Gravitational waves as a Big Bang thermometer

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JCAP 03 (2021) 054, arXiv:2011.04731 [hep-ph]

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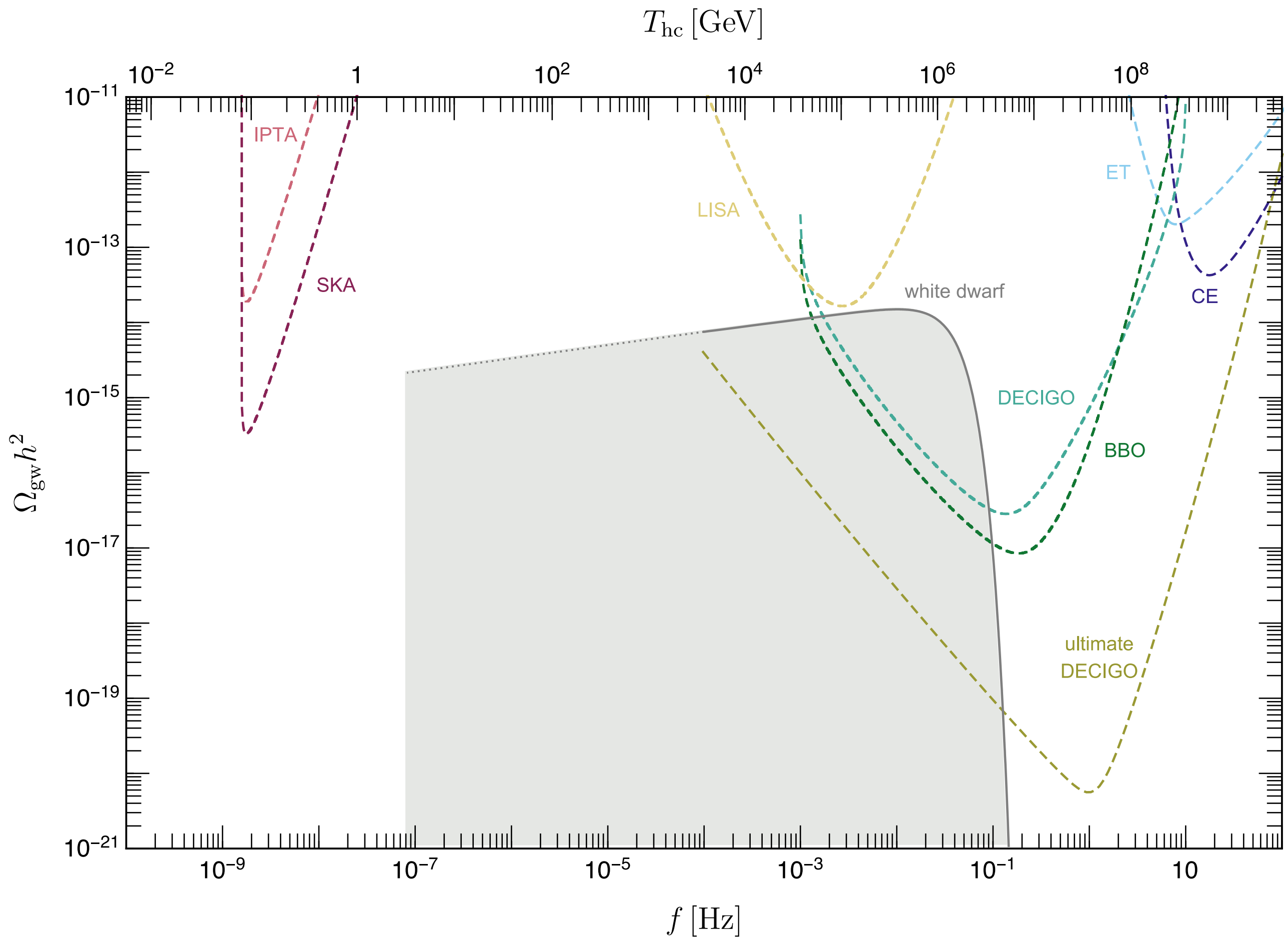
The experimental context



What about high frequencies??

High frequencies interesting as they probe **high-energy processes** in early universe

Weak interactions of gravity ensure that waves **travel undisturbed**

The aim:

Obtain state-of the art predictions for the spectrum of **gravitational waves** sourced by **thermal fluctuations** in the hot Big Bang plasma, and discuss current **bounds** and **prospects** for **experimental detection**

The novelty:

We **generalized** theoretical **predictions** for the SM to **arbitrary models**

Study of **experimental prospects** to detect high frequency GWs from

GW/EMW conversion in axion experiments and in proposal with Gauβian beam

The plan:

Primordial gravitational waves from thermal fluctuations

Predicted spectra for the SM, SMASH, nuMSM

Current bounds

Future prospects

Gravity waves from the thermal plasma

What are gravitational waves?

Excitations of the **metric** field **sourced** by **anisotropies** in the **stress-energy** momentum tensor. In local Minkowski frame

$$ds^2 \supset -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j ,$$
$$\partial^i h_{ij} = 0$$

$$\Box \left(h_{ij} - \frac{1}{2}\delta_{ij} h \right) = \frac{2}{M_P^2} T_{ij}^{\mathrm{TT}},$$
$$T^{\mathrm{TT}}{}^i_i = 0,$$
$$\partial^i T_{ij}^{\mathrm{TT}} = 0$$

In the **Big Bang plasma,** transverse traceless stress-energy coming from:

Viscosity effects in the plasma at large length scales
Plasma **absorption-emission** effects at small scales

A guaranteed background from thermal effects!

Other sources of stochastic backgrounds of **gravitational waves**:

inflation and reheating
phase transitions

Thermal emission relies on **better known physics** and is a **guaranteed** background

Despite Planck suppresion, emission accumulates since Big Bang

Peak of emission at $(\nu \sim T)$ redshifts at approx same rate as frequencies of previously emitted waves: **energy at peak adds up constructively**.

Peak in **microwave** regime: **Cosmic Microwave Gravitational Background (CGMB)**

Could a measurement of this background inform us of the temperature reached by the Hot Big Bang?

