The classical evolution of binary black hole systems in scalar-tensor theories

arXiv:2011.03547

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Scalar-tensor (Horndeski) gravity theories

Candidate theory: scalar Gauss-Bonnet gravity (sGB gravity)

Numerical relativity results

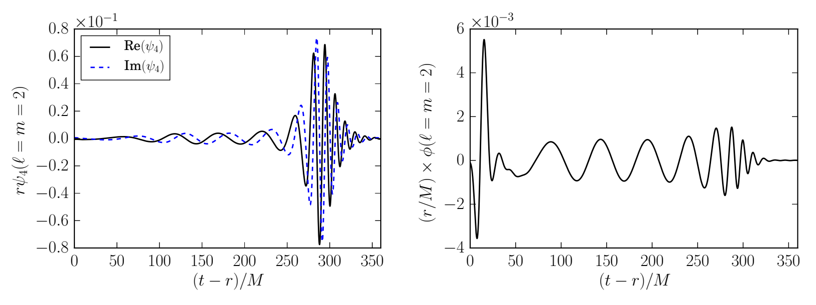
Main technical challenge

Conclusion

Outline and Summary

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left(R + X - V(\phi) + \alpha(\phi)X^2 + \beta(\phi)\mathcal{G}\right),$$

$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \qquad \mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}$$



Goals: understand why we choose to study the above theory, and understand how we made these plots!

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Scalar-tensor (Horndeski) gravity

Theories that have a tensor ($g_{\mu\nu}$) field and scalar (ϕ) field, and have second order equations of motion

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_1 \equiv \frac{1}{2}R + X - V(\phi),$$

$$\mathcal{L}_2 \equiv G_2\left(\phi, X\right),$$

$$\mathcal{L}_3 \equiv G_3\left(\phi, X\right)\Box\phi,$$

$$\mathcal{L}_4 \equiv G_4\left(\phi, X\right)R + \partial_X G_4\left(\phi, X\right)\delta^{\mu\nu}_{\alpha\beta}\nabla^\alpha\nabla_\mu\phi\nabla^\beta\nabla_\nu\phi,$$

$$\mathcal{L}_5 \equiv G_5\left(\phi, X\right)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\partial_X G_5\left(\phi, X\right)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla^\beta\phi\nabla_\rho\nabla^\gamma\phi,$$

$$X \equiv -\frac{1}{2}\left(\nabla\phi\right)^2,$$

Why study scalar-tensor gravity?

- Find a complete theory of quantum gravity
- Model the dynamics of the (early and late) universe
  - Quintessence, $f(R)$ gravity, Brans-Dicke gravity, Galileons, ...
- Test GR for sake of basic science (model selection tests)

Test GR for the sake of basic science: gravitational waves

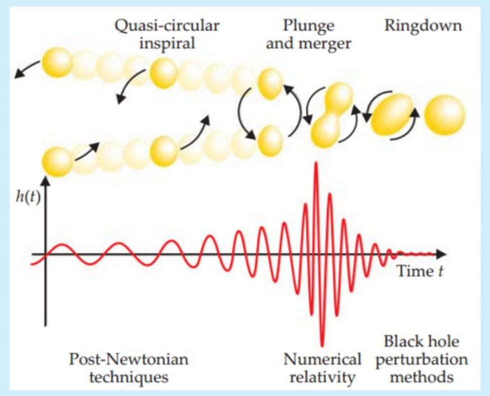

Figure: Numerical Relativity; Baumgarte and Shapiro

Employ *matched filtering* to extract gravitational wave signals: need to accurately model the physics

Guiding principles

We want to look at classical field theories that

1. Has a mathematically sensible interpretation
2. Matches all current observations
3. Can be tested/constrained with new observations
4. (Ideally) addresses a current problem in physics

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scalar Gauss-Bonnet gravity (sGB gravity)

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left(R + X - V(\phi) + \alpha(\phi) X^2 + \beta(\phi) \mathcal{G} \right),$$

where

$$X \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi,$$

$\mathcal{G}$: the *Gauss-Bonnet scalar*

$$\mathcal{G} \equiv R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\alpha\nu\beta} R^{\mu\alpha\nu\beta}.$$

