Hierarchy in double SU(2) models

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Contents

Introduction

- **Motivation**: to introduce new scalar particles, we need to have a mass hierarchy $\rightarrow$ understand the origin of these energy scales
- Is it possible to obtain hierarchy in a *natural* way? (i.e. general model/models & wide range of parameters)
- Scales originated by quantum corrections

- **Proposal**: toy model to study large separation of scales from quantum origin
- Start from a massless lagrangian and make use of the Coleman & Weinberg mechanism to obtain SSB; effective potential formalism (1-loop)

The model

$SU(2)_L \times SU(2)_H \times U(1)_X$

$$\mathcal{L}_0 = |D_\mu \Phi|^2 + |D_\mu \Theta|^2 - V_0(\Phi, \Theta)$$

$$\mathrm{D}^\mu_{L,H} = \partial^\mu - \frac{i}{2} g_{L,H} \sigma_a W^{a\mu}_{L,H} - \frac{i}{2} g_X Q_{L,H} X^\mu$$

Kinetic term $\longrightarrow$ boson mass terms

Scalars: classical fields and potential

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}, \qquad \Theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \eta \end{pmatrix}$$

$$V_0(\varphi, \eta) = \frac{1}{4!} \lambda_L \varphi^4 + \frac{1}{4!} \lambda_H \eta^4 + \frac{1}{4!} \lambda_{LH} \varphi^2 \eta^2$$

From $\mathcal{L}_0$ we obtain the mass matrix $G(\varphi, \eta)$ of the model $\rightarrow$ diagonalize

Particle content of the model: $W_{L,H}$, $Z_{L,H}$, $\hat{\gamma}$, φ, η

2 scalars (φ, η) and 7 gauge bosons whose masses m_j depend on the scalar backgrounds and gauge couplings:

$L - H$ decoupled limit ($g_X = 0$):

- $m_{W_{L,j}} = g_L \varphi/2$ $(j = 1, 2, 3)$
- $m_{W_{H,j}} = g_H \eta/2$ $(j = 1, 2, 3)$
- $m_X = 0$

General case ($g_X \neq 0$), more involved expressions:

- $W^\mu_{L,1}$, $W^\mu_{L,2}$, $W^\mu_{H,1}$ and $W^\mu_{H,2}$: the same m as with $g_X = 0$
- mixing between $W^\mu_{L,3}$, $W^\mu_{H,3}$ and $X^\mu \to Z^\mu_L$, Z^μ_H and $\hat{\gamma}^\mu$
- $\hat{\gamma}^\mu$ is always massless
- Z^μ_L and Z^μ_H masses: combination of the three gauge couplings $g_{L,H,X}$

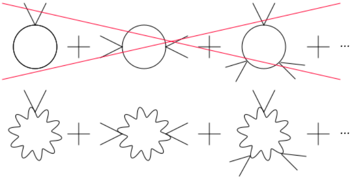
Effective potential

Tree level + 1-loop:

$$V(\varphi,\eta) = V_0(\varphi,\eta) + \frac{3}{64\pi^2}\sum_{j=1}^{7} m_j^4 \left[\ln\left(\frac{m_j^2}{\mu^2}\right) - \frac{5}{6}\right]$$

Restrictions:

Coleman & Weinberg hypothesis $\rightarrow |\lambda_j| < \epsilon_{CW} \cdot g_j^2$
Perturbativity $\rightarrow g_j^2 < \epsilon_{g^2} \cdot 4\pi \equiv g^2_{max}$



Coleman, Weinberg, Phys. Rev. D7 1888 (1973)

Casas, Espinosa, Quirós, arXiv:hep-ph/9409458v1

Phenomenology: hierarchy?

