

# Cosmological bubble friction in local equilibrium

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in collaboration with...

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### The aim:

Provide **new understanding** and **explicit calculations** for **bubble friction in local equilibrium**

### The novelty:

We **relate previous results** in the literature and provide a new **understanding** in terms of **entropy conservation**

We confirm directly the friction-like effect by studying time-dependent solutions, and **relate local friction** to the **field-dependence of enthalpy**

We also illustrate the effect for **detonations** in the wall frame

### The plan:

Usual understanding of bubble friction

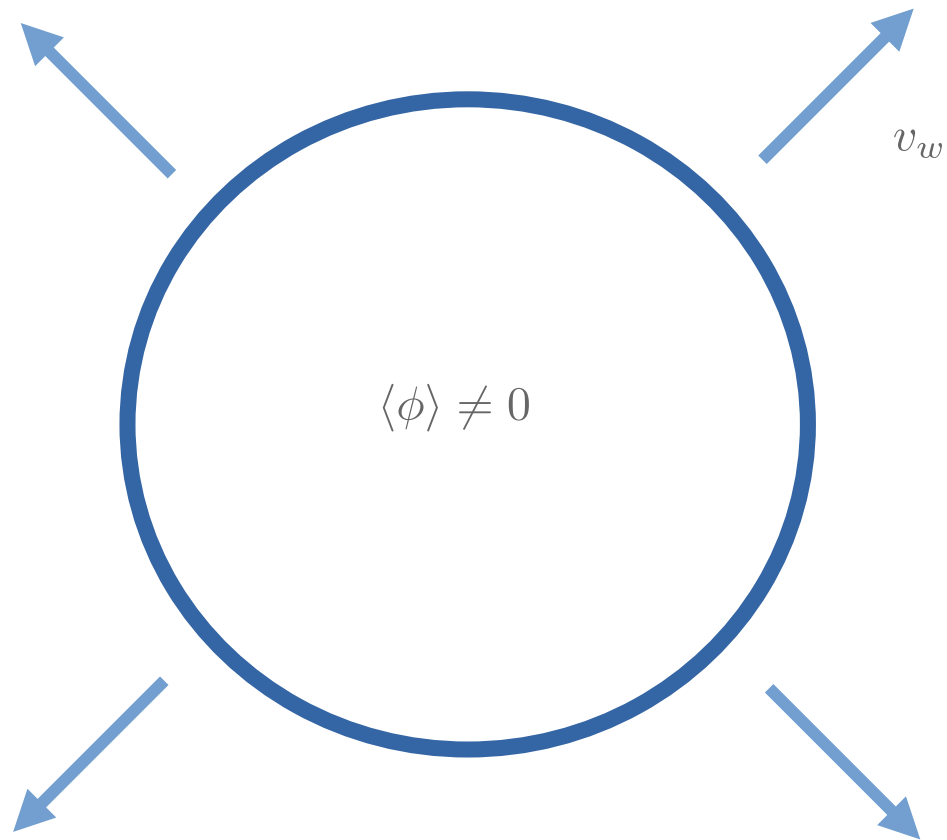
Friction in local equilibrium: previous literature