This theory contains the leading order (in derivatives) corrections to GR[1]

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[1] up to field redefinitions and conformal transformations; e.g. Weinberg, Phys.Rev.D 77 (2008) 123541

Shift symmetric sGB gravity

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 2\lambda\phi\mathcal{G}\right),$$

This theory does not admit **stationary** Schwarzschild black hole solutions[2]; instead "hairy" scalar black holes should be end states in this theory

$$\Box\phi + \lambda\mathcal{G} = 0,$$

Hairy black holes $\implies$ scalar radiation $\implies$ will give different predictions for gravitational waves from GR black holes

[2] Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

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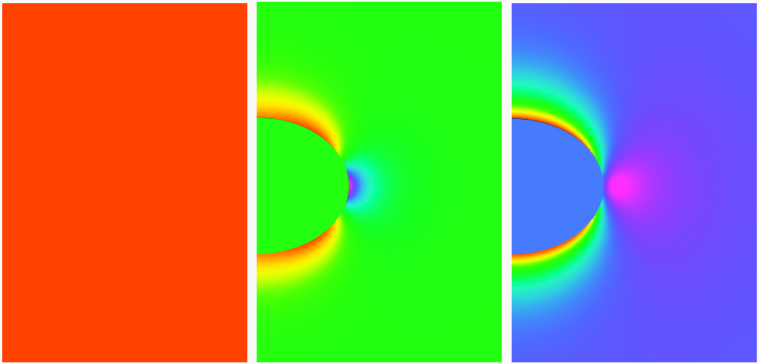
Numerical relativity simulations with this theory[3]

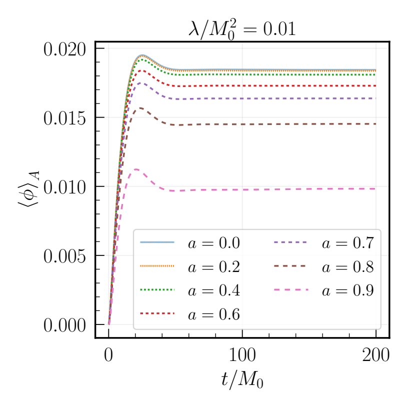
- We reformulate the equations of motion in **modified generalized harmonic** formulation
- We solve the **full** equations of motion using a numerical relativity code
- Results in the paper:
  - Consider spinning black hole evolution (axisymmetric spacetime)
  - Consider head on black hole collisions (axisymmetric spacetime)
  - Consider binary black hole merger (no symmetry assumptions)

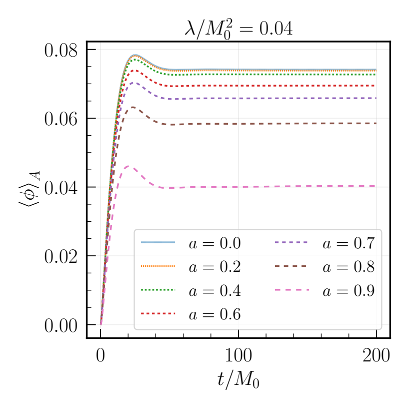
[3] A collaboration with Will East; see arXiv:2011.03547

Scalar hair growth around spinning black holes



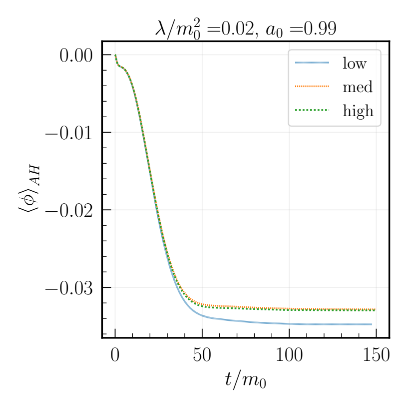
Scalar hair growth around spinning black holes





