PROBING PRIMORDIAL FEATURES WITH THE STOCHASTIC GWs BACKGROUND

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With Sebastien Renaux-Petel & Lukas Witkowski

arXiv 2012.02761 JCAP and 2105.06481 JCAP

OUTLINE

- STOCASTIC BACKGROUND OF GRAVITATIONAL WAVES

  &

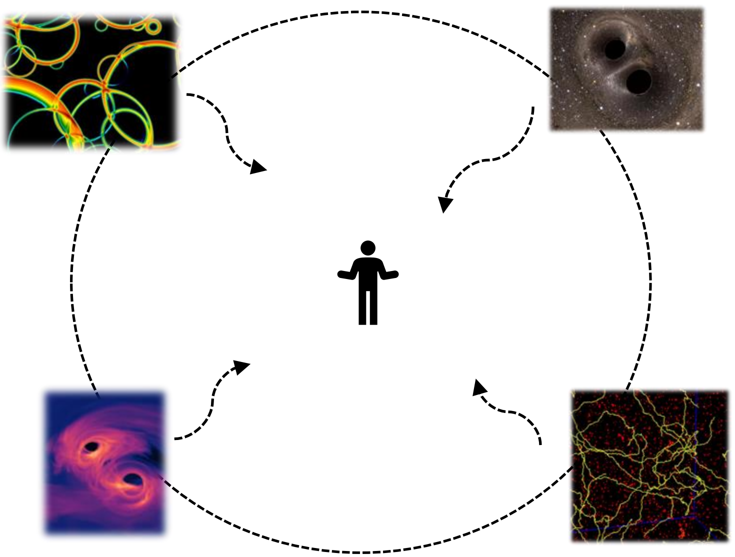
  INFLATION AT SMALL-SCALES

- FEATURES IN THE PRIMORDIAL POWER SPECTRUM

- OSCILLATIONS IN THE SGWB

STOCHASTIC BACKGROUND OF GWS

RANDOM SIGNAL PRODUCED BY MANY WEAK, INDEPENDENT AND UNRESOLVED SOURCES.



- ASTROPHYSICAL

CBCS, CORE COLLAPSE SUPERNOVAE, EARLY INSPIRALS ETC..

SEARCHES IN GROUND BASED INTERFEROMETERS (LVK)

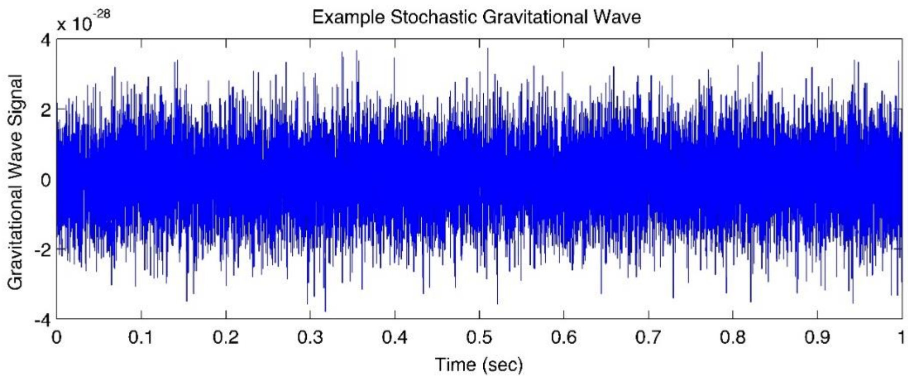
- COSMOLOGICAL

EARLY UNIVERSE PROCESSES,

FIRST ORDER PHASE TRANSITIONS, TOPOLOGICAL DEFECTS, INFLATION

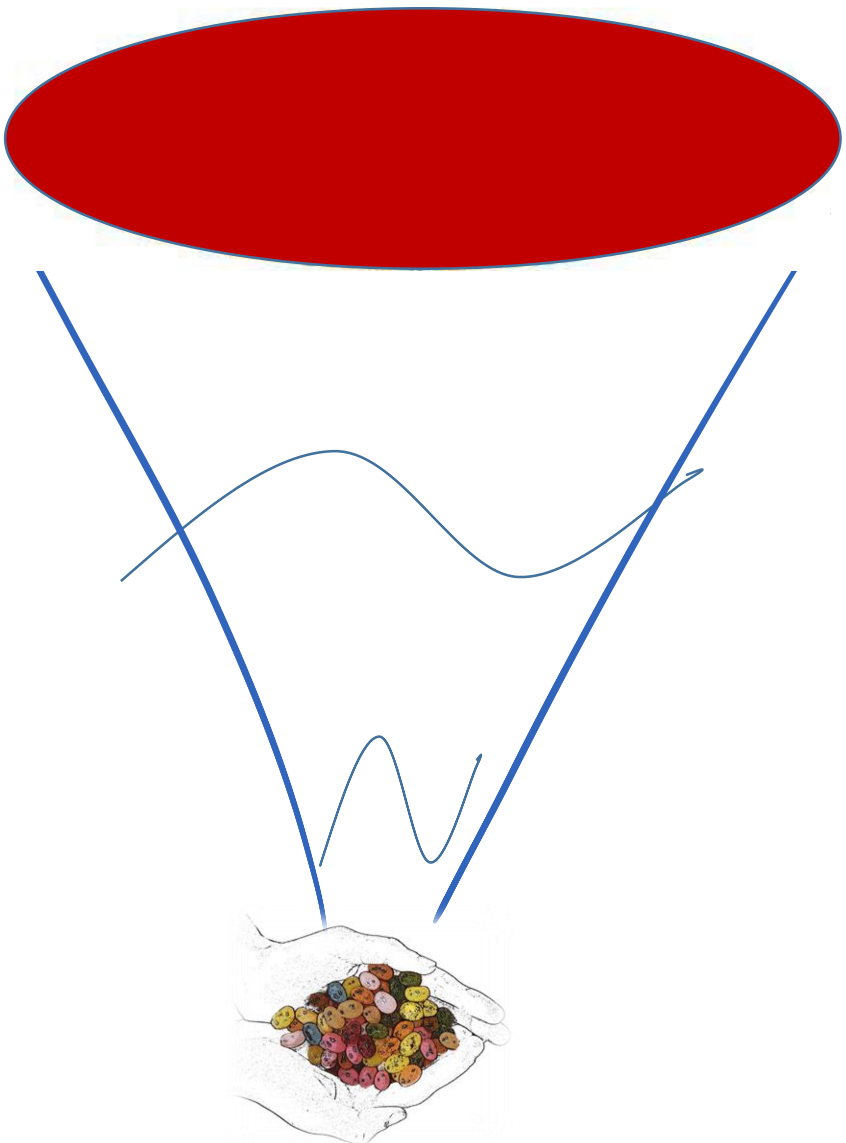
SEARCHES IN GROUND AND FUTURE SPACE INTERFEROMETERS (LISA ETC.)

Caprini and Figueroa '18 for a review

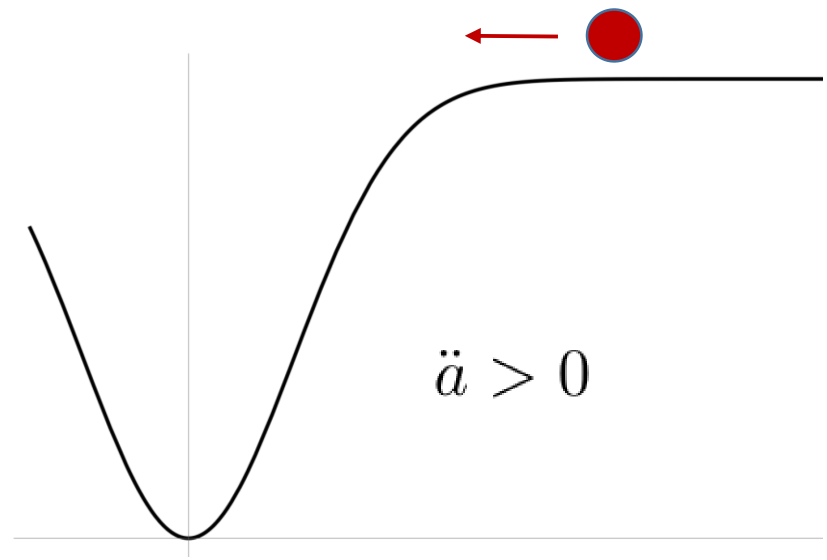


[Image:A. Stuver/LIGO]

COSMIC INFLATION



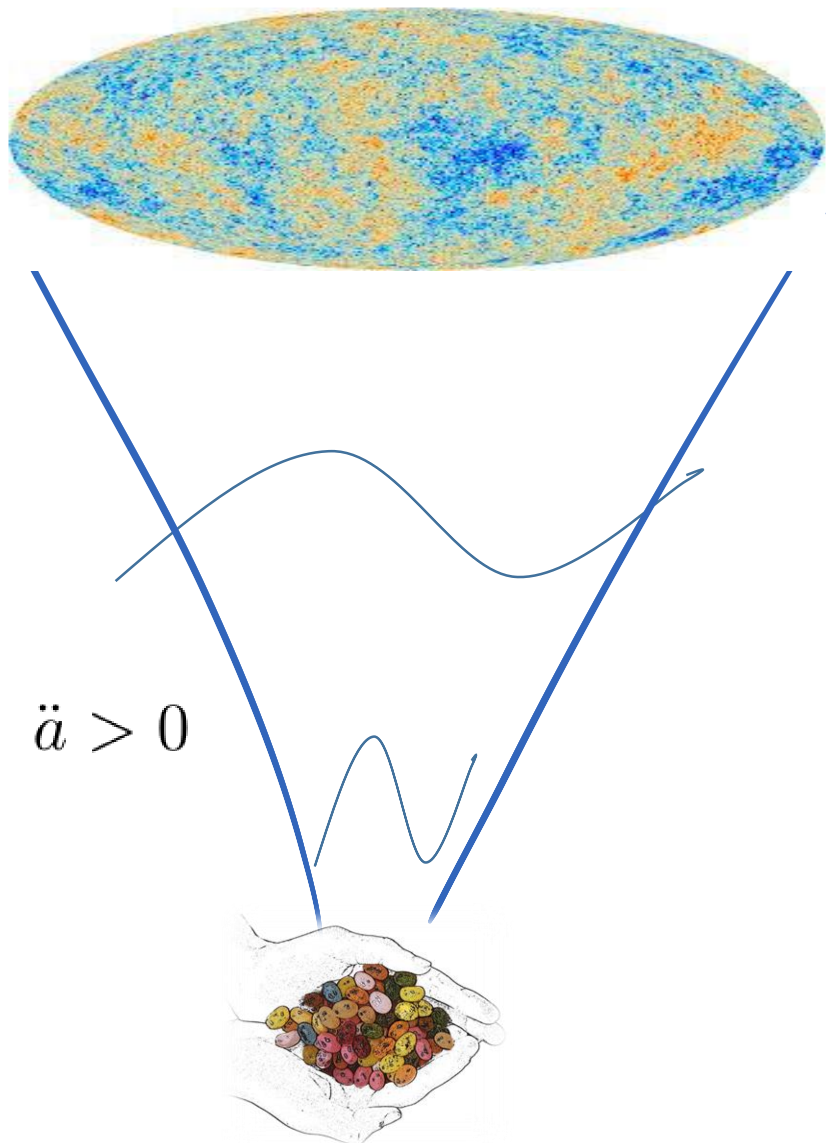
Toy Single-field

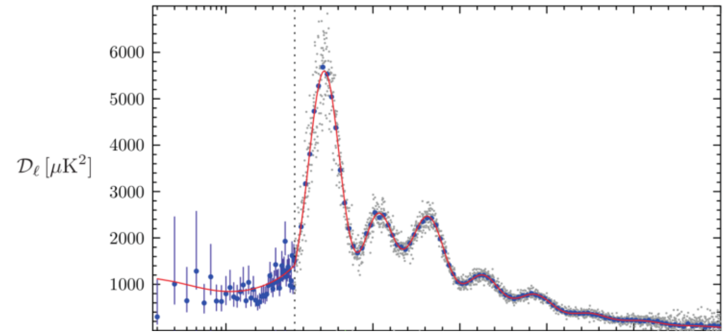


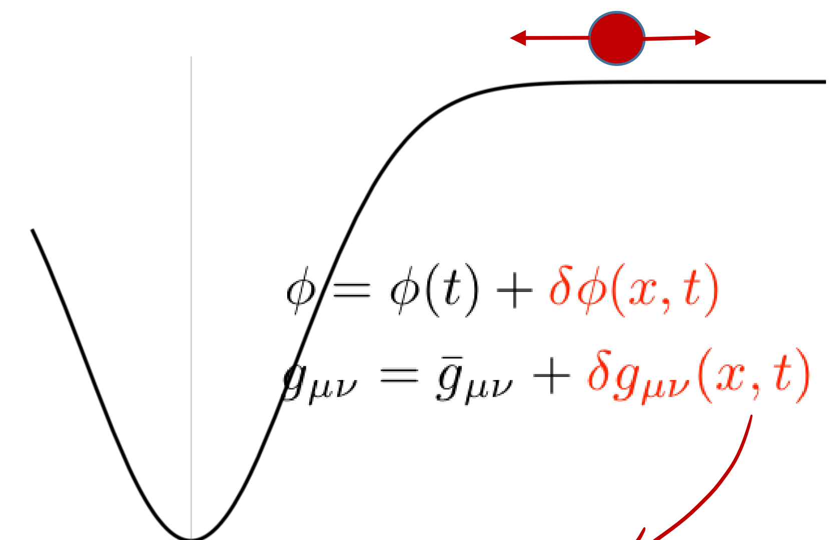
$$\mathcal{S}_\phi = \mathtt{GR} - \frac{1}{2}\int \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

COSMIC INFLATION







$$\phi = \phi(t) + \delta\phi(x,t)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}(x,t)$$

Comoving curvature perturbation $\zeta \Longrightarrow \delta T$

WHY GOING BEYOND THE SIMPLE SCENARIO?