Friction in local equilibrium from local stress-energy conservation

Numerical studies

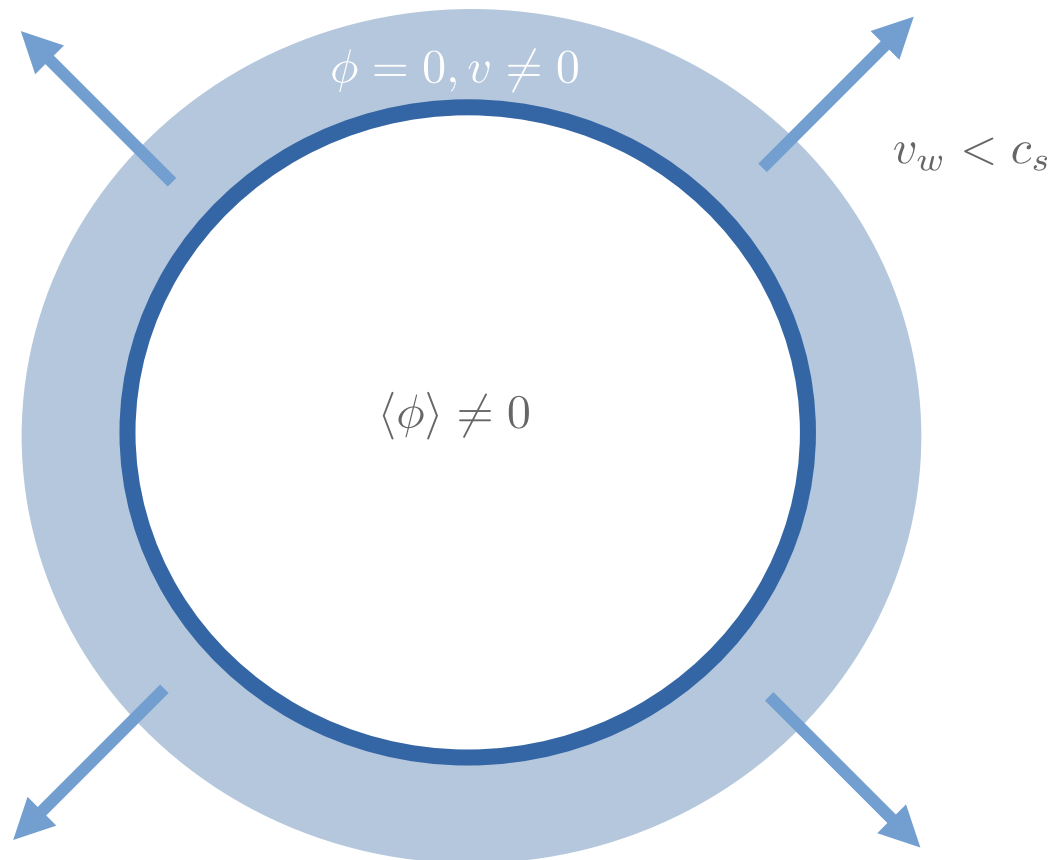
Usual understanding of bubble friction

# Bubble basics

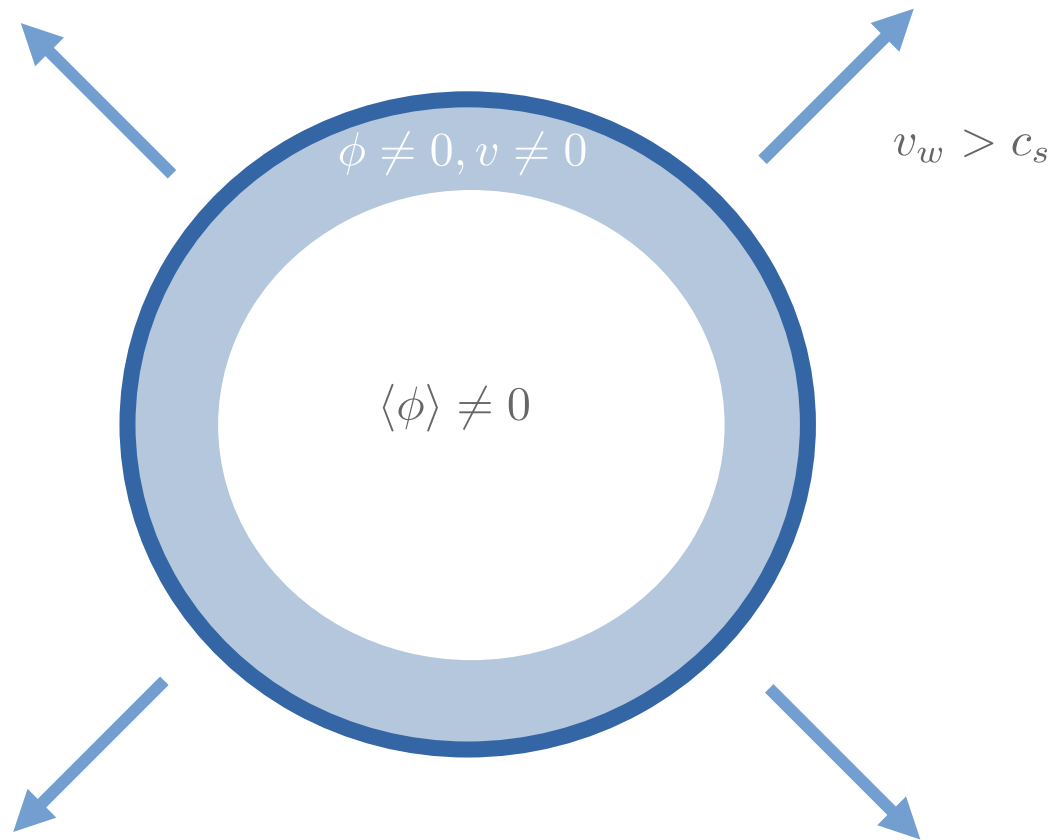


$$\langle \phi \rangle = 0, \quad T = T_{\text{nuc}}$$

# Bubble basics: deflagration



# Bubble basics: detonation



$$\langle \phi \rangle = 0, \quad T = T_{\text{nuc}}$$

# Bubble basics

- **Without friction** effects, latent heat converts to bulk motion and bubbles are expected to **accelerate** towards luminal speeds (**runaway** bubbles)
- **Colliding bubbles** source **gravitational waves**

Large bubble velocities imply more energy available for conversion into gws

- **Higgs bubbles** could lead to **electroweak baryogenesis**, usually requiring **low speeds** (see however [Cline & Kainulainen])

It is crucial to understand friction!



# Friction in the scalar equation of motion

- The usual treatment is based on the scalar equation of motion, averaged in plasma

$$f_i(\mathbf{p}, x) = f_i^{\text{eq}}(\mathbf{p}) + \delta f_i(\mathbf{p}, x)$$

[Prokopec-Moore '95]

$$\square\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} \delta f_i(\mathbf{p}, \mathbf{x}) = 0$$

finite T effective potential

Friction from deviations of equilibrium

- Going to wall frame, the previous eq. implies

$$\Delta V_T = - \sum_i \int d\phi \frac{dm_i^2}{d\phi} \int \frac{d^3\mathbf{p}}{(s\pi)^3 2E_i} \delta f_i(\mathbf{p}, x)$$

Driving force (pressure from change in potential energy at equilibrium)

Friction per unit area, out of eq. [Bödeker-Moore]

# Friction in local equilibrium?

- It would seem that **constant  $v_w$**  for  $T < T_c$  requires **non-equilibrium effects**. For **relativistic bubbles**:
  - **Leading order** friction  $v_w$ -independent: allows **runaways** [Bödeker-Moore]
  - Higher order effects  $v_w$ -dependent: ultrarelativistic but **subluminal** speeds  
[Bödeker-Moore] [Höche, Kozaczuk, Long, Turner, Wang]
- It is **commonly assumed** that there is **no friction in local equilibrium**