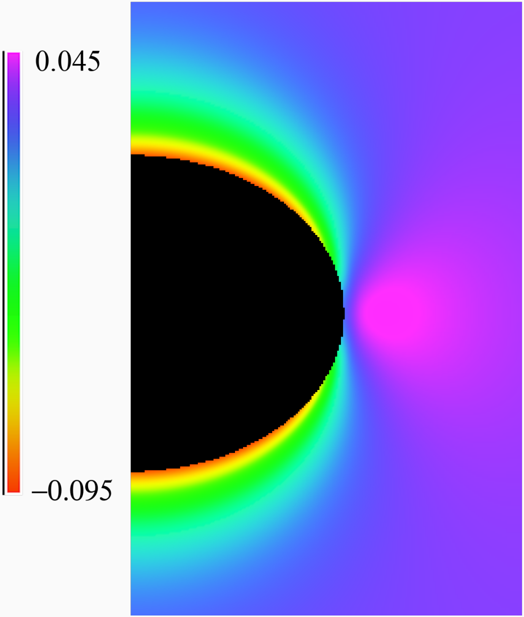
- $\langle\phi\rangle_A$: average scalar field value on black hole horizon
- a: initial dimensionless black hole spin

Scalar hair growth around spinning black holes

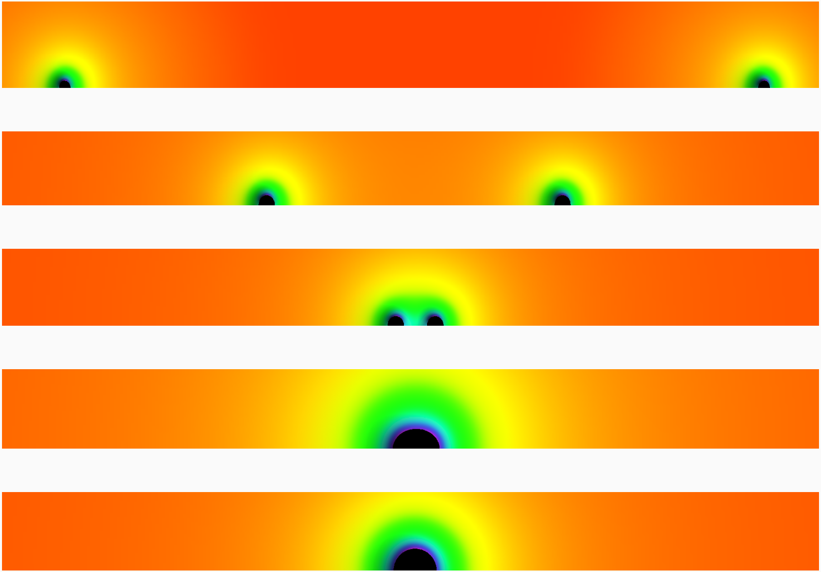


- $\langle\phi\rangle_A$: average scalar field value on black hole horizon, at three different resolutions (convergence study)

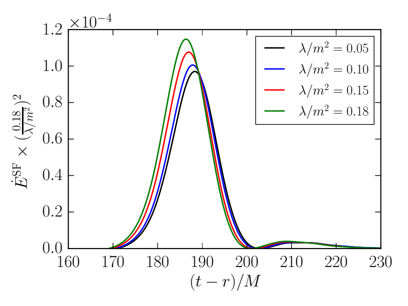
Scalar field density around a spinning black hole

