EPS 2021

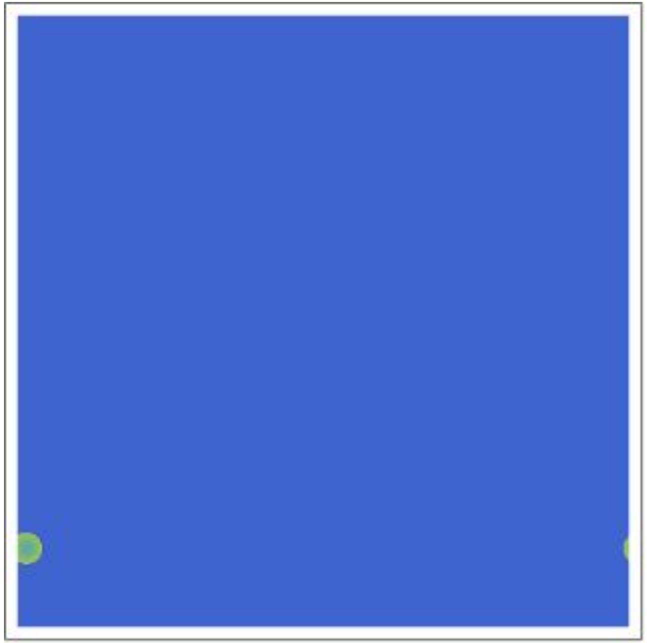
# A hybrid simulation of gravitational wave production in first-order phase transitions

Ryusuke Jinno, Thomas Konstandin and
**Henrique Rubira**

henrique.rubira@desy.de

(2010.00971 JCAP 04 (2021) 014)

# Before anything, the most important... Cool stuff always come with videos



# The message to take home...

New simulation scheme for
sound-shell contribution in 1st order
PT

# The message to take home...

New simulation scheme for sound-shell contribution in 1st order PT

Advantages:

1) fast (easier to explore parameter dependence);

2) Don't need to include the scalar field (solve particle physics scale in a cosmo simulation);

3) Incorporate shock front easily.

# The message to take home...

New simulation scheme for sound-shell contribution in 1st order PT

Advantages:

1) fast (easier to explore parameter dependence);

2) Don't need to include the scalar field (solve particle physics scale in a cosmo simulation);

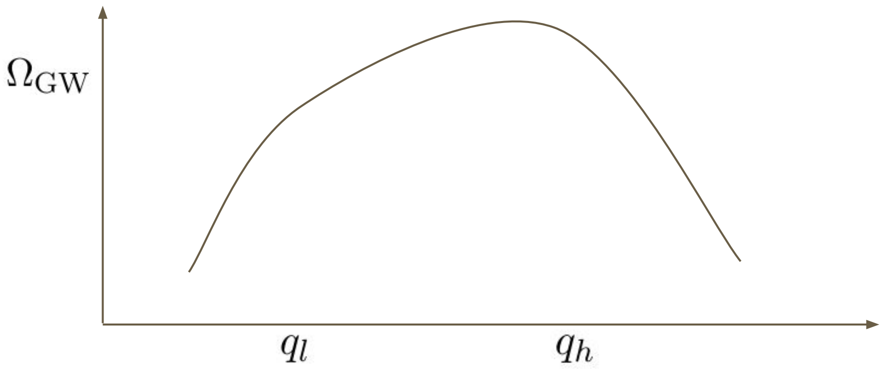
3) Incorporate shock front easily.

A user-friendly parametrization...

$$\Omega_{\rm GW} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

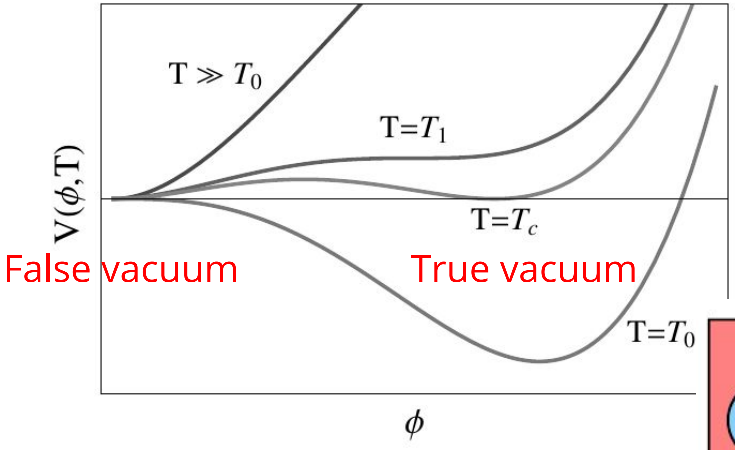
$$q_l \simeq 1, \qquad q_h \simeq 1/\xi_{\rm shell},$$

$$n_l \in [2,4]\,, \quad n_m \in [-1,0]\,, \quad n_h \in [-4,-3]\,,$$



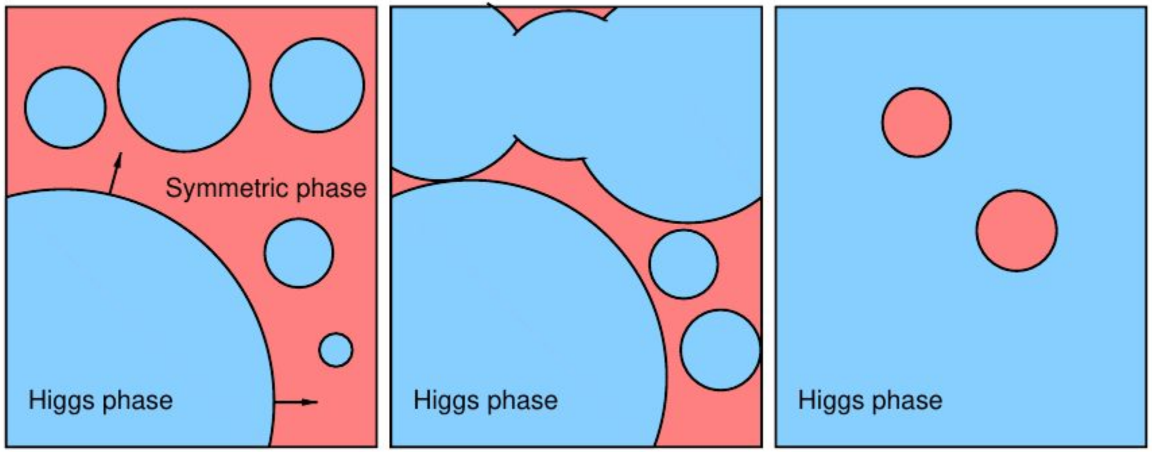
Henrique Rubira

# 1st Order Phase Transition (PT)




$$\Gamma(t) = \Gamma_* e^{\beta(t-t_*)}$$

Disjoint regions of space can make the transition, generating bubbles at different places

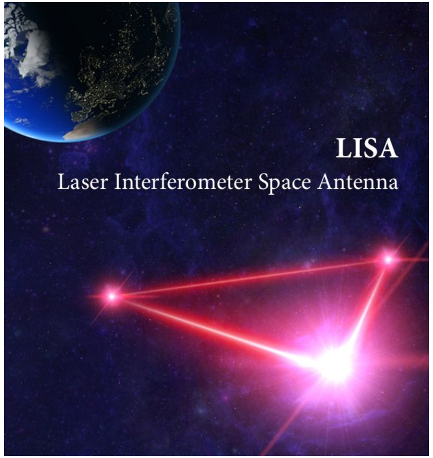



For a review, see Maggiore's book or (2008.09136)

Henrique Rubira

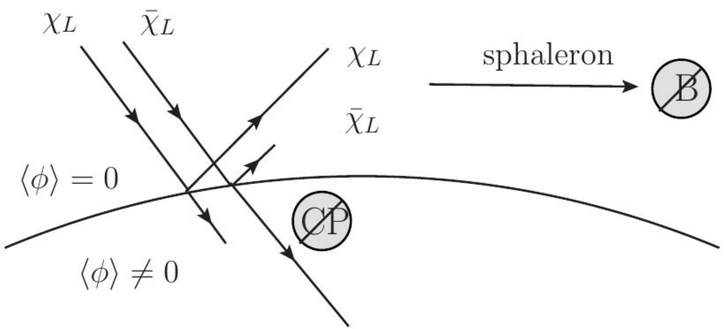
# Motivations for 1st Order PT

## 1) LISA is flying in next decade



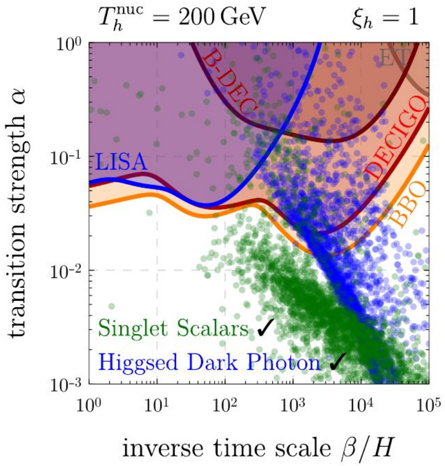

(1702.00786)

## 2) Electroweak Baryogenesis




(1302.6713)

## 3) BSM physics

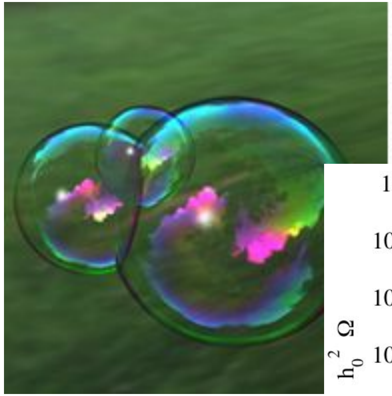



(1811.11175)

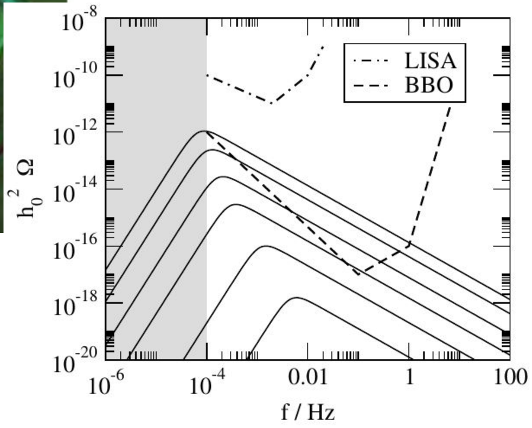
# GW from 1st Order PT -- State of the art

# GW from 1st Order PT -- State of the art

**Envelope approximation**



Konstandin, Huber (08)

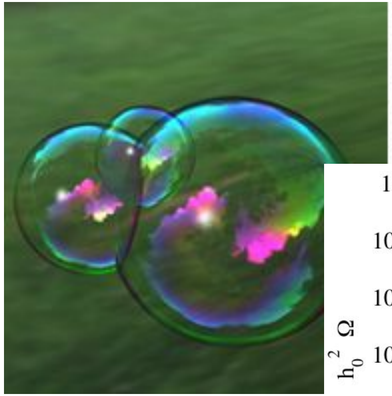



Energy contained in a thin non-collided yet shell (fluid or scalar)

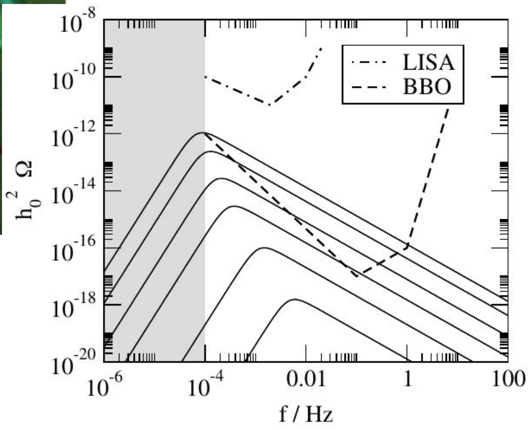
See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