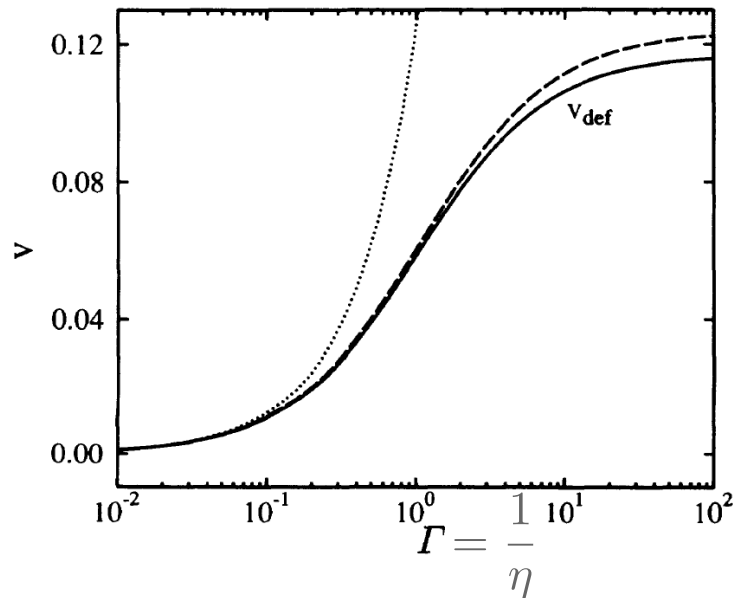
Local equilibrium: previous literature

# Friction in local equilibrium?

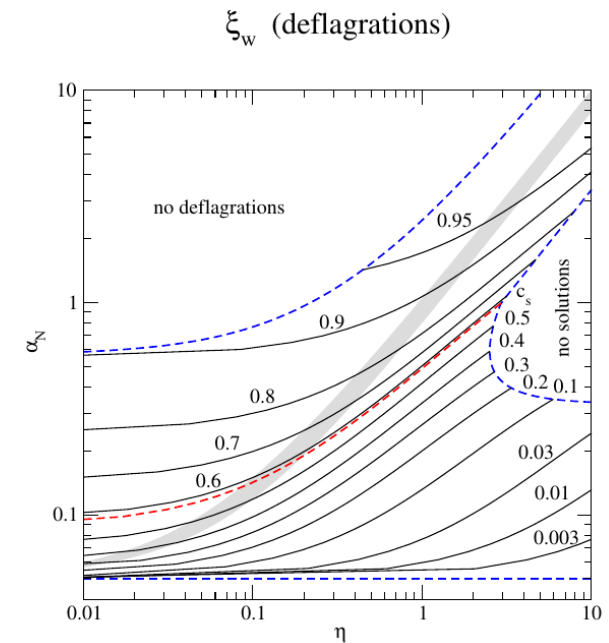
Phenomenological friction term

$$\square\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} + \eta u^\mu \partial_\mu \phi = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine '93]



[Espinosa, Konstandin, No, Servant '10]



Subluminal velocity for deflagrations without friction?

# Friction in local equilibrium

[Konstandin, No '10]

- First direct study of **bubble velocity** in **local equilibrium**.
- Subluminal velocities as a result of hydrodynamic equations causing the **fluid** to **heat up** in front of the bubbles, which **reduces driving force**
- Effect thought to happen **only** in **deflagrations**

# Friction in local equilibrium

[Barroso Mancha, Prokopec, Świeżewska '20]

- **Stress-energy conservation** plus **Lorentz invariance**, away from bubble wall

$$T_{\text{plasma}}^{\mu\nu} = (\rho + p)u^\mu u^\nu - p\eta^{\mu\nu}$$

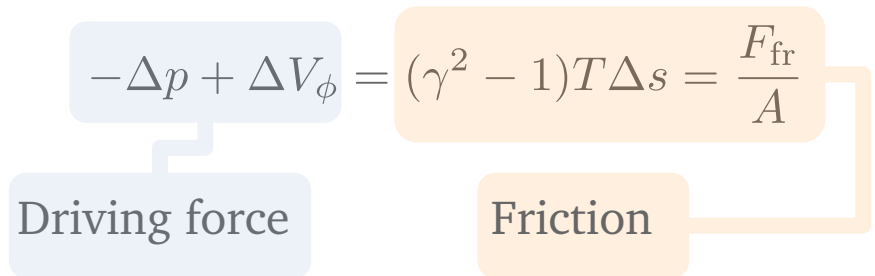
$$= Tsu^\mu u^\nu - p\eta^{\mu\nu}$$

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$$T_\phi^{\mu\nu} = \eta^{\mu\nu}V(\phi)$$


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$$\langle \Delta T_\phi^{zz} \rangle + \langle \Delta T_{\text{plasma}}^{zz} \rangle = 0$$



- **No distinction** between **detonations** and **deflagrations**
- **Friction grows with  $v_w$ : no runaway** behaviour
- D.o.f. in local equilibrium lead to larger friction than usually expected

# Open questions

- Is the **hydrodynamic obstruction** of [Konstandin, No] the **same** effect as the **friction force** of [Barroso Mancha, Prokopec, Świeżewska] ?
- If so, can one **extend results** of [Konstandin, No] to **detonations**?
- **Where is friction encoded** in the time-dependent, **differential equations** for the scalar and plasma?

# Friction in equilibrium from local stress-energy conservation



# Local stress-energy conservation

$$T^{\mu\nu} = T_{\phi}^{\mu\nu} + T_p^{\mu\nu}$$

$$T_{\phi}^{\mu\nu} = \partial^{\mu}\phi\partial^{\nu}\phi - \eta^{\mu\nu}\left(\frac{1}{2}\partial_{\rho}\phi\partial^{\rho}\phi - V(\phi)\right)$$

$$T_p^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - \eta^{\mu\nu}p = \omega u^{\mu}u^{\nu} - \eta^{\mu\nu}p$$

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$$\nabla_{\mu}T^{\mu\nu} = 0$$

$$\square\phi + \frac{\partial}{\partial\phi}(V(\phi) - p) = 0,$$

$$\partial_{\mu}(\omega u^{\mu}u^{\nu} - \eta^{\mu\nu}p) + \frac{\partial p}{\partial\phi}\partial^{\nu}\phi = 0.$$

[Ignatius, Kajantie, Kurki-Suonio, Laine '93]

Friction-like behaviour comes from field-dependence of  $\omega = Ts$

$p, \omega = Ts$  all calculable from the thermal effective potential!

