

# **Inflation with strongly non-geodesic motion: theoretical motivations and observational imprints**

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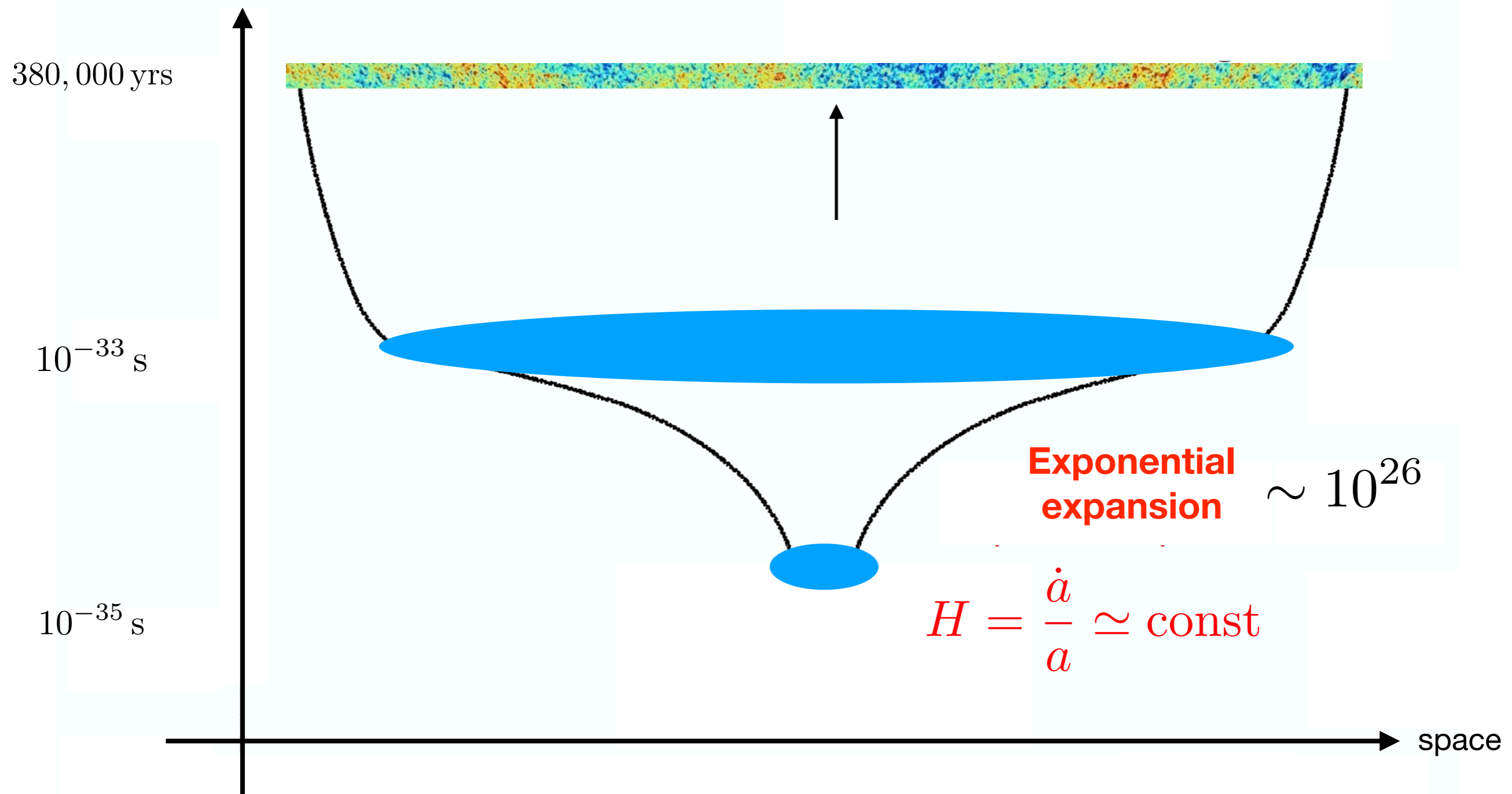
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**GEO****DESI**