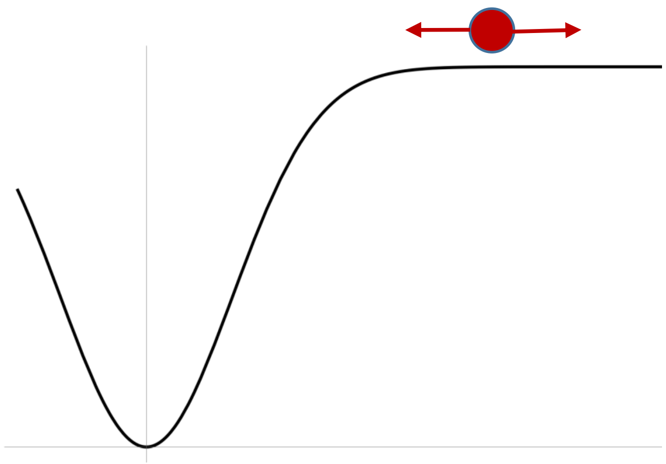
2 COMMENTS:

1. THEORETICALLY: HARD TO BELIEVE IT IS NOT JUST A PHENOMENOLOGICAL EFFECTIVE DESCRIPTION.

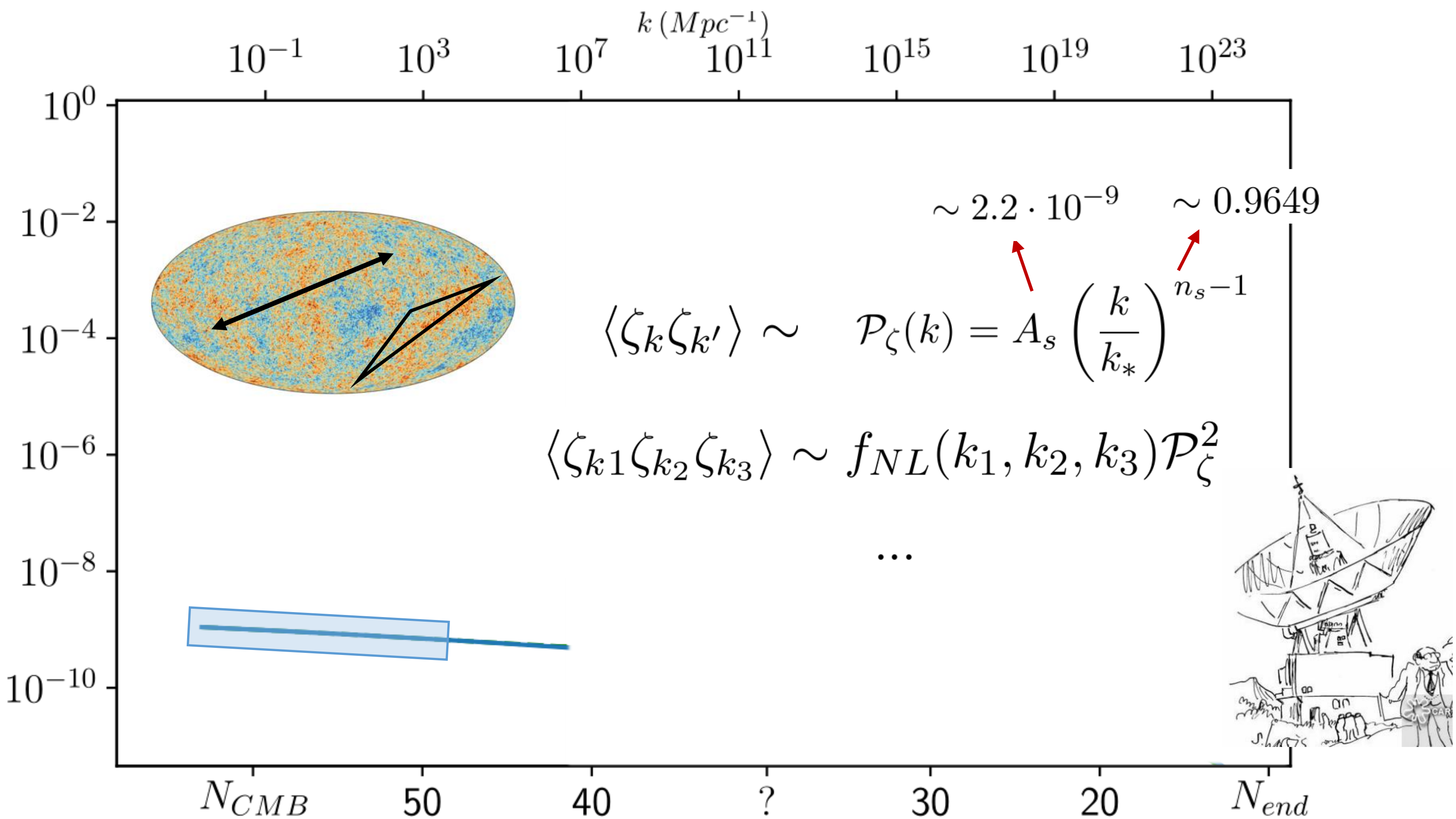
   SEVERAL ISSUE WHEN UV EMBEDDING (ETA-PROBLEM, ETC.)

2. EXPERIMENTALLY: THE PICTURE IS CONSISTENT WITH DATA BUT

   DATA CONSTRAIN ONLY A SMALL PART OF THE INFLATIONARY HISTORY



PRIMORDIAL SCALAR FLUCTUATIONS



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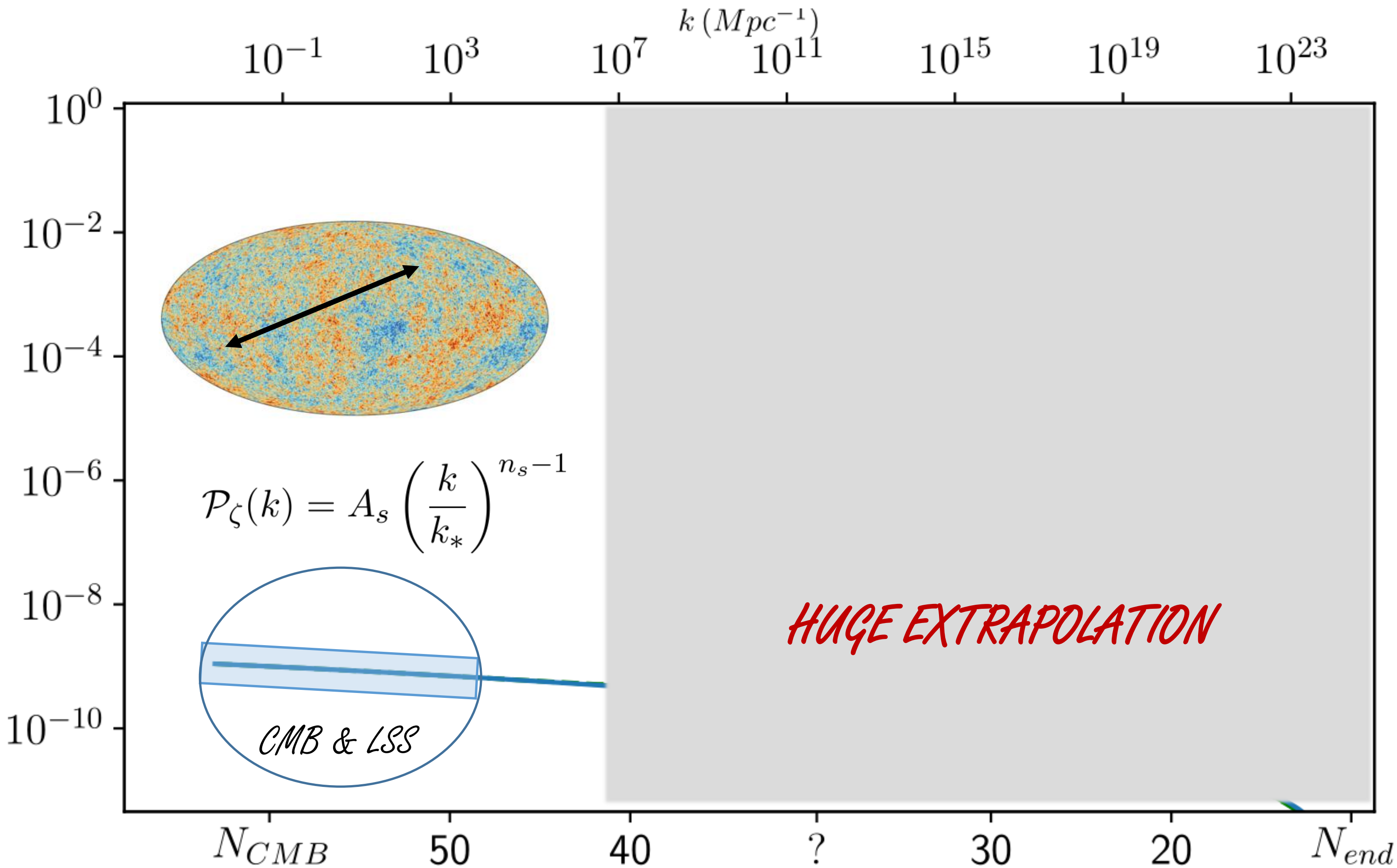