# Total entropy conservation

- The previous equations can be seen to imply the conservation of the total entropy:

$$\frac{dS}{dt} = 0, \quad S = \int d^3x s \gamma(v)$$

- Entropy density** dominated by **relativistic degrees of freedom**

$$s = \frac{2\pi^2}{45} g_{*s} T^4$$

- Phase transition** makes some d.o.f heavy: **local decrease in entropy density** from decrease in  $g_{*s}$
- This has to be **compensated by a heating effect** in front or behind the bubble wall

→ Connection to [Konstandin, No], but **should also apply to detonations**

# Planar wall frame

Assuming stationary regime in the wall frame  $v^z \equiv \mathfrak{v}$

$$\begin{aligned} -\phi''(z) + \frac{\partial}{\partial \phi}(V(\phi, T)) &= 0, \\ \omega\gamma^2\mathfrak{v}^2 + \frac{1}{2}(\phi'(z))^2 - V(\phi, T) &= c_1, \\ \omega\gamma^2\mathfrak{v} &= c_2, \end{aligned}$$

Also solved in [Konstandin, No]  
cf [Espinosa, Konstandin, No, Servant]

From the second equation, comparing 2 sides of the wall where  $\phi' = 0$

$$\Delta V(\phi, T) = -\Delta p + \Delta V(\phi) = \Delta((\gamma^2 - 1)Ts) = \frac{F_{\text{fr}}}{V}$$

**Friction force** of [Barroso Mancha, Prokopec, Świeżewska] **recovered** when  
**assuming constant  $v, T$  across wall**

→ Same effect as hydrodynamic obstruction of [Konstandin, No]

# Reduction to single scalar equation

$$\begin{array}{l}
 -\phi''(z) + \frac{\partial}{\partial \phi}(V(\phi, T)) = 0, \\
 \hline
 \omega\gamma^2 \mathbf{v}^2 + \frac{1}{2} (\phi'(z))^2 - V(\phi, T) = c_1, \\
 \hline
 \omega\gamma^2 \mathbf{v} = c_2,
 \end{array}
 \quad \longrightarrow \quad
 \begin{array}{l}
 T = T(c_1, c_2, \phi, \phi') \rightarrow T(\mathbf{v}_+, T_+, \phi, \phi'), \\
 \mathbf{v} = \mathbf{v}(c_1, c_2, \phi, \phi') \rightarrow (\mathbf{v}_+, T_+, \phi, \phi'),
 \end{array}$$

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 \end{array}$$

$$-\phi''(z) + \frac{\partial}{\partial \phi} \hat{V}(\phi, T(\mathfrak{v}_+, T_+, \phi, \phi')) = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine]

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[Ignatius, Kajantie, Kurki-Suonio, Laine]

## Boundary conditions

$$\phi(z) \rightarrow 0, \quad z \rightarrow \infty, \quad \phi'(z) \rightarrow 0, \quad |z| \rightarrow \infty,$$

Additionally, expect that in **broken phase** field goes to a **minimum** [Konstandin, No]

$$\phi''(z) \rightarrow 0, \quad z \rightarrow -\infty$$

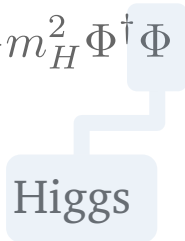
These conditions fix  $\mathbf{v}_+$  in terms of  $T_+$ . Latter fixed by nucleation temperature away from wall (accounting from extra hydrodynamic profile for deflagrations)

# Numerical studies

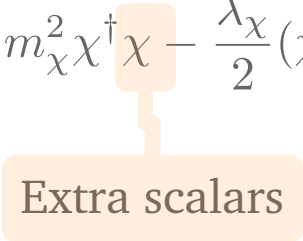
# Example model

**SM extension** by  $N$  additional **complex singlets** allowing for **first order phase transition** for the Higgs

$$\mathcal{L} \supset -m_H^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 - m_\chi^2 \chi^\dagger \chi - \frac{\lambda_\chi}{2} (\chi^\dagger \chi)^2 - \lambda_{H\chi} \Phi^\dagger \Phi \chi^\dagger \chi.$$



