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WEIZMANN INSTITUTE OF SCIENCE

MINERVA STIFTUNG

Gravitational Wave Echo of Relaxion Trapping

Abhishek Banerjee, E.M., Gilad Perez, Wolfram Ratzinger and Pedro Schwaller
arXiv:2105.12135

Eric Madge

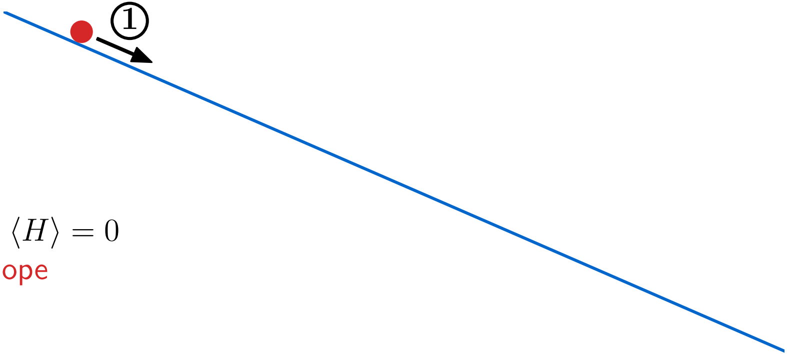
Weizmann Institute of Science

EPS-HEP – July 30, 2021

Relaxion (during inflation)

[Graham, Kaplan, Rajendran (2015)]

$$V(H,\phi) = \underbrace{\left(\Lambda^2 - g\Lambda\phi\right)}_{\mu_H^2(\phi)} |H|^2 + \lambda |H|^4 - cg\Lambda^3\phi - \Lambda_{\text{br}}^4 \frac{|H|^2}{v_H^2} \cos\frac{\phi}{f_\phi}$$



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   only linear slope

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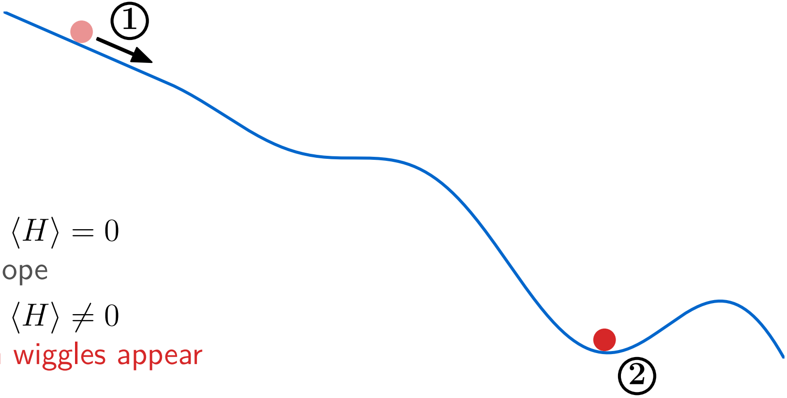

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2. $\mu_H^2 < 0 \implies \langle H \rangle \neq 0$
   backreaction wiggles appear

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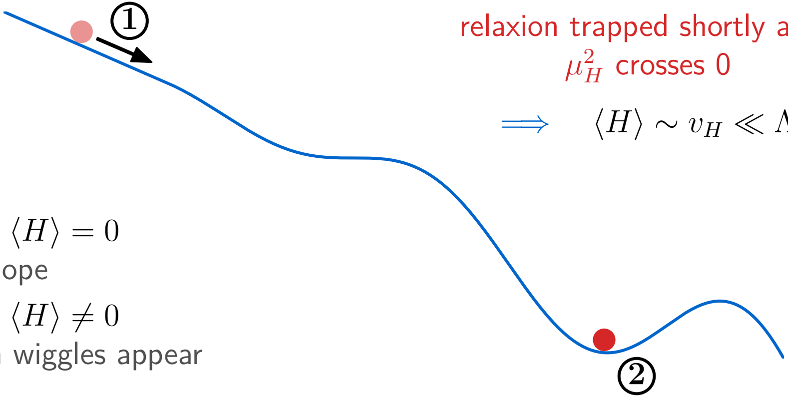