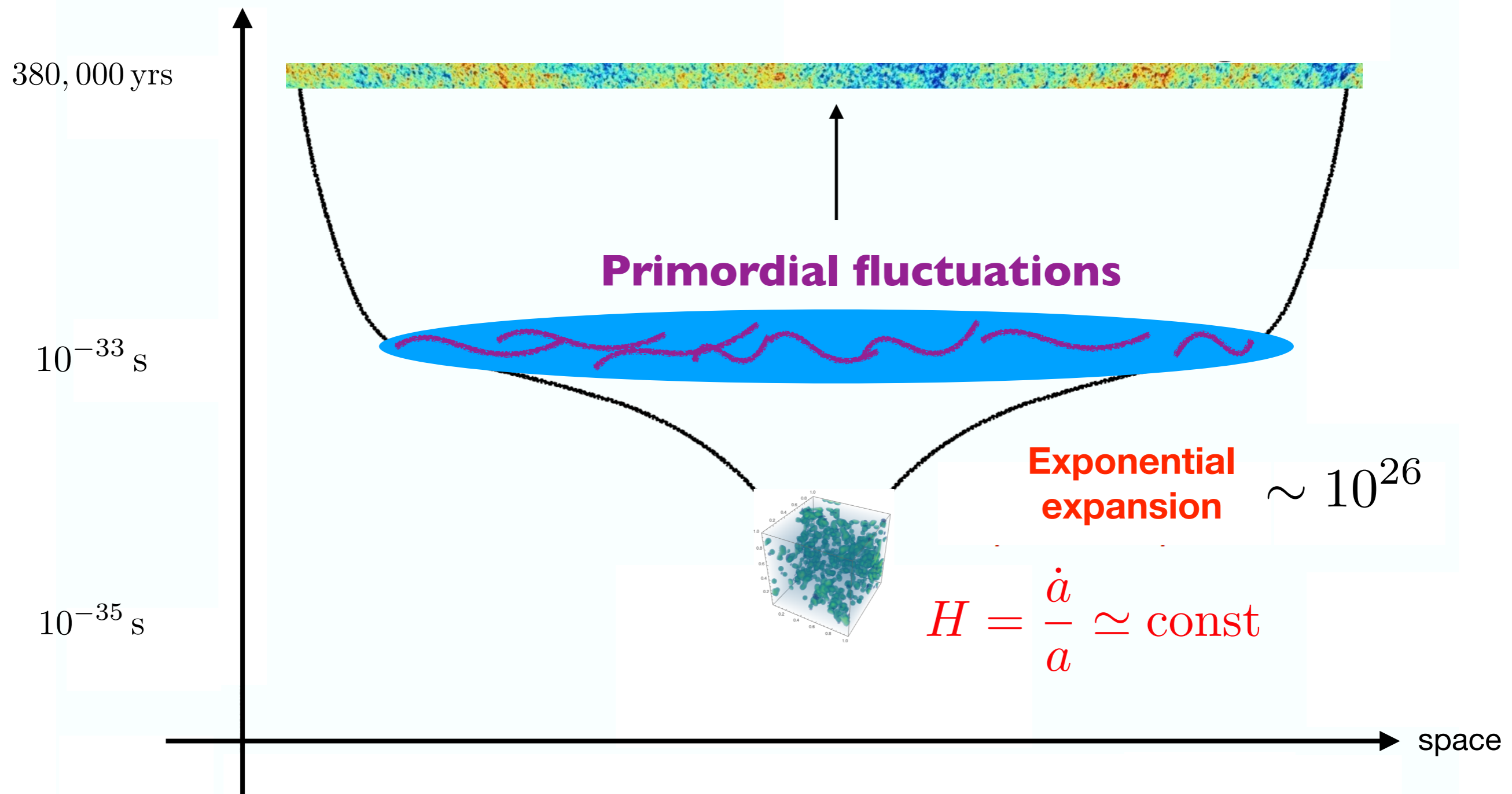
# ***Inflation: a giant microscope***

a tiny patch of space becomes the entire observable universe

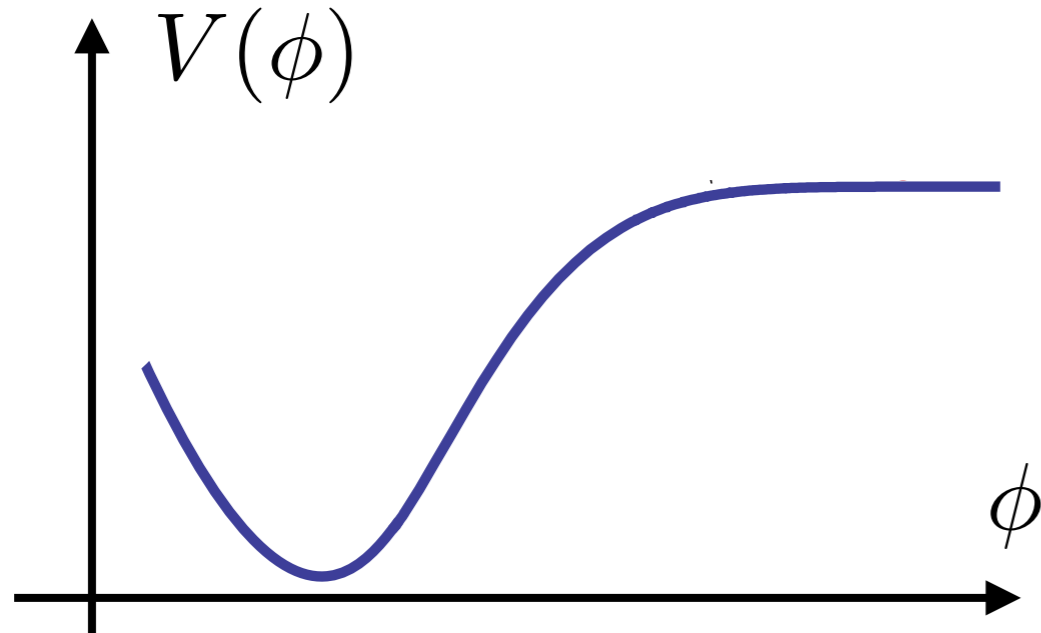


# ***Inflation: a giant microscope***

vacuum quantum fluctuations stretched to cosmological scales



# The Eta problem



$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{\text{Pl}}^2}{2} \left( \frac{V_{,\phi}}{V} \right)^2 \ll 1$$

$$\eta \equiv M_{\text{pl}}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Prolonged phase of inflation

Why is the inflaton so light?

$$\eta \sim \frac{m_\phi^2}{H^2} \ll 1$$

$$m_\phi^2 \sim \Lambda_{\text{cut-off}}^2 \gg H^2$$

like the Higgs hierarchy problem

# ***UV-sensitivity of inflation***

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_{\delta} \frac{\mathcal{O}_{\delta}(\phi)}{M^{\delta-4}}$$

Slow-roll action

Corrections to the low-energy  
effective potential



$$\frac{\Delta m_{\phi}^2}{H^2} \sim \left( \frac{M_{\text{Pl}}}{M} \right)^2$$

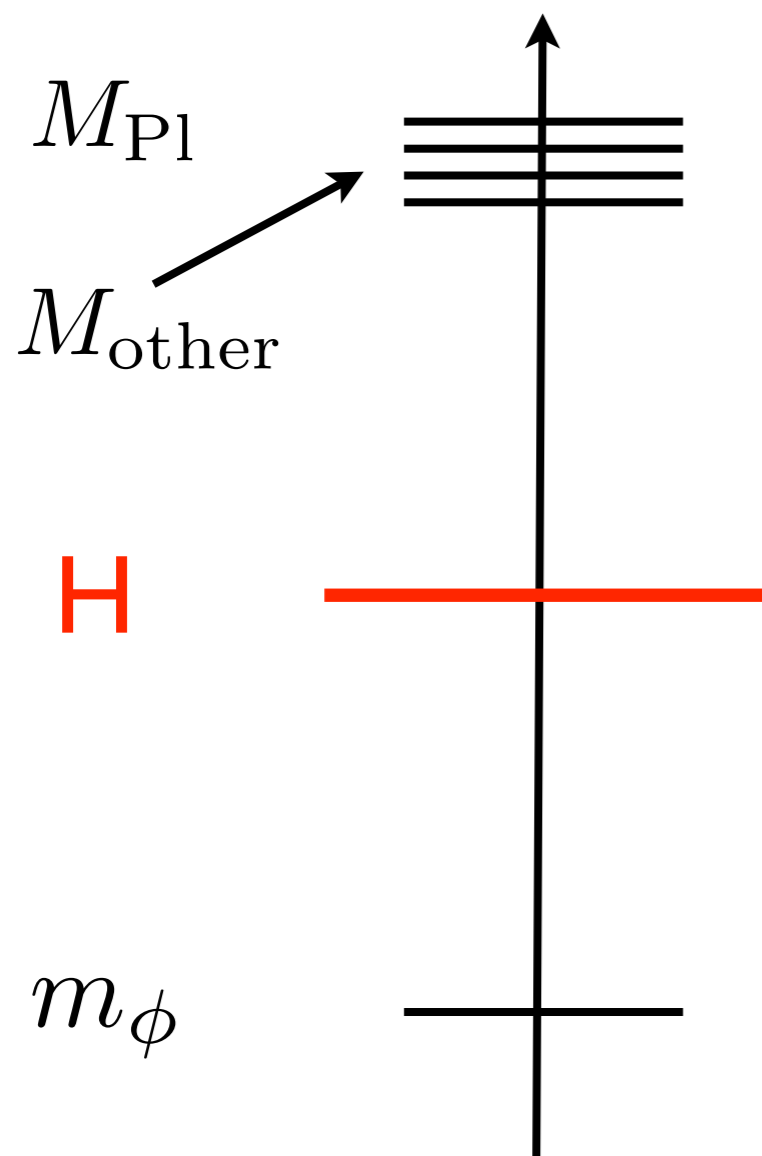


$$\Delta\eta \gtrsim 1$$

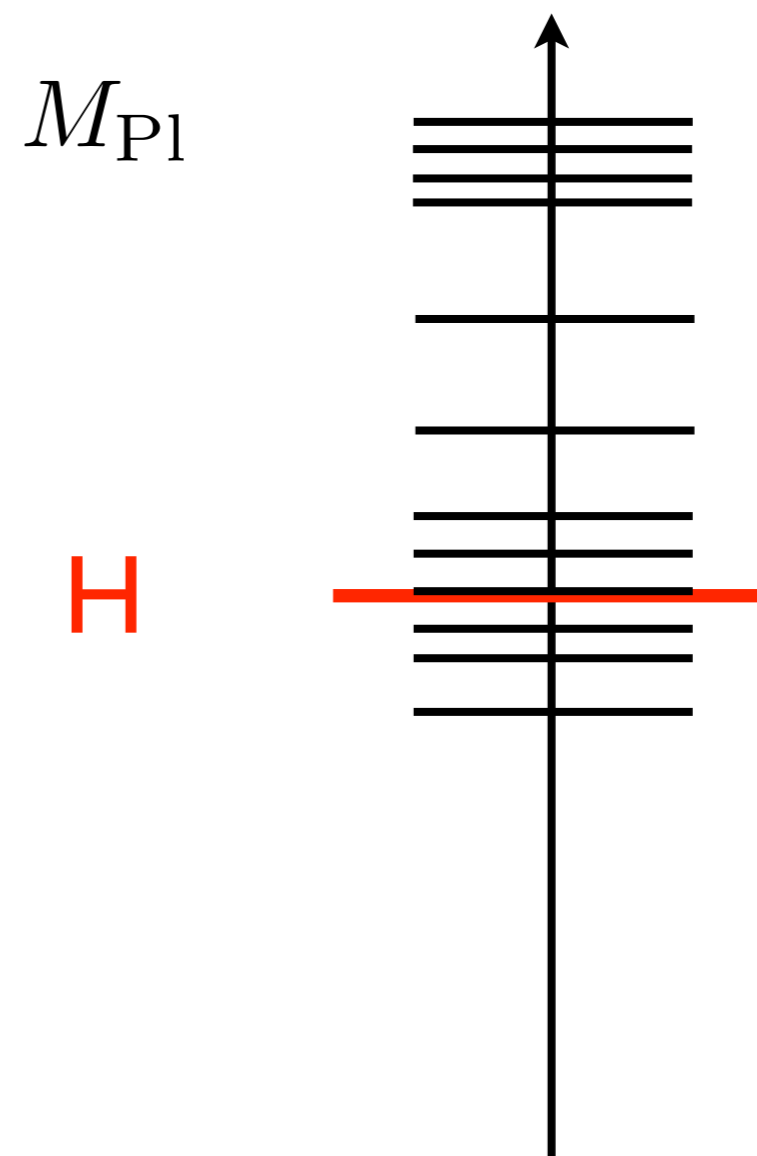
Planck-scale physics  
does not decouple