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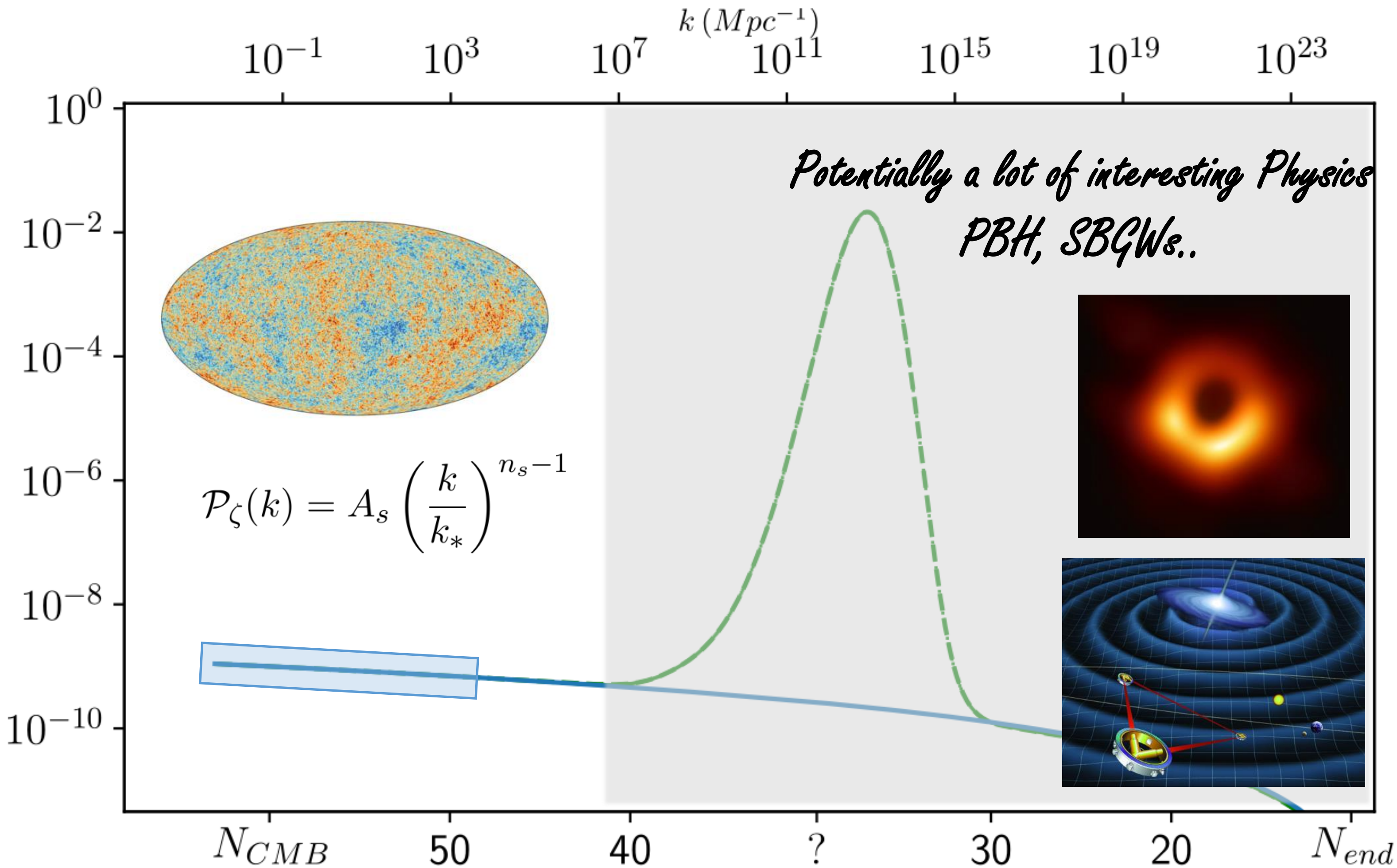

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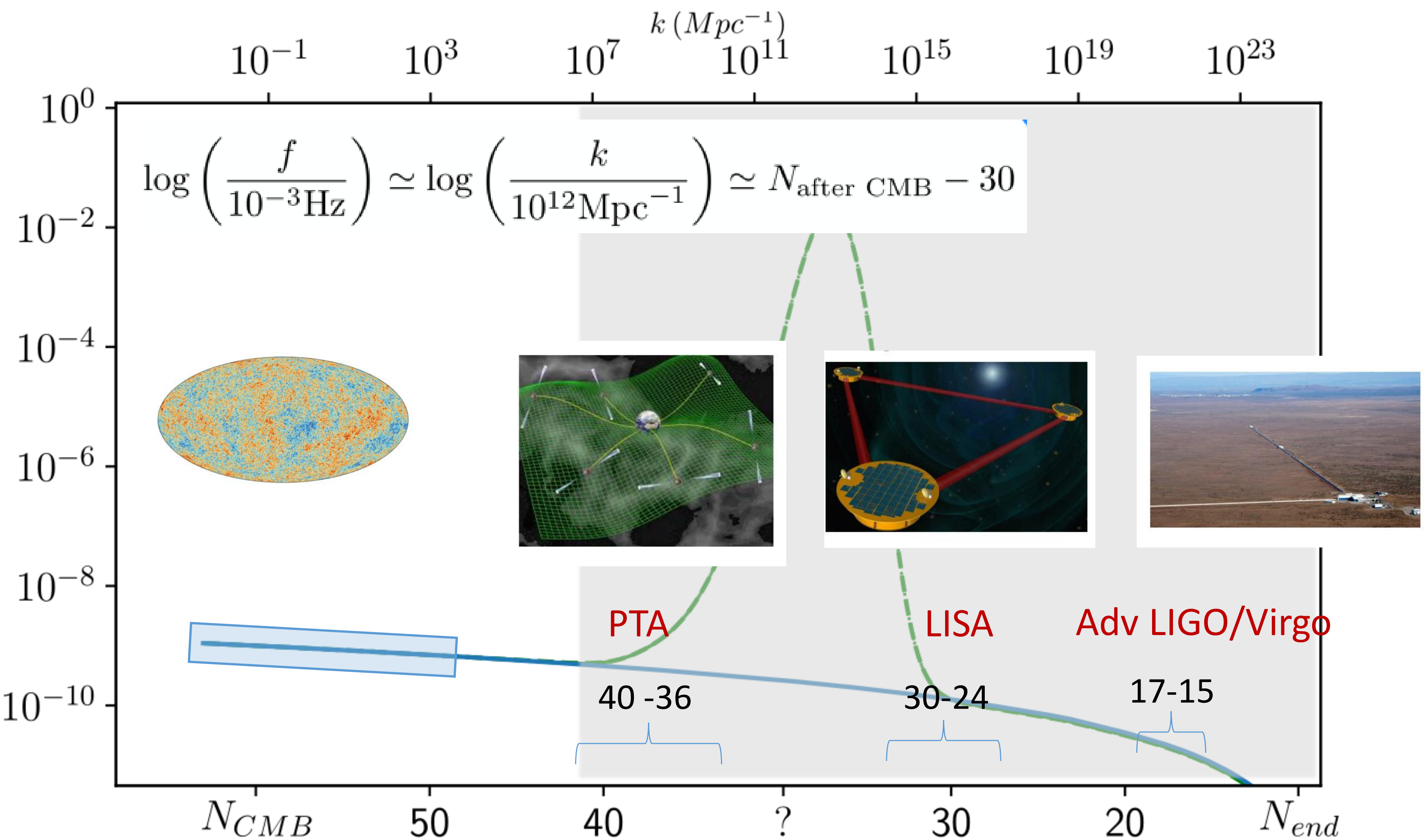
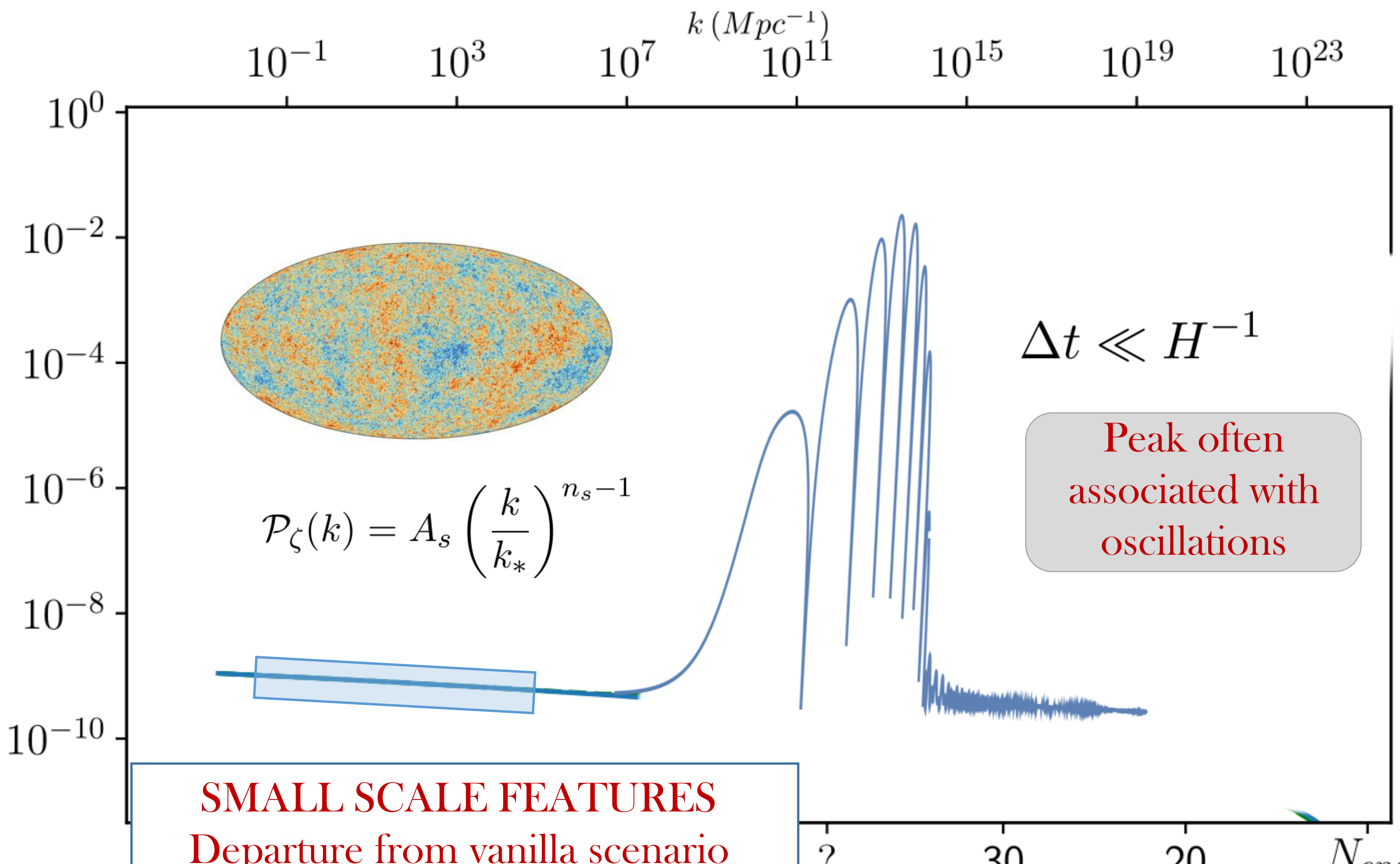


FEATURES IN THE PRIMORDIAL SPECTRUM



SMALL SCALE FEATURES
Departure from vanilla scenario leading to oscillation in $\mathcal{P}_\zeta(k)$

JF, S.Renaux-Petel, L.Witkowski 2012.02761
JF, S.Renaux-Petel, L.Witkowski 2105.06481

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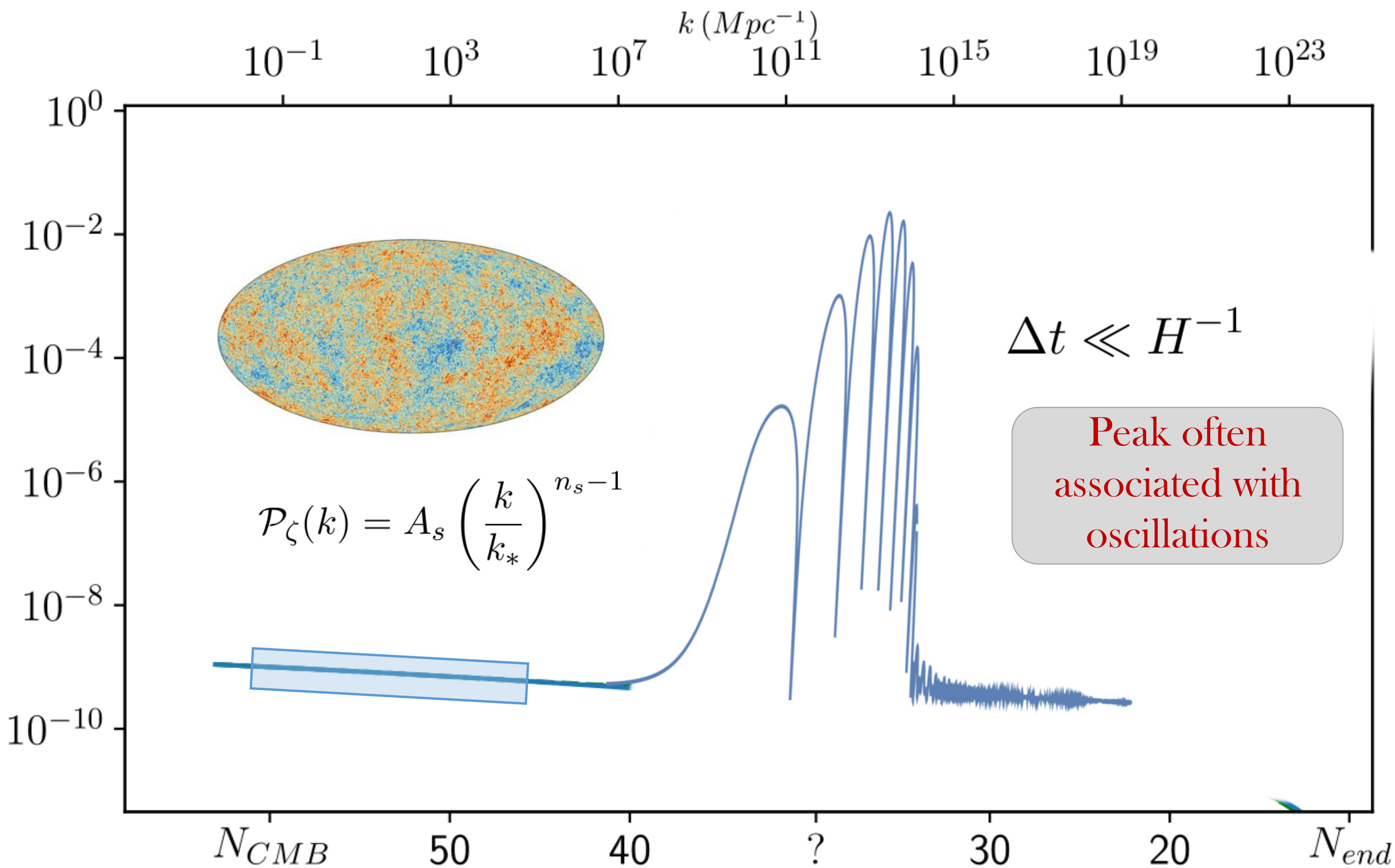
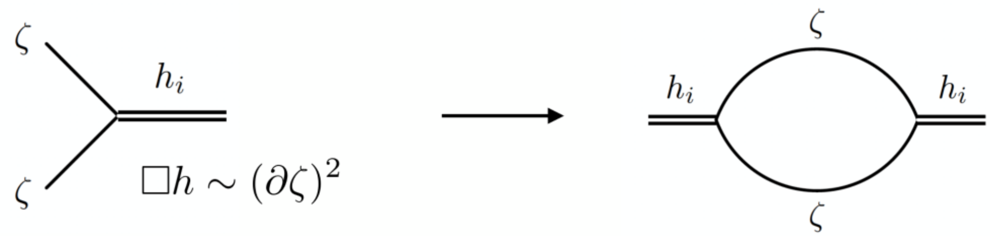