We can define the hierarchy between the L and H sectors as:

$$\mathfrak{R} = \frac{m_{W_H}^2}{m_{W_L}^2} = \frac{g_H^2 \langle \eta \rangle^2}{g_L^2 \langle \varphi \rangle^2}$$

- L: light particles $\longrightarrow m_{W_L,Z_L}^2$
- H: particles at a higher mass scale m_{W_H,Z_H}^2
- Large hierarchy $\mathfrak{R} \gg 1 \longrightarrow$ final probability expressions can be simplified
- $m_\eta^2 / m_\varphi^2 \sim \mathfrak{R}$

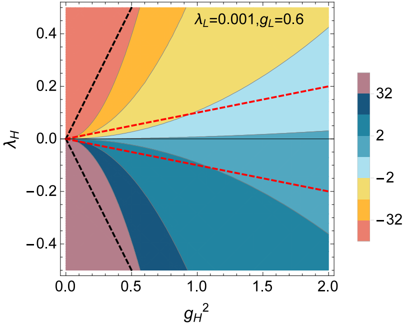
If $g_X = 0, \lambda_{LH} = 0 \rightarrow$ simple analytical expressions.
If $g_X \neq 0, \lambda_{LH} = 0 \rightarrow$ analytical expressions only if $g_X \ll 1$.
If $\lambda_{LH} \neq 0 \rightarrow$ involved case without analytical expressions $\rightarrow$ numerical.

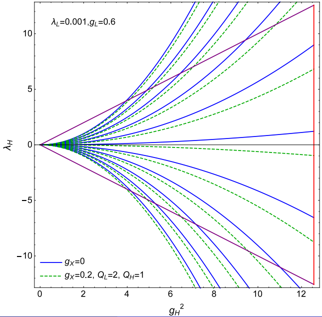
Fixed L sector

Constant hierarchy $\mathfrak{R}$ lines in the (g_H^2, λ_H) plane $(\lambda_{LH} = 0)$

$$\mathfrak{R} = Exp\left[\frac{128\pi^2}{27}\left(\frac{\lambda_L}{g_L^4} - \frac{\lambda_H}{g_H^4}\right)\right] \qquad (g_X = 0)$$

$$\mathfrak{R} = Exp\left[\frac{128\pi^2}{27}\left(\frac{\left(\lambda_L + \frac{9}{128\pi^2} g_L^2 Q_L^2 g_X^2\right)}{\left(g_L^4 + \frac{2}{3} g_L^2 Q_L^2 g_X^2\right)} - (L \leftrightarrow H)\right)\right]$$



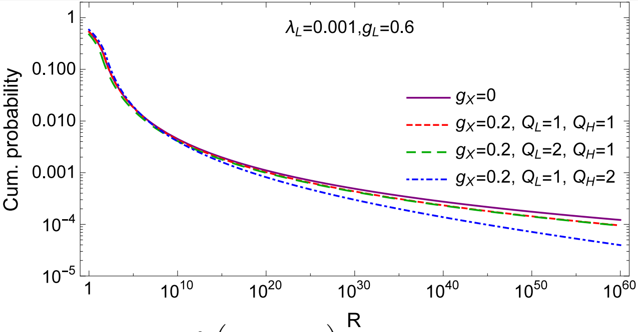


Fixed L sector

Probability $\mathfrak{P}^{(\alpha_L)}_{\rm cumul}$: ratio of the area with $\mathfrak{R} \in [\mathfrak{R}_0, \infty]$ and the total allowed area in the (g_H^2, λ_H) plane (*CW-triangle*)

Cumulative probability for $\mathfrak{R}$ from $\mathfrak{R}_0$ to ∞ for fixed $\alpha_L = (g_L^2, \lambda_L)$

$$\mathfrak{P}^{(\alpha_L)}_{\rm cumul} = \frac{1}{6}\left(\frac{27\ln \mathfrak{R}_0}{32\pi} - \frac{4\pi\lambda_L}{g_L^4}\right)^{-2} \overset{\mathfrak{R}_0 \gg 1}{\simeq} \frac{0.44}{(\log_{10}\mathfrak{R}_0)^2}$$