relaxion trapped shortly after μ_H^2 crosses 0

$$\Longrightarrow \quad \langle H\rangle \sim v_H \ll \Lambda$$

1. $\mu_H^2 > 0 \Longrightarrow \langle H\rangle = 0$
   only linear slope
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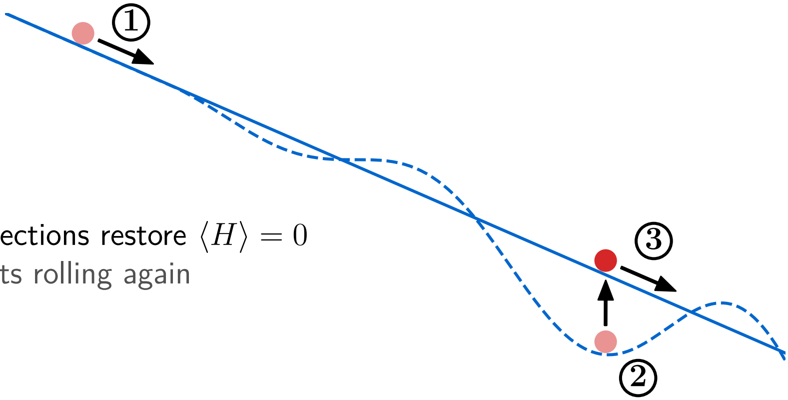

3. thermal corrections restore $\langle H \rangle = 0$
   relaxion starts rolling again

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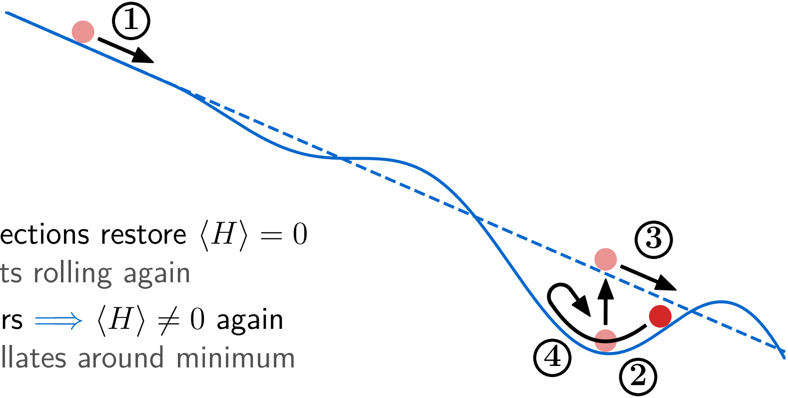

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   relaxion oscillates around minimum

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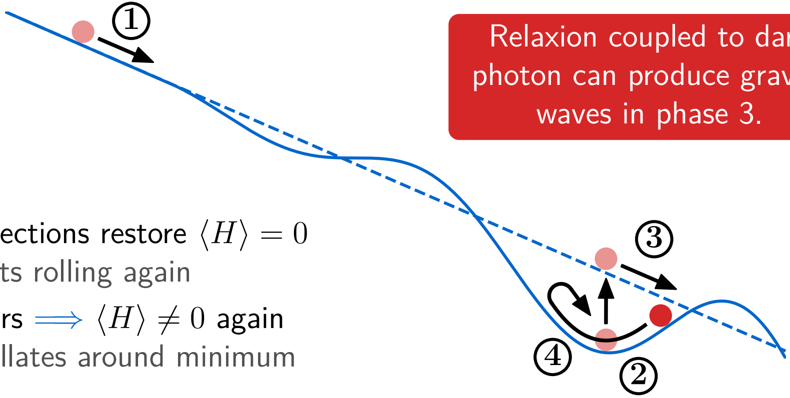

Relaxion coupled to dark photon can produce gravity waves in phase 3.

3. thermal corrections restore $\langle H \rangle = 0$
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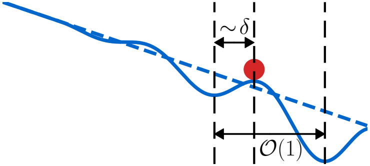
Why should we couple the relaxion to a dark photon?



