Two-loop QCD corrections to $Wb\bar{b}$ production at hadron colliders

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based on **arXiv:2102.02516**
with Simon Badger and Simone Zoia

EPS-HEP 2021
July 26, 2021

Science & Technology Facilities Council

UNIVERSITY OF CAMBRIDGE

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Precise prediction for the LHC

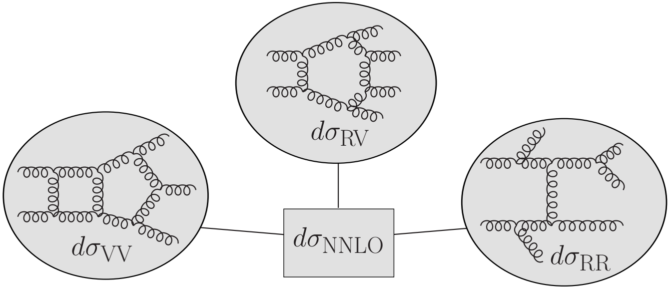
⇒ QCD corrections are important at the LHC

$$d\sigma = d\sigma^{\mathrm{LO}} + \underbrace{d\sigma^{\mathrm{NLO}}}_{10\text{–}30\%} + \underbrace{d\sigma^{\mathrm{NNLO}}}_{1\text{–}10\%} + \dots$$

NNLO frontier: 2 to 3 scattering

- $pp \to jjj$: $R_{3/2}$, $m_{jjj} \Rightarrow \alpha_s$ determination at multi-TeV range
- $pp \to \gamma\gamma j$: background to Higgs p_T, signal/background interference effects
- $pp \to Hjj$: Higgs p_T, background to VBF (probes Higgs coupling)
- $pp \to Vjj$: Vector boson p_T, W^+/W^- ratios, multiplicity scaling
- $pp \to VVj$: background for new physics

NNLO cross sections for 2 → 3 processes

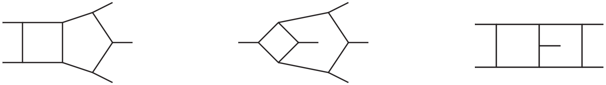


$$\text{loop amplitude} = \sum (\text{rational coefficients}) \times (\text{integral/special functions})$$

$$\text{finite remainder} = \text{loop amplitude} - \text{poles}$$

Massive progress in massless 2-loop 5-particle scattering

- All 2-loop 5-particle integrals are known



[Papadopoulos,Tommasini,Wever(2015)] [Gehrmann,Henn,Lo Presti(2015,2018)] [Abreu,Page,Zeng(2018)]
[Abreu,Dixon,Herrmann,Page,Zeng(2018,2019)] [Chicherin,Gehrmann,Henn,Wasser,Zhang,Zoia(2018,2019)][Chicherin,Sotnikov(2020)]

- Many 2-loop 5-particle QCD amplitudes known analytically

Leading colour $\Rightarrow$ $5g$, $2q3g$, $4q1g$, $2q3\gamma$, $2q1g2\gamma$

[Abreu,Agarwal,Badger,Brønnum-Hansen,Buccioni,Chawdhry,Czakon,Dormans,Febres Cordero, Gehrmann, HBH,Henn,Ita,Lo Presti,Mitov,Page,Peraro,Poncelet,Sotnikov,Tancredi,von Manteuffel,Zeng(2015-2021)]

Full colour $\Rightarrow$ $5g$ all-plus, $2q1g2\gamma$, $3g2\gamma$

[Badger,Chicherin,Gehrmann,Heinrich,Henn,Peraro,Wasser,Zhang,Zoia(2019)] [Agarwal,Buccioni,Tancredi,von Manteuffel(2021)]
[Badger,Bronnum-Hansen,Chicherin,Gehrmann,HBH,Henn,Marcoli,Moodie,Peraro,Zoia(2021)]

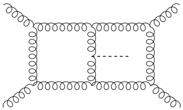
- NNLO QCD calculations for $2 \to 3$ processes

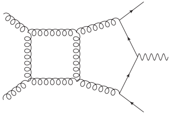
$pp \to \gamma\gamma\gamma$ [Chawdhry,Czakon,Mitov,Poncelet(2019)][Kallweit,Sotnikov,Wiesemann(2020)]

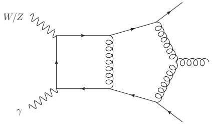
$pp \to \gamma\gamma j$ [Chawdhry,Czakon,Mitov,Poncelet(2021)] $pp \to jjj$ [Czakon,Mitov,Poncelet(2021)]