Ingredients to compute thermal spectrum

The **CGMB spectrum** can be **computed** for an arbitrary theory **knowing**:

Dynkin indices of particle representations R

$$\mathrm{Tr} T^a_{n,R} T^b_{n,R} = T_{n,R} \delta^{ab} \quad a, b = 1, \ldots, N_n$$

Index labelling gauge group

Number of generators in group *n*

Gauge and Yukawa couplings and their running

Three **functions counting the degrees of freedom** in the plasma, which can be derived from thermal effective potential

$$g_{*\rho}(T), g_{*s}(T), g_{*c}(T)$$

Ingredients to compute thermal spectrum

Energy density

$$\rho = \frac{\partial U}{\partial V} \equiv \frac{\pi^2}{30} g_{*\rho}(T) T^4$$

Entropy density

$$s = \frac{\partial S}{\partial V} = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

Heat capacity

$$c = \frac{1}{V} \frac{\partial U}{\partial T}\bigg|_V = T \frac{\partial s}{\partial T}\bigg|_V = \frac{2\pi^2}{15} g_{*c}(T) T^3$$

Relation to **effective potential** at finite T, $\Delta V(T)$

$$g_{*\rho} = \frac{30}{\pi^2 T^4}\left(\Delta V(T) - T\frac{\partial \Delta V(T)}{\partial T}\right),$$

$$g_{*s} = -\frac{45}{2\pi^2 T^3}\frac{\partial \Delta V(T)}{\partial T},$$

$$g_{*c} = -\frac{15}{2\pi^2 T^2}\frac{\partial^2 \Delta V(T)}{\partial T^2},$$

Rate of gravitational wave production

$$\rho_{\rm gw} = \frac{M_P^2}{4} \langle \dot{h}_{ij}(t,\mathbf{x})\, \dot{h}_{ij}(t,\mathbf{x}) \rangle$$

Production rate neglecting backreaction related to 2-point correlator of stress-energy tensor [Laine, Ghiglieri]

$$(\partial_t + 4H)\rho_{\rm CGMB} = \frac{1}{2M_P^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int d^4x\, e^{ikx} \langle T_{ij}^{\rm TT}(x) T_{ij}^{\rm TT}(0) \rangle \equiv \frac{4T^4}{M_p^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \hat{\eta}\left(T, \frac{k}{T}\right)$$

Small k (large length scales): source from viscosity effects dominated by hypercharge interactions of right-handed leptons

$$\hat{\eta}(T,\hat{k}) = \frac{\eta_{\rm shear}}{T^3 g_1(T)^4 \log(5/\hat{m}_1)} \qquad \hat{k} \equiv \frac{k}{T}$$

$$\hat{m}_n^2(T) = \frac{m_n^2(T)}{T^2} = g_n^2(T) \left(\frac{1}{3} T_{n,\rm Ad} + \frac{1}{6} \sum_{\hat{\imath}} T_{n,\hat{\imath}} + \frac{1}{6} \sum_{\hat{\alpha}} T_{n,\hat{\alpha}} \right)$$

Thermal Debye mass

Dynkin indices of gauge fields, real scalars, Weyl fermions

Rate of gravitational wave production

Large k (short length scales): Source from thermal excitations in the plasma. Generalized results of [Laine & Ghiglieri] to arbitrary theories

$$\hat{\eta}_{\mathrm{HTL}}(T,\hat{k}) + \sum_{n=1}^{N_g} g_n(T)^2 N_n \left(\frac{1}{2} T_{n,\mathrm{Ad}}\, \eta_{gg}(\hat{k}) + \sum_{\hat{\imath}} T_{n,\hat{\imath}}\, \eta_{sg}(\hat{k}) + \frac{1}{2} \sum_{\hat{\alpha}} T_{n,\hat{\alpha}}\, \eta_{fg}(\hat{k}) \right) + \frac{1}{4} \sum_{i\alpha\beta} |y^i_{\alpha\beta}(T)|^2\, \eta_{sf}(\hat{k}),$$

Leading log contribution

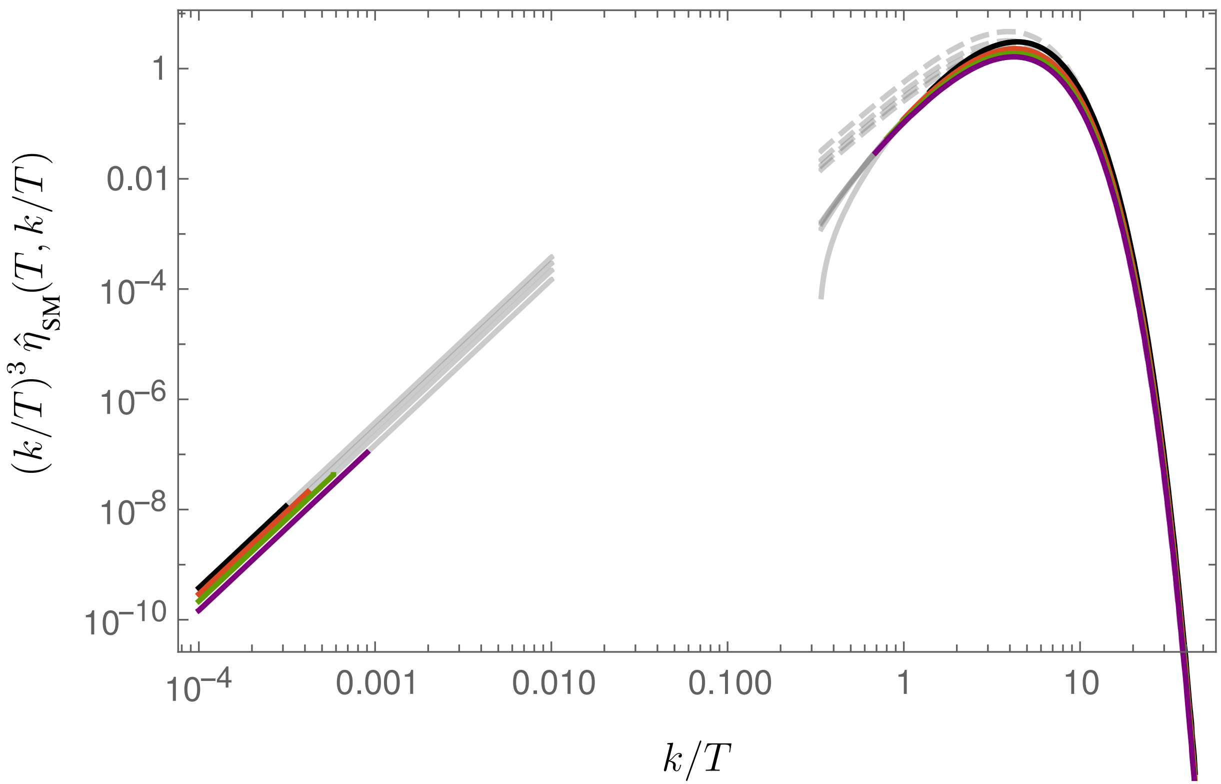
$$\hat{\eta}_{\mathrm{HTL}}(T,\hat{k}) = \frac{\hat{k}}{16\pi(e^{\hat{k}}-1)} \sum_n N_n \hat{m}_n^2(T) \log\left(1 + 4\frac{\hat{k}^2}{\hat{m}_n^2(T)}\right).$$