# GW from 1st Order PT -- State of the art

**Envelope approximation**



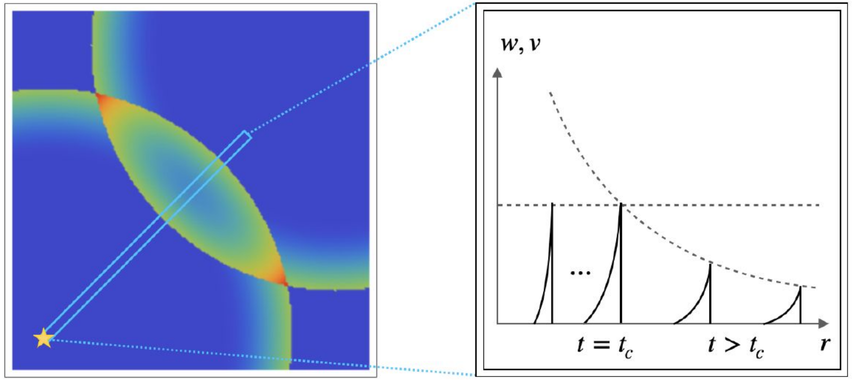
Konstandin, Huber (08)




Energy contained in a thin non-collided yet shell (fluid or scalar)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

Latter, it became clear that the **sound shell** contribution is larger than the scalar

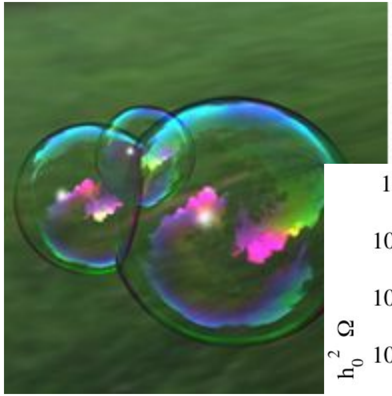



Enhanced by $$\left(\frac{\beta}{H_*}\right)$$ Nucleation rate O(100)

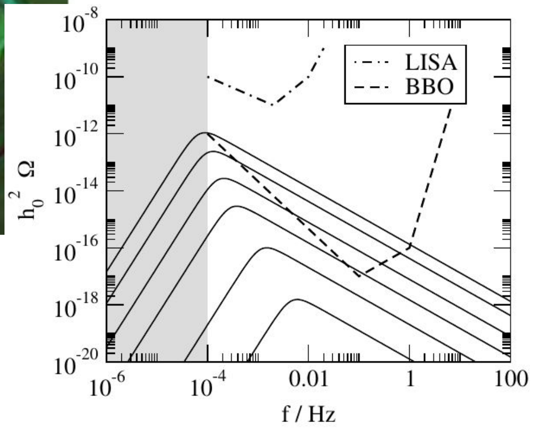
Henrique Rubira

# GW from 1st Order PT -- State of the art

## Envelope approximation



Konstandin, Huber (08)

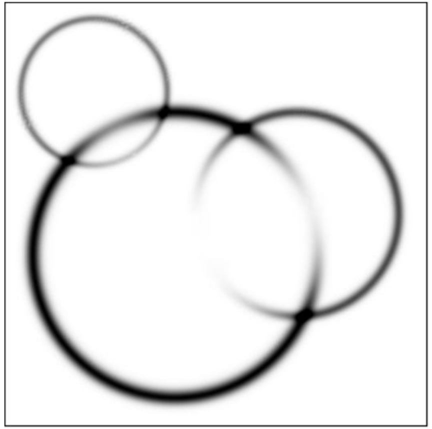



Energy contained in a thin non-collided yet shell (fluid or scalar)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

## Bulk flow

Konstandin (17) and Jinno, Takimoto (17b)



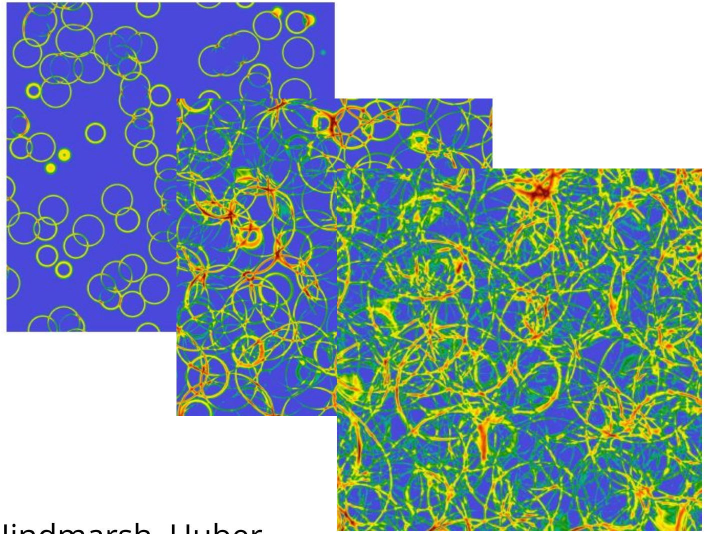
## Sound shell model

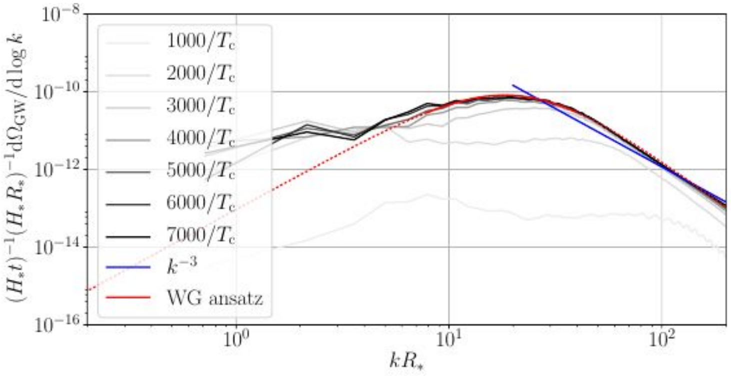
Hindmarsh (16)

$$\frac{d\Omega_{\rm GW}(k)}{d\ln(k)} \sim \begin{cases} (kR_*)^5, & k\Delta R_*, kR_* \ll 1, \\ (kR_*)^1, & k\Delta R_* \ll 1 \ll kR_*, \\ (kR_*)^{-3}, & 1 \ll k\Delta R_*, R_*. \end{cases}$$

# GW from 1st Order PT -- State of the art

**Lattice simulations**






Hindmarsh, Huber, Rummukainen, Weir (13,15,17)

Scalar Field (HEP scale) + bubble size (cosmo scale)