- distance between first minimum and maximum:

$$\Delta\phi_1 \sim 2\delta f_\phi \ll f_\phi \qquad \left(\delta = \frac{m_\phi^2 f_\phi^2}{\Lambda_{\text{br}}^4} \lesssim 10^{-3}\right)$$

- relaxion displaced from initial minimum

if $\Delta\theta = \Delta\phi/f_\phi \gtrsim 2\delta$: additional friction required! $\longrightarrow$ dark photon

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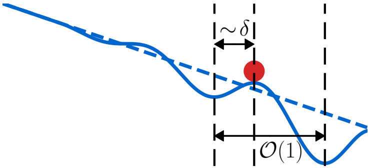
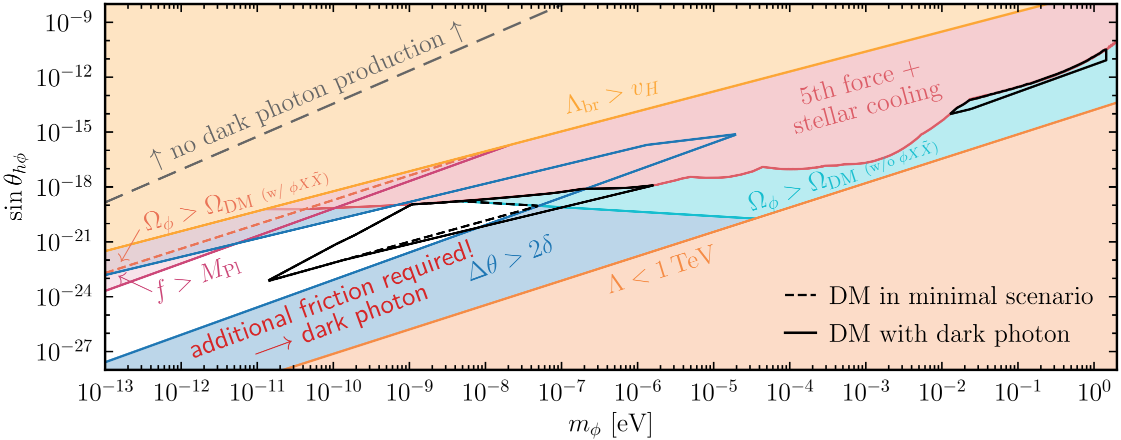




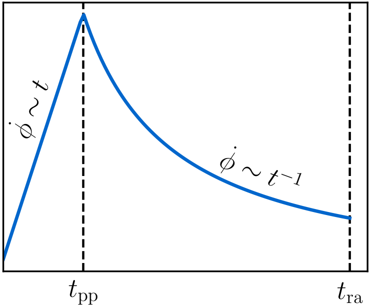
Relaxion and dark photon evolution

- relaxion coupled to dark photon X: $\mathcal{L} \supset -\frac{r_X}{4}\frac{\phi}{f_\phi}X_{\mu\nu}\tilde{X}^{\mu\nu}$

$$\Longrightarrow \qquad \ddot{\phi} + 3H\dot{\phi} - \frac{\Lambda_{\text{br}}^4}{f_\phi} + \frac{r_X}{f_\phi}\frac{\langle \tilde{X}_{\mu\nu}X^{\mu\nu}\rangle}{4\,a^4} = 0$$

- relaxion reaches terminal velocity when $\frac{r_X}{4a^4}\langle \tilde{X}X\rangle \sim \Lambda_{\text{br}}^4$:

$$\frac{\dot{\phi}}{f_\phi} = \frac{\xi H}{r_X} \qquad\qquad \xi \sim \mathcal{O}(10-100)$$



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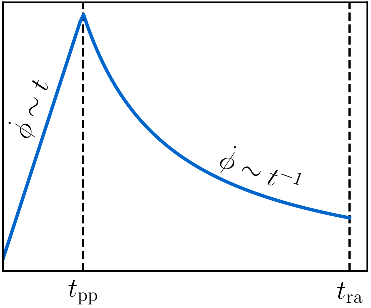
$$X_\lambda''(\tau, k) + \left(k^2 - \lambda\, k \frac{r_X \phi'(\tau)}{f_\phi}\right) X_\lambda(\tau, k) = 0$$