Gauge and Yukawa couplings

Dynkin indices of representations

Thermal loop functions computed by [Laine & Ghiglieri]

State of the art calculation of $\hat{\eta}$ in the SM



Production from **shear viscosity** effects

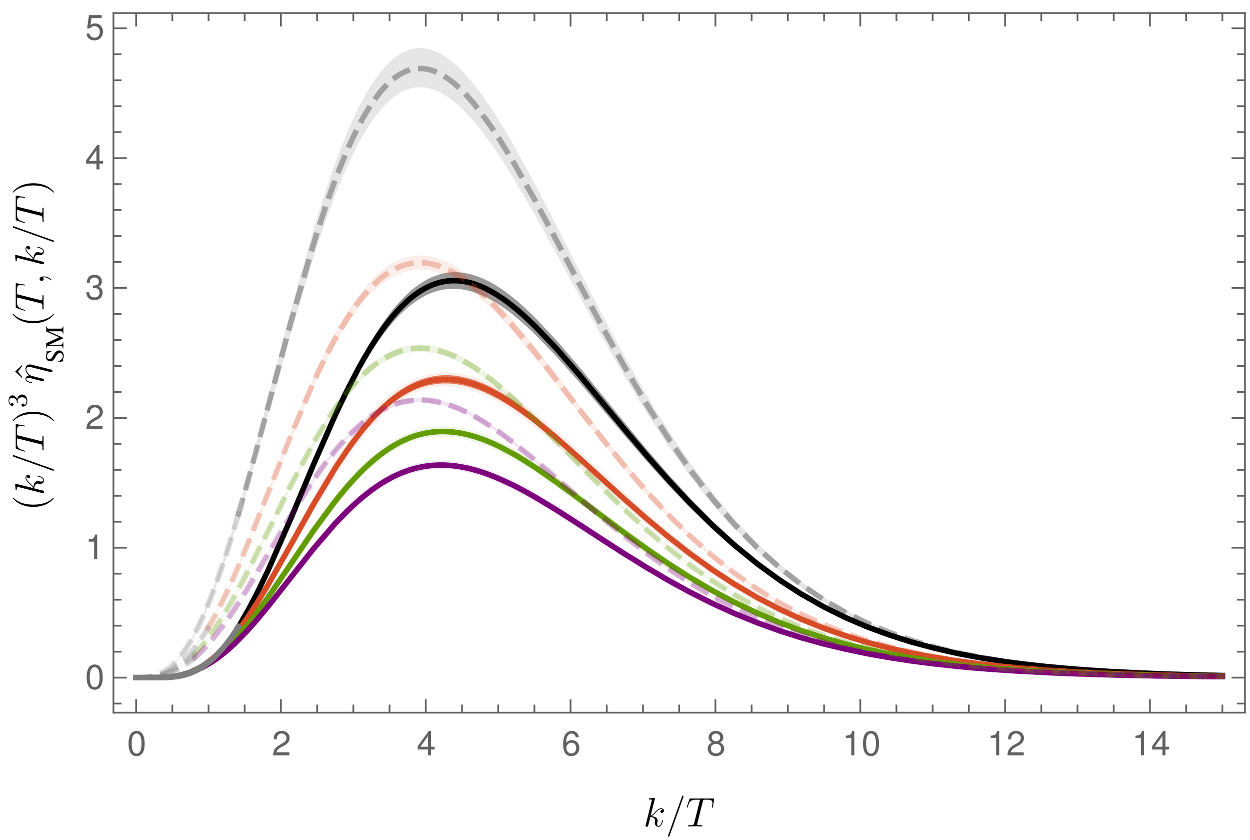
Production from **particle excitations** in the plasma

Leading log vs full-leading order

10^3 GeV , 10^8 GeV, 10^{13} GeV, Mp

State of the art calculation of $\hat{\eta}$ in the SM

Production from **particle excitations** in the plasma



Leading log vs full-leading order

10^3 GeV , 10^8 GeV, 10^{13} GeV, Mp

Integrating the rate until today

To get current spectrum one has to **integrate over time** and **account for expansion** of the universe, using:

Relation between **temperature** and **time** from entropy conservation and Friedmanns equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{\pi}{\sqrt{90}} [g_{*\rho}(T)]^{1/2} \frac{g_{*s}(T)}{g_{*c}(T)} \frac{T^3}{M_P}$$

Redshifting of frequencies from entropy conservation

$$\hat{k} = \frac{2\pi f}{T_0} \left(\frac{g_{*s}(T)}{g_{*s}(\mathrm{fin})} \right)^{1/3}$$

Integrating the rate until today

$$\Omega_{\rm CGMB}(f) = \frac{1}{\rho_{\rm tot}} \frac{\rho_{\rm CGMB}}{d\log k}$$

$$\simeq \frac{1440\sqrt{10}}{2\pi^2 M_P}\, \Omega_\gamma\, \left[g_{*s}(\text{fin})\right]^{1/3} \frac{f^3}{T_0^3} \times \int_{T_{\rm ewco}}^{T_{\rm max}} \mathrm{d}T\, \frac{g_{*c}(T)}{\left[g_{*s}(T)\right]^{4/3} \left[g_{*\rho}(T)\right]^{1/2}}\, \hat{\eta}\left(T, 2\pi \left[\frac{g_{*s}(T)}{g_{*s}(\text{fin})}\right]^{1/3} \frac{f}{T_0}\right) .$$

