

Two-loop QCD corrections to $Wb\bar{b}$ production at hadron colliders

Heribertus Bayu Hartanto

based on **arXiv:2102.02516**
with Simon Badger and Simone Zoia

EPS-HEP 2021
July 26, 2021



Science & Technology
Facilities Council



UNIVERSITY OF
CAMBRIDGE



European Research Council
Established by the European Commission

Precise prediction for the LHC

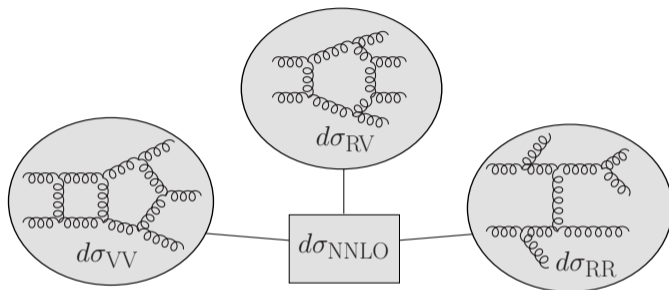
⇒ QCD corrections are important at the LHC

$$d\sigma = d\sigma^{\text{LO}} + \underbrace{d\sigma^{\text{NLO}}}_{10-30\%} + \underbrace{d\sigma^{\text{NNLO}}}_{1-10\%} + \dots$$

NNLO frontier: 2 to 3 scattering

- ▶ $pp \rightarrow jjj$: $R_{3/2}$, $m_{jjj} \Rightarrow \alpha_s$ determination at multi-TeV range
- ▶ $pp \rightarrow \gamma\gamma j$: background to Higgs p_T , signal/background interference effects
- ▶ $pp \rightarrow Hj j$: Higgs p_T , background to VBF (probes Higgs coupling)
- ▶ $pp \rightarrow Vj j$: Vector boson p_T , W^+/W^- ratios, multiplicity scaling
- ▶ $pp \rightarrow VVj$: background for new physics

NNLO cross sections for $2 \rightarrow 3$ processes

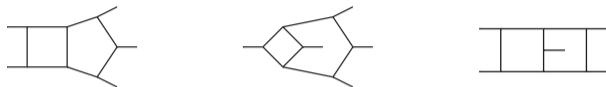


$$\text{loop amplitude} = \sum (\text{rational coefficients}) \times (\text{integral/special functions})$$

$$\text{finite remainder} = \text{loop amplitude} - \text{poles}$$

Massive progress in massless 2-loop 5-particle scattering

- ▶ All 2-loop 5-particle integrals are known



[Papadopoulos, Tommasini, Wever(2015)] [Gehrmann, Henn, Lo Presti(2015,2018)] [Abreu, Page, Zeng(2018)]
 [Abreu, Dixon, Herrmann, Page, Zeng(2018,2019)] [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia(2018,2019)][Chicherin, Sotnikov(2020)]

- ▶ Many 2-loop 5-particle QCD amplitudes known analytically

Leading colour $\Rightarrow 5g, 2q3g, 4q1g, 2q3\gamma, 2q1g2\gamma$

[Abreu, Agarwal, Badger, Brønnum-Hansen, Buccioni, Chawdhry, Czakon, Dormans, Febres Cordero, Gehrmann, HBH, Henn, Ita, Lo Presti, Mitov, Page, Peraro, Poncelet, Sotnikov, Tancredi, von Manteuffel, Zeng(2015-2021)]

Full colour $\Rightarrow 5g$ all-plus, $2q1g2\gamma, 3g2\gamma$

[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia(2019)] [Agarwal, Buccioni, Tancredi, von Manteuffel(2021)]
 [Badger, Bronnum-Hansen, Chicherin, Gehrmann, HBH, Henn, Marcoli, Moodie, Peraro, Zoia(2021)]

- ▶ NNLO QCD calculations for $2 \rightarrow 3$ processes

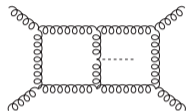
$pp \rightarrow \gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet(2019)][Kallweit, Sotnikov, Wiesemann(2020)]

$pp \rightarrow \gamma\gamma j$ [Chawdhry, Czakon, Mitov, Poncelet(2021)]

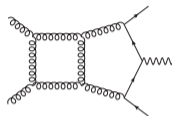
$pp \rightarrow jjj$ [Czakon, Mitov, Poncelet(2021)]