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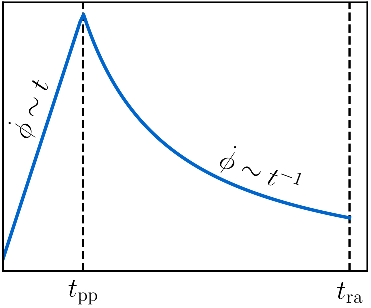

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- anisotropic stress in dark photon energy-momentum tensor sources GWs

$\Longrightarrow$ stochastic GW background

Gravitational Wave Production

GW in TT gauge

$$(\partial_\tau^2 + k^2) a(\tau) h_{ij}(\tau, \mathbf{k}) = \frac{2\, a(\tau)}{M_{\mathsf{Pl}}^2} \Pi_{ij}(\tau, \mathbf{k})$$

anisotropic stress tensor

$$\Pi_{ij}(\mathbf{k}) \sim \int d^3q\, E(\mathbf{q}) E(\mathbf{k} - \mathbf{q}) + (E \leftrightarrow B)$$

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GW spectrum generated at reappearance:

- peak frequency:

$$f_{\mathrm{peak}} \sim 2\, \frac{k^X_{\mathrm{peak}}}{a} \sim \frac{a_{\mathrm{ra}}}{a_0} \xi H_{\mathrm{ra}} \sim 1\,\mu\mathrm{Hz} \left(\frac{\xi}{25}\right) \left(\frac{T_{\mathrm{ra}}}{1\,\mathrm{GeV}}\right)$$

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  $\Pi_{ij} \sim \rho_X$

- peak amplitude: $$\Omega^{\text{peak}}_{\text{GW}} \sim \frac{\left(\rho_X^{\text{ra}}/f^{\text{ra}}_{\text{peak}}\right)^2}{\rho_c\, M_{\text{Pl}}^2}\left(\frac{a_{\text{ra}}}{a_0}\right)^4 \sim 10^{-10}\left(\frac{25}{\xi}\right)^2\left(\frac{m_\phi f_\phi}{T^2_{\text{ra}}}\right)^4$$

  $\Omega_{\text{GW}}(f) \sim \frac{1}{\rho_c}\frac{d\,\rho_{\text{GW}}}{d\log f}$

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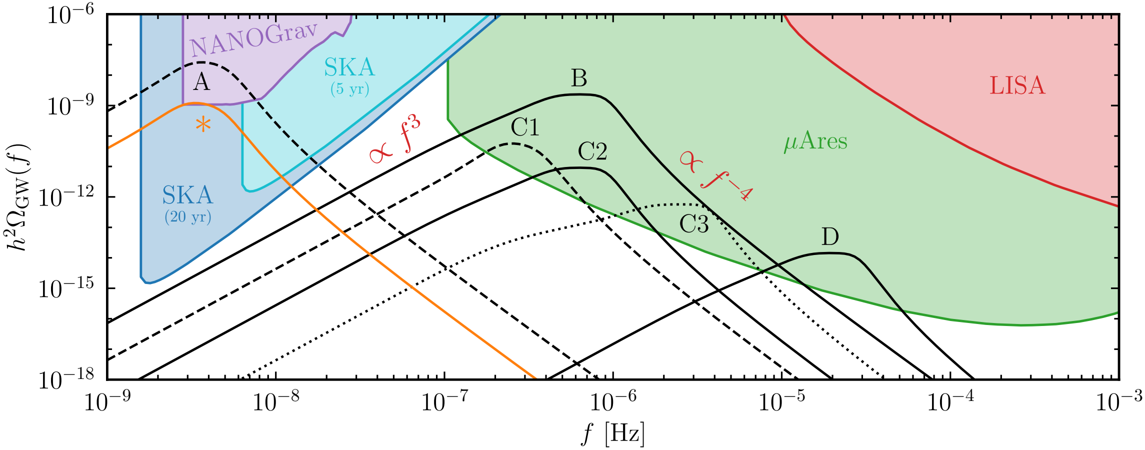
GW spectra