Integral dominated by high T in which all g functions tend to a common limit

$$\Omega_{\rm CGMB}(f) \approx 4.03 \times 10^{-12} \left[\frac{T_{\rm max}}{M_P}\right] \left[\frac{g_{*s}(T_{\rm max})}{106.75}\right]^{-5/6} \left[\frac{f}{\rm GHz}\right]^3 \hat{\eta}\left(T_{\rm max}, 2\pi \left[\frac{g_{*s}(T_{\rm max})}{g_{*s}(\text{fin})}\right]^{1/3} \frac{f}{T_0}\right)$$

Linear effect with $T_{\rm max}$: A true thermometer!

It is common to introduce **dimensionless strain**

$$h_c(f) = 1.26 \times 10^{-18} \left[\frac{\rm Hz}{f}\right] \sqrt{h^2\, \Omega_{\rm GW}^{(0)}(f)}$$

Main features

Peak frequencies

$$f_{\rm peak}^{\Omega_{\rm CGMB}}(T_{\rm max}) \approx 79.8\,{\rm GHz}\left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{1/3},$$

$$f_{\rm peak}^{h_c^{\rm CGMB}}(T_{\rm max}) \approx 40.5\,{\rm GHz}\left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{1/3},$$

Measuring peak value and position can lead to estimate of $T_{\rm max}$ and $g_{*s}(T_{\rm max})$

$$\begin{aligned}\Omega_{\rm CGMB}(f_{\rm peak}^{\Omega}(T_{\rm max})) &\approx \left(\frac{g_{*s,{\rm SM}}(T_{\rm max})}{g_{*s}(T_{\rm max})}\right)^{11/6}\Omega_{\rm CGMB,SM}(f_{\rm peak,SM}^{\Omega}(T_{\rm max})),\\ h_c^{\rm CGMB}(f_{\rm peak}^{h_c}(T_{\rm max})) &\approx \left(\frac{g_{*s,{\rm SM}}(T_{\rm max})}{g_{*s}(T_{\rm max})}\right)^{7/12} h_c^{\rm CGMB,SM}(f_{\rm peak,SM}^{h_c}(T_{\rm max})),\end{aligned} \qquad (1)$$

SM leads to **higher power** in gravitational waves

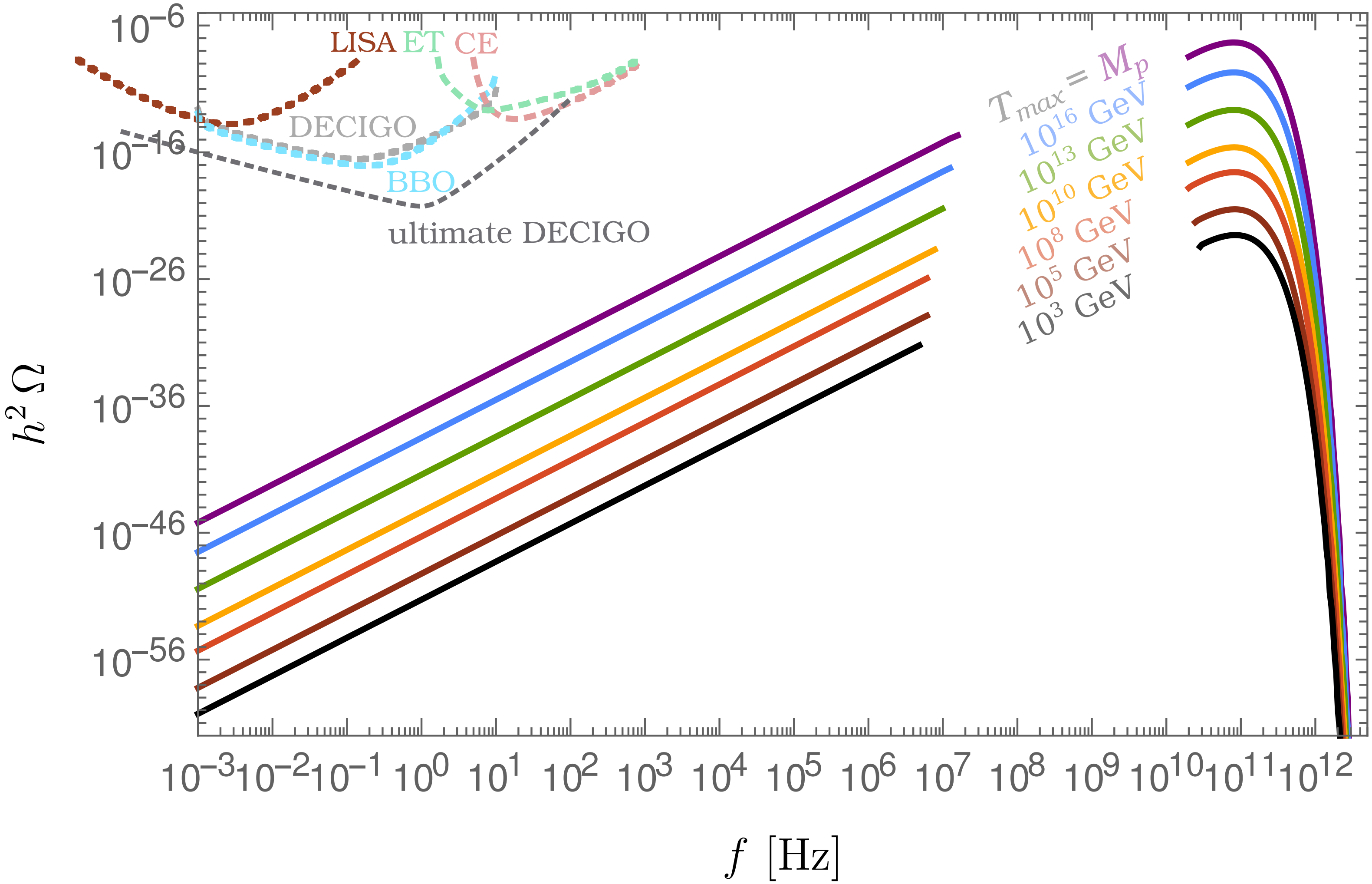
What if $T_{\max}$ goes above M_P?