Scattering with an off-shell leg

$pp \to H + 2j$ | $pp \to W/Z + 2j$ | $pp \to W/Z + \gamma j$







- rich potential phenomenology
- massless internal particles, focus on QCD corrections
- high algebraic and analytic complexity
  $\Rightarrow$ six independent variables
  $\Rightarrow$ 3 square roots
- two-loop planar master integrals are available (using differential equations method)
  [Abreu,Ita,Moriello,Page,Tschernow,Zeng(2020)]: numerical solution using generalised series expansion [Moriello(2019)]
  [Canko,Papadopoulos,Syrrakos(2020)][Syrrakos(2020)]: analytic solution in terms of GPLs

Leading colour $Wb\bar{b}$ amplitude

$$\bar{d}(p_1) + u(p_2) \to b(p_3) + \bar{b}(p_4) + W^+(p_5)$$

- colour decomposition at leading colour $\to$ only planar contribution

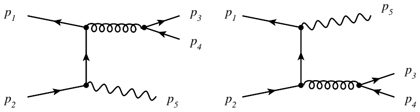
$$\mathcal{A}^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W) \sim g_s^6 g_W \, N_c^2 \, \delta_{i_1}^{\;\bar{i}_4} \delta_{i_3}^{\;\bar{i}_2} \, A^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W)$$

- massless b quarks, $p_3^2 = p_4^2 = 0$
- onshell W boson

$$p_5^2 = m_W^2, \qquad \sum_\lambda \varepsilon_W^{\mu *}(p_5, \lambda) \varepsilon_W^{\nu}(p_5, \lambda) = -g^{\mu\nu} + \frac{p_5^\mu p_5^\nu}{m_W^2}$$

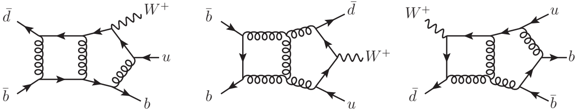
Invariants:

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 - p_3)^2, \quad s_{34} = (p_3 + p_4)^2,$$
$$s_{45} = (p_4 + p_5)^2, \quad s_{15} = (p_1 - p_5)^2, \quad s_5 = p_5^2,$$
$$\mathrm{tr}_5 = 4 i \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma .$$



Integrand construction

Feynman diagrams generated using QGRAF [Nogueira(1993)]



MATHEMATICA+FORM to process the numerator topologies and interfere with tree level

$$M^{(2)} = \sum_{\text{spin}} A^{(0)*} A^{(2)} = M^{(2)}_{\text{even}} + \text{tr}_5\, M^{(2)}_{\text{odd}}$$

Numerators containing: $\text{tr}(\cdots)$ and $\text{tr}(\cdots\gamma_5\cdots\gamma_5\cdots)$ $\Rightarrow$ anti-commuting γ_5 prescription

$\text{tr}(\cdots\gamma_5\cdots)$ $\Rightarrow$ Larin's prescription [Larin(1993)]

Amplitudes in terms of scalar integrals

$$M^{(2)}_k(\{p\}) = \sum_i c_{k,i}(\epsilon, \{p\})\, \mathcal{I}_{k,i}(\epsilon, \{p\}), \qquad k \in \{\text{even}, \text{odd}\}$$

Reconstructing the finite remainders

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i c_{k,i}(\{p\},\epsilon)\,\mathcal{I}_{k,i}(\{p\},\epsilon)$$

$\downarrow$ IBP reduction

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i d_{k,i}(\{p\},\epsilon)\,\mathrm{MI}_{k,i}(\{p\},\epsilon)$$

$\downarrow$ map to special function basis

$\downarrow$ subtract UV/IR poles

$\downarrow$ ϵ expansion

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\})\; m_{k,i}(f) + \mathcal{O}(\epsilon)$$

$$k \in \{\text{even}, \text{odd}\}$$

FiniteFlow[Peraro(2019)] LiteRed[Lee(2012)]