DO OSCILLATIONS LEAVE OBSERVATIONAL IMPRINTS IN THE SGWB ?

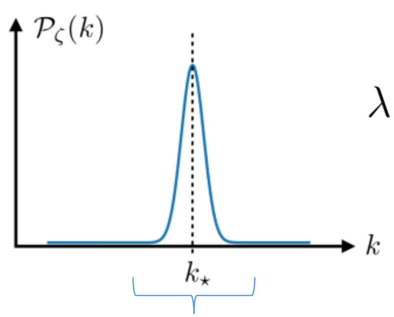
SCALAR INDUCED SGWB

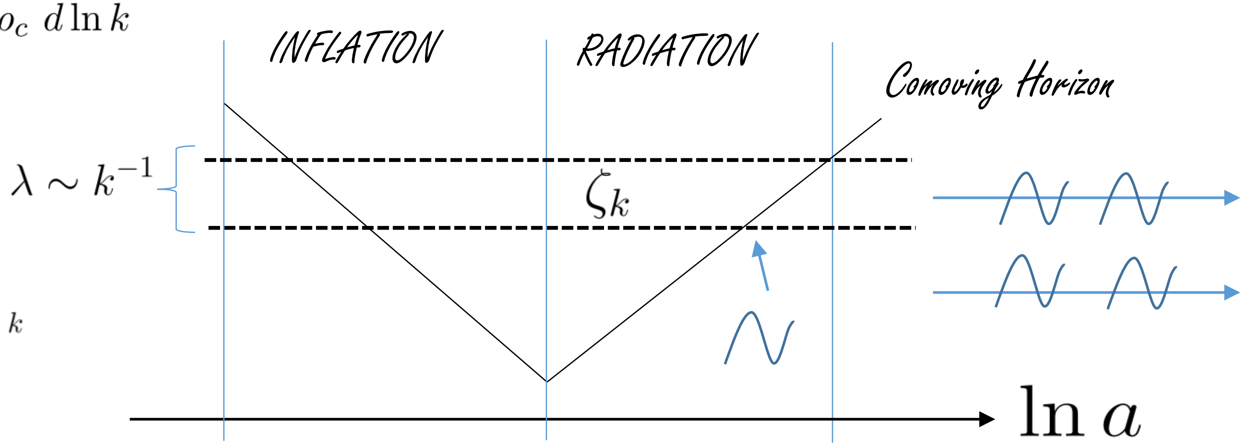


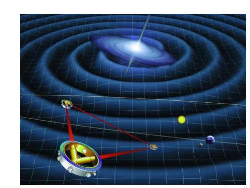
Acquaviva et al. '02;
Mollerach, Harari, Matarrese '03;
Ananda, Clarkson, Wands '06;
Baumann et al. '07

$$\Omega_{\rm GW}(k) = c_g \Omega_{\rm r,0} \int_0^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s\, \mathcal{T}_{\rm RD}(d,s)\, \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s+d)\right) \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s-d)\right)$$

$$\frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$







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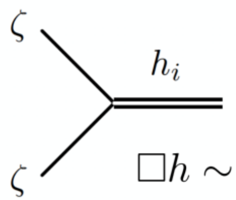
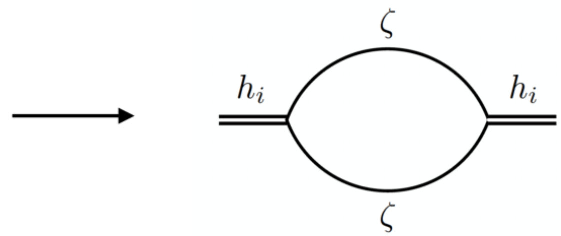


$\Box h \sim (\partial\zeta)^2$



$$\Omega_{\rm GW}(k) = c_g \Omega_{\rm r,0} \int_0^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s\, \mathcal{T}_{\rm RD}(d,s)\, \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s+d)\right) \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s-d)\right)$$

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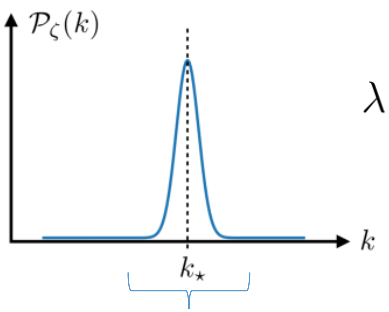

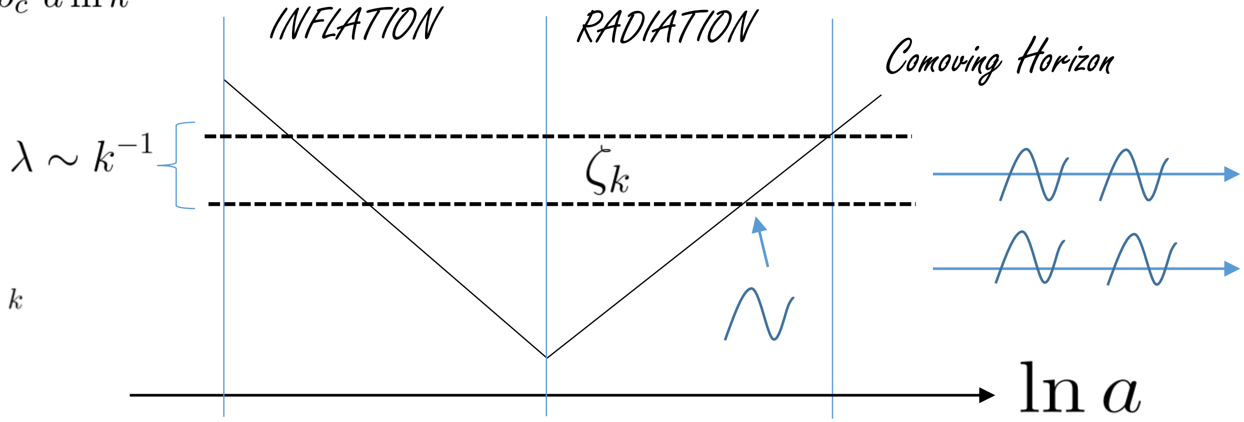

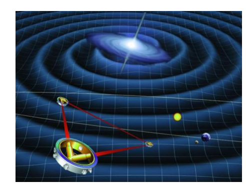

$\Omega_{\rm GW}(k) \simeq 10^{-5}\mathcal{P}^2 \implies \mathcal{P}/\mathcal{P}_0 \simeq 10^5 - 10^6$ ENHANCEMENT TO BE SEEN IN GWs DETECTORS

OUTLINE

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  &
  INFLATION AT SMALL-SCALES

- FEATURES IN THE PRIMORDIAL POWER SPECTRUM

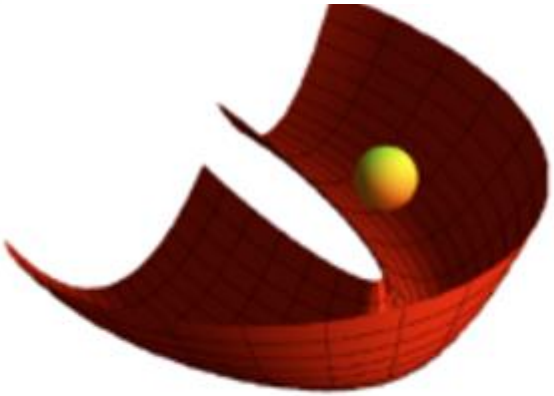
- OSCILLATIONS IN THE SGWB

PRIMORDIAL FEATURES

- SHARP FEATURE – Localized Event
  (Step in the potential / 2-stage / turn in field-space etc..)

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_{\rm lin}\cos\left(\omega_{\rm lin}k + \phi_{\rm lin}\right)\Big)$$

K periodic and a preferred scale selected $2/k_f$



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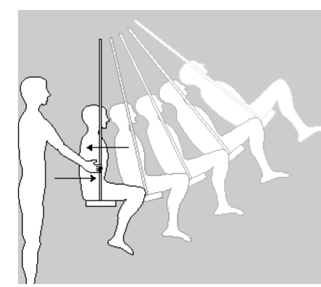
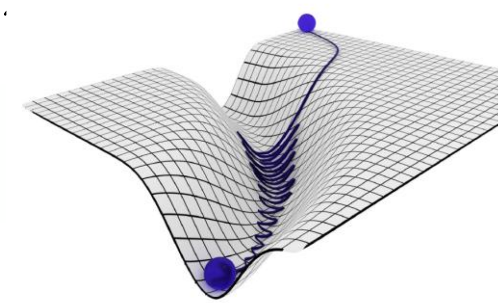


- RESONANT FEATURE – Oscillations of BkG
  (Ex. Monodromy inflation, double turn, in-out horizon..)

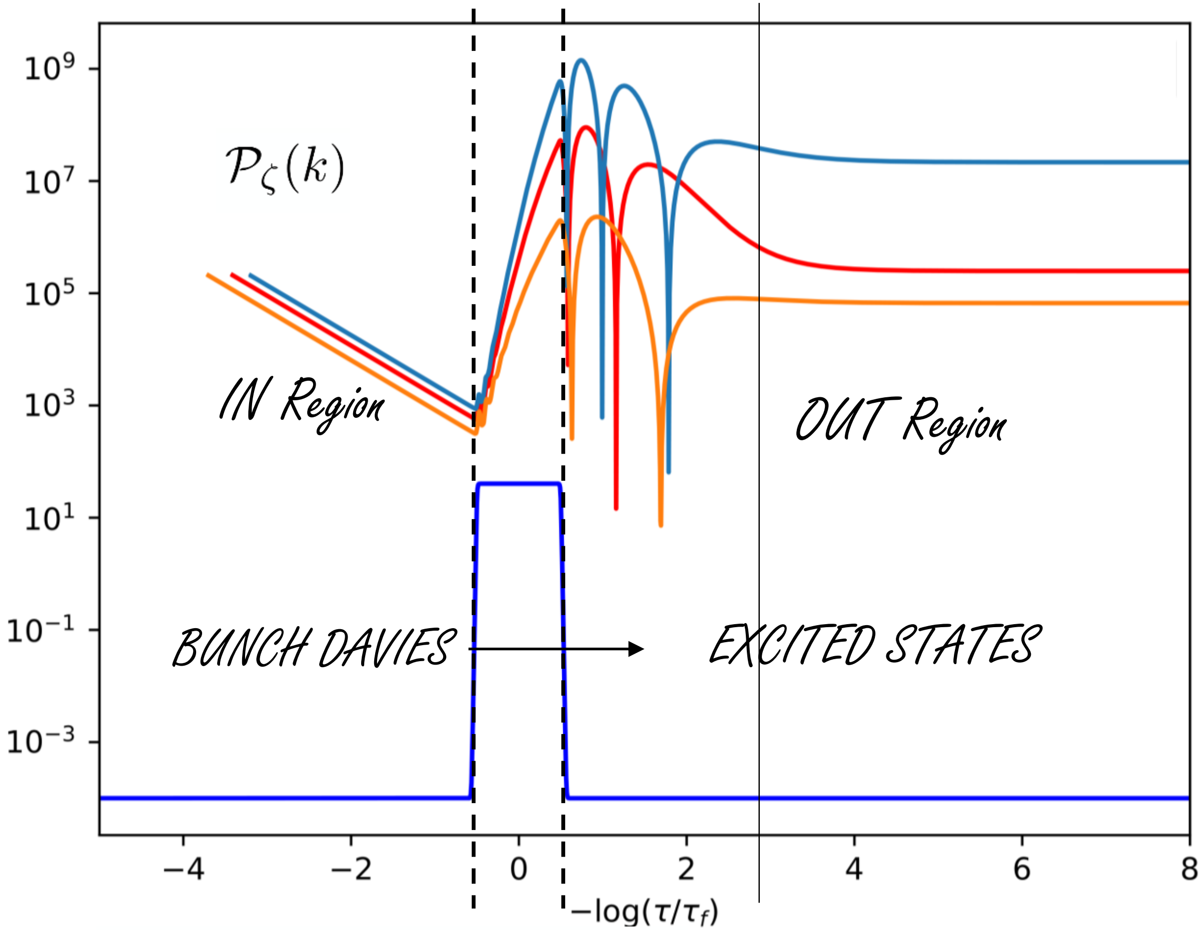
$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_{\log}\cos\left(\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log}\right)\Big)$$