# Guidance from string theory?

Hope:

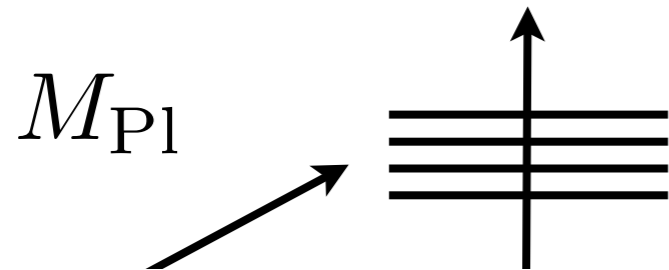


Find:



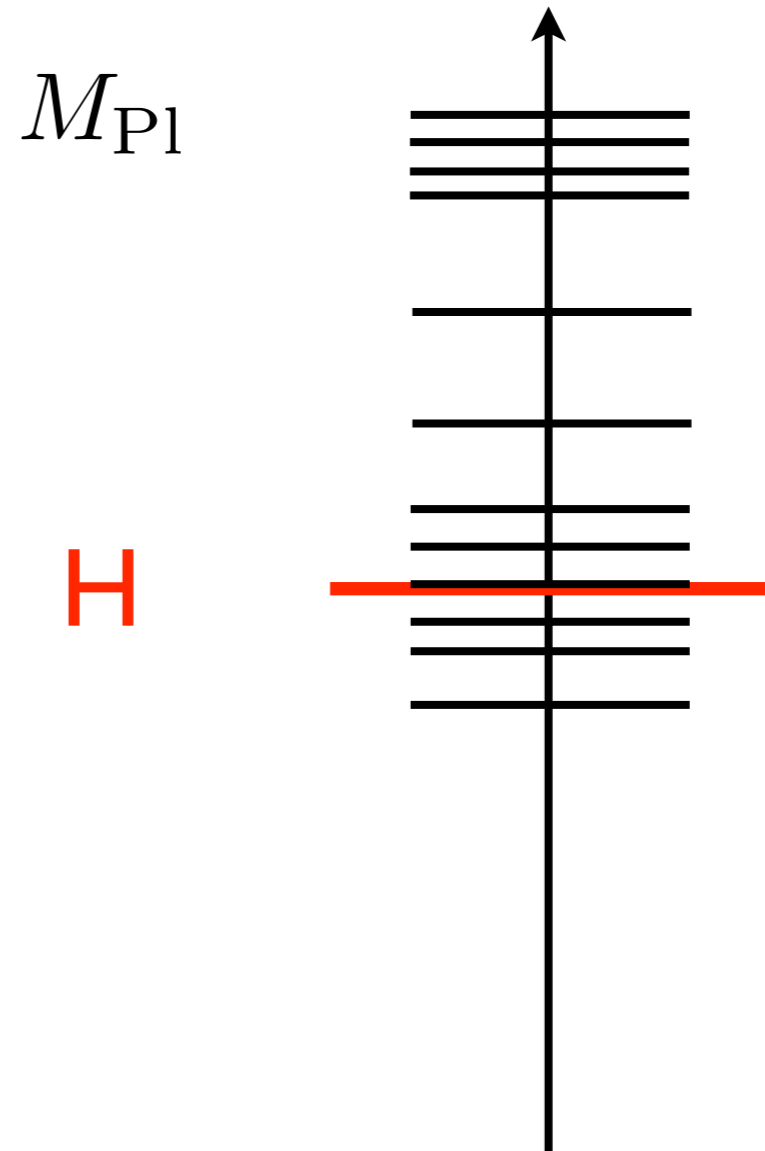
# Guidance from string theory?

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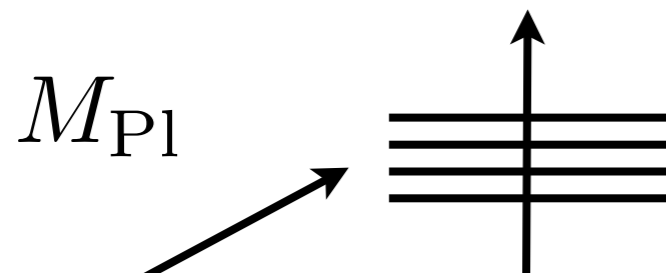
- Multiple degrees of freedom
- Steep potentials
- Large couplings

Find:

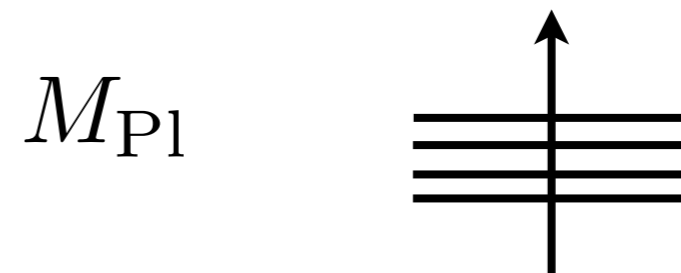


# Guidance from string theory?

Hope:



Find:



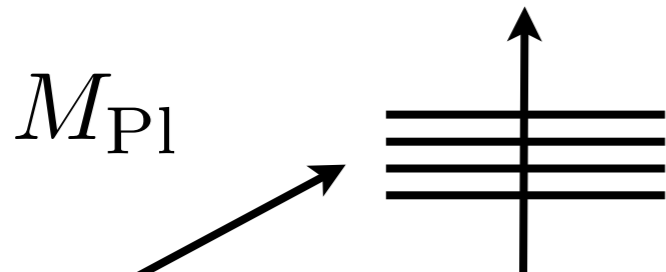
- Multiple degrees of freedom
- Steep potentials
- Large couplings

Single-field slow-roll:  
at best **emergent approximate description**

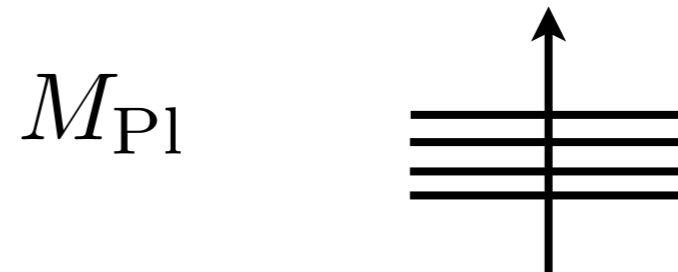
$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} G_{IJ}(\phi^K) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^I) \right)$$

# Guidance from string theory?

Hope:



Find:



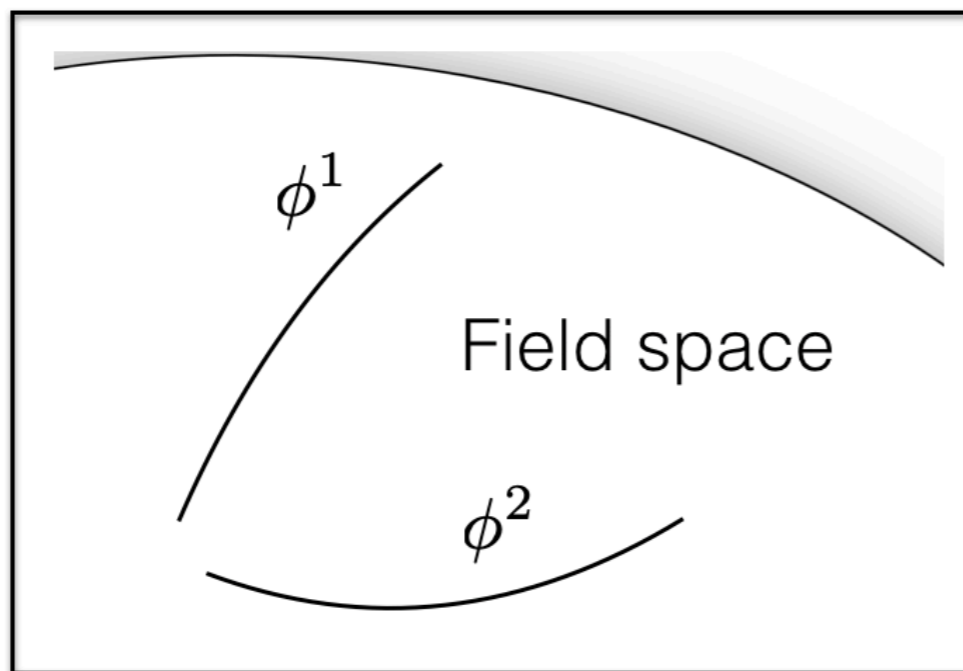
- Encompass **large class of top-down constructions**
- Useful test-bed to **sharpen our understanding**
- Reveals **new mechanisms to inflate** and **new EFT of fluctuations**