Cannot ignore backraction of produced gravitons. Expect gravitons to **reach thermal equilibrium** and **decouple at $T=M_P$**

Spectrum given by the equilibrium distribution redshifted from $T=M_P$ to today

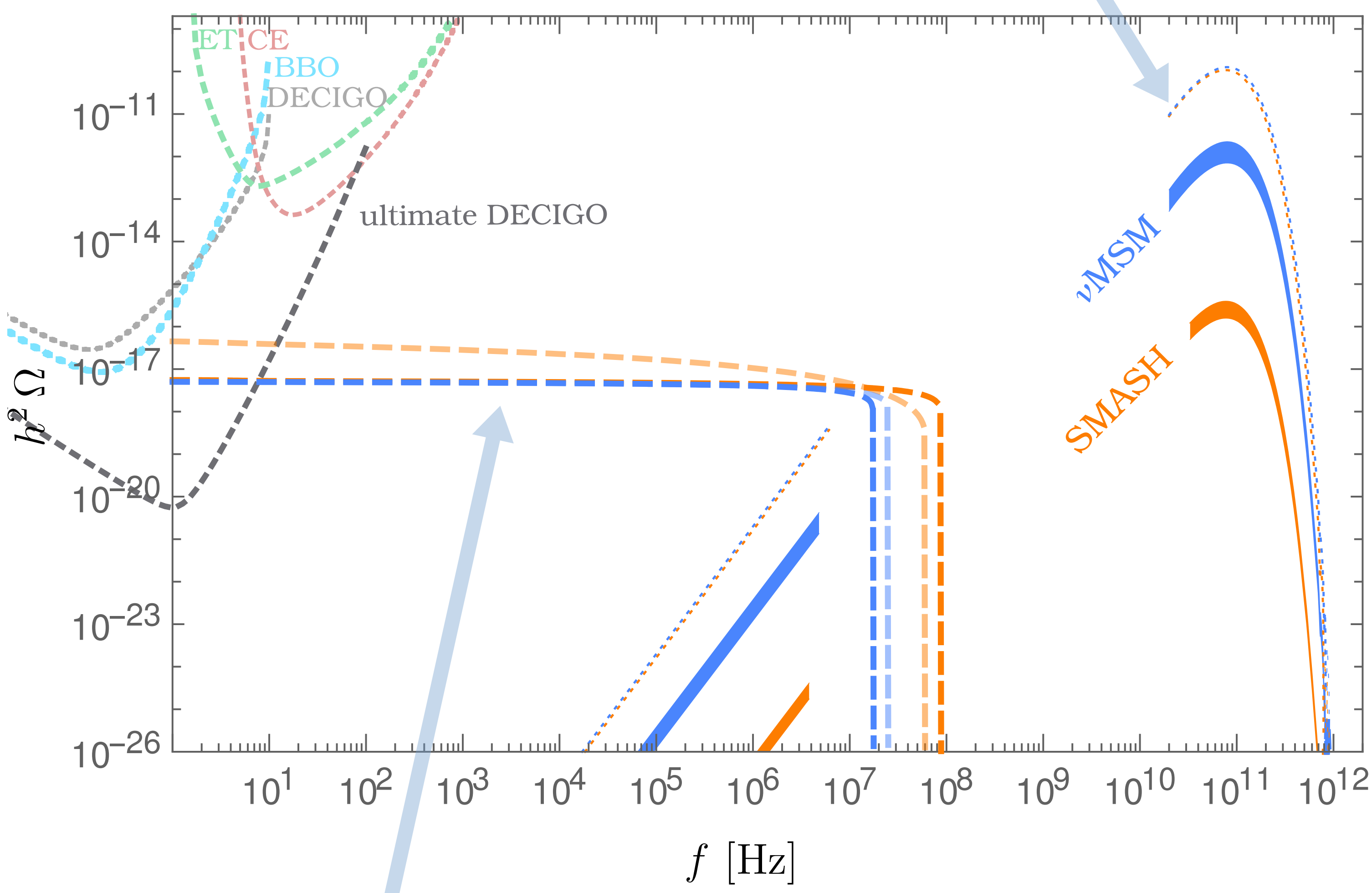
$$\Omega_{\rm Eq.CGMB}(f) = \frac{16\pi^2}{3M_P^2H_0^2}\frac{f^4}{e^{2\pi f/T_{\rm grav}}-1}, \qquad T_{\rm grav} = \left(\frac{g_{*s}({\rm fin})}{g_{*s}(M_P)}\right)^{1/3} T_0.$$

Spectrum in the SM



Spectrum in SMASH/νMSM

Single-field inflation upper bound to reheating temperature



Gravitational waves generated during inflation

Current experimental bounds

How to detect high-freq gravitational waves?

Direct searches

Interferometers [Nishizawa et al, Akutsu et al, Chou et al]

Cavity experiments measuring **rotation of polarization** [Cruise & Ingley]

Experiments measuring **resonant spin precession** of electrons [Ito et al]