- construct a basis of special function using master integral components

$$\mathrm{MI}_i(s) = \sum_{w\geq 0} \epsilon^w \mathrm{MI}_i^{(w)}(s)$$

- Finite remainders

$$F_k^{(2)} = M_k^{(2)} - \sum_{j=1}^{2} I^{(j)} M_k^{(2-j)}$$

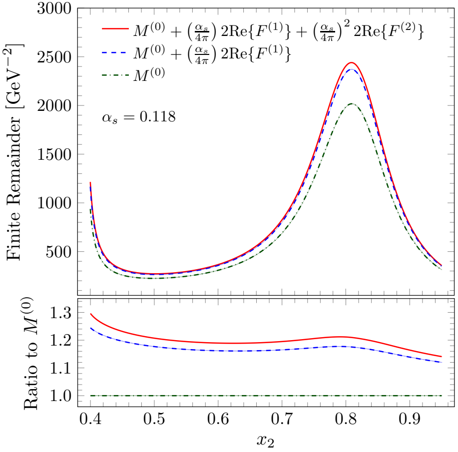
- Numerically compute $e_{k,i}$ over finite fields
- Reconstruct analytic expressions of $e_{k,i}$ from several numerical evaluations [Peraro(2016)]
- Define a new basis $\Rightarrow$ only functions that appear in the finite remainder
- Evaluate using generalised series expansion method implemented in DiffExp [Hidding(2020)]

Numerical evaluation

Evaluation on a univariate slice of the physical phase space

$$
\begin{aligned}
p_1 &= \frac{\sqrt{s}}{2}(1,0,0,1),\\
p_2 &= \frac{\sqrt{s}}{2}(1,0,0,-1),\\
p_3 &= \frac{x_1\sqrt{s}}{2}\left(1,1,0,0\right),\\
p_4 &= \frac{x_2\sqrt{s}}{2}\left(1,\cos\theta,-\sin\phi\sin\theta,-\cos\phi\sin\theta\right),\\
p_5 &= \sqrt{s}\,(1,0,0,0)-p_3-p_4
\end{aligned}
$$

$$s=1,\; m_W^2=0.1,\; \phi=0.1,\; x_1=0.6$$



- 1100 points → average 260 s/point
- Reasonable evaluation time with basic DiffExp setup
- further optimisation is possible
  [Abreu,etal(2020)][Becchetti,etal(2020)]

Summary

- ✓ First analytic result for 2-loop 5-point amplitude with one massive leg
  $\Rightarrow$ leading colour $u\bar{d} \to W^+ b\bar{b}$
- ✓ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops
- ✗ Include W-boson decay
- ✗ Application to other processes
- ✗ Full colour (need non-planar integrals)

Summary

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THANK YOU!!!

Back-up Slides

Reconstructing the finite remainders

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\})\, m_{k,i}(f) + \mathcal{O}(\epsilon), \qquad k \in \{\text{even}, \text{odd}\}$$

- set $s_{12} = 1$
- Not all $e_{k,i}$ coefficients independent
  $\Rightarrow$ find linear relations between coefficients and reconstruct the simpler ones

$$\sum_i y_i e_i = 0, \qquad y_i \in \mathbb{Q}$$

$\Rightarrow$ allow to supply known/candidate coefficients $\tilde{e}_j$

$$\sum_i y_i e_i + \sum_j \tilde{y}_j \tilde{e}_j = 0, \qquad y_i, \tilde{y}_j \in \mathbb{Q}$$

- guess the denominator $\rightarrow$ from letters [Abreu,etal(2019)][Abreu,etal(2020)]
- partial fraction in one variable (s_{23}) and reconstruct in the remaining variables ($s_{34}, s_{45}, s_{15}, s_5$)

$\Rightarrow \sim 4$ times speed up $\qquad \Rightarrow$ 2 prime fields needed

- Reconstructed analytic expressions are simplified using MultivariateApart[Heller,von Manteuffel(2021)]

Numerical evaluation

- Only 19 linear combinations of $f_i^{(4)}$ appear in the two-loop finite remainder

  $\Rightarrow$ define a new basis $g_i^{(w)}$

$$\left\{ f_i^{(w)}(s) \right\} \Longrightarrow \left\{ g_i^{(w)}(s) \right\}$$

- Apply generalised series expansion method directly to the $g_i^{(w)}$ basis

$$\vec{g} = \begin{pmatrix} \epsilon^4 g_i^{(4)} \\ \epsilon^3 g_i^{(3)} \\ \epsilon^2 g_i^{(2)} \\ \epsilon g_i^{(1)} \\ 1 \end{pmatrix}$$

$$d\vec{g} = \epsilon d\tilde{B} \cdot \vec{g}$$

- Much simpler than the DEs for the master integrals
- Use generalised series expansion approach [Moriello(2019)] as implemented in DiffExp [Hidding(2020)]