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} G_{IJ}(\phi^K) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^I) \right)$$

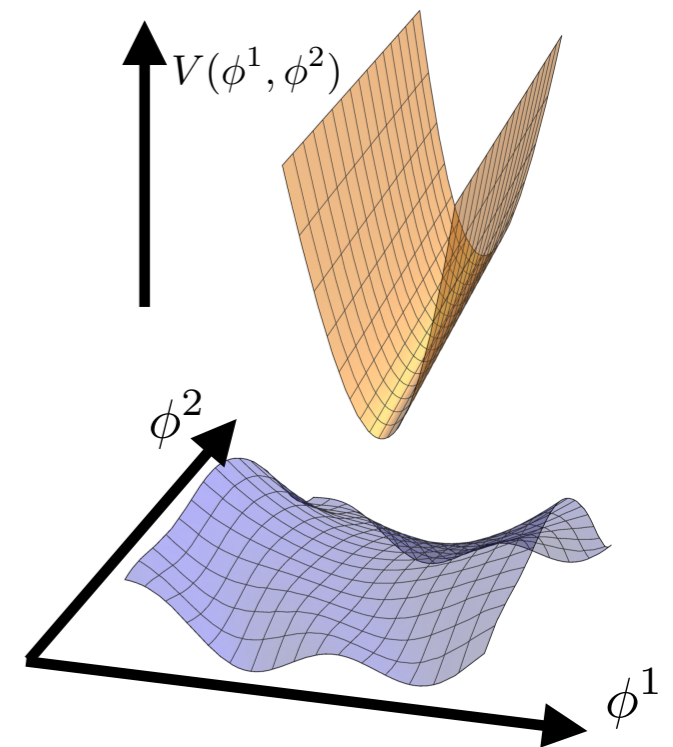
# Inflation in curved field space

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} G_{IJ}(\phi^K) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^I) \right) \quad \text{Top-down (e.g. sugra) or bottom-up (EFT)}$$

**Curved field space is generic**

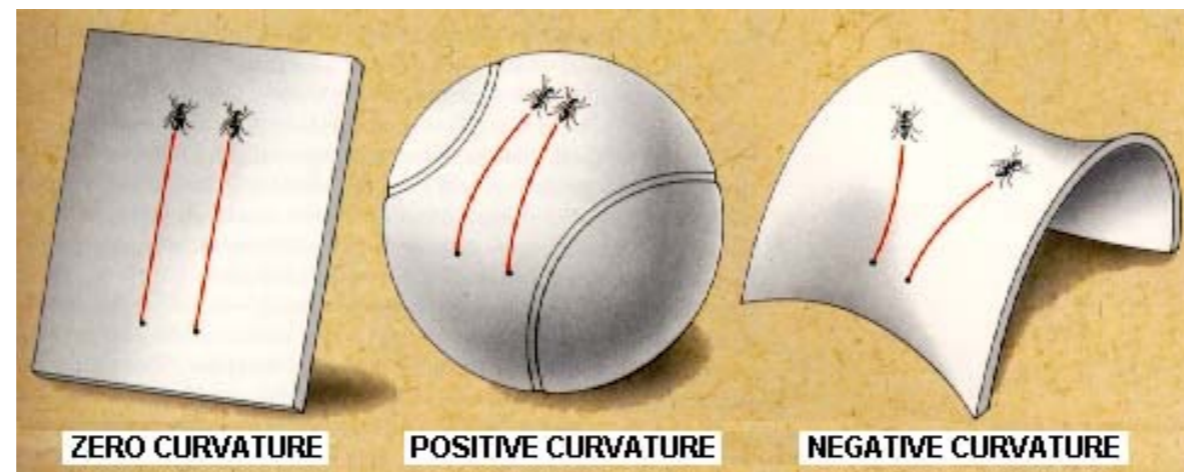


Invariance under field redefinitions:  
fields are coordinates on a 'field space',  
with metric  $G_{IJ}$



# ***Geometrical destabilization of inflation***

Initially neighboring geodesics tend to fall away from each other in the presence of **negative curvature**.

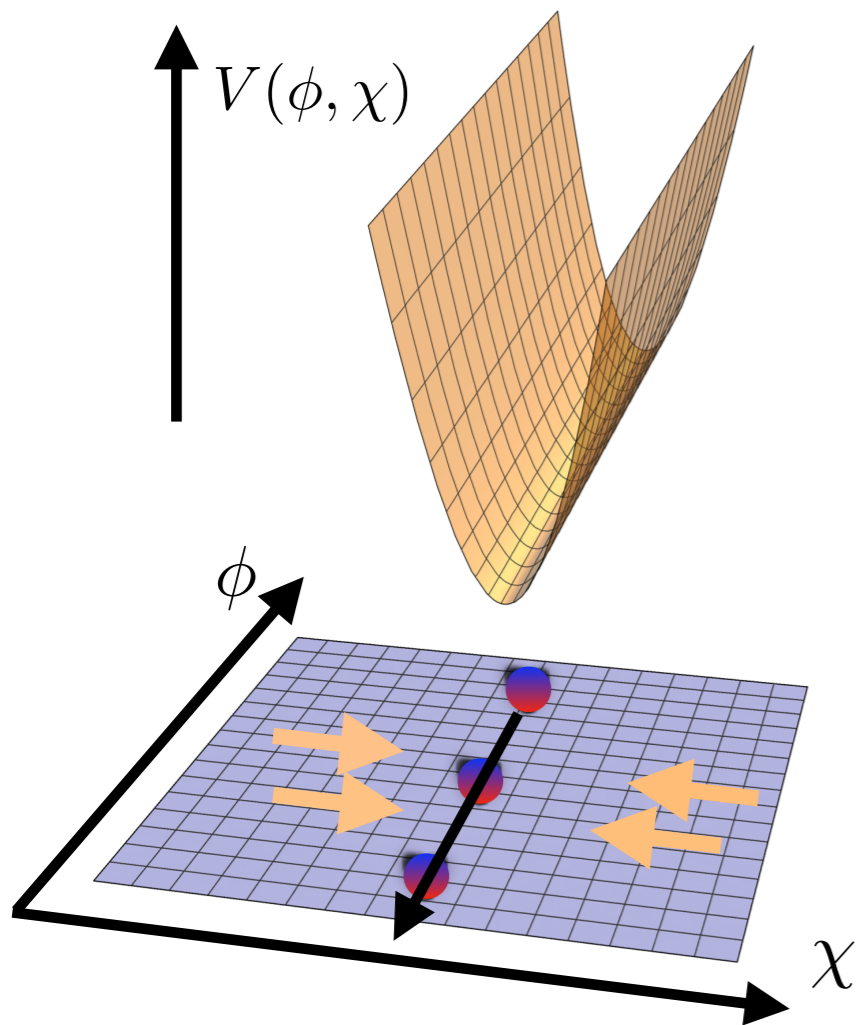


This effect applies during inflation, it can overcome the effect of the potential, and can destabilize inflationary trajectories.