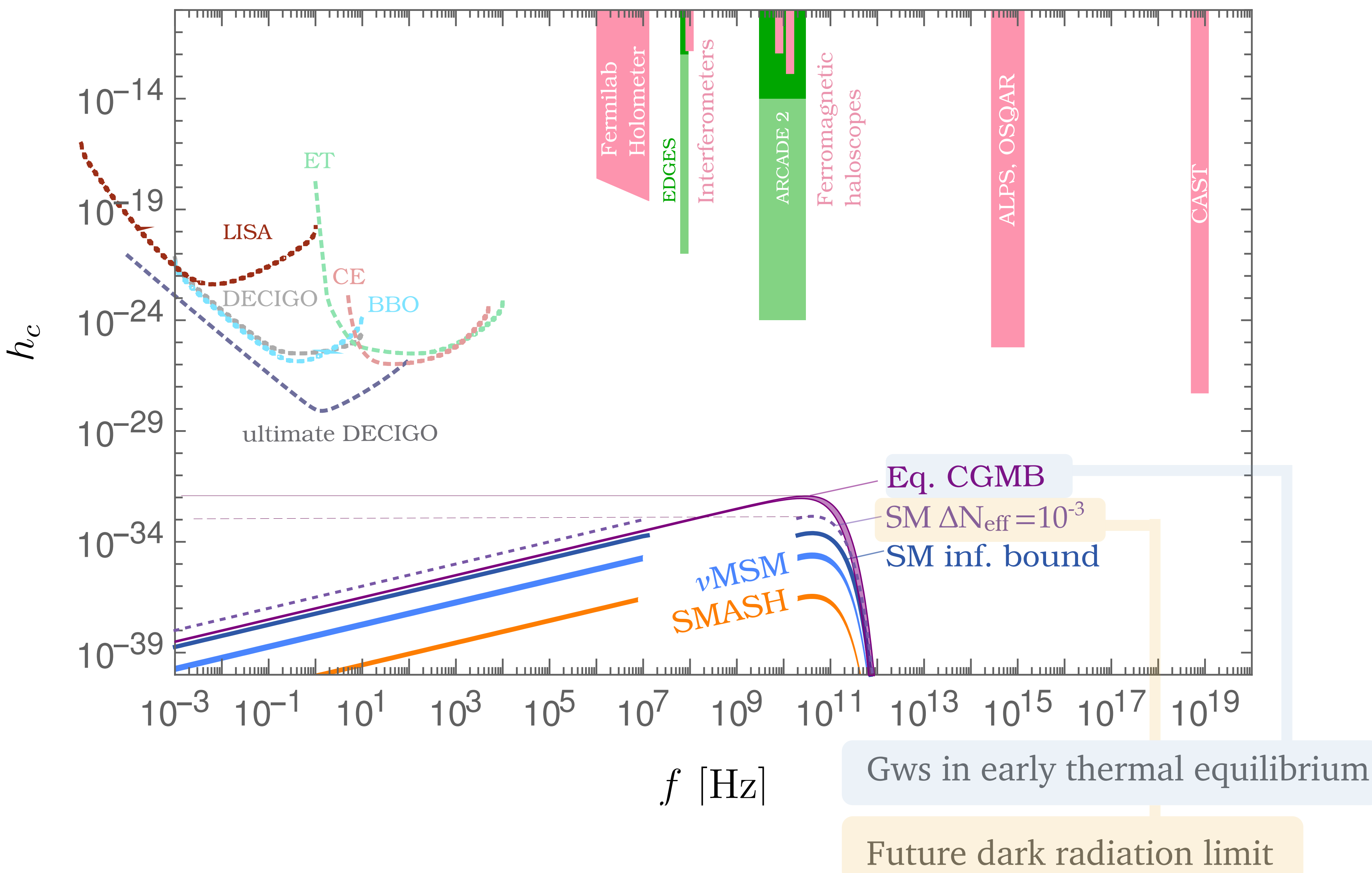
Inverse Gertsenshtein effect: Partial conversion of gravitational waves into electromagnetic radiation

Can use **axion experiments** featuring photon-axion conversion! [Cruise, Ellji et al]

Indirect searches

Gertsenshtein effect can modify the **CMB** spectrum! [Domcke & Cely]

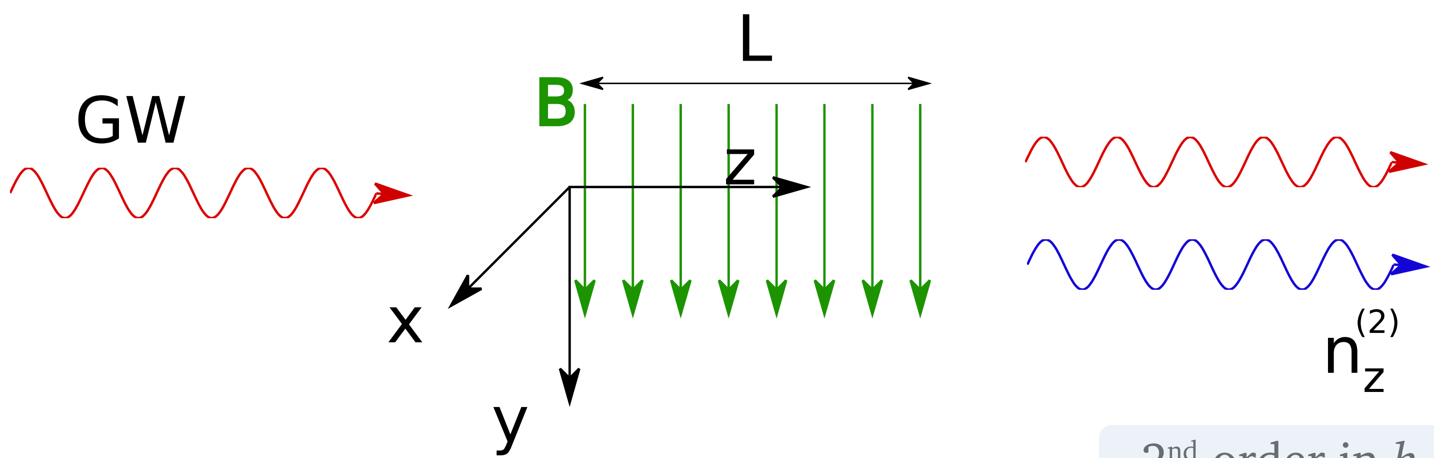
Current direct and indirect bounds



Future prospects in the laboratory

Magnetic GW-EMW conversion in vacuum

Gertsenshtein effect



2^{nd} order in h_c

Average power at terminal position of magnetic field (assuming coherence)

$$f \frac{\mathrm{d}P^{(2)}_{\mathrm{EMW}}}{\mathrm{d}f} \simeq \pi^2 f^2 h_c^2(f) B^2 L^2 A = 4.20 \times 10^{-23}\,\mathrm{W} \left[\frac{f}{40\,\mathrm{GHz}}\right]^2 \left[\frac{h_c(f)}{10^{-21}}\right]^2 \left[\frac{B}{\mathrm{T}}\right]^2 \left[\frac{L}{\mathrm{m}}\right]^2 \left[\frac{A}{\mathrm{m}^2}\right].$$