Huge hierarchy between those scales

# Our set up in a nutshell

Motivation: construct a simulation that doesn't need to solve the Higgs

Important: Higgs is only (indirectly) as a boundary condition

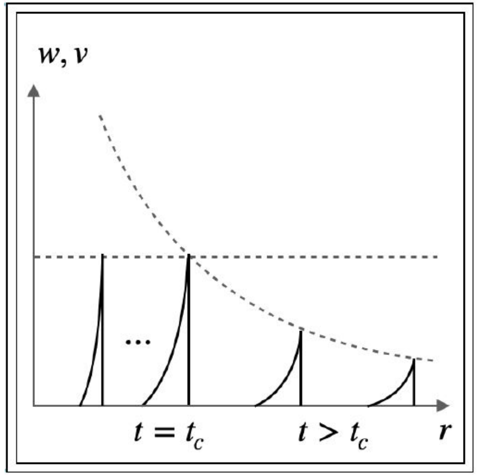
Henrique Rubira

# Our set up in a nutshell

Plasma velocity and enthalpy

Simulate (1d + spherical sym) how the velocity and enthalpy evolve

1d simulation



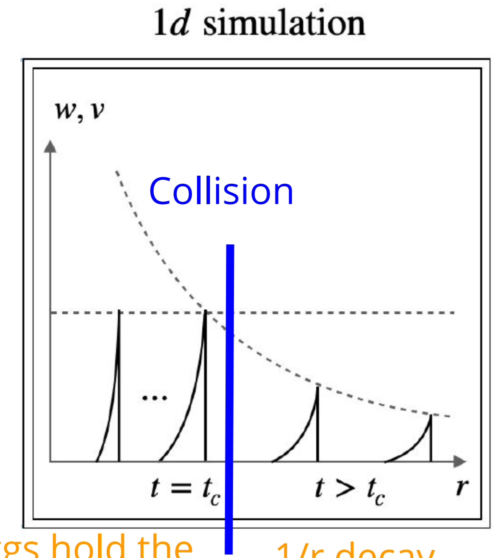

Motivation: construct a simulation that doesn't need to solve the Higgs

Important: Higgs is only (indirectly) as a boundary condition

# Our set up in a nutshell

Plasma velocity and enthalpy

Simulate (1d + spherical sym) how the velocity and enthalpy evolves



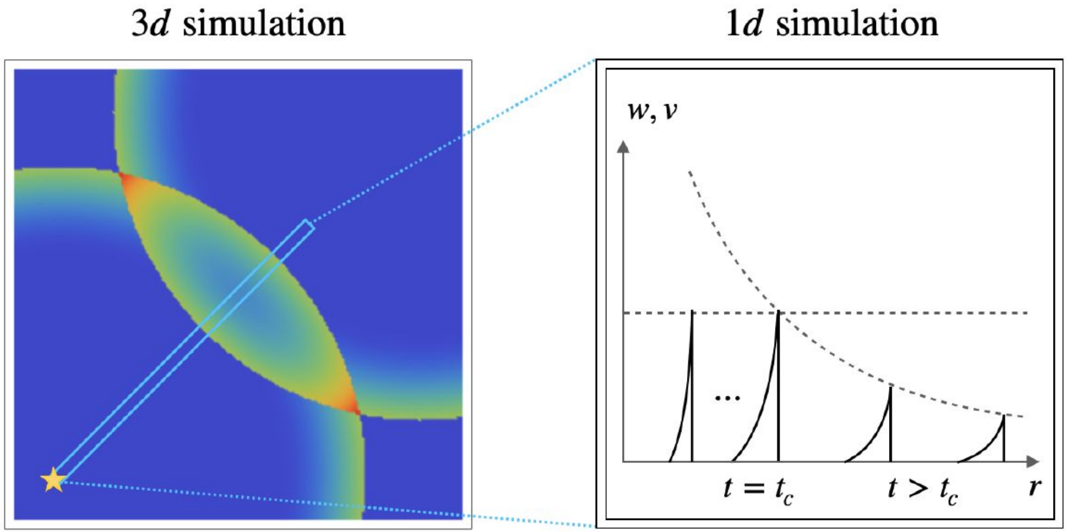

Higgs hold the bubble wall

1/r decay

Motivation: construct a simulation that doesn't need to solve the Higgs

Important: Higgs is only (indirectly) as a boundary condition

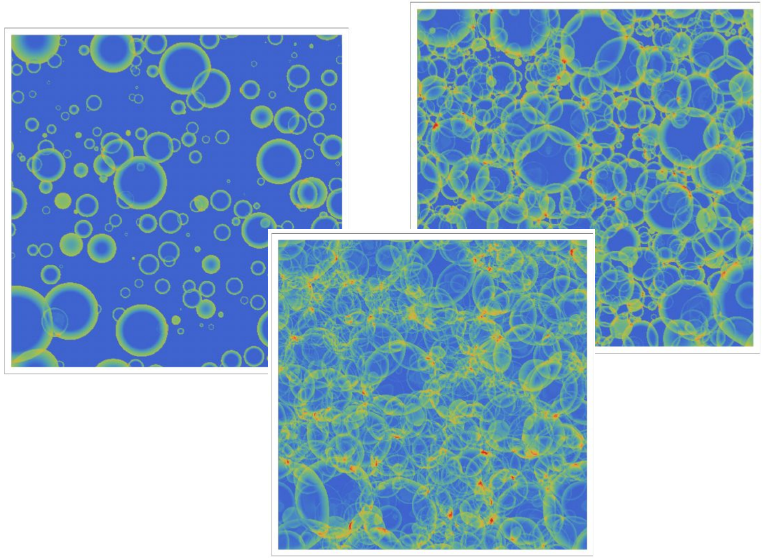
# Our set up in a nutshell

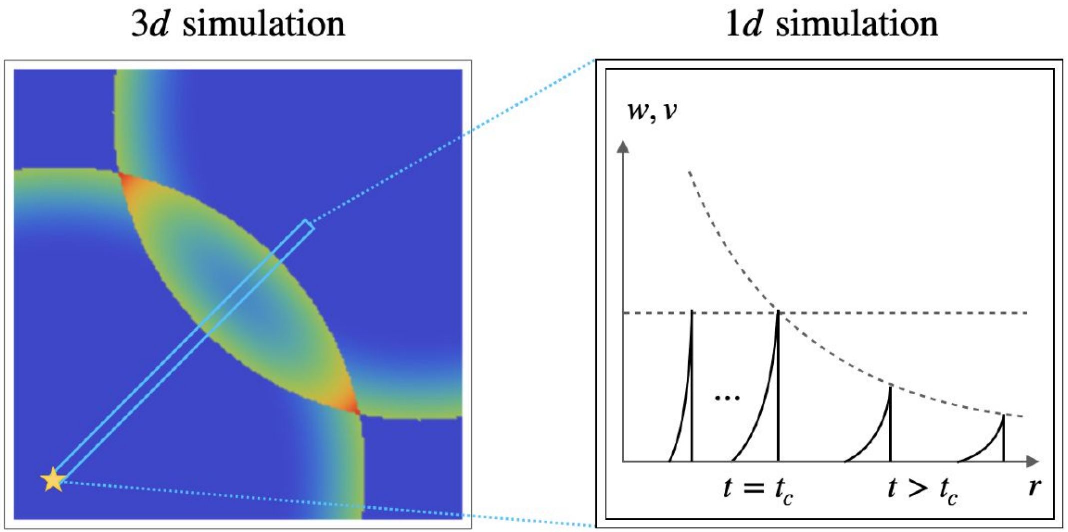



Embed a 1d hydro simulation (fast to run) into a 3d lattice

Henrique Rubira

# Our set up in a nutshell

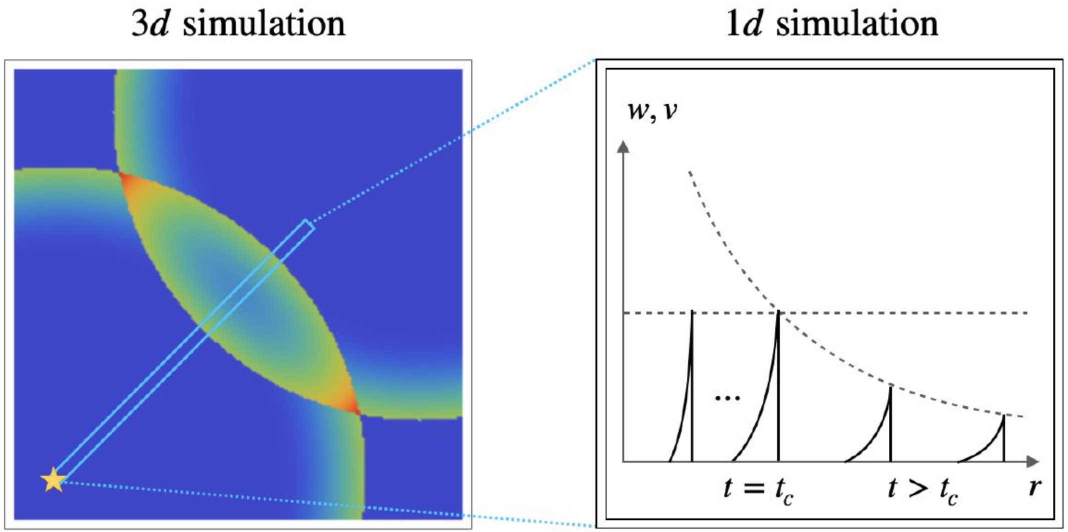


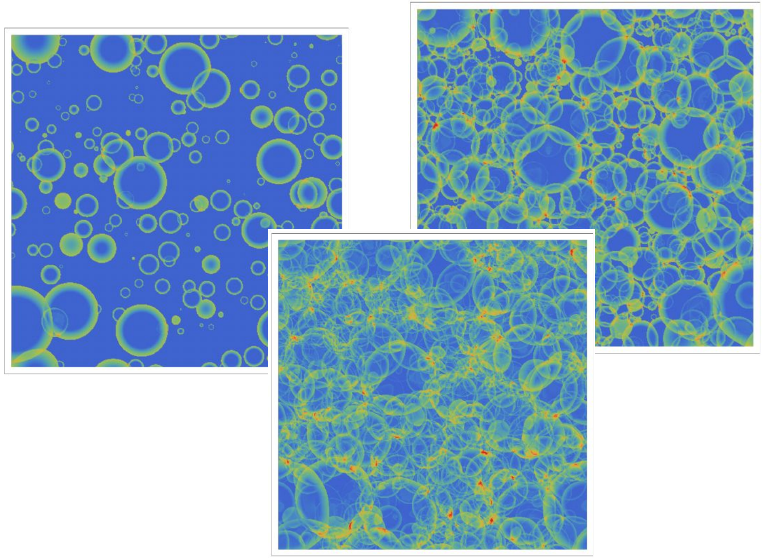



Embed a 1d hydro simulation (fast to run) into a 3d lattice

Henrique Rubira

# Our set up in a nutshell

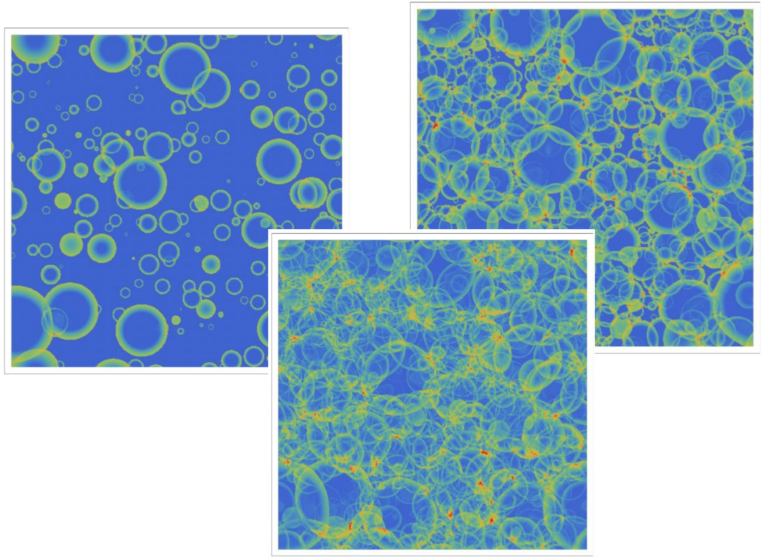


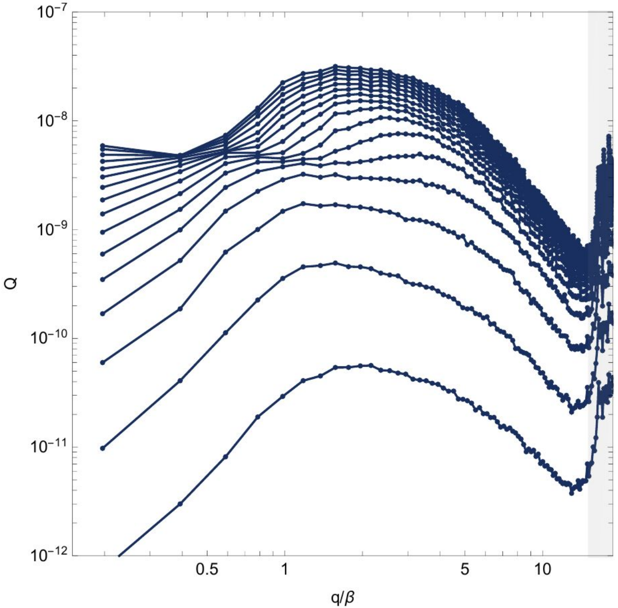



Embed a 1d hydro simulation (fast to run) into a 3d lattice

Other advantages:
More bubbles -- O(2500)
Realistic nucleation (not simultaneous)

