - \epsilon^{1+2c})^{-1/2}$$

Dual picture:
Extra Dimension

UV brane:
elementary
sector



Froggatt-Nielsen-like
mass matrices



$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

Anomalous Dimensions γ ↔ XD Localization c ↔ FN U(1) charge