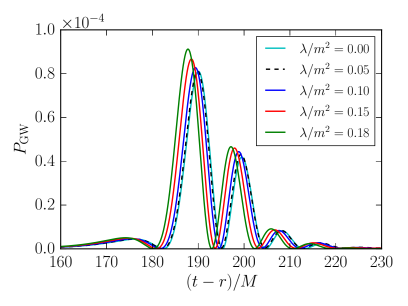


Head on black hole collisions



Head on black hole collisions: gravitational and scalar radiation





Flux of scalar field vs flux of gravitational waves

Binary black hole collisions

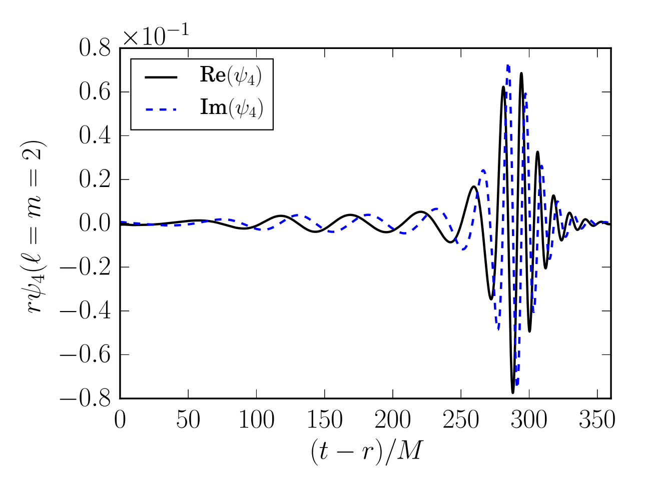


Figure: $\lambda/M^2 = 0.01$

Gravitational wave strain from two ESGB binary black holes merging

Binary black hole collisions

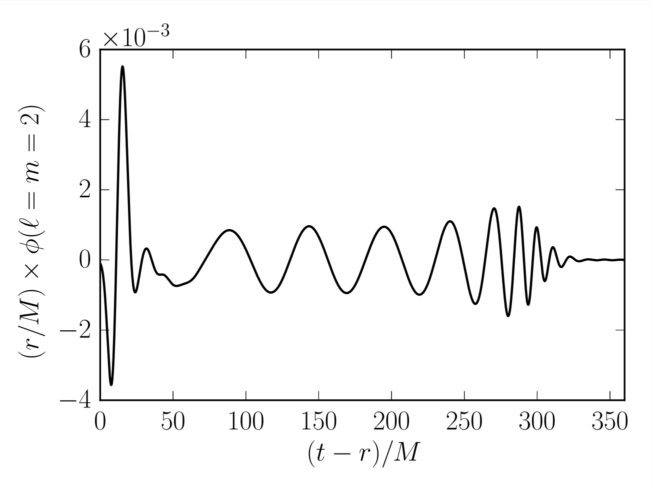


Figure: $\lambda/M^2 = 0.01$

Radiated scalar waves

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Challenge: Well-posed initial value formulation

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left(R + X - V(\phi) + \alpha(\phi) X^2 + \beta(\phi) \mathcal{G} \right).$$

- We need a **well-posed** system of equations to solve on a computer
- Terms like $\beta(\phi)\mathcal{G}$ can **change** the structure of the equations of motion in a way that can potentially spoil the hyperbolic character of the Einstein equations
- The hyperbolic character depends not only on the theory, but on ones choice of coordinates and how one **formulates** the equations of motion

Solution: modified generalized harmonic (MGH) formulation[4]

- Specify two auxiliary Lorentzian metrics $\hat{g}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ in addition to the spacetime metric $g^{\mu\nu}$
- Specify the gauge/coordinate condition with:

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu x^\gamma = H^\gamma, \tag{1}$$

  where H^γ is source function
- Free parameters: $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$, H^γ (more details given at end of talk)
- Besides using the MGH formulation, we begin with GR initial data, and use standard techniques from numerical relativity

[4]Kovacs and Reall, Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

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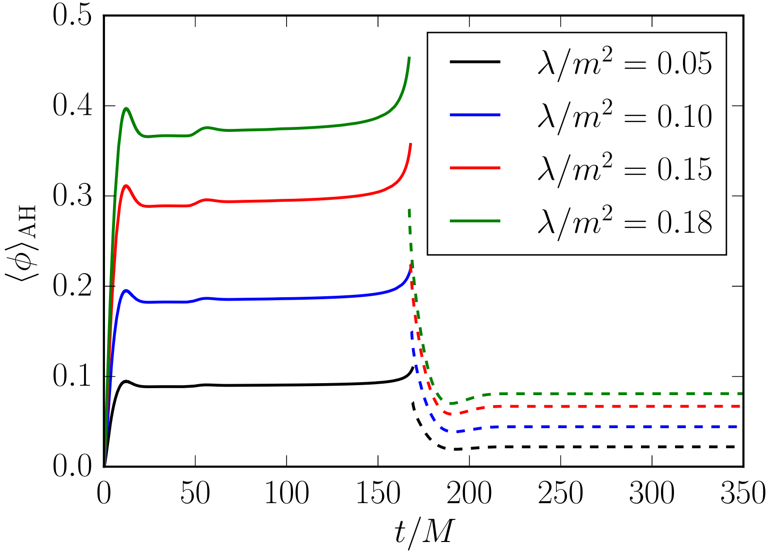
Conclusion