# Our set up in a nutshell

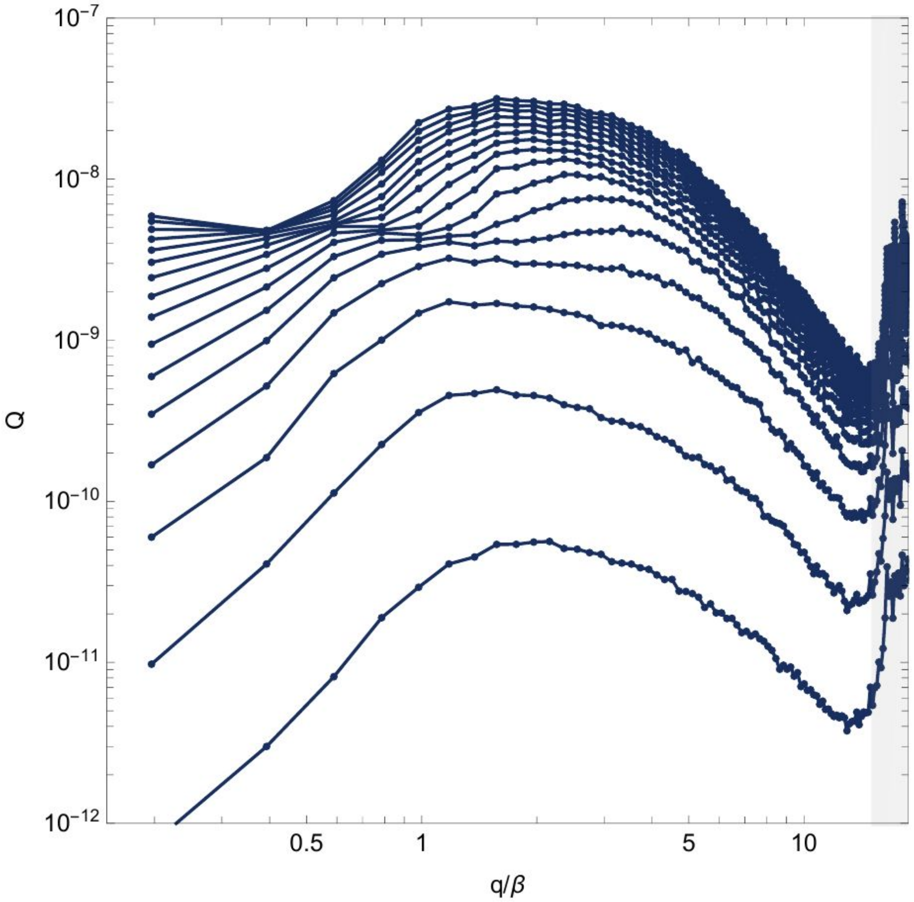





Calculate the GW at each time step in the 3d lattice

# Results

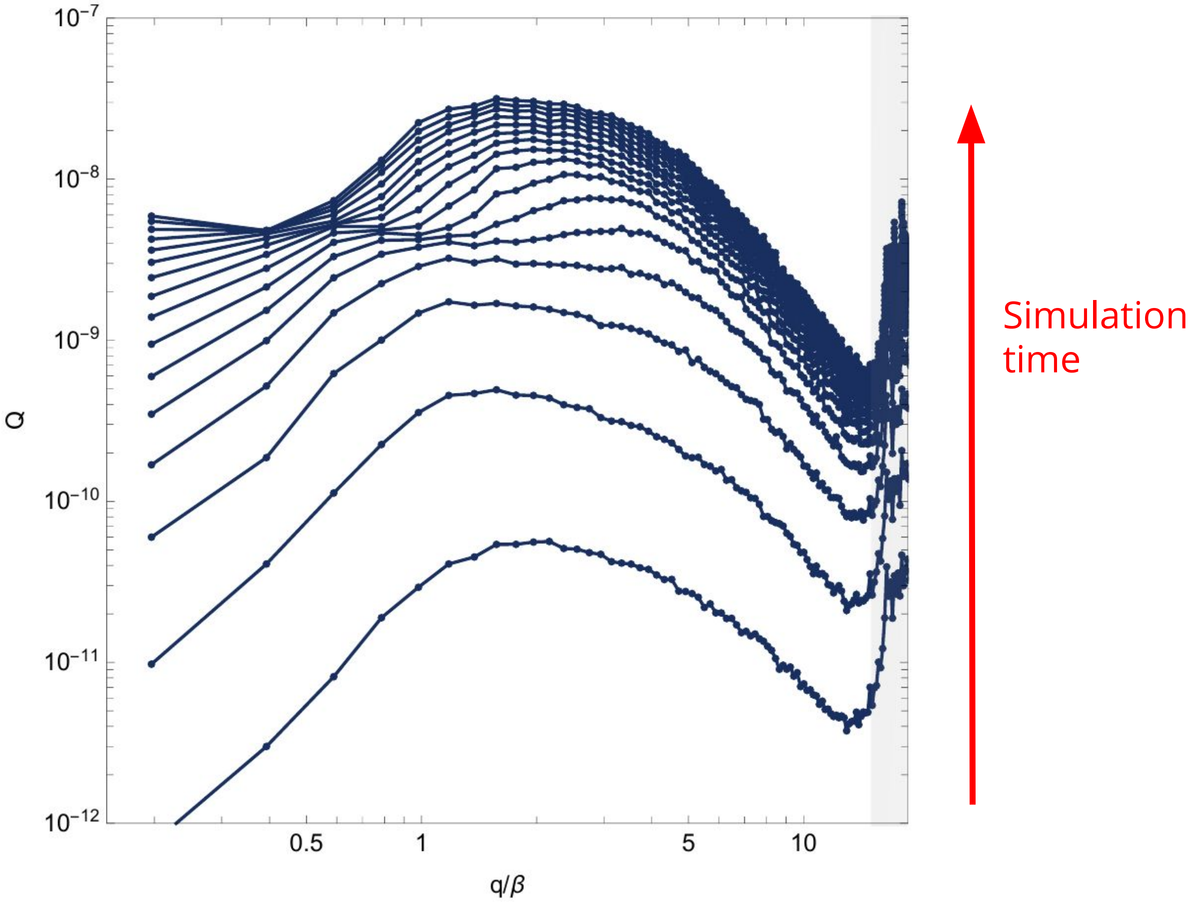
$$\Omega_{\rm GW} = \frac{w^2\,\tau}{4\pi^2 \rho_{\rm tot} M_P^2 \beta} \times Q',$$



Henrique Rubira

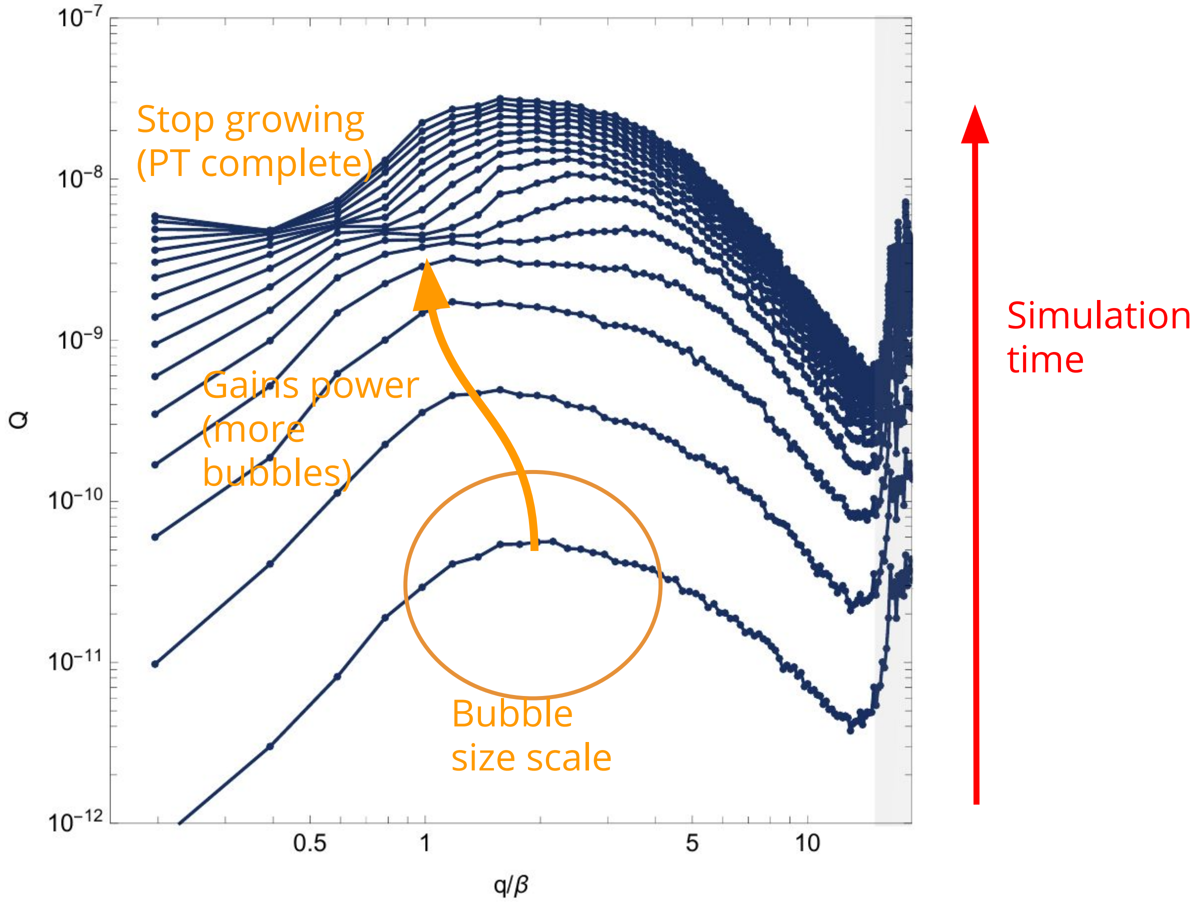
# Results

$$\Omega_{\rm GW} = \frac{w^2\,\tau}{4\pi^2\rho_{\rm tot}M_P^2\beta} \times Q',$$



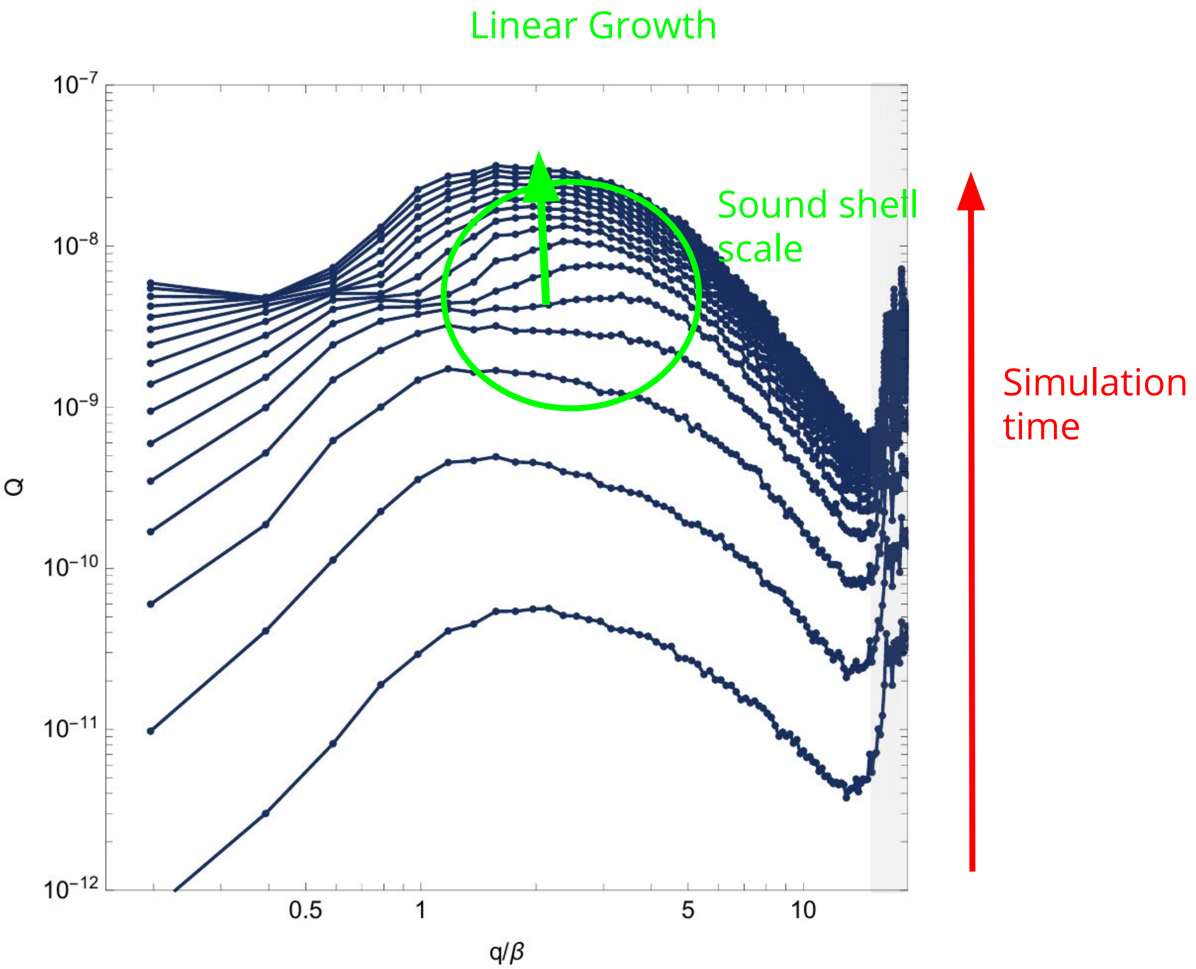
# Results

$$\Omega_{\rm GW} = \frac{w^2\,\tau}{4\pi^2 \rho_{\rm tot} M_P^2 \beta} \times Q',$$



# Results

$$\Omega_{\rm GW} = \frac{w^2\,\tau}{4\pi^2 \rho_{\rm tot} M_P^2 \beta} \times Q',$$



Henrique Rubira

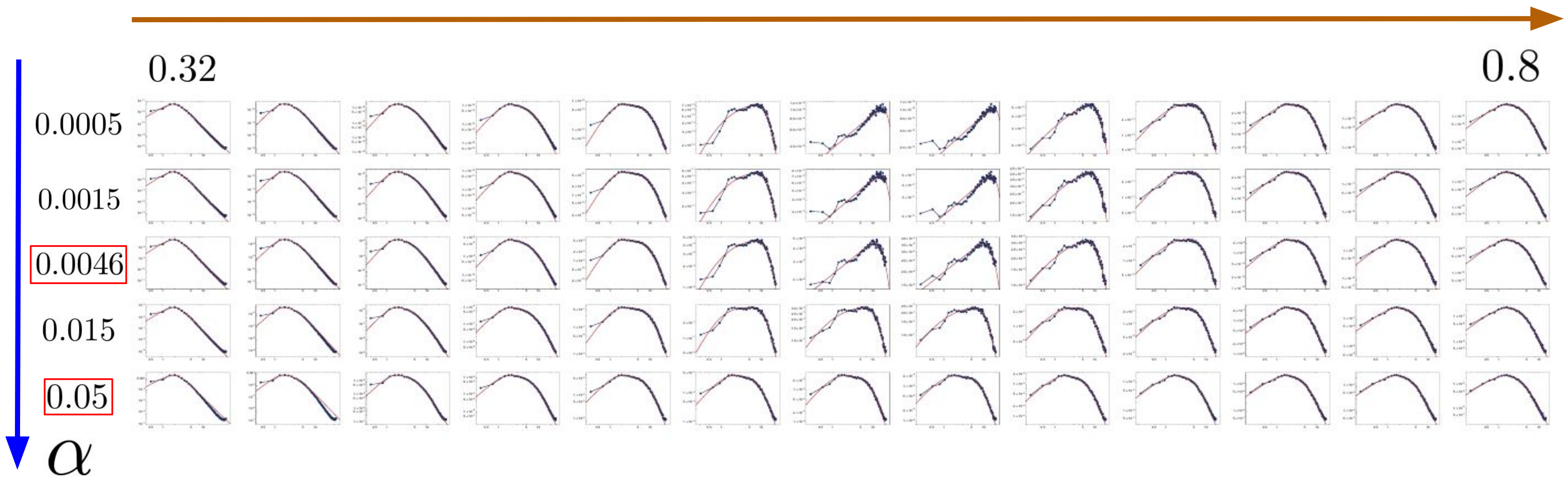
# Results

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

65 simulations (13x5)
+ 26 with larger box and high resolution

$\xi_w$



# Results

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

# Results

**Amplitude** of the spectrum normalized by the kinetic energy measured in the simulation times the sound-shell width

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

Henrique Rubira

# Results

We also show that the amplitude of the spectrum is ( relatively) well parametrized by $\kappa\alpha$

**Amplitude** of the spectrum normalized by the kinetic energy measured in the simulation times the sound-shell width

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

# Results

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

Low freq.