$$f_{\text{peak}} \propto \xi T_{\text{ra}}$$

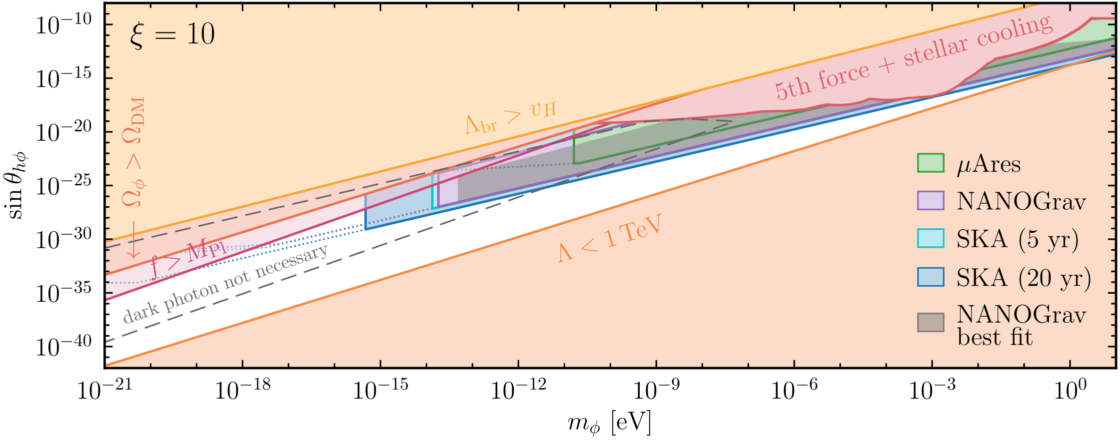
$$\Omega_{\text{GW}}^{\text{peak}} \propto \frac{m_\phi^4 f_\phi^4}{\xi^2 T_{\text{ra}}^8}$$

	$*$	A	B	C1	C2	C3	D
m_ϕ [eV]	10^{-9}	3×10^{-9}	2×10^{-3}	5	5×10^{-3}	5×10^{-6}	0.1
f_ϕ [GeV]	10^{14}	10^{14}	10^{12}	10^{8}	10^{11}	10^{14}	10^{12}
T_{ra} [GeV]	0.02	0.02	1	1	1	1	30
ξ	10	10	25	10	25	100	25



Results ($\xi = 10$)

$$f_{\text{peak}} \propto \xi\, T_{\text{ra}} \qquad h^2\Omega_{\text{GW}}^{\text{peak}} \propto \xi^{-2}\, m_\phi^{-12}\, (\sin\theta_{h\phi})^{12}\, \Lambda^{-8}\, T_{\text{ra}}^{-8}$$



$$T_{\text{BBN}} \sim 10\text{ MeV} \lesssim T_{\text{ra}} \lesssim v_H \sim 170\text{ GeV}$$

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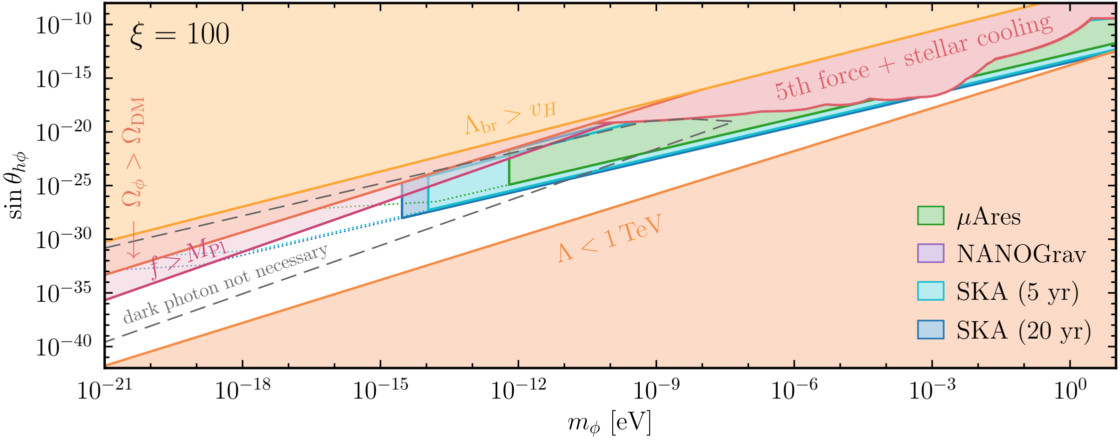