Higgs



Extra scalars

**Pressure** from **thermal corrections to potential** in high- $T$  expansion

$$p(h, T) =$$

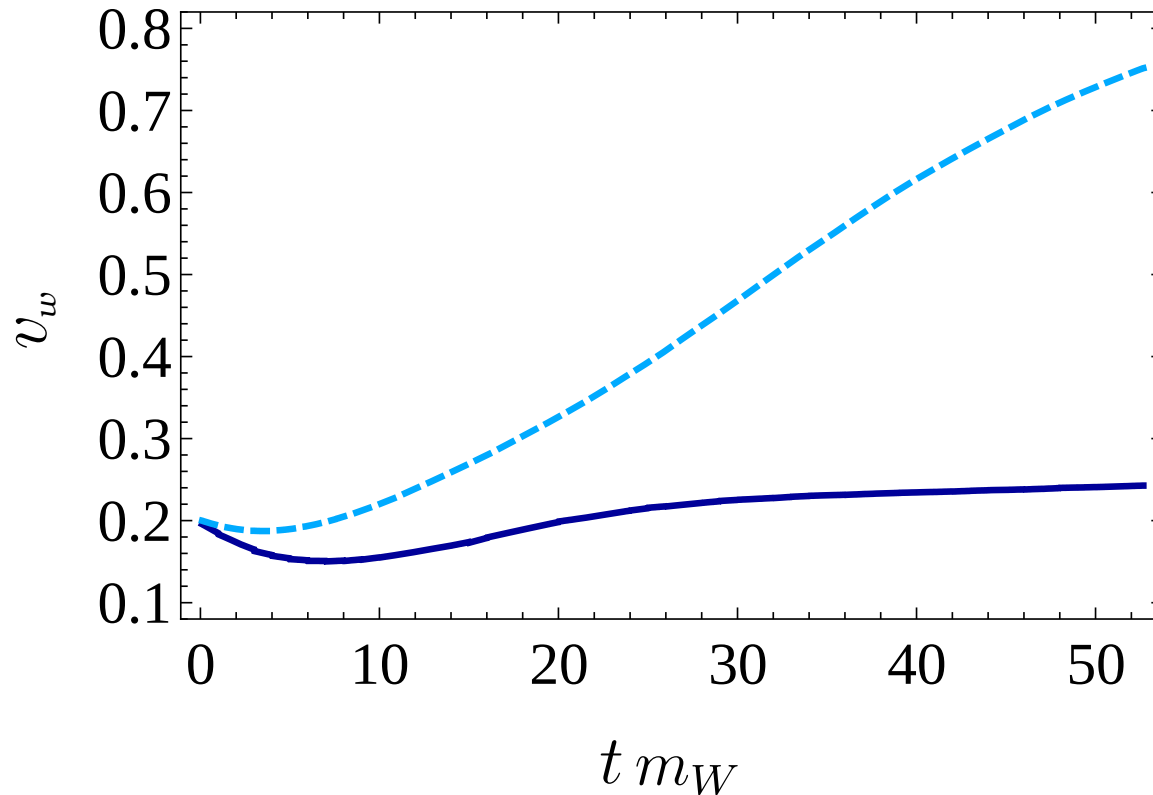
$$\frac{\pi^2 T^4}{90} (g_{*,\text{SM}} + 2N) - T^2 \left( h^2 \left( \frac{y_b^2}{8} + \frac{3g_1^2}{160} + \frac{3g_2^2}{32} + \frac{\lambda}{8} + \frac{N\lambda_{H\chi}}{24} + \frac{y_t^2}{8} \right) + \frac{m_H^2}{6} + \frac{Nm_\chi^2}{12} \right)$$

$$- \frac{T}{12\pi} \left( -\frac{3}{4} (g_2 h)^3 - \frac{3h^3}{8} \left( \frac{3g_1^2}{5} + g_2^2 \right)^{3/2} - 3 \left( \frac{h^2 \lambda}{2} + m_H^2 \right)^{3/2} - \left( \frac{3h^2 \lambda}{2} + m_H^2 \right)^{3/2} \right.$$

$$\left. - 2N \left( \frac{h^2 \lambda_{H\chi}}{2} + m_\chi^2 \right)^{3/2} \right)$$



# Time-dependent deflagrations



Ignoring  $\frac{\partial \omega}{\partial \phi} = \frac{\partial(Ts)}{\partial \phi}$