$\boxed{q_l \simeq 1,}$

$\boxed{n_l \in [2,4]\,,}$

Typical bubble size at collision!

$$n_m \in [-1, 0]$$

# Results

A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

Low freq.

$\boxed{q_l \simeq 1,}$

$\boxed{n_l \in [2,4],}$

Typical bubble size at collision!

$n_m \in [-1,0]$

High freq.

$\boxed{q_h \simeq 1/\xi_{\rm shell}}$

$\boxed{n_h \in [-4,-3]}$

peak at sound shell scale

# Comparison to other works

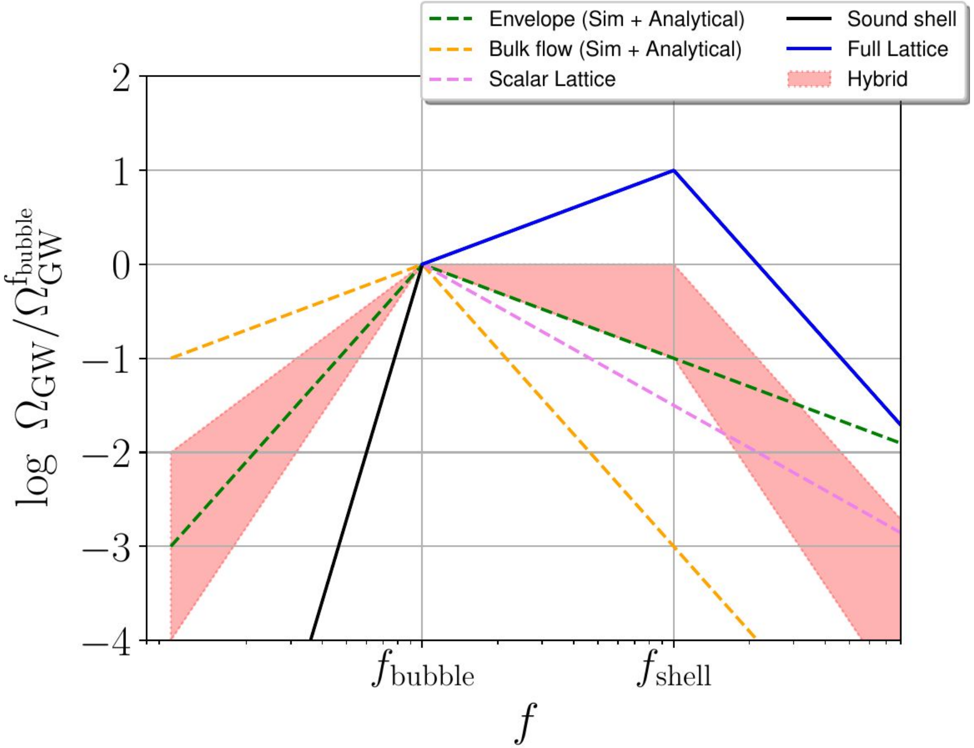
When we compare to simulations with the scalar field, we have found:

- Similar scaling
- IR peak shifted to lower freq. (Realistic nucleation)
- Factor ~ 2 in overall amplitude (More bubbles)

# Comparison to other works

When we compare to simulations with the scalar field, we have found:

- Similar scaling
- IR peak shifted to lower freq. (Realistic nucleation)
- Factor ~ 2 in overall amplitude (More bubbles)



# Conclusions

New simulation scheme (free of scalar field scale) to calculate sound-shell contribution

$$Q'_{\rm int} \simeq 9 \times \xi_{\rm shell} \times \left(\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty\right)^2$$

Sound shell size

Kinetic energy

Parametrized as

$$\Omega_{\rm GW} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$q_l \simeq 1, \qquad q_h \simeq 1/\xi_{\rm shell},$$

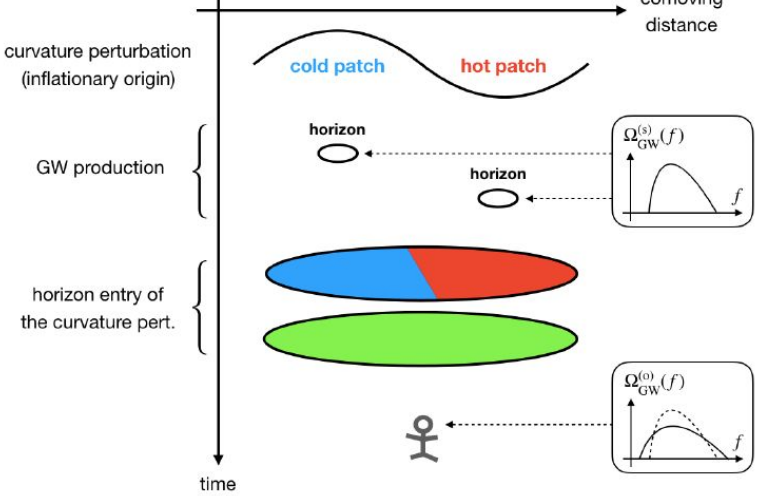
$$n_l \in [2,4]\,, \quad n_m \in [-1,0]\,, \quad n_h \in [-4,-3]\,,$$

Henrique Rubira

# Stay tuned

2002.11083, Domcke, Jinno and **Rubira**

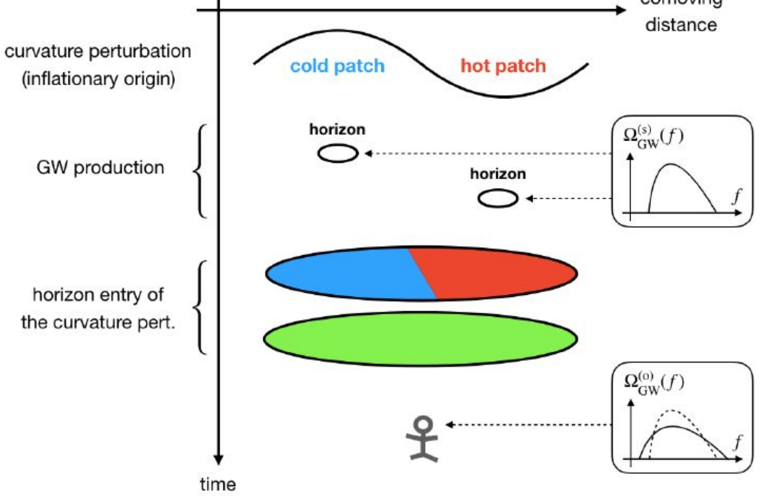
We have shown in 2002.11083 that temperature fluctuations in the line-of-sight can affect the GW spectrum



Henrique Rubira

# Stay tuned

2002.11083, Domcke, Jinno and **Rubira**




We have shown in 2002.11083 that temperature fluctuations in the line-of-sight can affect the GW spectrum

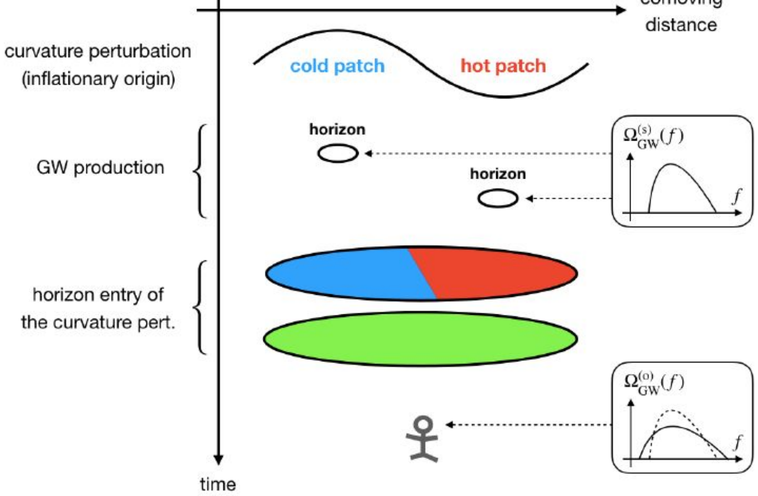
What if the bubbles nucleate on top of a Universe with temperature fluctuations?

Jinno, Konstandin, **Rubira**, van de Vis
(To appear)

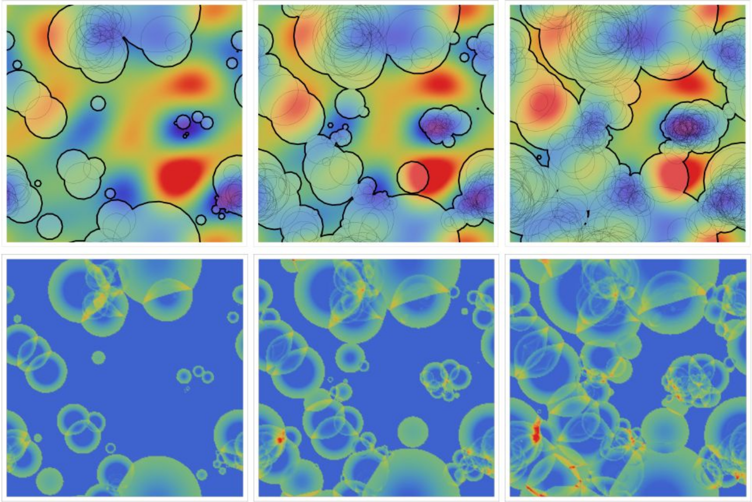
Henrique Rubira

# Stay tuned

2002.11083, Domcke, Jinno and **Rubira**




We have shown in 2002.11083 that temperature fluctuations in the line-of-sight can affect the GW spectrum



What if the bubbles nucleate on top of a Universe with temperature fluctuations?