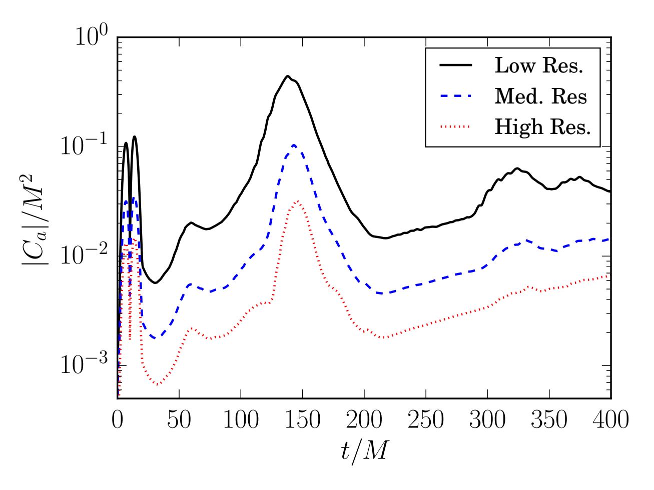
- Can test GR with gravitational waves
  - for that you need gravitational waveform templates to compare to data
- **Claim:** We now have the tools to produce gravitational waveforms produced during the merger of two black holes for a whole class of scalar-tensor gravity theories
- **Future work:** More astrophysically realistic initial data; generate waveform template

Backup slides

Head on black hole collisions: scalar field on horizon



Head on black hole collisions: convergence



Convergence of "constraint violation":

$$C^{\alpha} \equiv H^{\alpha} + \tilde{g}^{\mu\nu}\Gamma^{\alpha}_{\mu\nu} \qquad (2)$$

Shift symmetric sGB gravity: equations of motion

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2\lambda\phi\mathcal{G}\right),$$

$$\begin{aligned} E^{(g)}_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^\alpha\nabla_\gamma\phi\right)\delta^\beta{}_{(\mu}g_{\nu)\delta} \\ -\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 = 0, \\ E^{(\phi)} \equiv \Box\phi + \lambda\mathcal{G} = 0. \end{aligned}$$

Well-posed initial value problem: strongly hyperbolic formulation

$$\partial_t \mathsf{v} + \sum_{i=1}^{3} \hat{A}^i \partial_{x^i} \mathsf{v} + \mathsf{F} = 0.$$

- **Strongly hyperbolic:** Matrix $\hat{A}$ has real eigenvalues, and has a complete set of eigenvectors[5]
- Hyperbolicity of Einstein equations and Horndeski equations depends on the **formulation** of the equations of motion

[5] More technically, has a symmetrizer that one can bound independently of the derivatives of v; e.g. Sarbach and Tiglio, Living Rev.Rel. 15 (2012) 9

Why does the MGH formulation work?[6]

$$\partial_t \mathsf{v} + \sum_{i=1}^{3} \hat{A}^i \partial_{x^i} \mathsf{v} + \mathsf{F} = 0.$$

- In generalized harmonic formulation, $\hat{A}^i$ has all real eigenvalues (light speed): for GR the symbol can be diagonalized, but when adding modified gravity terms the matrix forms Jordan blocks due to ϕ, $g_{\mu\nu}$ couplings in the principal part
- In MGH formulation, eigenvalues have different values (depending on $g^{\mu\nu}$, $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$); a matrix with different real eigenvalues is more robust to remaining diagonalizable when adding modified gravity "perturbations" to the principal symbol

[6] For more discussion, see Kovacs and Reall, Phys. Rev. D 101, 124003 (2020)