Log-K Periodic M/H



SHARP FEATURE

(Step in the potential / 2-stage / turn in field-space etc..)



$$\mathrm{ph}\left(\frac{\beta_k}{\alpha_k}\right) \sim e^{2ik/k_\mathrm{f}}$$

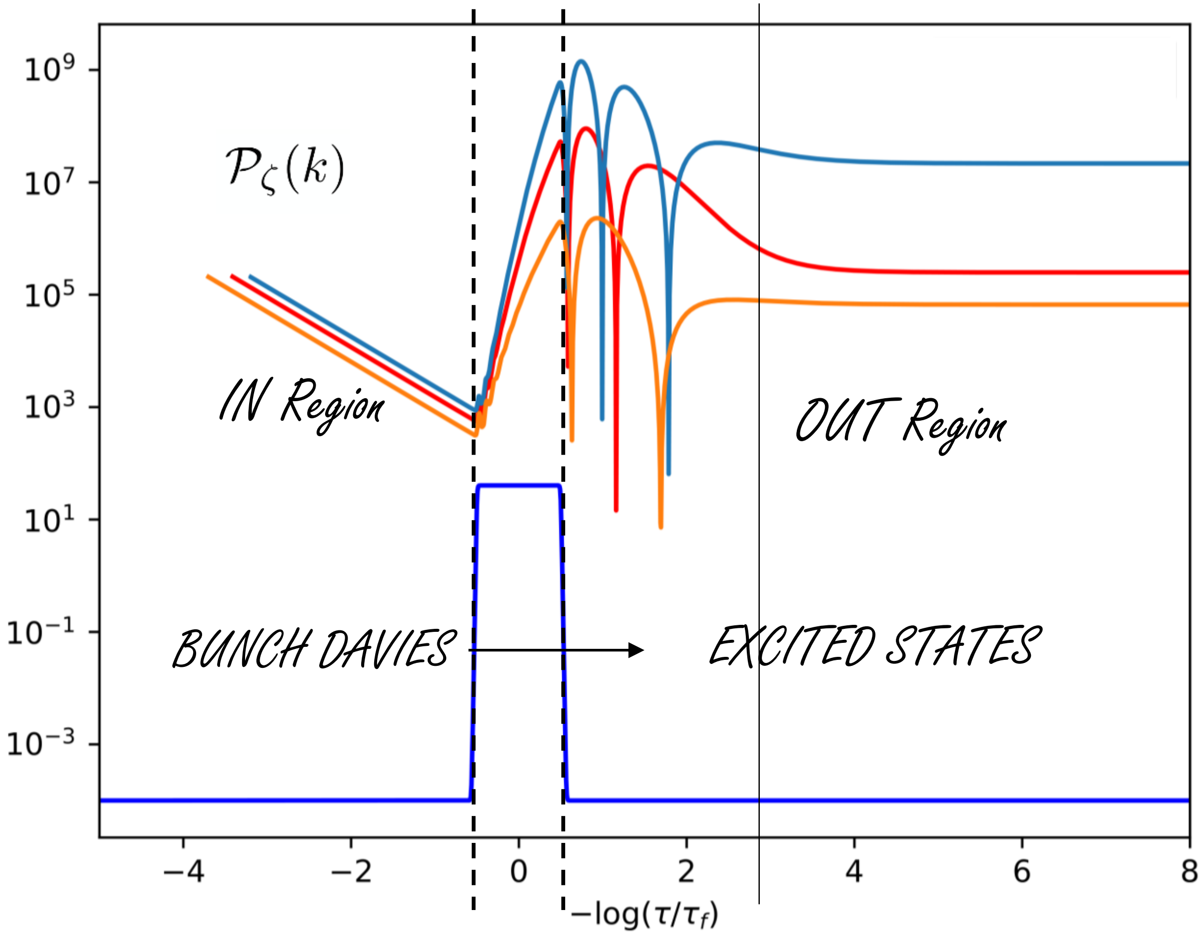
$$\hat{\zeta}_k(\tau) = \zeta_k^{\mathrm{BD}}(\tau)\hat{a}(\boldsymbol{k}) + \mathrm{h.c.}(-\boldsymbol{k}) \longrightarrow \hat{\zeta}_k(\tau) = \left[\alpha_k\, \zeta_k^{\mathrm{BD}}(\tau) + \beta_k\, \zeta_k^{*\mathrm{BD}}(\tau)\right]\hat{a}(\boldsymbol{k}) + \mathrm{h.c.}(-\boldsymbol{k})$$

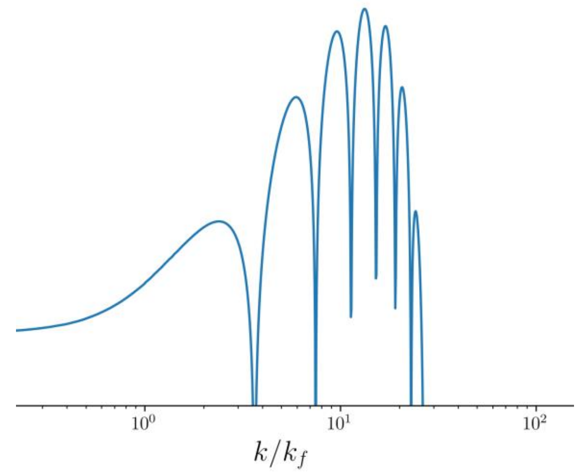
$$\mathcal{P}_0^{1/2} e^{-ik\tau}(1 + ik\tau)$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

SHARP FEATURE

(Step in the potential / 2-stage / turn in field-space etc..)





$$\omega_{\rm lin} = 2/k_f$$

$$\mathrm{ph}\left(\frac{\beta_k}{\alpha_k}\right) \sim e^{2ik/k_{\rm f}}$$

$$\mathcal{P}_\zeta(k) \sim \mathcal{P}_0(k)|\alpha_k|^2\left(1 + \frac{|\beta_k|^2}{|\alpha_k|^2} + 2\frac{|\beta_k|}{|\alpha_k|}\cos\left(\frac{2k}{k_{\rm f}}\right)\right) \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

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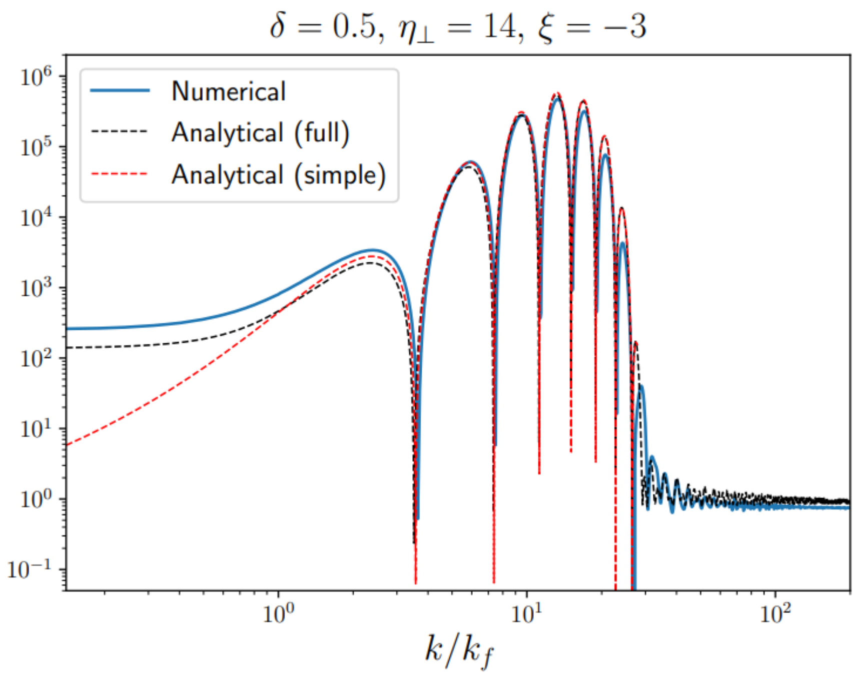
SHARP FEATURES

N - PEAKS IN $\mathcal{P}_\zeta(k)$

$$\mathcal{P}_\zeta(k)$$



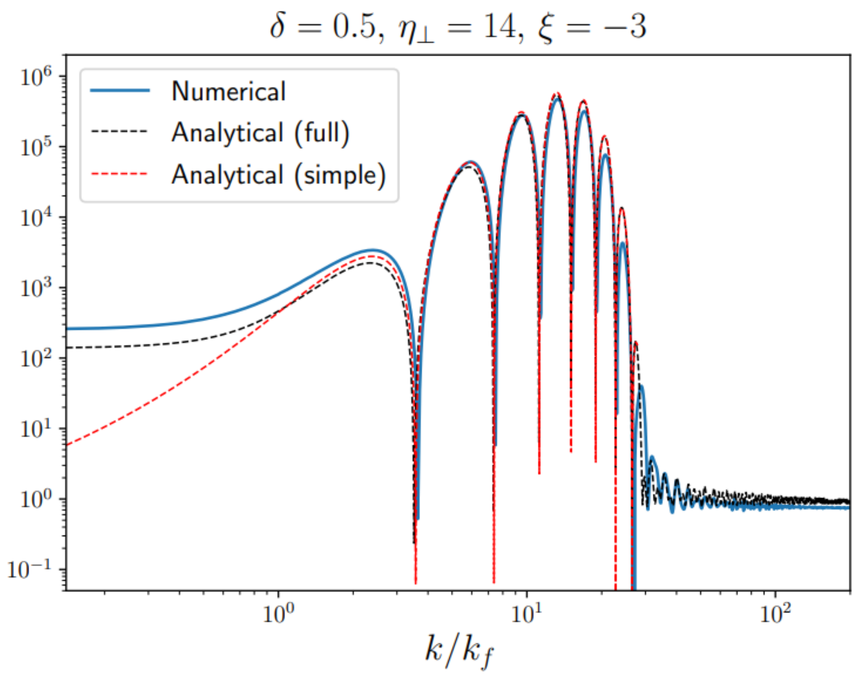
SHARP FEATURES

N - PEAKS IN $\mathcal{P}_\zeta(k)$ $\Longrightarrow$ N(N+1)/2 PEAKS IN $\Omega_{\rm GW}$