Jinno, Konstandin, **Rubira**, van de Vis
(To appear)

# Thanks

# Extra slides

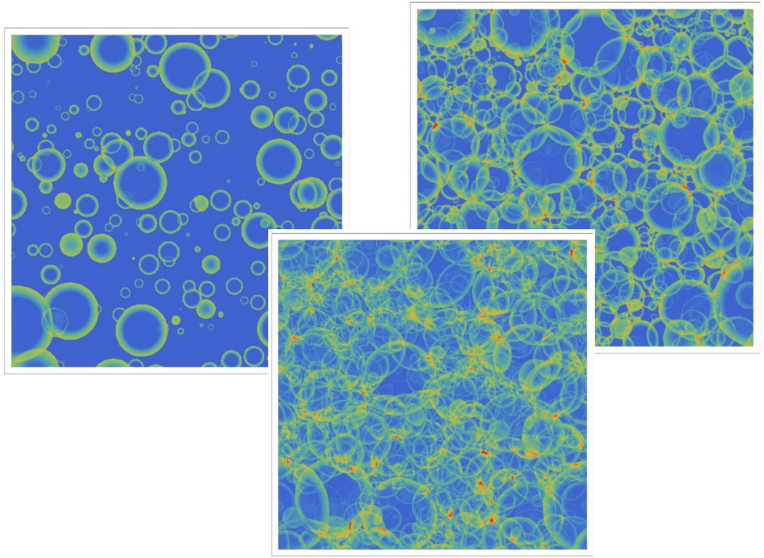
# Our set up

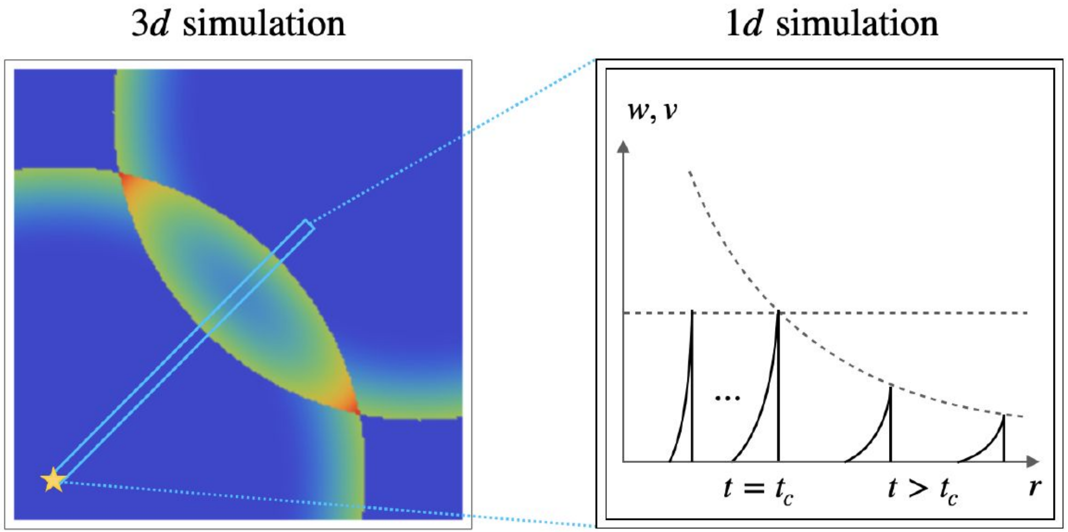
Now slowly and with more details...

A 5 steps calculation

Henrique Rubira

# Our set up






1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

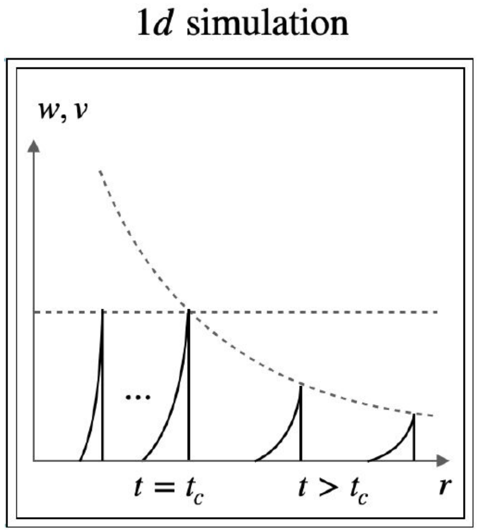
2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

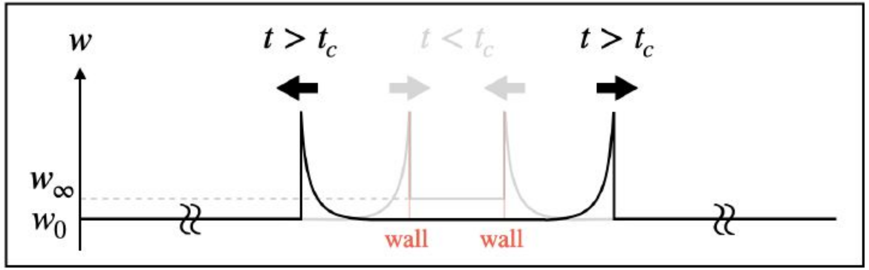
4) Construct a velocity grid embedding the 1d simulation;

5) Calculate GW from stress-energy tensor.

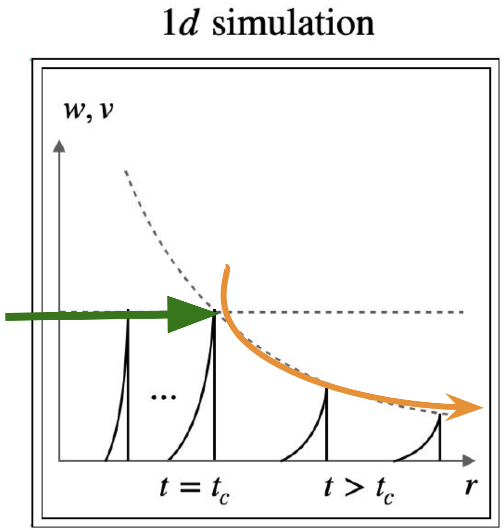
# The 1d simulation




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;



# The 1d simulation




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

Decays as 1/r respecting $\partial_\mu T^{\mu\nu} = 0$ (1905.00899)

Henrique Rubira

# The 1d simulation

Given an alpha (vacuum/plasma)

$$\alpha_+ \equiv \frac{\epsilon}{a_+ T_+^4} \; ;$$

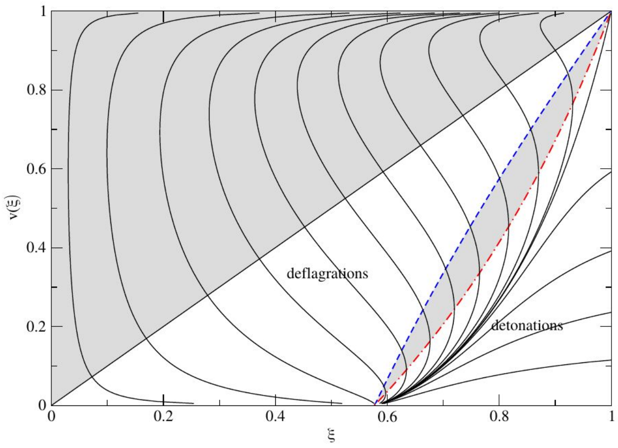
Calculate fluid velocity inside and outside bubble

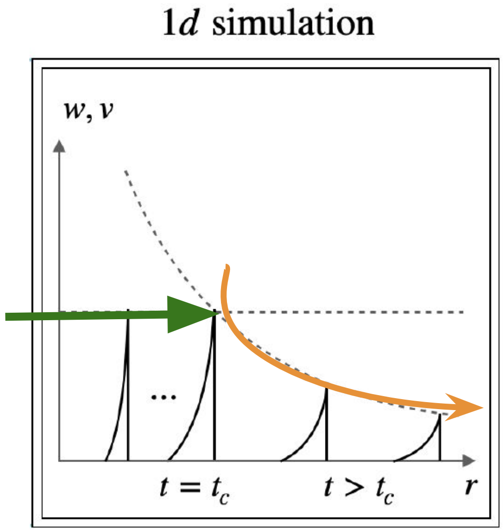
$$v_+ v_-$$

By stress-energy conservation

$$\frac{dv}{d\tau} = 2vc_s^2(1-v^2)(1-\xi v) \; ,$$
$$\frac{d\xi}{d\tau} = \xi[(\xi - v)^2 - c_s^2(1-\xi v)^2] \; ,$$

Any information about microphysics (scalar potential) is encapsulated in alpha




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

**Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)**

Decays as 1/r respecting $\partial_\mu T^{\mu\nu} = 0$ (1905.00899)

Henrique Rubira

# The 1d simulation

$$p_- = \frac{1}{3}a_- T_-^4$$

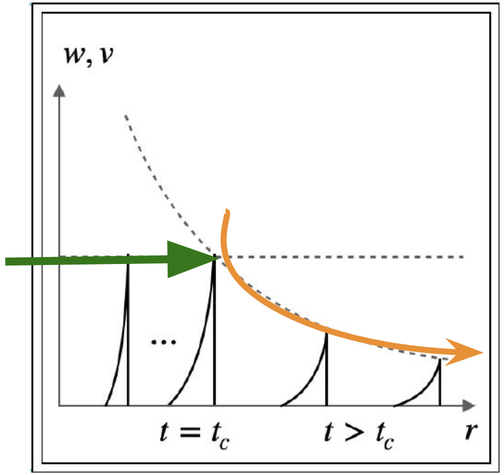
$$e_- = a_- T_-^4$$

$$e_+ = a_+ T_+^4 + \epsilon$$

$$p_+ = \frac{1}{3}a_+ T_+^4 - \epsilon$$

Higgs is incorporated as a boundary condition

1d simulation




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

**Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)**

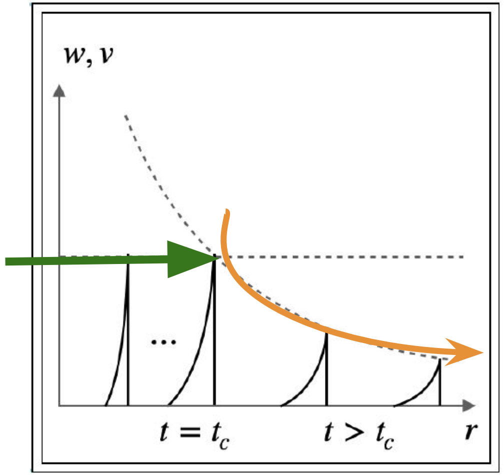
Decays as 1/r respecting $\partial_\mu T^{\mu\nu} = 0$ (1905.00899)

# The 1d simulation

After collision: we remove Higgs energy injection and solve for the fluid system (with spherical sym)

$$\partial_\mu T^{\mu\nu} = 0$$

1d simulation




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

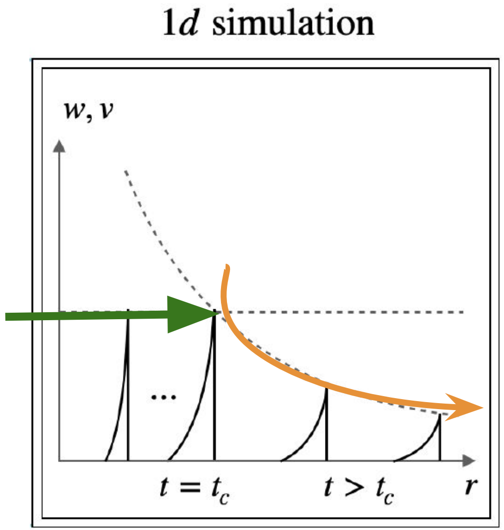
Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

**Decays as 1/r respecting** $\partial_\mu T^{\mu\nu} = 0$ **(1905.00899)**

# The 1d simulation

$$T_{\mu\nu} = w u_\mu u_\nu + p g_{\mu\nu}$$

$$w = \rho + p$$




$$\partial_t \begin{pmatrix} \rho \\ v \end{pmatrix} + A\, \partial_r \begin{pmatrix} \rho \\ v \end{pmatrix} + h = 0,$$

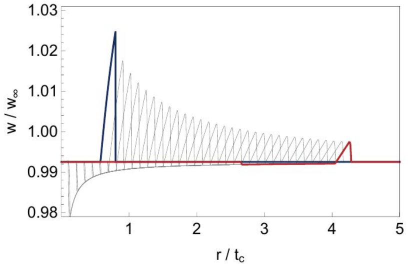
$$A = \frac{1}{1 - c_s^2 v^2} \begin{pmatrix} (1-c_s^2)v & \rho + p \\ \frac{c_s^2(1-v^2)^2}{\rho + p} & (1-c_s^2)v \end{pmatrix} \qquad h = \frac{d-1}{r} \begin{pmatrix} \frac{(\rho+p)v}{1-c_s^2 v^2} \\ -\frac{c_s^2 v^2 (1-v^2)}{1-c_s^2 v^2} \end{pmatrix}$$

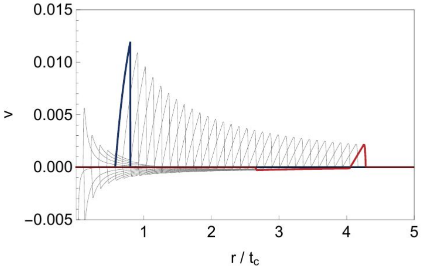
Need to solve shock fronts!
Intricate discretization scheme
called Kurganov-Tadmor