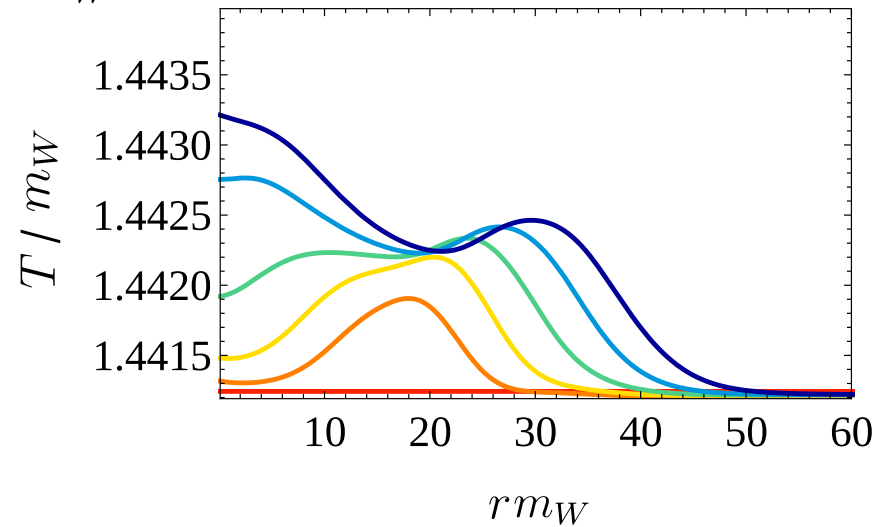
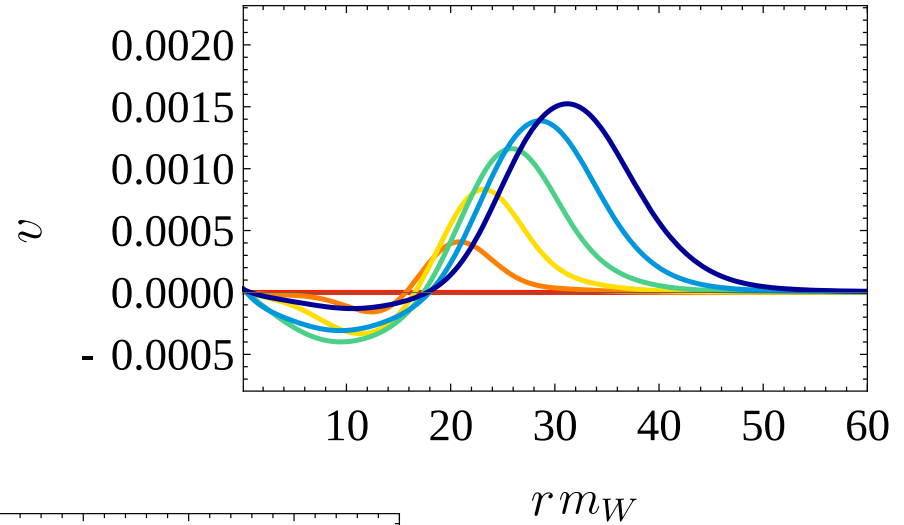
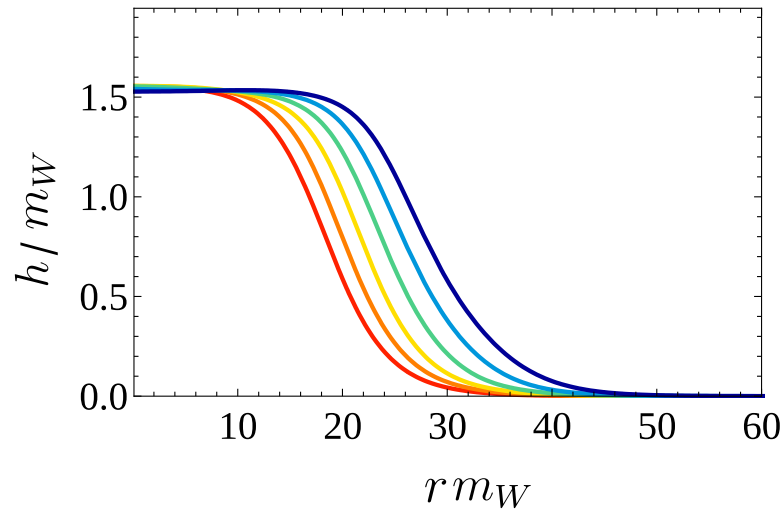
Accounting for  $\frac{\partial \omega}{\partial \phi} = \frac{\partial(Ts)}{\partial \phi}$