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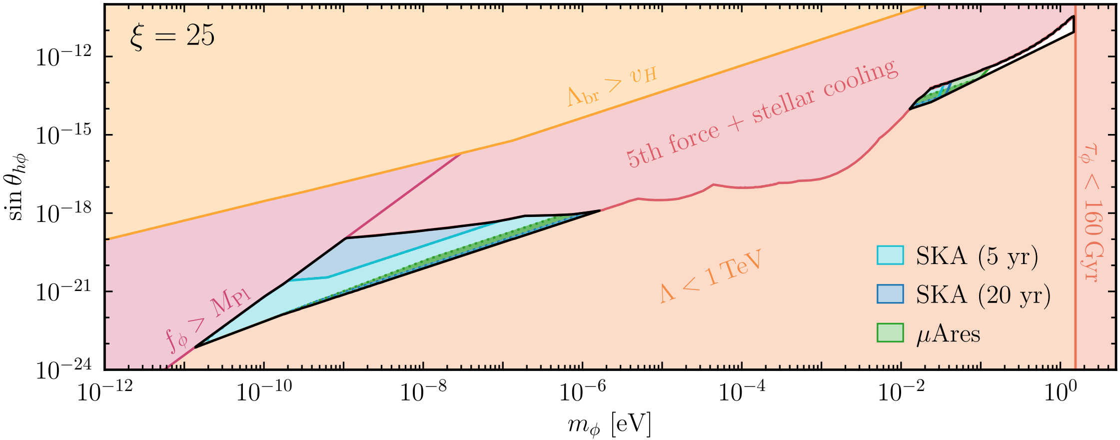
Relaxion Dark Matter

displaced relaxion oscillates $\Longrightarrow$ ultra-light dark matter [Banerjee, Kim, Perez (2018)]



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Conclusion

- Relaxion coupled to dark photon can produce gravitational waves:

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- SKA will be able to probe a part of the relaxion dark matter parameter space

- large portion of the parameter space will be accessible to μHz observatories such as μAres

- if the relaxion is not DM, we obtain constraints from NANOGrav

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Thank you for your attention!

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Dark photon production

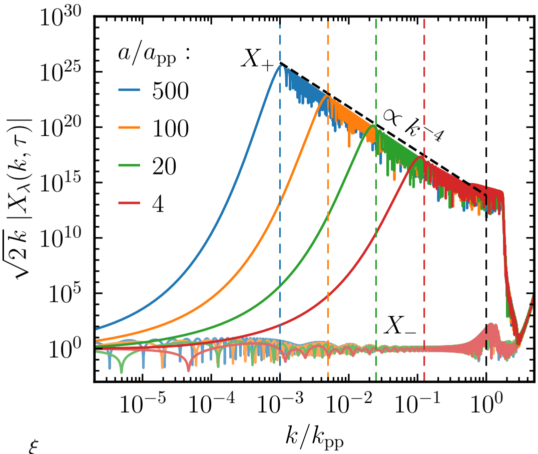
dark photon EoM: $$X''_\lambda(\tau,k) + \left(k^2 - \lambda\, k \frac{r_X \phi'(\tau)}{f_\phi}\right) X_\lambda(\tau,k) = 0$$

- '+' helicity modes with $k < \frac{r_X \phi'(\tau)}{f_\phi}$ experience tachyonic instability

  $\Longrightarrow$ exponential production of some k-modes in one helicity

- energy predominantly transferred to most tachyonic mode: $k = \frac{r_X \phi'}{2 f_\phi} = \frac{\xi a H}{2}$

- after exiting the tachyonic band: $X(k,\tau) \propto \cos(k\tau)/\sqrt{2k}$



$\Longrightarrow$ $X_+(k,\tau) \sim k^{-9/2}\cos(k\tau - \xi)$ for $k > \frac{\xi}{2\tau}$