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

**Decays as 1/r respecting** $\partial_\mu T^{\mu\nu} = 0$ **(1905.00899)**

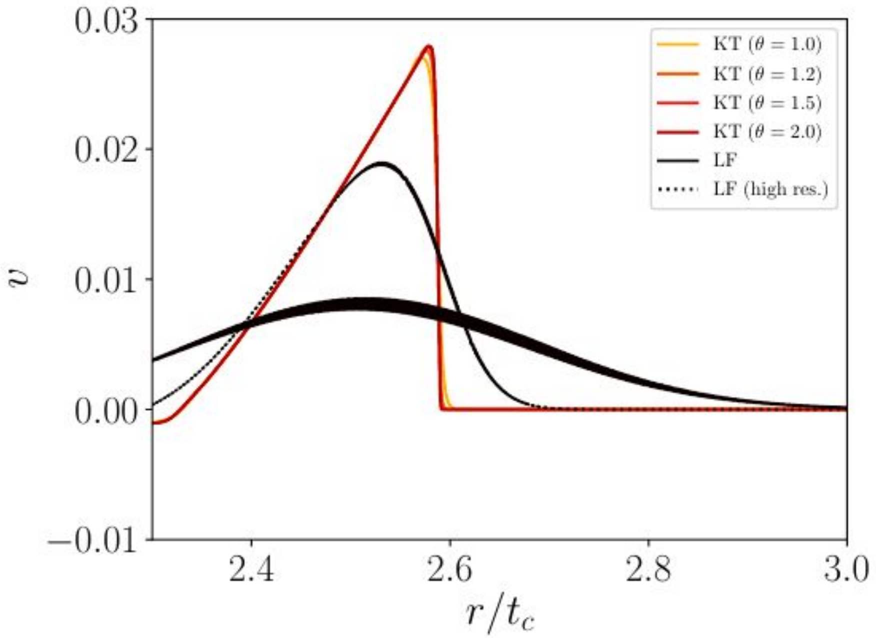


# The 1d simulation

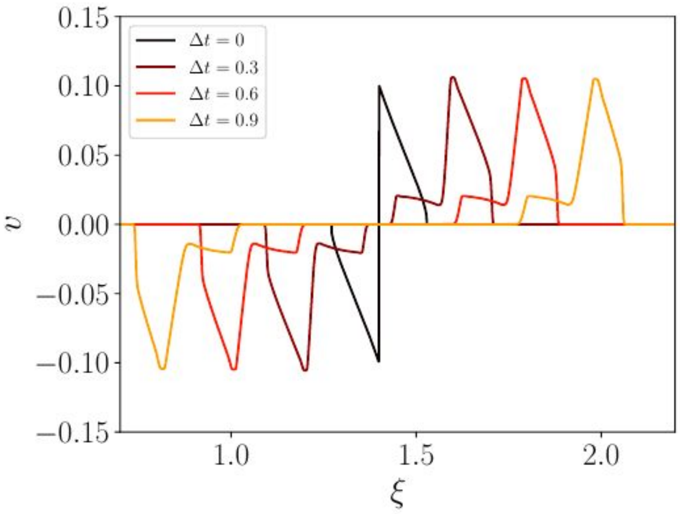
A detail for those that like numerical schemes

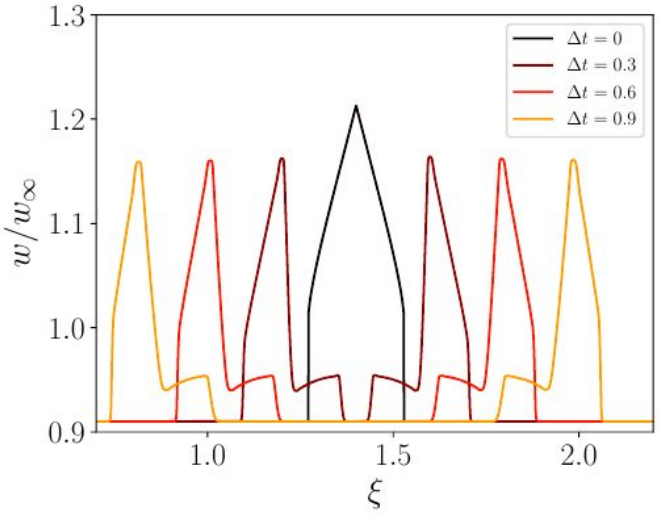
If one try to solve the shocks with a standard numerical schemes, it wont work (see Appendix A)



# The 1d simulation

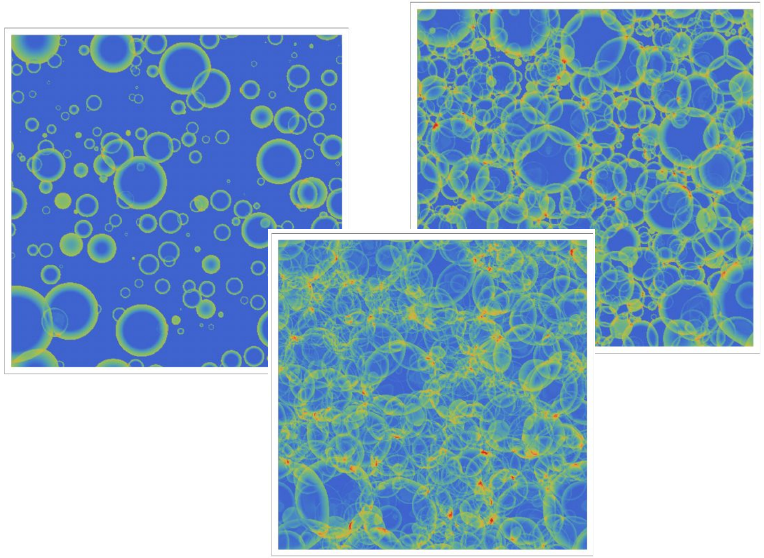
The first collision

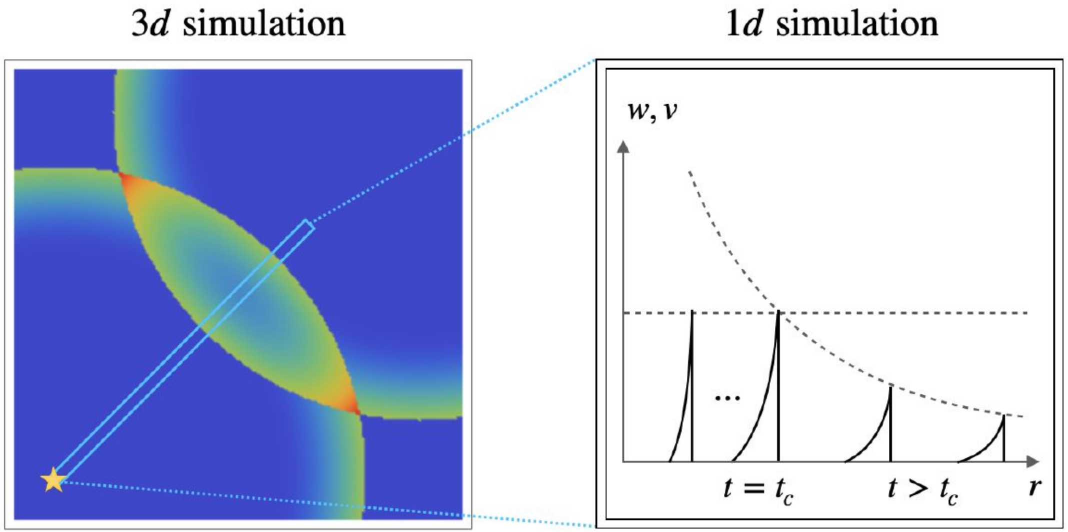


Henrique Rubira

# Our set up






1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

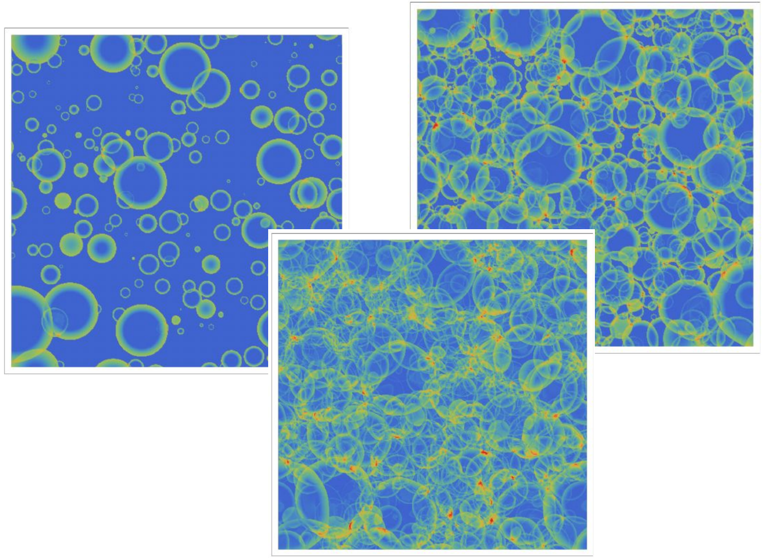
2) Nucleate bubbles and let them grow in a 3d lattice;

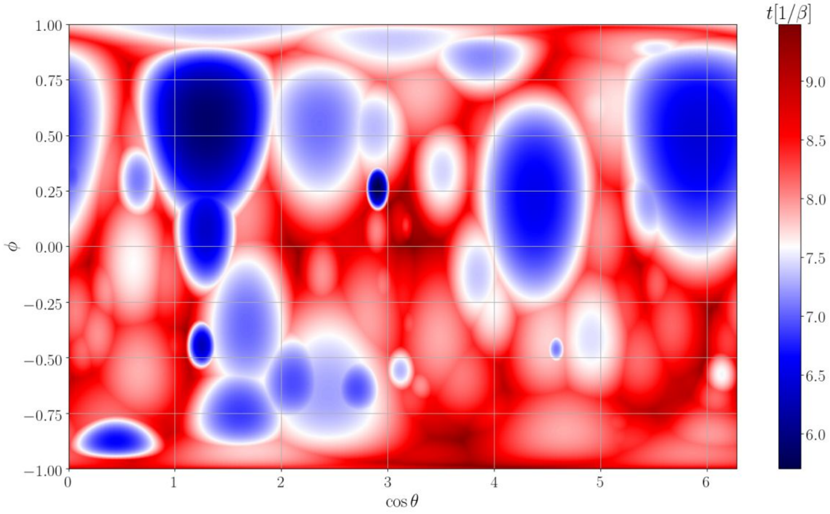
$$\Gamma \propto e^{\beta t}$$

Nucleation rate per volume parameterized by beta

Henrique Rubira

# Our set up



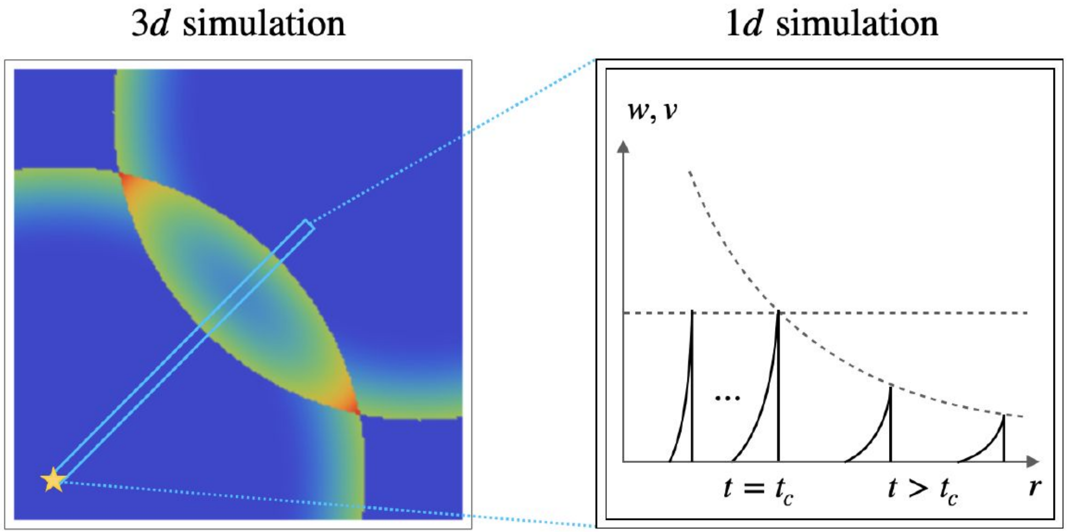



1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

# Our set up




$$\frac{\Delta w}{w_0} \simeq \sum_{i:\text{bubbles}} \frac{\Delta w^{(i)}}{w_0}, \qquad \vec{v} \simeq \sum_{i:\text{bubbles}} \vec{v}^{(i)},$$

$$\frac{\Delta w^{(i)}}{w_0}(t,\vec{x}) \simeq \frac{\Delta w^{(1\text{d})}}{w_0}$$

$$\vec{v}^{(i)}(t,\vec{x}) \simeq \hat{n}^{(i)} v^{(1\text{d})}$$

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

4) Construct a velocity grid embedding the 1d simulation;