More on the Modified generalized harmonic (MGH) formulation[7]

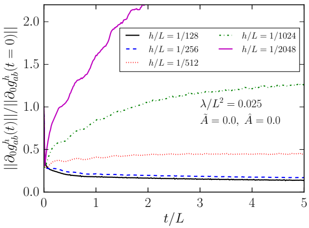
$$
\begin{aligned}
C^\gamma &\equiv H^\gamma - \tilde{g}^{\alpha\beta}\nabla_\alpha\nabla_\beta x^\gamma \\
&= H^\gamma + \tilde{g}^{\alpha\beta}\Gamma^\gamma_{\alpha\beta} = 0, \\
E^{\alpha\beta} - \hat{P}_\delta{}^{\gamma\alpha\beta}\nabla_\gamma C^\delta - \frac{1}{2}\kappa\left(n^\alpha C^\beta + n^\beta C^\alpha + \rho n^\gamma C_\gamma g^{\alpha\beta}\right) &= 0, \\
\hat{P}_\delta{}^{\gamma\alpha\beta} &\equiv \frac{1}{2}\left(\delta^\alpha_\delta \hat{g}^{\beta\gamma} + \delta^\beta_\delta \hat{g}^{\alpha\gamma} - \delta^\gamma_\delta \hat{g}^{\alpha\beta}\right).
\end{aligned}
$$

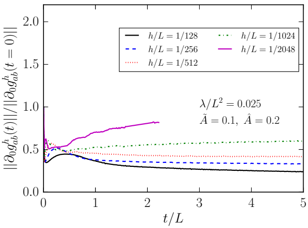
Divergence of equations of motion and use $\nabla_\alpha E^{\alpha\beta} = 0$, get propagation of constraint violation:

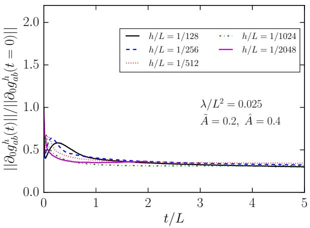
$$
-\frac{1}{2}\hat{g}^{\alpha\gamma}\nabla_\alpha\nabla_\gamma C^\beta - \hat{g}^{\gamma\beta}R_{\delta\gamma}C^\delta - \cdots = 0.
$$

[7] Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

Hyperbolicity test: Self-convergence in harmonic vs modified harmonic gauge







Order reduction approach for ESGB gravity[8]

Assume $\epsilon \sim \lambda$ and $|\epsilon| \ll 1$

$$\begin{aligned} g_{\mu\nu} =& g^{(0)}_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + \cdots \\ \phi =& \phi^{(0)} + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots \end{aligned} \tag{3a}$$

$$\phi^{(0)} = 0, \tag{4a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{4b}$$

$$\Box \phi^{(1)} = \lambda \mathcal{G}\left[g^{(0)}_{\alpha\beta}\right], \tag{5a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{5b}$$

$$R_{\mu\nu}[g^{(2)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(2)}_{\alpha\beta}] = \lambda \times F\left[\phi^{(1)}\right] \tag{6}$$

[8] Okounkova, *Phys. Rev. D* 100 (2019)

Initial conditions

- For technical reasons, we always start with a GR solution (e.g. one spinning black hole, two boosted black holes), and then let the black holes grow scalar hair as we evolve in time
- After a finite amount of evolution, the black holes stop growing scalar hair (growth saturates)

$$S_{ESGB} = \frac{1}{2} \int d^4x \sqrt{-g} \left(R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2\lambda \phi \mathcal{G} \right),$$

More questions/future work

- Further develop the MGH formulation of general relativity and scalar-tensor gravity theories
  - What are "good" choices for the auxiliary metrics?
  - Make contact with the BSSN-type formulations
- Evolution of other Horndeski gravity theories
  - Binary black hole waveform catalogues for other kinds of scalar-tensor gravity theories
  - Consider early universe cosmological simulations in these theories
- More systematic study of the binary black hole problem in MGH formulation
  - Better initial data
  - Compare waveforms of GR vs. modified gravity theories