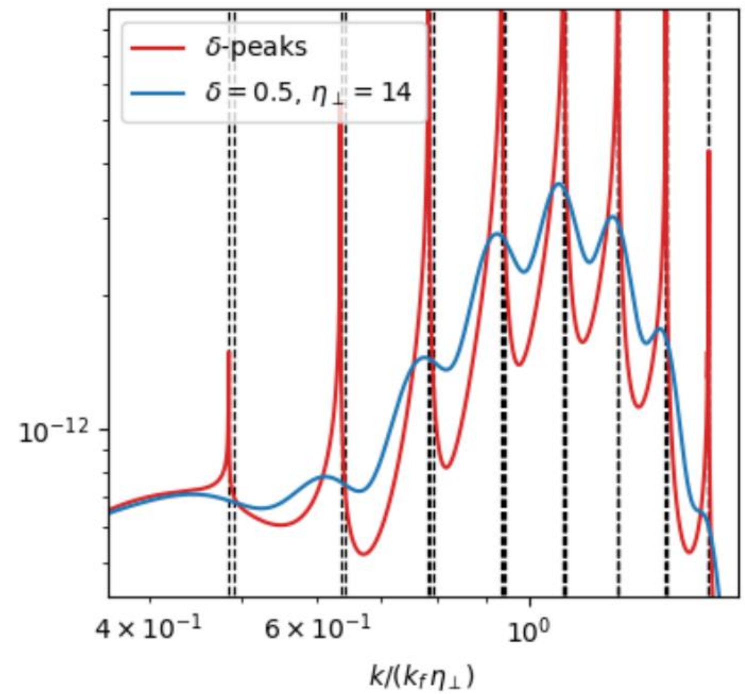
$$k_{\max,ij} = \frac{1}{\sqrt{3}}(k_{\star i} + k_{\star j})\,, \quad \text{with} \quad k_{\max,ij} > |k_{\star i} - k_{\star j}|$$

R. Cai, S. Pi, S. Wang, &
X. Yang '19

$\mathcal{P}_\zeta(k)$



$\Omega_{\rm GW}$

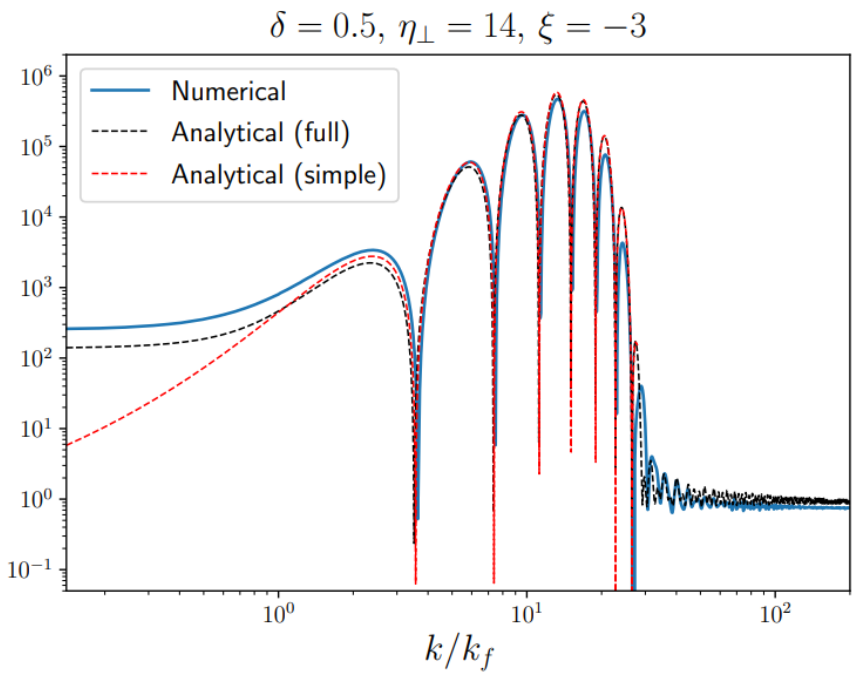


J. F. , S. Renaux-Petel, L. Witkowski, '20.

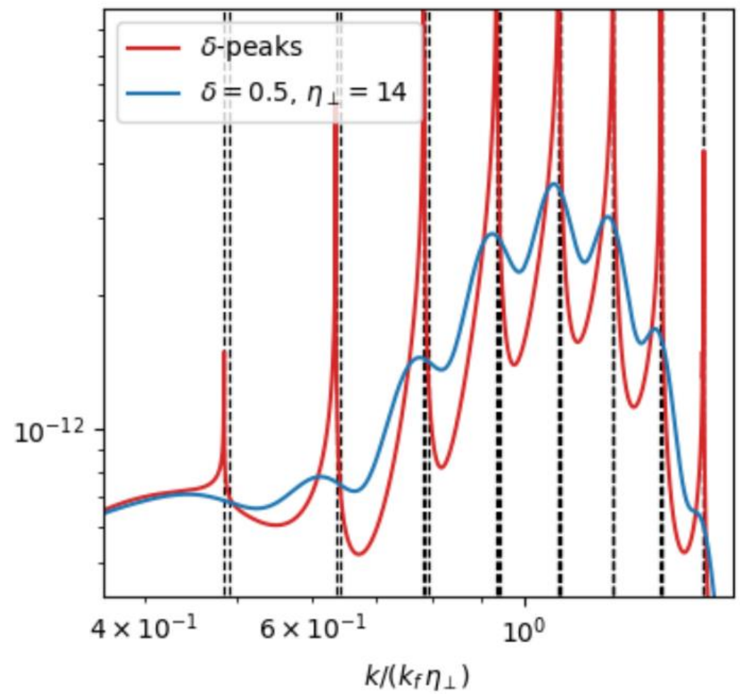
SHARP FEATURES

$$\omega_{\rm lin} \longrightarrow \omega_{\rm lin}^{\rm GW} = \sqrt{3}\,\omega_{\rm lin}$$

$\mathcal{P}_\zeta(k)$



$\Omega_{\rm GW}$



J. F. , S. Renaux-Petel, L. Witkowski, '20.

SHARP FEATURE - TEMPLATE

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_{\rm lin}\cos\left(\omega_{\rm lin}k + \phi_{\rm lin}\right)\Big)$$

$$\omega_{\rm lin}^{\rm GW} = \sqrt{3}\,\omega_{\rm lin}$$

$$\Omega_{\rm GW}(k) = \overline{\Omega}_{\rm GW}\Big(1 + \mathcal{A}_{\rm lin}\cos\left(\omega_{\rm lin}^{\rm GW}k + \varphi_{\rm lin}\right)\Big)$$

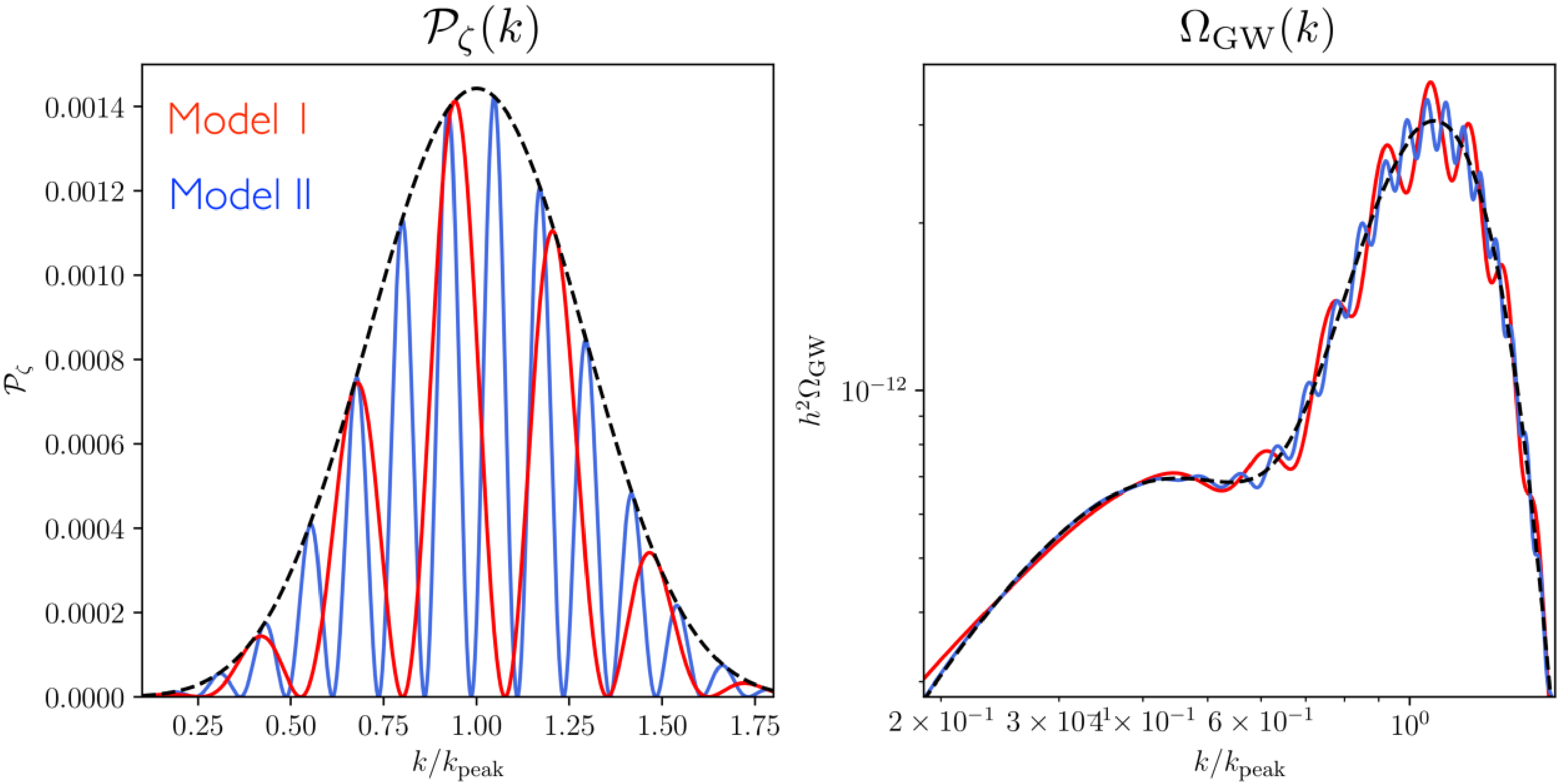
OVERALL SHAPE → ENVELOPE OF THE POWER SPECTRUM