Scattering with an off-shell leg

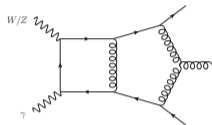
$$pp \rightarrow H + 2j$$



$$pp \rightarrow W/Z + 2j$$



$$pp \rightarrow W/Z + \gamma j$$



- ▶ rich potential phenomenology
- ▶ massless internal particles, focus on QCD corrections
- ▶ high algebraic and analytic complexity
 - \Rightarrow six independent variables
 - \Rightarrow 3 square roots
- ▶ two-loop planar master integrals are available (using differential equations method)
 - [Abreu,Ita,Moriello,Page,Tschernow,Zeng(2020)]: numerical solution using generalised series expansion [Moriello(2019)]
 - [Canko,Papadopoulos,Syrrakos(2020)][Syrrakos(2020)]: analytic solution in terms of GPLs

Leading colour $Wb\bar{b}$ amplitude

$$\bar{d}(p_1) + u(p_2) \rightarrow b(p_3) + \bar{b}(p_4) + W^+(p_5)$$

- colour decomposition at leading colour \rightarrow only planar contribution

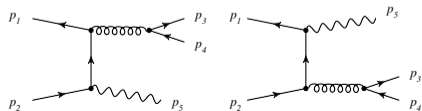
$$\mathcal{A}^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W) \sim g_s^6 g_W N_c^2 \delta_{i_1}^{\bar{i}_4} \delta_{i_3}^{\bar{i}_2} A^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W)$$

- massless b quarks, $p_3^2 = p_4^2 = 0$
- onshell W boson

$$p_5^2 = m_W^2, \quad \sum_{\lambda} \varepsilon_W^{\mu*}(p_5, \lambda) \varepsilon_W^{\nu}(p_5, \lambda) = -g^{\mu\nu} + \frac{p_5^{\mu} p_5^{\nu}}{m_W^2}$$

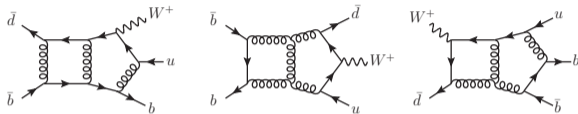
Invariants:

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 - p_3)^2, \quad s_{34} = (p_3 + p_4)^2, \\ s_{45} = (p_4 + p_5)^2, \quad s_{15} = (p_1 - p_5)^2, \quad s_5 = p_5^2, \\ \text{tr}_5 = 4i \epsilon_{\mu\nu\rho\sigma} p_1^{\mu} p_2^{\nu} p_3^{\rho} p_4^{\sigma}.$$



Integrand construction

Feynman diagrams generated using QGRAF [Nogueira(1993)]



MATHEMATICA+FORM to process the numerator topologies and interfere with tree level

$$M^{(2)} = \sum_{\text{spin}} A^{(0)*} A^{(2)} = M_{\text{even}}^{(2)} + \text{tr}_5 M_{\text{odd}}^{(2)}$$

Numerators containing: $\text{tr}(\dots)$ and $\text{tr}(\dots \gamma_5 \dots \gamma_5 \dots)$ \Rightarrow anti-commuting γ_5 prescription

$\text{tr}(\dots \gamma_5 \dots)$ \Rightarrow Larin's prescription [Larin(1993)]

Amplitudes in terms of scalar integrals

$$M_k^{(2)}(\{p\}) = \sum_i c_{k,i}(\epsilon, \{p\}) \mathcal{I}_{k,i}(\epsilon, \{p\}), \quad k \in \{\text{even, odd}\}$$

Reconstructing the finite remainders

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i c_{k,i}(\{p\}, \epsilon) \mathcal{I}_{k,i}(\{p\}, \epsilon)$$

↓ IBP reduction

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i d_{k,i}(\{p\}, \epsilon) \text{MI}_{k,i}(\{p\}, \epsilon)$$

↓ map to special function basis

↓ subtract UV/IR poles

↓ ϵ expansion

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\}) m_{k,i}(f) + \mathcal{O}(\epsilon)$$

$k \in \{\text{even, odd}\}$

FINITEFLOW[Peraro(2019)] LITERED[Lee(2012)]

- ▶ construct a basis of special function using master integral components

$$\text{MI}_i(s) = \sum_{w \geq 0} \epsilon^w \text{MI}_i^{(w)}(s)$$

- ▶ Finite remainders

$$F_k^{(2)} = M_k^{(2)} - \sum_{j=1}^2 I^{(j)} M_k^{(2-j)}$$

- ▶ Numerically compute $e_{k,i}$ over finite fields
- ▶ Reconstruct analytic expressions of $e_{k,i}$ from several numerical evaluations [Peraro(2016)]
- ▶ Define a **new basis** \Rightarrow only functions that appear in the finite remainder
- ▶ Evaluate using **generalised series expansion method** implemented in DIFFEXP [Hidding(2020)]