# Minimal realization

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2$$

Inflaton  $\phi$  + Heavy field  $\chi$



Hope



Effective  
single-field dynamics

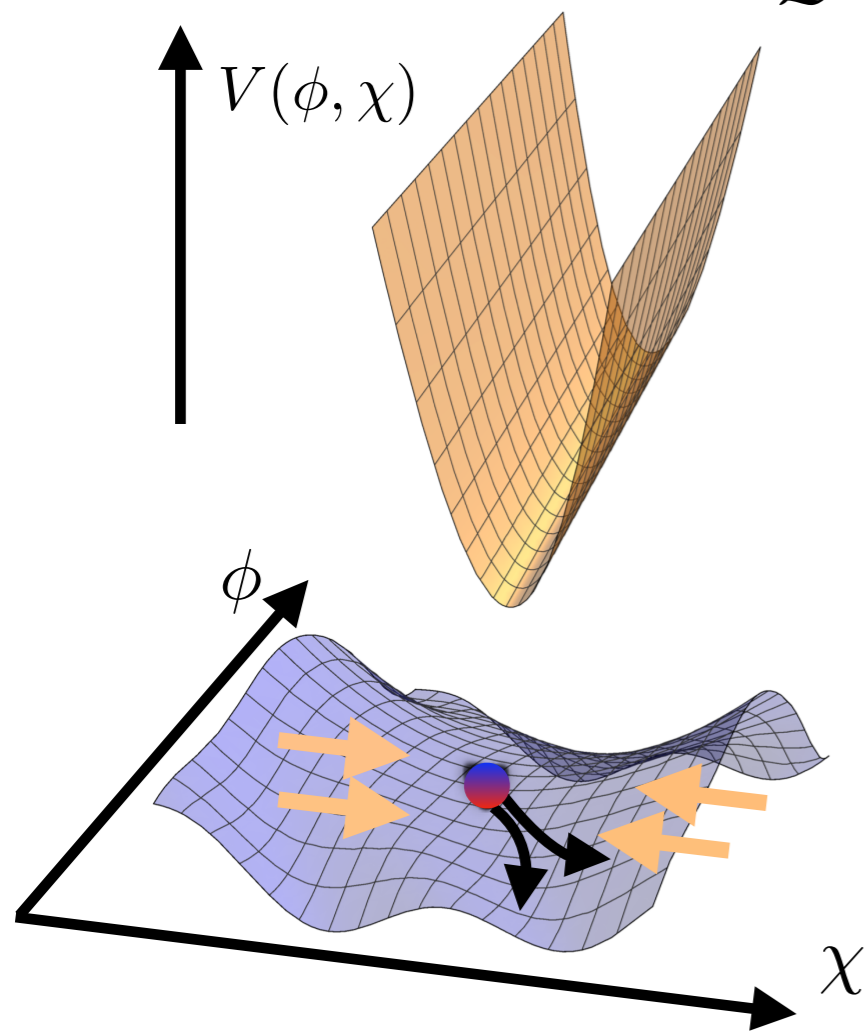
(valley with steep walls)

# Minimal realization

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \left( 1 + 2\frac{\chi^2}{M^2} \right) - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2$$

Inflaton  $\phi$  + Heavy field  $\chi$

+ curved field space / derivative interactions



More realistic



$$m_{\chi,\text{eff}}^2 = m^2 - 2\frac{\dot{\phi}^2}{M^2}$$

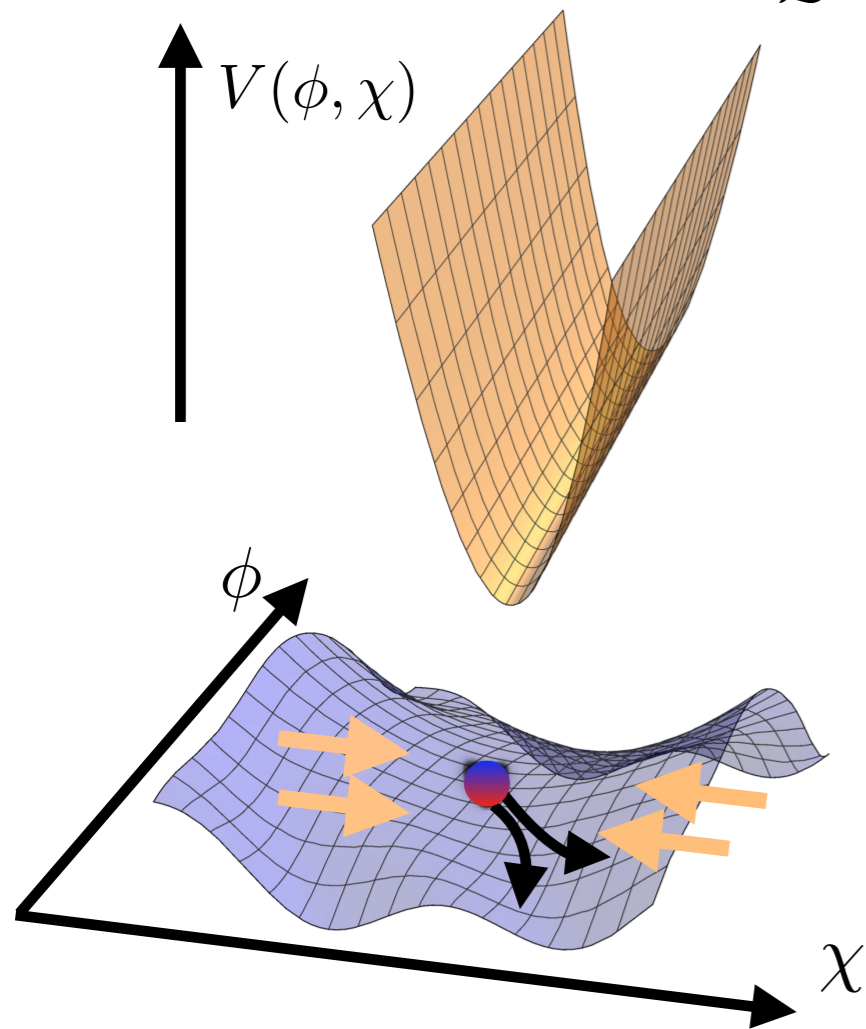
Competing effects of  
potential and geometry

# Minimal realization

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \left( 1 + 2\frac{\chi^2}{M^2} \right) - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2$$

Inflaton  $\phi$  + Heavy field  $\chi$

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More realistic



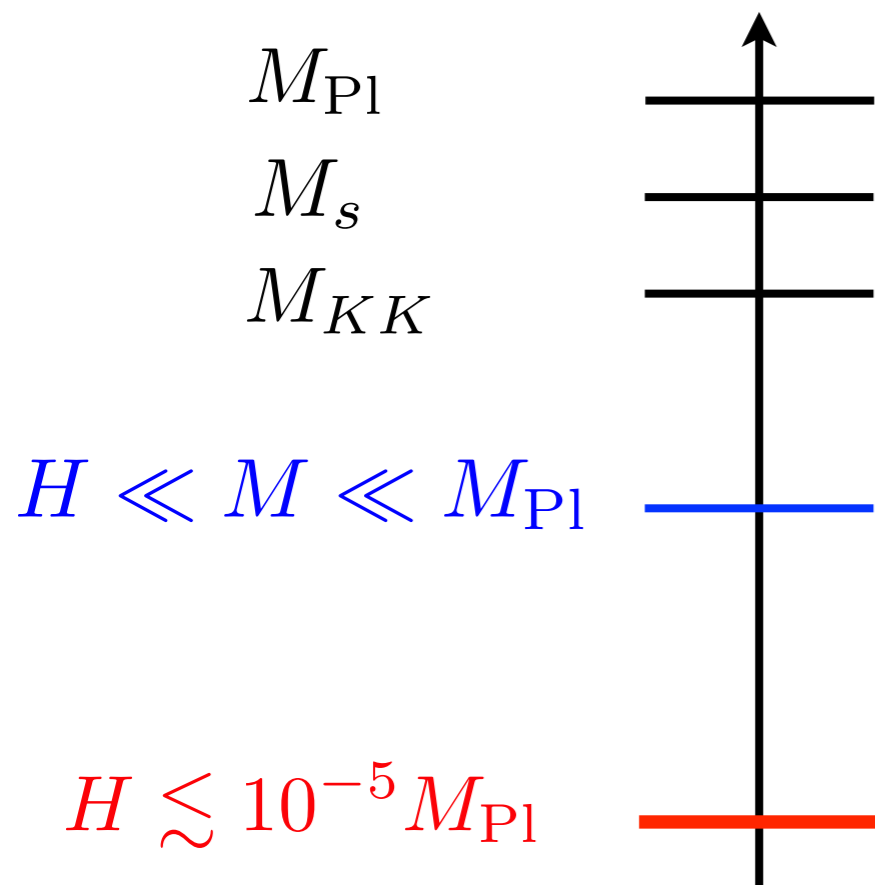
$$\frac{m_{\chi,\text{eff}}^2}{H^2} = \frac{m^2}{H^2} - 4\epsilon(t) \left( \frac{M_{\text{Pl}}}{M} \right)^2$$

Competing effects of  
potential and geometry

# Geometrical destabilization of inflation

$$\frac{m_{\chi, \text{eff}}^2}{H^2} = \frac{m^2}{H^2} - 4\epsilon(t) \left( \frac{M_{\text{Pl}}}{M} \right)^2 \quad \text{Particular case of general result:}$$

Rolling of the inflation in a negatively curved field space tends to induces an instability