If B field surrounded by tube, **coherence** safe from waveguide effect for

$$f \gg f_c \equiv \frac{c_{11}}{\pi^2} \frac{L}{d^2} \simeq 5.5 \times 10^7\,\mathrm{Hz} \left[\frac{L}{\mathrm{m}}\right] \left[\frac{\mathrm{m}}{d}\right]^2$$

Heterodyne/single photon detectors

Sensitivity of Heterodyne radio wave receiver

$$\left[h_c^{\rm CGMB}\right]^{\rm HET}_{\rm sens} \simeq 9.65\times10^{-21}\left[\frac{\rm S/N}{2}\right]^{1/2}\left[\frac{\Delta t}{\pi\times10^{7}\ {\rm s}}\right]^{-1/4}\left[\frac{f}{40\ {\rm GHz}}\right]^{-3/4}\left[\frac{\Delta f}{f}\right]^{-1/4}\times$$

$$\times K_{\rm rec}^{1/2}\left[\frac{T_{\rm sys}}{4\,{\rm K}}\right]^{1/2}\left[\frac{B}{\rm T}\right]^{-1}\left[\frac{L}{\rm m}\right]^{-1}\left[\frac{A}{{\rm m}^2}\right]^{-1/2}.$$

Factor order 1

System noise temperature

Sensitivity of single photon detectors

$$\left[h_c^{\rm CGMB}\right]^{\rm SPD}_{\rm sens} \simeq 7.71\times10^{-24}\left[\frac{\rm S/N}{2}\right]^{1/2}\left[\frac{\Delta t}{\pi\times10^{7}\ {\rm s}}\right]^{-1/4}\left(\frac{\Delta\omega}{10^{-4}\,{\rm eV}}\right)^{-1/2}\times$$

$$\times\epsilon^{-1/2}\left[\frac{\Gamma_{\rm D}}{10^{-3}\ {\rm Hz}}\right]^{1/4}\left[\frac{B}{\rm T}\right]^{-1}\left[\frac{L}{\rm m}\right]^{-1}\left[\frac{A}{{\rm m}^2}\right]^{-1/2}.$$

Photon detection efficiency

Dark detection rate

Heterodyne/single photon detectors

$BLA^{1/2}$ sensitivty shared by **light-shining-through wall** experiments with optical cavities, **helioscopes** searching for magnetic conversion of axions

	B [T]	L [m]	d [m]	n_{tubes}	$BLA^{1/2}$	f_c [Hz]	$[h_c^{CGMB}]^{HET}_{sens}$	$[h_c^{CGMB}]^{SPD}_{sens}$
ALPS IIc	5.3	211	0.05	1	49.6 Tm^2	4.6×10^{12}	–	–
BabyIAXO	2.5	10	0.7	2	21.9 Tm^2	1.1×10^{9}	4.41×10^{-22}	3.52×10^{-25}
MADMAX	4.83	6	1.25	1	32.1 Tm^2	1.9×10^{8}	3.01×10^{-22}	2.40×10^{-25}
IAXO	2.5	20	0.7	8	87.7 Tm^2	2.2×10^{9}	1.10×10^{-22}	8.79×10^{-26}

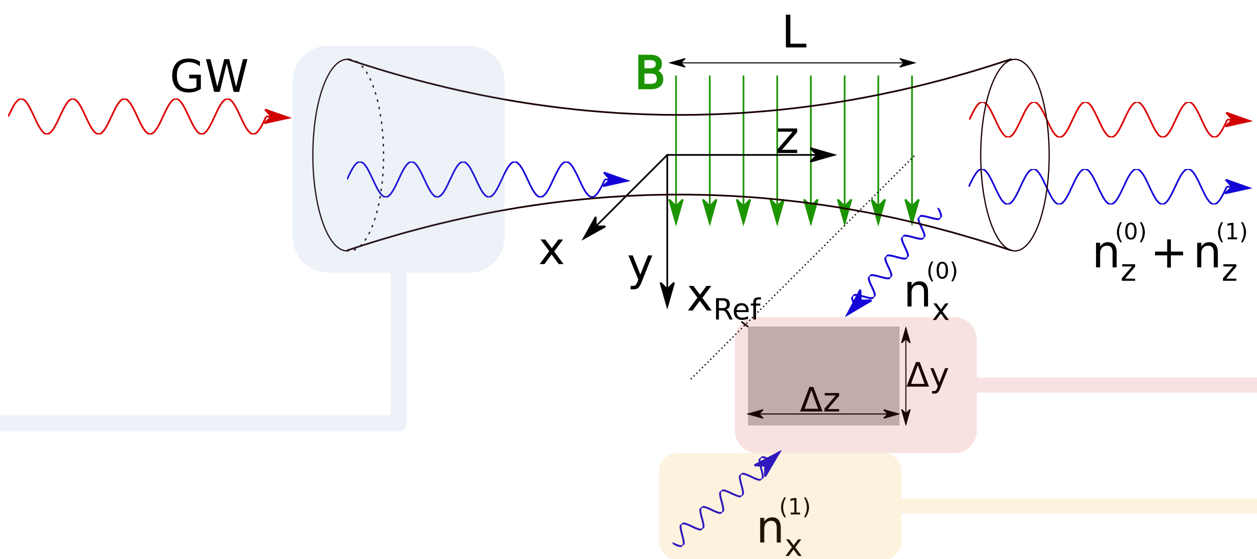
ALPS cannot access CGMB due to f_c above CGMB peak

Pretty hopeless with respect to:

CGMB with **early time equilibration** $h_c \sim 10^{-32}$
future **dark radiation** constraints $h_c \sim 10^{-33}$

GW-EMW conversion with Gaußian beam

Li et Al Gaußian beam proposal:



Powerful transverse **Gaußian beam**, polarized in x direction, tuned to GW freq.

Orthogonal photon flux, **first-order in h_c**!