Numerical evaluation

Evaluation on a univariate slice of the physical phase space

$$p_1 = \frac{\sqrt{s}}{2}(1, 0, 0, 1),$$

$$p_2 = \frac{\sqrt{s}}{2}(1, 0, 0, -1),$$

$$p_3 = \frac{x_1\sqrt{s}}{2}(1, 1, 0, 0),$$

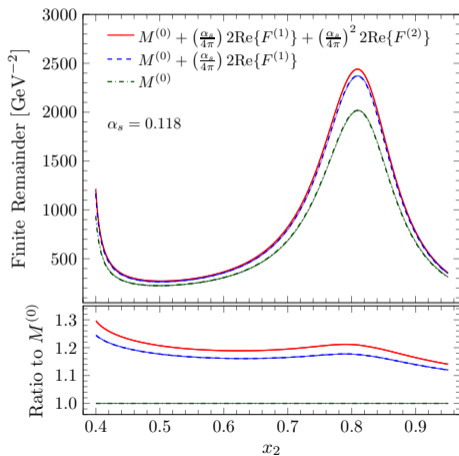
$$p_4 = \frac{x_2\sqrt{s}}{2}(1, \cos\theta, -\sin\phi\sin\theta, -\cos\phi\sin\theta),$$

$$p_5 = \sqrt{s}(1, 0, 0, 0) - p_3 - p_4$$

$$s = 1, m_W^2 = 0.1, \phi = 0.1, x_1 = 0.6$$

- ▶ 1100 points \rightarrow average 260 s/point
- ▶ Reasonable evaluation time with **basic** DIFFEXP setup
- ▶ further optimisation is possible

[Abreu,etal(2020)][Becchetti,etal(2020)]



Summary

- ✓ First analytic result for 2-loop 5-point amplitude with one massive leg
⇒ leading colour $u\bar{d} \rightarrow W^+ b\bar{b}$
- ✓ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops
- ✗ Include W -boson decay
- ✗ Application to other processes
- ✗ Full colour (need non-planar integrals)

Summary

- ✓ First analytic result for 2-loop 5-point amplitude with one massive leg
⇒ leading colour $u\bar{d} \rightarrow W^+ b\bar{b}$
- ✓ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops
- ✗ Include W -boson decay
- ✗ Application to other processes
- ✗ Full colour (need non-planar integrals)

THANK YOU!!!

Back-up Slides

Reconstructing the finite remainders

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\}) m_{k,i}(f) + \mathcal{O}(\epsilon), \quad k \in \{\text{even, odd}\}$$

- ▶ set $s_{12} = 1$
- ▶ Not all $e_{k,i}$ coefficients independent
 \Rightarrow find linear relations between coefficients and reconstruct the simpler ones

$$\sum_i y_i e_i = 0, \quad y_i \in \mathbb{Q}$$

\Rightarrow allow to supply known/candidate coefficients \tilde{e}_j

$$\sum_i y_i e_i + \sum_j \tilde{y}_j \tilde{e}_j = 0, \quad y_i, \tilde{y}_j \in \mathbb{Q}$$

- ▶ guess the denominator \rightarrow from letters [Abreu,etal(2019)][Abreu,etal(2020)]
- ▶ partial fraction in one variable (s_{23}) and reconstruct in the remaining variables ($s_{34}, s_{45}, s_{15}, s_5$)
 $\Rightarrow \sim 4$ times speed up $\Rightarrow 2$ prime fields needed

- ▶ Reconstructed analytic expressions are simplified using MULTIVARIATEAPART[Heller,von Manteuffel(2021)]

Numerical evaluation

- ▶ Only 19 linear combinations of $f_i^{(4)}$ appear in the two-loop finite remainder
 \Rightarrow define a new basis $g_i^{(w)}$

$$\left\{ f_i^{(w)}(s) \right\} \implies \left\{ g_i^{(w)}(s) \right\}$$

- ▶ Apply generalised series expansion method directly to the $g_i^{(w)}$ basis

$$\vec{g}^T = \begin{pmatrix} \epsilon^4 g_i^{(4)} \\ \epsilon^3 g_i^{(3)} \\ \epsilon^2 g_i^{(2)} \\ \epsilon g_i^{(1)} \\ 1 \end{pmatrix}$$

$$d\vec{g} = \epsilon d\tilde{B} \cdot \vec{g}$$

- Much simpler than the DEs for the master integrals
- Use generalised series expansion approach [Moriello(2019)] as implemented in DIFFEXP [Hidding(2020)]