**Friction-like behaviour!**

$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H_\chi} = 0.85$$

- Obtained with neural network pre-trained with Mathematica solution

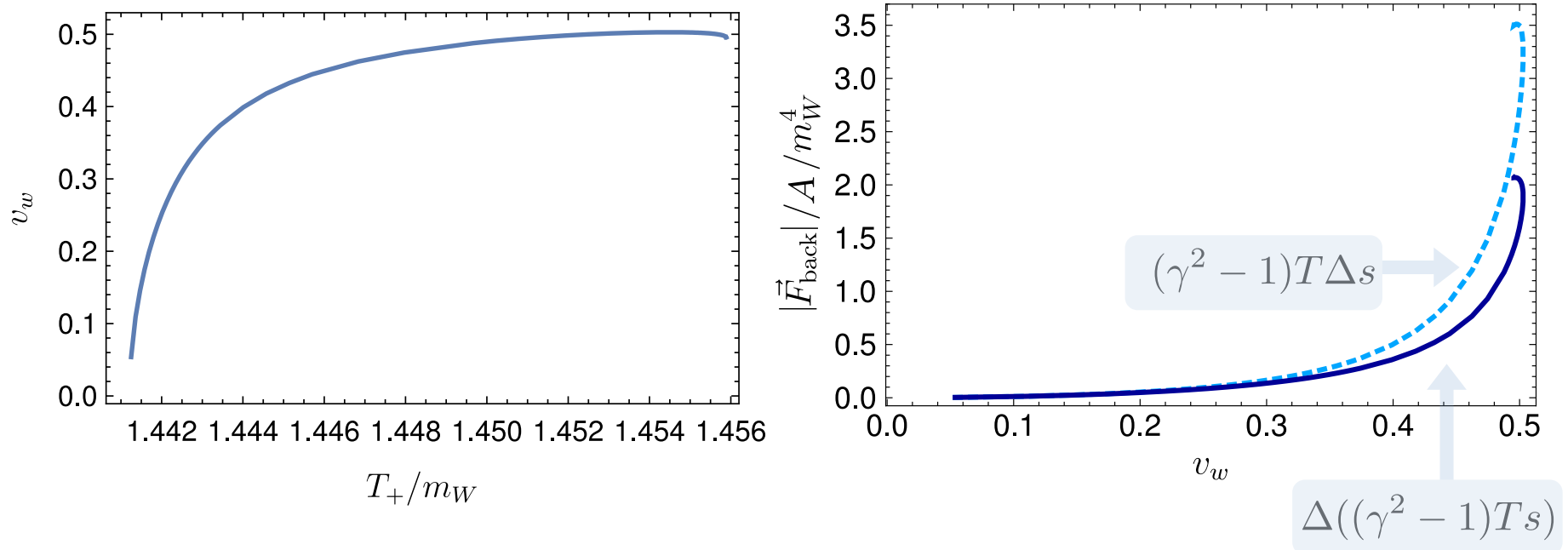
# Time-dependent deflagrations



$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

# Static deflagrations in wall frame

Family of solutions without necessarily imposing  $\phi''(z) \rightarrow 0, \quad z \rightarrow -\infty$



**Friction force grows with velocity!**

Physical case with  $\phi''(-\infty) \rightarrow 0$  corresponds to right endpoint of curves

$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

# Novel static detonations in wall frame

- The solutions  $T(\mathbf{v}_+, T_+, \phi, \phi'), \mathbf{v}(\mathbf{v}_+, T_+, \phi, \phi')$  are actually **multivalued**, and so is the “pseudopotential”  $\hat{V}(\phi, T(\mathbf{v}_+, T_+, \phi, \phi'))$
- We find that a branch of solutions with larger fluid velocities supports **static detonation solutions**
- We have found that the friction force can deviate from [Barroso Mancha et al] by a factor of 3

# Conclusions

Even in **local equilibrium**, there is a **non-dissipative**, friction-like **backreaction effect**

This effect is behind the runaway obstruction of [Konstandin, No] and the friction force of [Barroso Mancha, Prokopec, Świeżewska]

We provided an **intuitive understanding** based on **entropy conservation**

By solving the time-dependent equations for bubble propagation, we showed that the **backreaction** is **generated locally** by the **field-derivatives of the enthalpy**

We showed that, as expected from the results of [Barroso Mancha et al], the **backreaction exists for detonations**, yet **accurate estimates** of the friction force require **tracking changes** of  $v, T$  across the bubble.

Thank you!