**A large hierarchy** is generic in string theory constructions

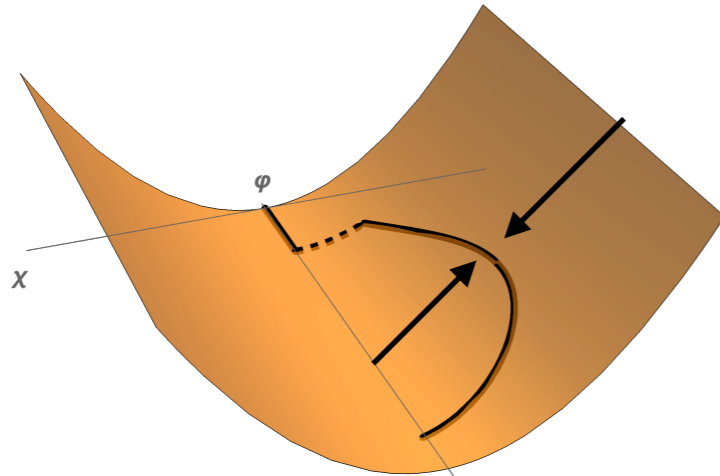
$$R^{\text{field space}} M_{\text{Pl}}^2 \sim (M_{\text{Pl}}/M)^2 \sim 10^5$$

Can easily compensate  $\epsilon$  suppression

**Destabilize** would-be stable trajectories

# Fate: sidetracked inflation

Garcia-Saenz, RP, Ronayne  
[1804.11279 JCAP](#)



Characteristic features  
 seen in minimal realization (toy model)

$$\ddot{\chi} + 3H\dot{\chi} - 2\frac{\dot{\varphi}^2}{M^2}\chi + V_{,\chi} = 0$$

Additional field: at **minimum of effective potential, depends on kinetic energy of inflaton**

like ‘gelaton’ Tolley, Wyman 2009

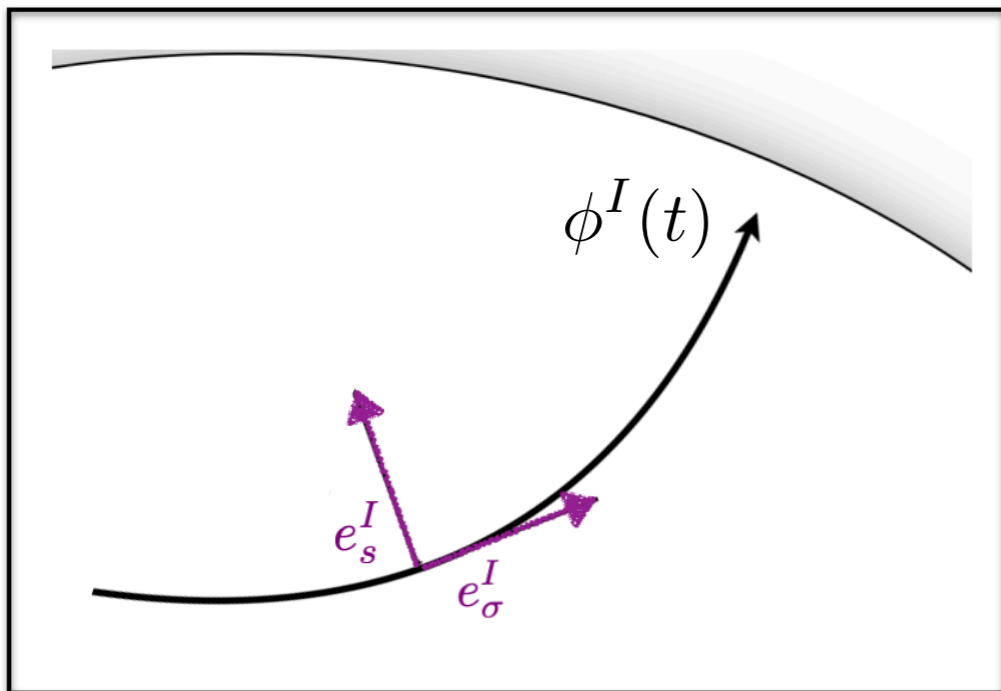
$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{4\chi}{M^2 \left(1 + \frac{2\chi^2}{M^2}\right)} \dot{\chi}\dot{\varphi} + \frac{V_{,\varphi}}{1 + \frac{2\chi^2}{M^2}} = 0$$

Inflaton: ‘standard’ but  
**modified effective potential: flattened compared to original V**

see also Dong et al 2011, McAllister et al 2014, Flauger et al 2014

# General dynamics

$$\mathcal{D}_t \dot{\phi}^I + 3H \dot{\phi}^I + V^{,I} = 0 \quad \text{with} \quad \mathcal{D}_t A^I \equiv \dot{A}^I + \Gamma_{JK}^I \dot{\phi}^J A^K \quad \begin{array}{l} \text{covariant derivative} \\ \text{along the trajectory} \end{array}$$



Standard “slow-roll” dynamics:

$$\dot{\phi}^I \simeq -V^{,I} / 3H$$

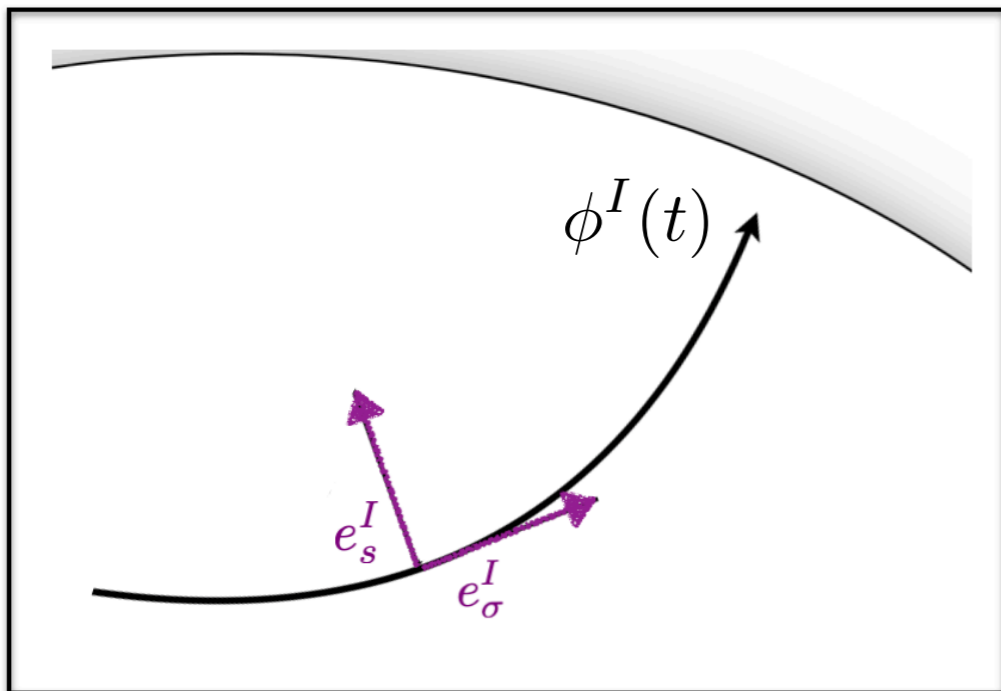
following gradient  
of potential

$$\mathcal{D}_t \dot{\phi}^I \simeq 0$$

trajectory  $\simeq$   
field space geodesic

# General dynamics

$$\mathcal{D}_t \dot{\phi}^I + 3H \dot{\phi}^I + V^{,I} = 0 \quad \text{with} \quad \mathcal{D}_t A^I \equiv \dot{A}^I + \Gamma_{JK}^I \dot{\phi}^J A^K \quad \begin{array}{l} \text{covariant derivative} \\ \text{along the trajectory} \end{array}$$