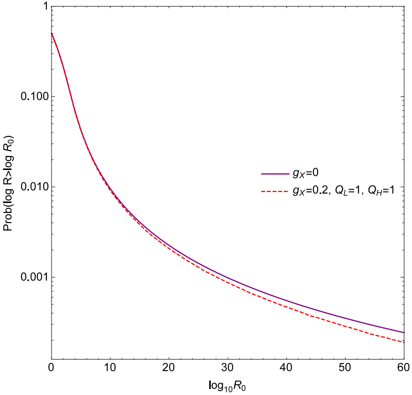


Note: the expression is valid for $\ln \mathfrak{R}_0 > \frac{128\pi^2}{27}\left(\frac{\lambda_L}{g_L^4} + \frac{\epsilon_{CW}}{4\pi\epsilon_{g^2}}\right)$ (for smaller $\mathfrak{R}$ the expression is more invloved)

Both L and H sectors integrated out

Cumulative probability for $\mathfrak{R}$ from $\mathfrak{R}_0$ to ∞

$$\mathfrak{P}_{\text{cumul}} \overset{\mathfrak{R}_0 \gg 1}{\simeq} \frac{1}{3}\left(\frac{32\pi}{27 \ln \mathfrak{R}_0}\right)^2 \simeq \frac{0.87}{(\log_{10} \mathfrak{R}_0)^2}$$



Conclusions

Summary:

- model with $SU(2)_L \times SU(2)_H \times U(1)_X$ symmetry
- assumptions: CW and perturbativity
- 2 sectors of particles with possible hierarchy between them
- hierarchy: depends on the couplings
  $\rightarrow g_L, g_H, \lambda_L, \lambda_H, g_X(Q_L, Q_H), \lambda_{LH}$
- wide regions of parameter space giving place to very different hierarchies

Results & future work:

- probability of obtaining very large hierarchies is suppressed, but only logarithmically $\rightarrow$ not excluded
- $\mathfrak{R} \gtrsim (M_P/m_{EW})^2 \sim 10^{32}$ only suppressed by $\mathfrak{P} \sim 10^{-3}$–$10^{-4}$
- same results are obtained if more symmetry groups are included as $\mathcal{G} = \prod_\chi SU(2)_\chi$

Thank you for your attention

Gauge boson masses in coupled scenario

The masses of the Z_L^μ and Z_H^μ gauge bosons are given by,

$$m^2_{Z_{L,H}} = \frac{\overline{M}^2}{2}\left[1 \mp \sqrt{1 - 4\overline{m}^2/\overline{M}^2}\right] ,$$

$$\overline{M}^2 = (g_H^2 + g_X^2 Q_H^2)\eta^2 + (g_L^2 + g_X^2 Q_L^2)\varphi^2 ,$$

$$\overline{m}^2 = (g_H^2 g_L^2 + g_X^2(Q_H^2 g_L^2 + Q_L^2 g_H^2))\eta^2\varphi^2/(4\overline{M}^2) ,$$

with $m_{Z_L} \approx \overline{m}$ and $m_{Z_H} \approx \overline{M}$ for $\overline{m} \ll \overline{M}$.

Probability for $g_X \neq 0$

$$\mathfrak{P}^{(\alpha_L)}_{\rm cumul} = \frac{(1+\mathfrak{a})^3}{6\,(1+\mathfrak{b})}\left(\frac{27\ln\mathfrak{R}_0}{32\pi} - \frac{4\pi\lambda_L}{g_L^4} + \mathfrak{c}\right)^{-2}$$

$$\begin{aligned}
\mathfrak{a} =& \frac{Q_H^2 g_X^2}{g_L^2}\left[-\frac{9g_L^2}{128\pi^2}\left(1+2\ln\mathfrak{R}_0\right)+\frac{2}{3}\right.\\
&\left. -\frac{g_X^2}{64\pi^2}\left(19(Q_L^2-Q_H^2)+6Q_H^2\ln\mathfrak{R}_0\right)+\frac{2\lambda_L}{3g_L^2}\right]\\
\mathfrak{b} =& \frac{2Q_H^2 g_x^2}{3g_L^2}\\
\mathfrak{c} =& \frac{3g_X^2}{32\pi g_L^2}\left(19Q_L^2-22Q_H^2+6Q_H^2\ln\mathfrak{R}_0\right)
\end{aligned}$$

Possible hierarchy cases