Gravitational Wave Spectrum

- IR: $f \ll f_{\text{peak}}$

  $$|\mathbf{q}| \sim |\mathbf{k}-\mathbf{q}| \sim k^X_{\text{peak}}$$

  $$\Omega_{\text{GW}}(f) \sim \Omega_{\text{GW}}^{\text{peak}}\, \xi^2\, \frac{f^3}{f^3_{\text{peak}}}$$

- peak: $f \sim f_{\text{peak}}$

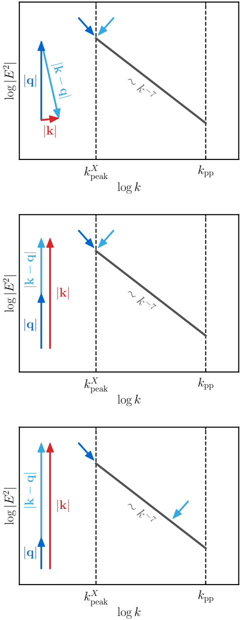
  $$|\mathbf{q}| \sim |\mathbf{k}-\mathbf{q}| \sim k^X_{\text{peak}}$$

  $$\Omega_{\text{GW}}(f_{\text{peak}}) = \Omega_{\text{GW}}^{\text{peak}}$$

- UV: $f \gg f_{\text{peak}}$

  $$|\mathbf{q}| \sim k^X_{\text{peak}}, \quad |\mathbf{k}-\mathbf{q}| \sim k$$

  $$\Omega_{\text{GW}}(f) \sim \Omega_{\text{GW}}^{\text{peak}}\, \frac{f^4_{\text{peak}}}{f^4}$$



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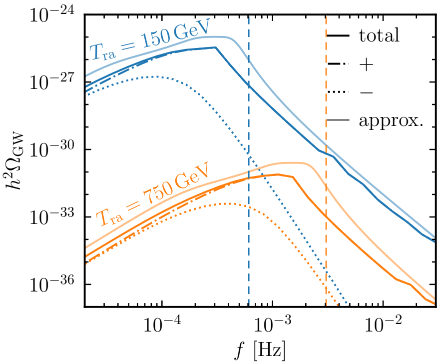
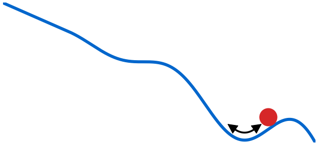


Relaxion Dark Matter

The second rolling phase displaces the relaxion from its minimum
$\Longrightarrow$ oscillating relaxion can constitute DM



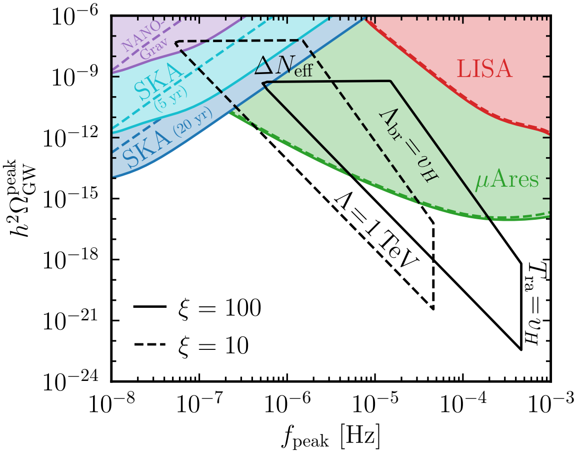
[Banerjee, Kim, Perez (2018)]

- relaxion starts to oscillate at T_{osc}:

  either T_{ra} or $3H = m_\phi$

- $\rho_\phi(T_{\text{osc}}) \approx \frac{1}{2} m_\phi^2 \Delta\phi^2$

  $$\Longrightarrow \quad \Omega_\phi \sim \frac{(m_\phi f_\phi)^{\frac{10}{3}}}{\Lambda^{\frac{4}{3}} T_{\text{osc}}^3}$$



$\Longrightarrow$ fixes f_ϕ in terms of m_ϕ, Λ and T_{ra}

NANOGrav “signal”

“..., the 12.5-year data set offers strong evidence for a spatially uncorrelated common-spectrum process across pulsars in the data set, but it favors only slightly the interpretation of this process as a GWB by way of HD inter-pulsar correlations.”

[NANOGrav (2020)]