Requirement of  
prolonged inflation

$$\longrightarrow e_{\sigma I} \mathcal{D}_t \dot{\phi}^I \simeq 0$$

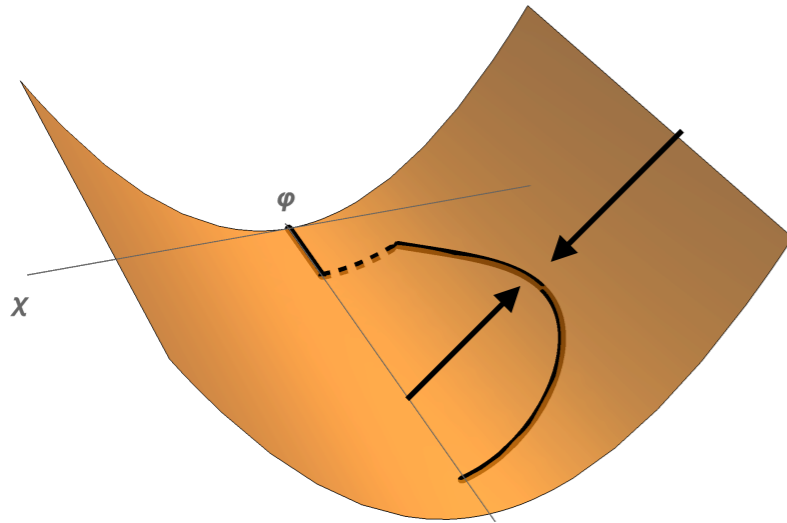
but

$$e_{sI} \mathcal{D}_t \dot{\phi}^I = -e_{sI} V^{,I} \quad \text{not constrained to be small}$$

$$e_{sI} \mathcal{D}_t e_{\sigma}^I / H \equiv \eta_{\perp} = \begin{array}{l} \text{dimensionless} \\ \text{perpendicular} \\ \text{acceleration} \end{array} = \begin{array}{l} \text{deviation from} \\ \text{geodesic motion} \end{array} = - \frac{V_s}{H |\dot{\phi}^I|}$$

# Sidetracked inflation

Garcia-Saenz, RP, Ronayne  
[1804.11279 JCAP](#)



Competition potential vs geometry:

$$\eta_{\perp}^2 = \mathcal{O}\left(\frac{m^2}{H^2}\right) \gg 1$$

Strongly non-geodesic motion

Requirement for sidetracked inflation:  
flat potentials wrt curvature scale

$$M \frac{V_{,\varphi}}{V} \ll 1, \quad M \frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

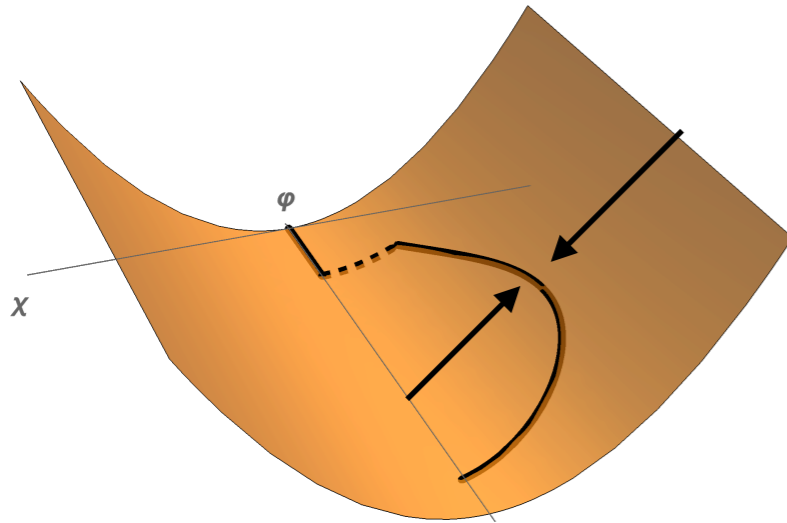
Sustained inflation with  
steep potential in Planck units  
iff strongly non-geodesic motion

$$\epsilon \simeq \frac{\frac{M_{\text{Pl}}^2 (\nabla V)^2}{2V^2}}{1 + \eta_{\perp}^2/9}$$

Hetz and Palma 2016  
Achucarro and Palma 2018

# Sidetracked inflation

Garcia-Saenz, RP, Ronayne  
1804.11279 JCAP



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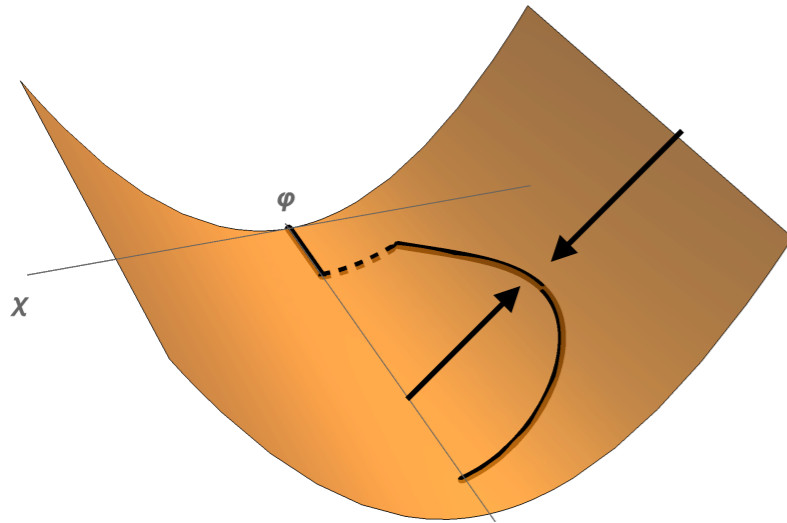
Strongly non-geodesic motion  
in negatively curved field space  
under scrutiny in recent years

sidetracked inflation  
hyperinflation  
angular inflation  
rapid-turn  
fat inflaton  
...

different names  
but overall  
similar  
mechanism

# Sidetracked inflation

Garcia-Saenz, RP, Ronayne  
1804.11279 JCAP



Competition potential vs geometry:

$$\eta_{\perp}^2 = \mathcal{O} \left( \frac{m^2}{H^2} \right) \gg 1$$

Strongly non-geodesic motion

Requirement for sidetracked inflation:  
flat potentials wrt curvature scale

$$M \frac{V_{,\varphi}}{V} \ll 1, \quad M \frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

But with cutoff  $M \ll M_{\text{Pl}}$

Natural expectation to have  
structures over distance  $M$

As tuned as slow-roll

# ***Inflation with strongly non-geodesic motion***

$$\mathcal{L}^{(2)} = a^3 \left[ M_{\text{Pl}}^2 \epsilon \left( \dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right) + 2 \underbrace{|\dot{\phi}| \eta_{\perp}} \dot{\zeta} \dot{Q}_s + \frac{1}{2} \left( \dot{Q}_s^2 - \frac{(\partial Q_s)^2}{a^2} - \underbrace{m_s^2 Q_s^2} \right) \right]$$

observable  
curvature perturbation

fluctuation orthogonal  
to the trajectory (entropic)

# Inflation with strongly non-geodesic motion

$$\mathcal{L}^{(2)} = a^3 \left[ M_{\text{Pl}}^2 \epsilon \left( \dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right) + 2|\dot{\phi}| \eta_{\perp} \dot{\zeta} Q_s + \frac{1}{2} \left( \dot{Q}_s^2 - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2 \right) \right]$$

Strongly  
non-geodesic  $\eta_{\perp}^2 \gg 1$

unless stabilization  
e.g. by potential

$$\frac{m_s^2}{H^2} = \frac{V_{;ss}}{H^2} - \eta_{\perp}^2 + \epsilon R^{\text{field space}} M_{\text{Pl}}^2$$