Henrique Rubira

# Our set up

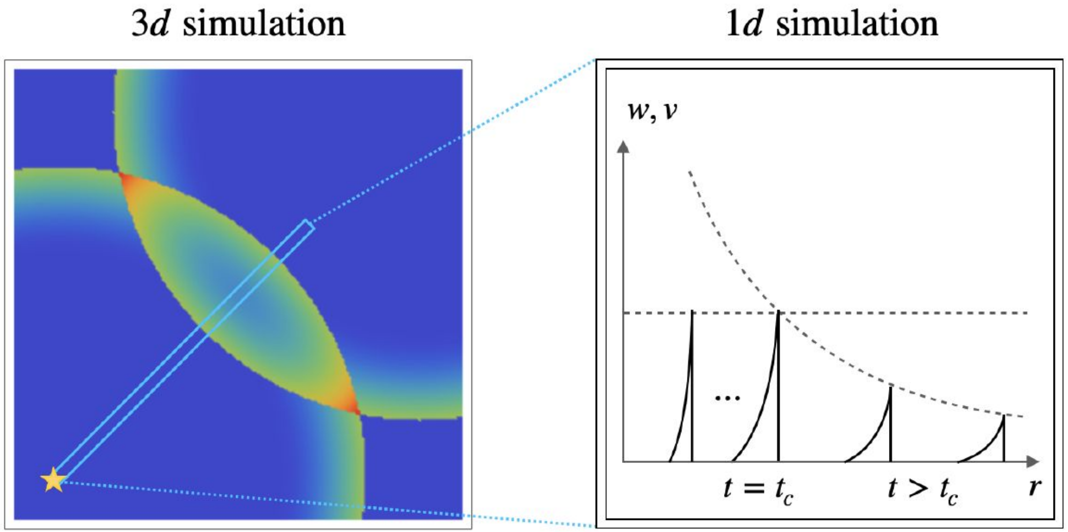
$$T^{ij}(\vec{x}) = v^i(\vec{x})v^j(\vec{x})\rho(\vec{x})$$

$$T_+(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left(\theta_i(\vec{k})\theta_j(\vec{k}) - \phi_i(\vec{k})\phi_j(\vec{k})\right),$$

$$T_\times(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left(\theta_i(\vec{k})\phi_j(\vec{k}) + \theta_i(\vec{k})\phi_j(\vec{k})\right).$$

$$T_{+,\times}(q,\vec{k},t) = \sum_{t'=t_{\rm init}}^{t} e^{iqt'} T_{+,\times}(t',\vec{k}),$$

$$\Omega(q,t) = C\, q^3 \langle T_+ T_+^* + T_\times T_\times^* \rangle|_{|\vec{k}|=q}.$$




1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

4) Construct a velocity grid embedding the 1d simulation;

5) Calculate GW from stress-energy tensor.

# Results detailed

## How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q) \,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2 .$$

Find some quantity that resembles it

1st attempt $\langle w\gamma^2 v^2 \rangle_{3\mathrm{d}}$

# Results detailed

How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

$$Q'_{\mathrm{int}} \equiv \int d\ln q \; Q'(q)\,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2.$$

Find some quantity that resembles it

Henrique Rubira

# Results detailed

How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

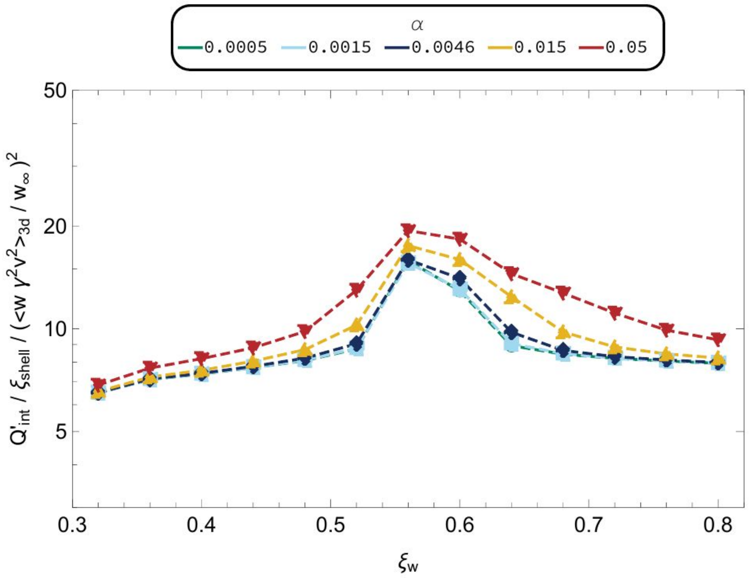
$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q)\,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2.$$

Find some quantity that resembles it

1st attempt $\langle w\gamma^2 v^2\rangle_{\rm 3d}$




Pretty good normalization!

$$Q'_{\rm int} \simeq 9 \times \xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2$$

Problem: not user-friendly (needs 3d simulations)

Henrique Rubira

# Results detailed

1st attempt $\langle w\gamma^2 v^2\rangle_{3\mathrm{d}}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{1\mathrm{d}}$

How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q) \,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2 .$$

Find some quantity that resembles it

Henrique Rubira

1st attempt $\langle w\gamma^2 v^2\rangle_{3\mathrm{d}}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{1\mathrm{d}}$

# Results detailed

How to parametrize the spectrum?

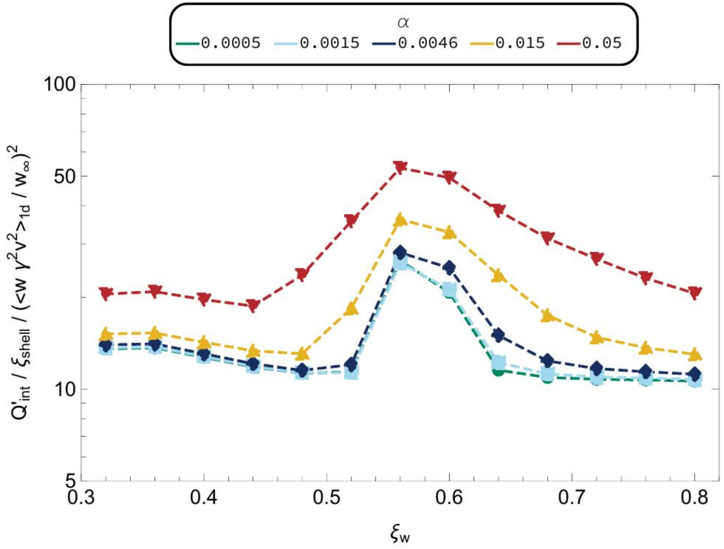
Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q)\,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2$$

Find some quantity that resembles it




good normalization!

$$Q'_{\rm int} \simeq 12 \times \xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{1\mathrm{d}}/w_\infty)^2 .$$

Problem: not user-friendly (needs 1d simulations)

# Results detailed

## How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q)\,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2$$

Find some quantity that resembles it

1st attempt $\langle w\gamma^2 v^2\rangle_{\rm 3d}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{\rm 1d}$

3rd attempt $\kappa\alpha = \dfrac{4}{\xi_w^3 w_\infty}\int d\xi \; w\gamma^2 v^2 \xi^2,$

Very user friendly!

# Results detailed

1st attempt $\langle w\gamma^2 v^2\rangle_{3\mathrm{d}}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{1\mathrm{d}}$

3rd attempt $\kappa\alpha = \frac{4}{\xi_w^3 w_\infty}\int d\xi\; w\gamma^2 v^2 \xi^2,$

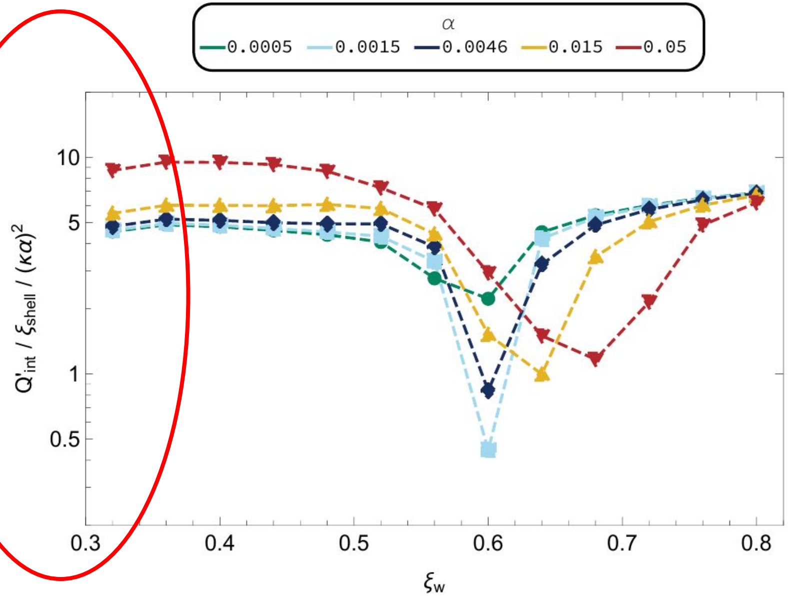
Very user friendly!

## How to parametrize the spectrum?

Good normalization also

Integrated spectrum gives better parameter dependence

$$Q'_{\mathrm{int}} \equiv \int d\ln q\; Q'(q)\,,$$




We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2$$

Find some quantity that resembles it

Henrique Rubira

# Results detailed

## How to parametrize the spectrum?

Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q)\,,$$

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2$$

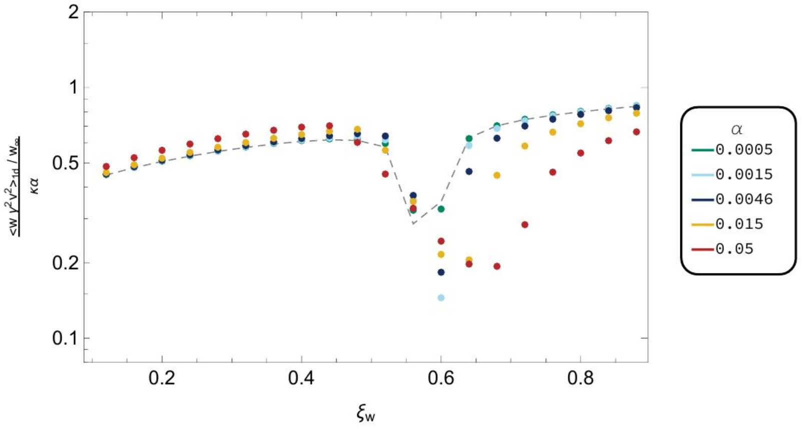
Find some quantity that resembles it

1st attempt $\langle w\gamma^2 v^2\rangle_{\rm 3d}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{\rm 1d}$

3rd attempt $\kappa\alpha = \dfrac{4}{\xi_w^3 w_\infty}\displaystyle\int d\xi \; w\gamma^2 v^2 \xi^2,$

We can also relate $\kappa\alpha$ to $\langle w\gamma^2 v^2\rangle_{\rm 1d}$ pretty easily




$$\boxed{Q'_{\rm int} \simeq 12 \times \xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 1d}/w_\infty)^2.}$$