Reflector

Pay the price of increased noise floor due to large EM fields

GW-EMW conversion with Gaußian beam

[Woods et al] argue that **main noise source** can be **dark count rate** of SPDs.

$$\left[h_c^{\mathrm{CGMB}}\right]^{\mathrm{GB}}_{\mathrm{sens}} \simeq 4.02\times10^{-29}\,\eta^{-1}\left[\frac{\mathrm{S/N}}{2}\right]\left[\frac{\Delta t}{10^4\,s}\right]^{-1/2}\left[\frac{\frac{\Delta f_0}{f_0}}{10^{-6}}\right]^{-1}\times$$
$$\times\,\epsilon^{-1}\left[\frac{\Gamma_{\mathrm{D}}}{10^{-3}\,\mathrm{Hz}}\right]^{1/2}\left[\frac{E_0}{5\times10^5\,\mathrm{V/m}}\right]^{-1}\left[\frac{B_y^{(0)}}{10\,\mathrm{T}}\right]^{-1}\left[\frac{L}{5\,\mathrm{m}}\right]^{-1}\left[\frac{\Delta y\Delta z}{0.01\,\mathrm{m}^2}\right]^{-1}\left[\frac{\mathcal{F}_x^{(1)}(x_{\mathrm{Ref}})}{10^{-5}}\right]^{-1}.$$

$0<\eta<1$ reflectivity of reflector

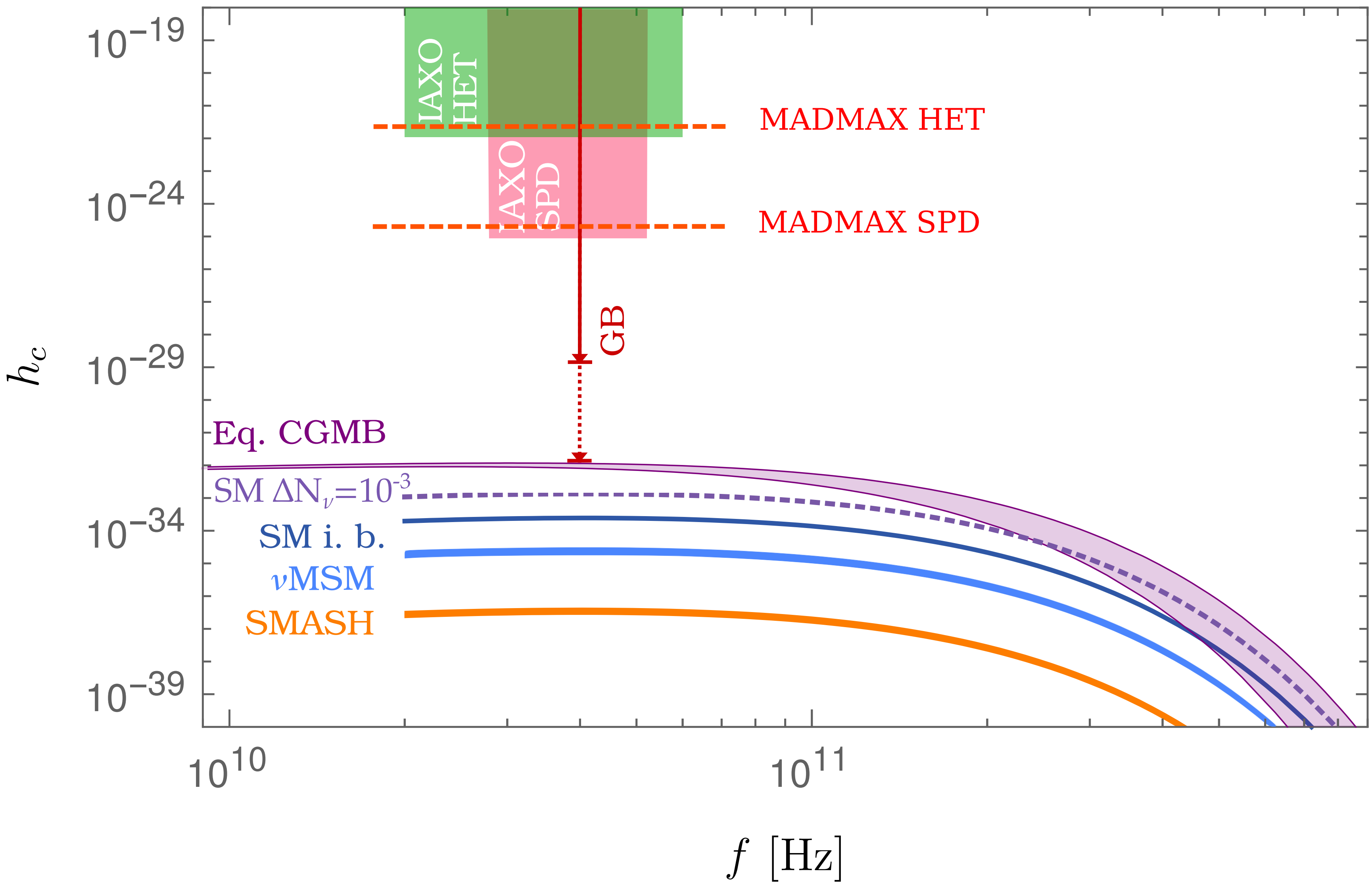
Quantity related to reflected flux reaching detector

$10^3\times$ (h_c with early time equilibration)

Possible 3 orders of mangitude increase from:

100× Higher power of gyrotron
100× stable running time
100× better dark count rate

GW-EMW conversion with Gaußian beam



Conclusions

There is a **guaranteed background** of gravitational waves of thermal origin, **peaking in the microwave region**: the CGMB

The **peak value** and **frequency** are directly **related** to the **maximum temperature of the Big Bang** and the **number** of **relativistic degrees** of **freedom** at **early times**

Current bounds are many **orders of magnitude above CGMB with early time equilibration** ($T_{\rm max} \sim M_P$) or reaching dark radiation limit $\Delta N_{\rm eff} \sim 10^{-3}$

Gertsenshtein effect of gravitational/electromagnetic wave conversion in a static magnetic field can be **exploited in axion experiments**

Exploiting Gaußian beams and **improvements** in **power** and **stability** of **gyrotrons**, and **dark count rate** of SPDs, could take us near **CGMB with early time equilibration**

Thank you!