PERIODIC STRUCTURE → PERIODIC STRUCTURE

AVERAGING OUT FROM 100% MODULATION TO 10% MODULATION

J. F., S. Renaux-Petel, L. Witkowski; 2012.02761

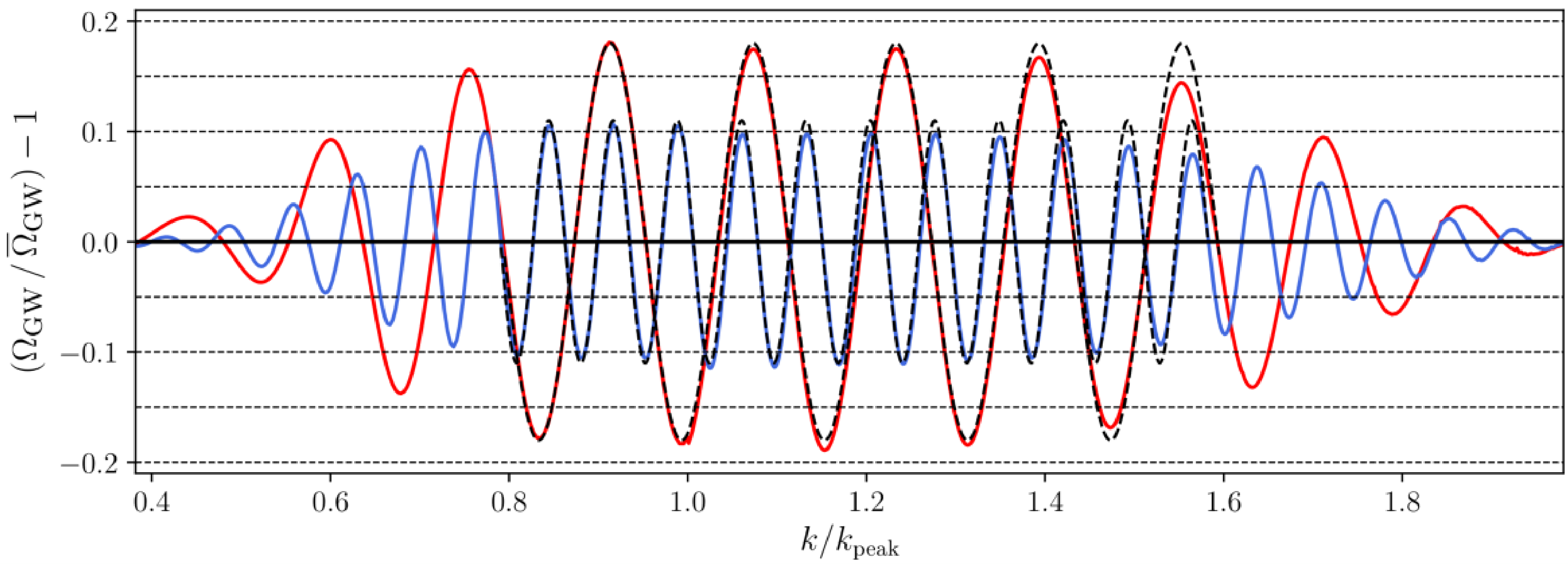
SHARP FEATURE - TEMPLATE



J. F., S. Renaux-Petel, L. Witkowski; 2012.02761

SHARP FEATURE - TEMPLATE

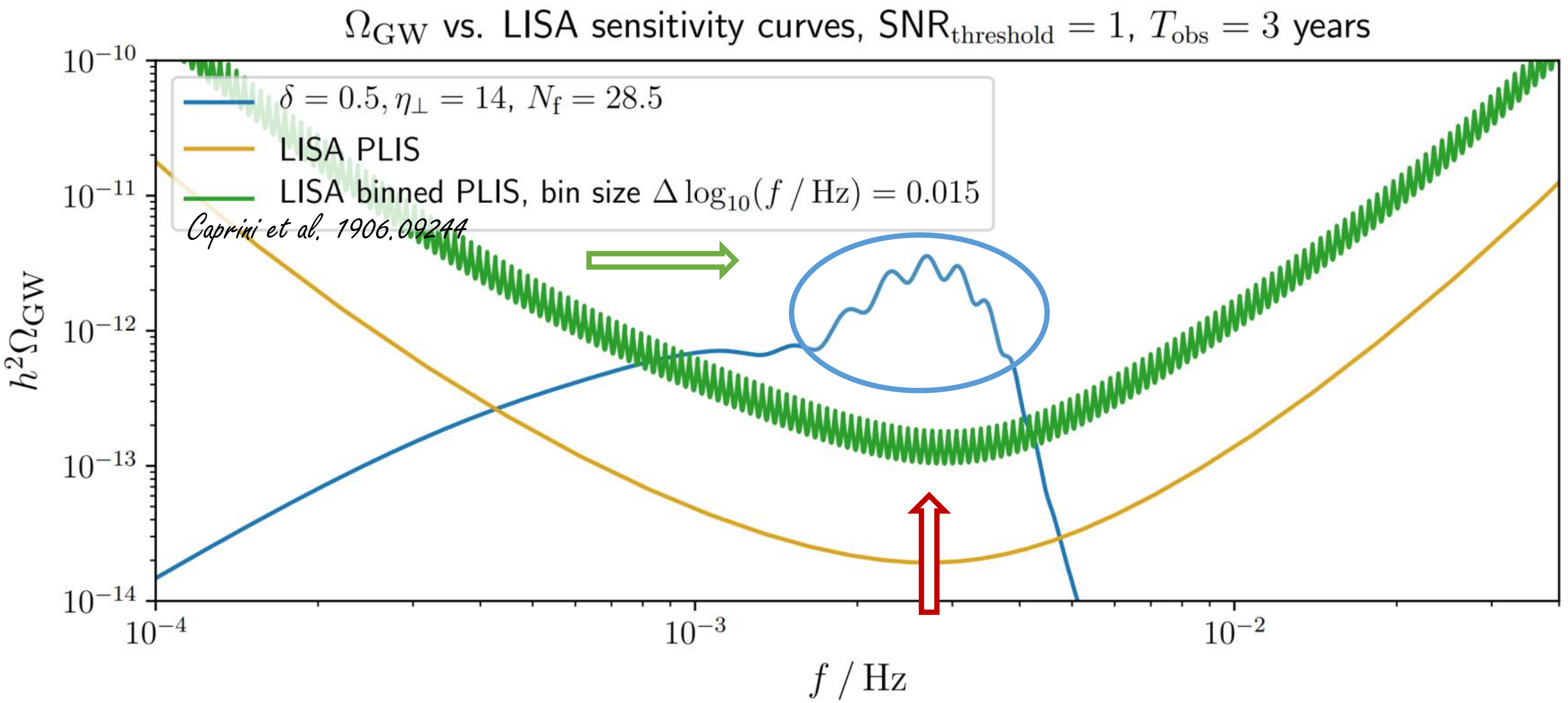
$\overline{\Omega}_{\rm GW}(k)$: SMOOTHED GWS SPECTRUM



AVERAGING OUT FROM 100% MODULATION TO 10% MODULATION

J. F., S. Renaux-Petel, L. Witkowski; 2012.02761

POTENTIAL DISCOVERY



- AMPLITUDE
- FREQUENCY PEAK
- OSCILLATIONS PERIOD

⇒

- ENERGY SCALE
- WHEN DURING INFLATION?
- FOR HOW LONG?

RESONANT FEATURE - TEMPLATE

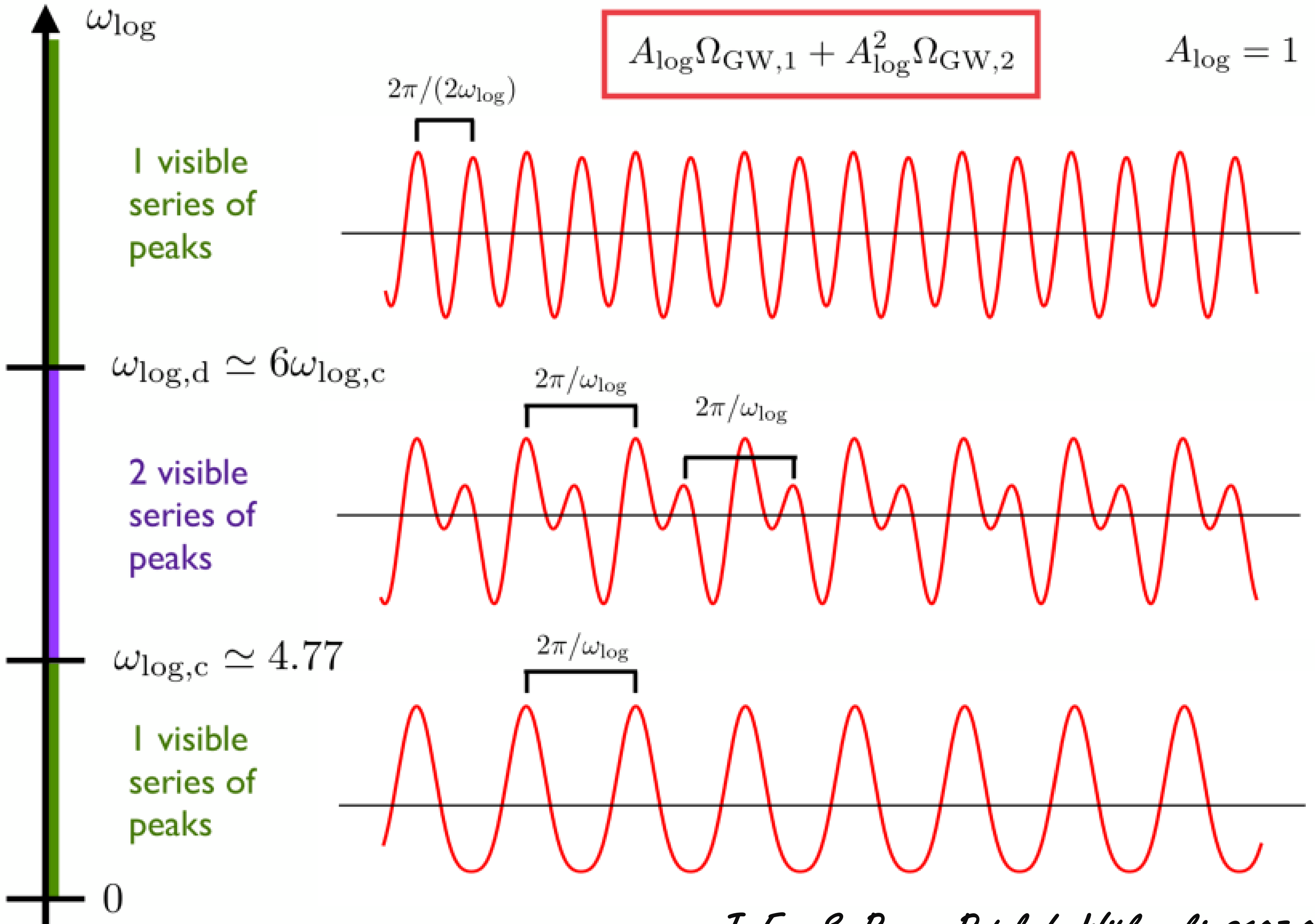
$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_{\log}\cos\left(\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log}\right)\Big)$$

$$\Omega_{\rm GW}(k) = \overline{\Omega}_{\rm GW}(k)\Big[1 + \mathcal{A}_{\log,1}\cos\left(\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log,1}\right) + \mathcal{A}_{\log,2}\cos\left(2\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log,2}\right)\Big]$$

- OVERALL SHAPE → ENVELOPE OF THE POWER SPECTRUM
- COMPLICATED PERIODIC STRUCTURE IN LOG(k)
- THE TWO CONTRIBUTIONS DROP WITH FREQUENCY BUT AT DIFFERENT RATE

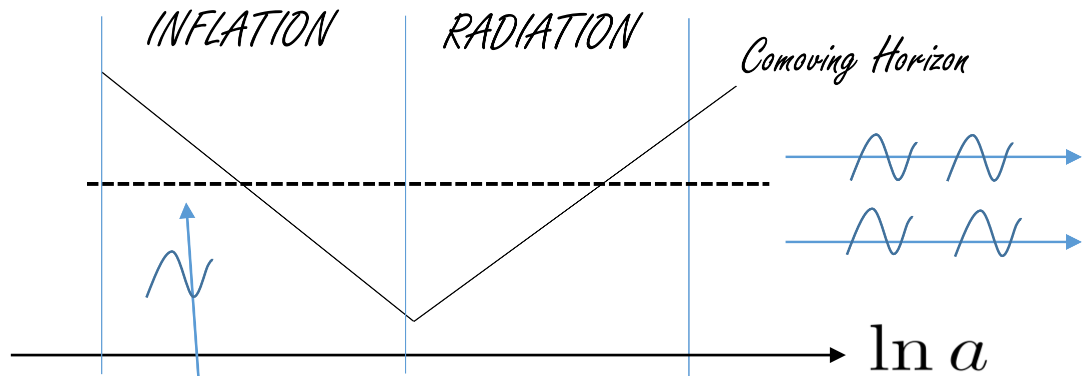
J. F., S. Renaux-Petel, L. Witkowski; 2105.06481

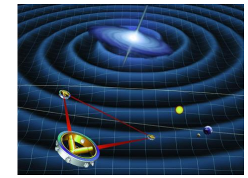
RESONANT FEATURE - TEMPLATE



J. F., S. Renaux-Petel, L. Witkowski; 2105.06481

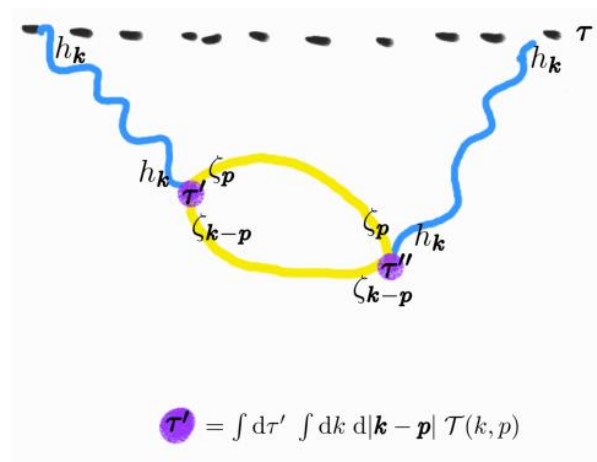
NOT THE END OF THE STORY ..GWs DURING INFLATION ...