$$m_s^2 < 0 \quad \& \quad \left| \frac{m_s^2}{H^2} \right| \gg 1$$

Transient instability

relevant sub-Hubble mass

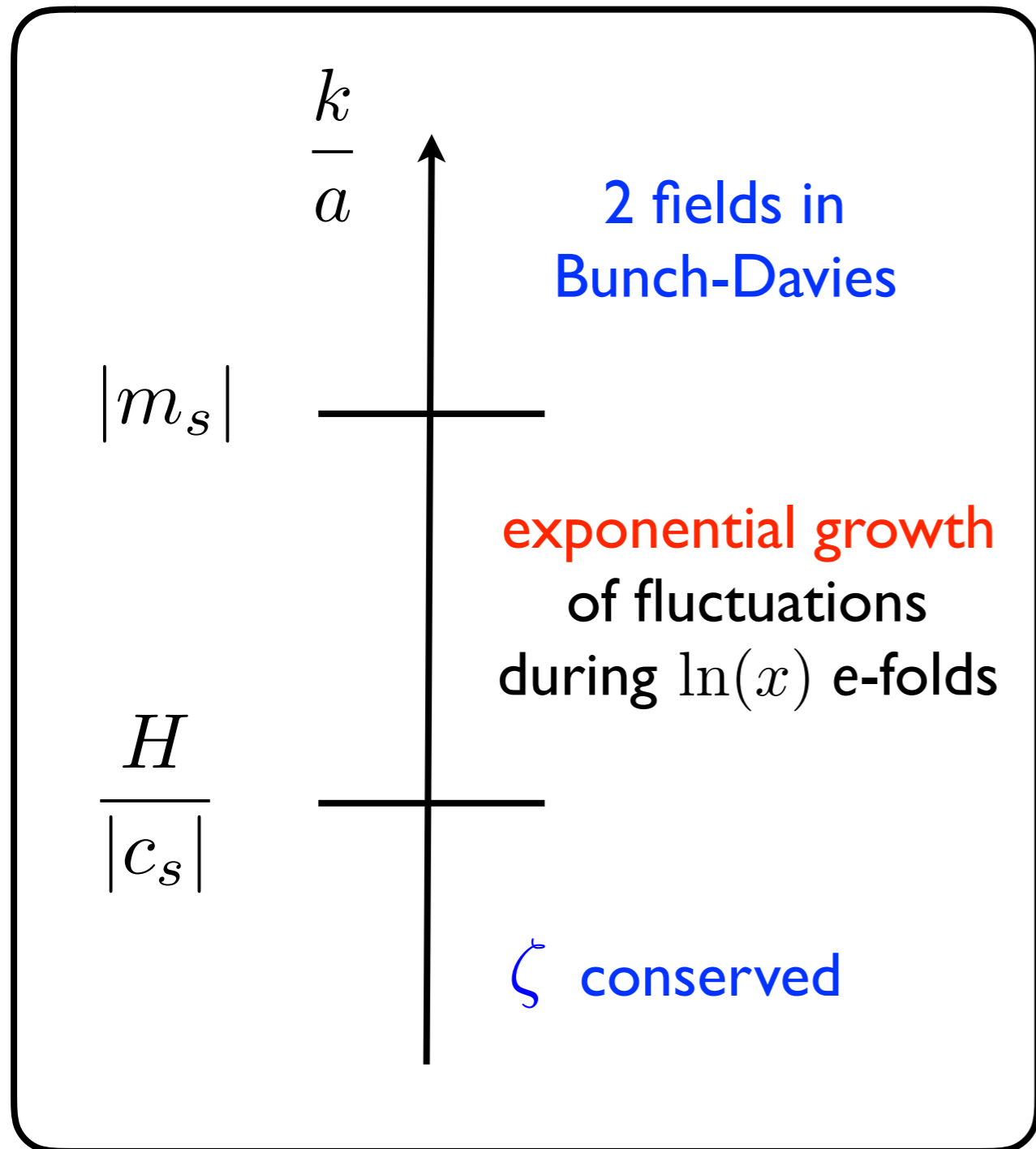
$$\frac{m_{s(\text{eff})}^2}{H^2} = \frac{V_{;ss}}{H^2} + 3\eta_{\perp}^2 + \epsilon R^{\text{field space}} M_{\text{Pl}}^2$$

$$\frac{m_{s(\text{eff})}^2}{H^2} \gg 1$$

Stable background

relevant super-Hubble mass

# Inflation with strongly non-geodesic motion



Cremonini et al 2010  
Brown 2018  
RP et al 2018, 2019  
Bjorkmo, Ferreira, Marsh 2019 ...

$$\omega^2 = c_s^2 k^2 \quad \& \quad \frac{1}{c_s^2} = \frac{m_{s(\text{eff})}^2}{m_s^2} < 0$$



$$\frac{\mathcal{P}_\zeta}{\mathcal{P}_{\text{standard}}} \sim e^{2x} \gg 1$$

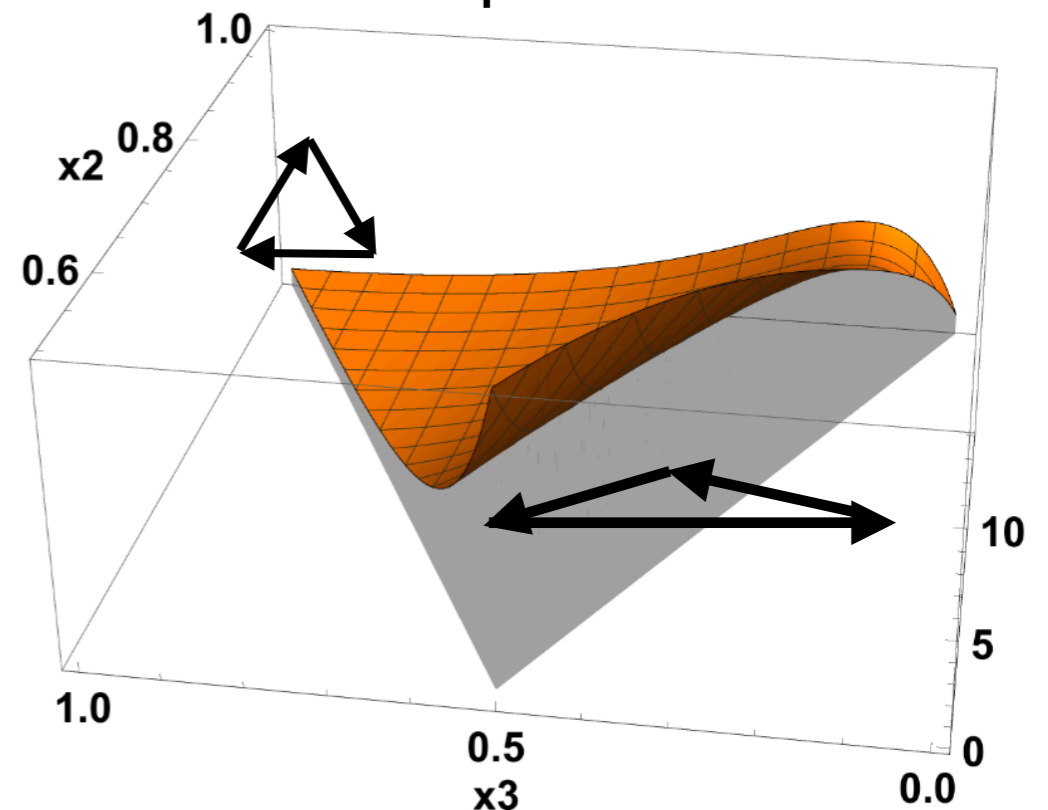
# EFT with imaginary sound speed

$$S^{\text{eff}} = \int d\tau d^3x a^2 \epsilon M_{\text{Pl}}^2 \left[ \frac{\zeta'^2}{c_s^2} - (\vec{\nabla} \zeta)^2 \right] + \int d\tau d^3x \frac{a \epsilon M_{\text{Pl}}^2}{H} \left( \frac{1}{c_s^2} - 1 \right) \left[ \zeta' (\vec{\nabla} \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right]$$

EFT valid for  $k|c_s|/a < xH$

- May appear as essentially classical instability but more subtle
- NGs come from **interactions between exponentially growing and decaying modes**
- Enhancement in flattened configurations, like non Bunch-Davies

Shape of NG:



$$f_{NL}^{\text{flat}} \sim \left( \frac{1}{|c_s^2|} + 1 \right) x^3$$

# ***EFT with imaginary sound speed***

- Perfect agreement between EFT and first-principle numerical computations in two-field models with PyTransport  
Mulryne & Ronayne
- **n-point functions** also enhanced in flattened configurations and perturbativity guaranteed

$$\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} \times \mathcal{P}_\zeta^{(n-2)/2} \sim \left( f_{NL}^{\text{flat}} \mathcal{P}_\zeta^{1/2} \right)^{n-2} \lesssim 1$$

Fumagalli, Garcia-Saenz, Pinol,  
RP, Ronayne [1902.03221](#) PRL

- Powerful **model-independent constraints** on inflation with strongly non-geodesic motion

# Conclusion

- Inflation with strongly non-geodesic motion: **new type of inflationary attractor**, common in negatively curved field space
- Inflate on steep potentials, but as tuned as slow-roll inflation
- Naturally gives rise to tachyonic instability around Hubble crossing, which can be described by **single field EFT with imaginary sound speed**.
- Specific non-Gaussian signatures: **n-point functions enhanced in flattened configurations**
- Limited period of strongly non-geodesic motion: **PBH generation mechanism**  
Fumagalli, RP, Ronayne, Witkowski [2004.08369](#)
- Strong sharp turn: order one oscillations in scalar power spectrum, and corresponding **oscillatory features in the scalar-induced stochastic gravitational wave background**  
Fumagalli, RP, Witkowski, [2012.02761 JCAP](#)