- SOURCE $\Box h_{\mu\nu} \propto (\partial Q)^2 \propto \epsilon(\partial\zeta)^2$

  Q : Scalar Fluctuation



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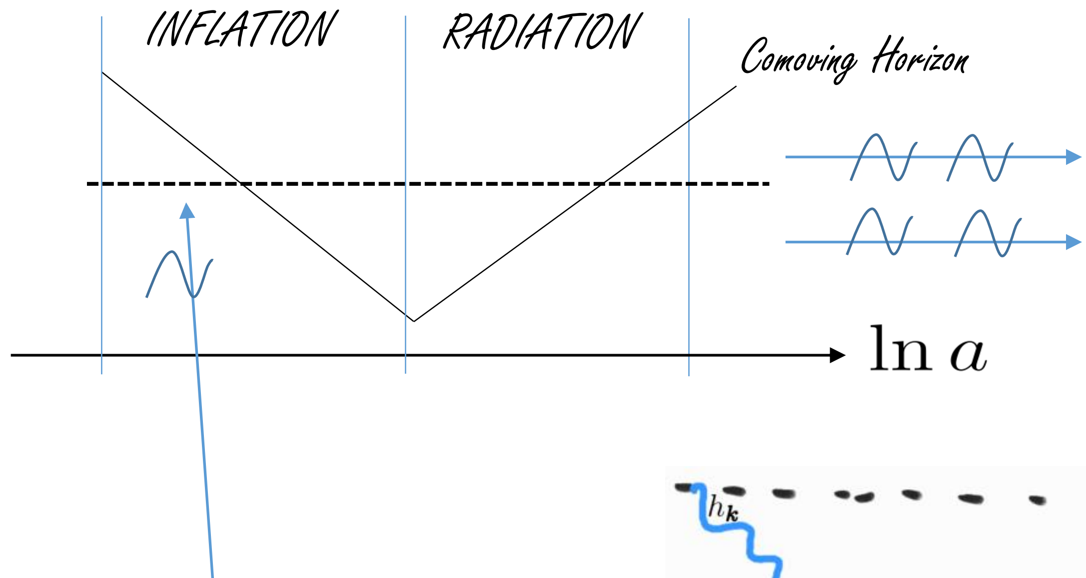

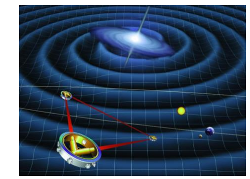

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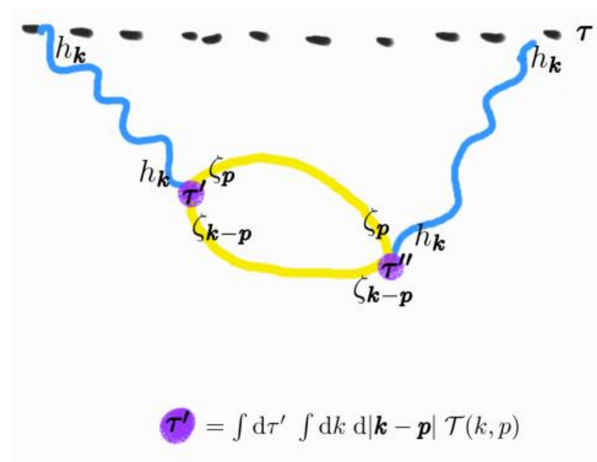
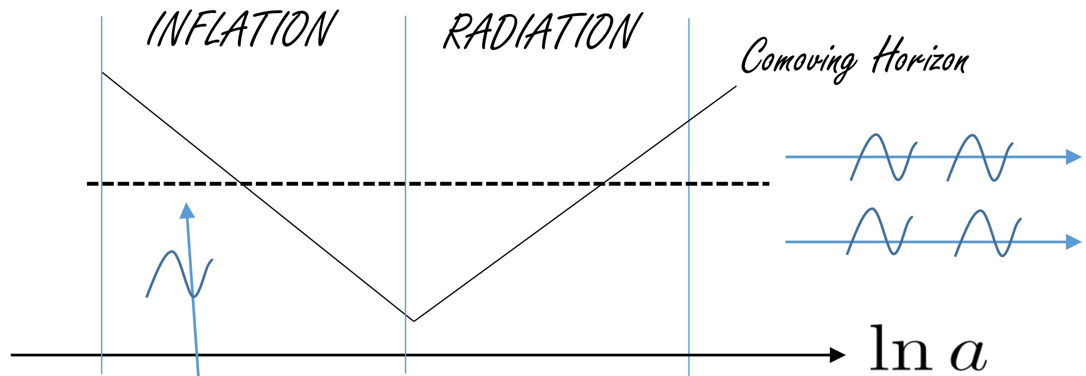
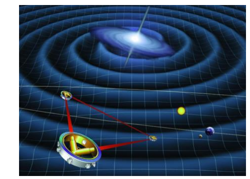


IF TRIVIAL TIME DEPENDECE ON SUB-HUBBLE SCALES

$$\Omega_{\rm GW}^{\rm inf}(k,\tau) \propto \epsilon^2 \int \int \mathcal{P}_\zeta^2 \simeq \epsilon^2 \Omega_{\rm GW}^{rad}$$

NOT THE END OF THE STORY ..GWs DURING INFLATION ...





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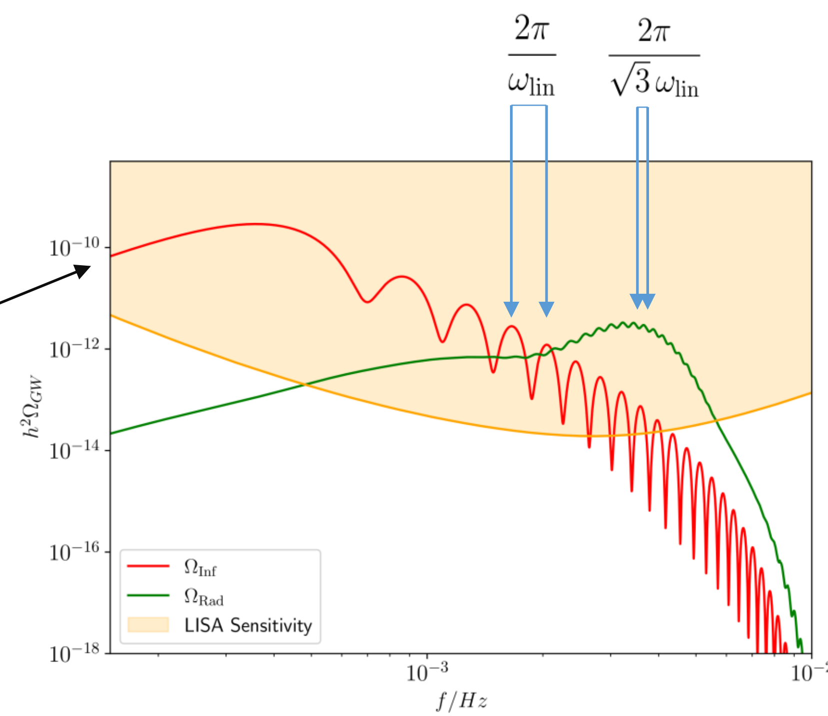
Q : Scalar Fluctuation

BUT ENHANCED for EXCITED STATES

$$\frac{\Omega_{\rm Inf}|_{k_{\rm max}}}{\Omega_{\rm Rad}|_{2k_*/\sqrt{3}}} = O(1)\epsilon^2 \left(\frac{k_*}{k_f}\right)^5$$



JF et al. In preparation

SUMMARY

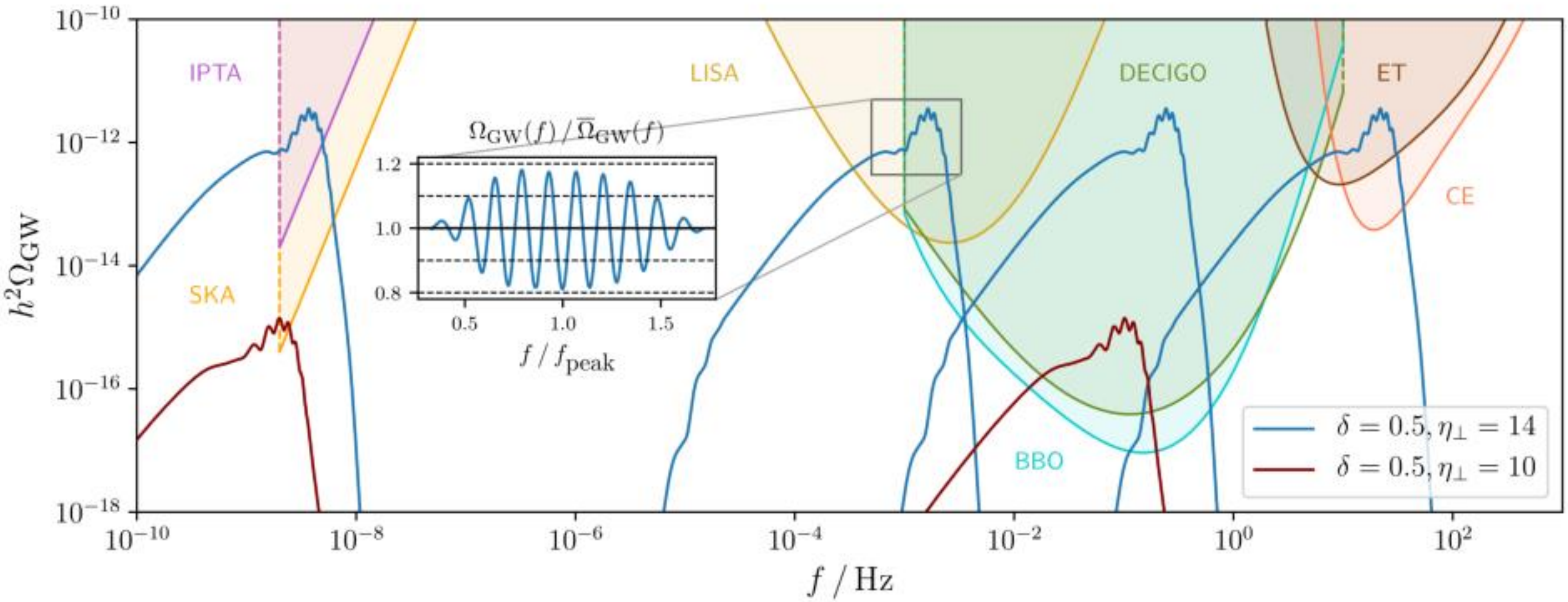
- CMB/LSS PROVIDE INFORMATION ON A LIMITED PART OF THE INFLATIONARY HISTORY

- MANY REASONS TO GO BEYOND THE VANILLA SCENARIO
  (leading to characteristic features in the primordial power spectrum)

SUMMARY

Oscillations in primordial scalar power spectrum → *Oscillations in frequency profile of GWs*

Probing inflation at small scales



FUTURE

- DETECTABILITY WITH LISA AND OTHER GWS OBSERVATORIES
  To what extend we can reconstruct 10% oscillations?

- STOCHASTIC BACKGROUND NEW WINDOW TO PROBE INFLATION
  Can be used to differentiate among different early universe scenarios?
  (Single field Vs Multi field inflation, Alternative to inflation , etc.)

- BACKREACTION/PERTURBATIVITY VS DETECTABILITY IN PRESENT AND FUTURE GWS DETECTORS To constraint scenarios from the theory

- WAY TO SEPARATE COSMOLOGICAL AND ASTROPHYSICAL BACKGROUND? (Speculative)

- NON- GAUSSIANITY Influence of the Trispectrum??

Back up

PEAK / PARTICLE PRODUCTION / OSCILLATIONS

$$\mathcal{P}_\zeta(k) \sim \mathcal{P}_0(k)|\alpha_k|^2\left(1 + \frac{|\beta_k|^2}{|\alpha_k|^2} + 2\frac{|\beta_k|}{|\alpha_k|}\cos\left(\frac{2k}{k_\mathrm{f}}\right)\right)$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

✖ ENHANCED POWER SPECTRUM

$$|\alpha_k|^2 \gg 1$$

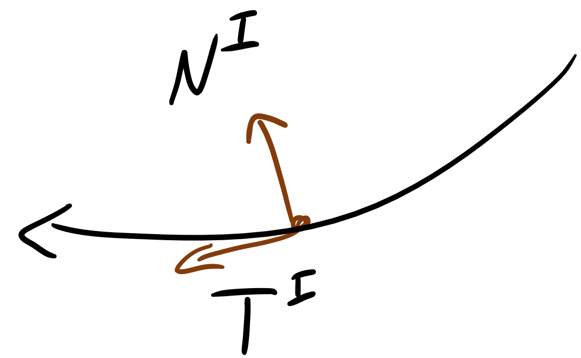
➕ QUANTIZATION CONDITIONS

$$\Rightarrow \left|\frac{\beta_k}{\alpha_k}\right| \simeq 1$$

LARGE AMOUNT OF PARTICLE PRODUCTION
&
ORDER ONE OSCILLATIONS

EXAMPLE: INFLATION IN NON-GEODESIC MOTION

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}G_{IJ}\partial^\mu\phi^I\partial_\mu\phi^J - V(\phi)\right]$$



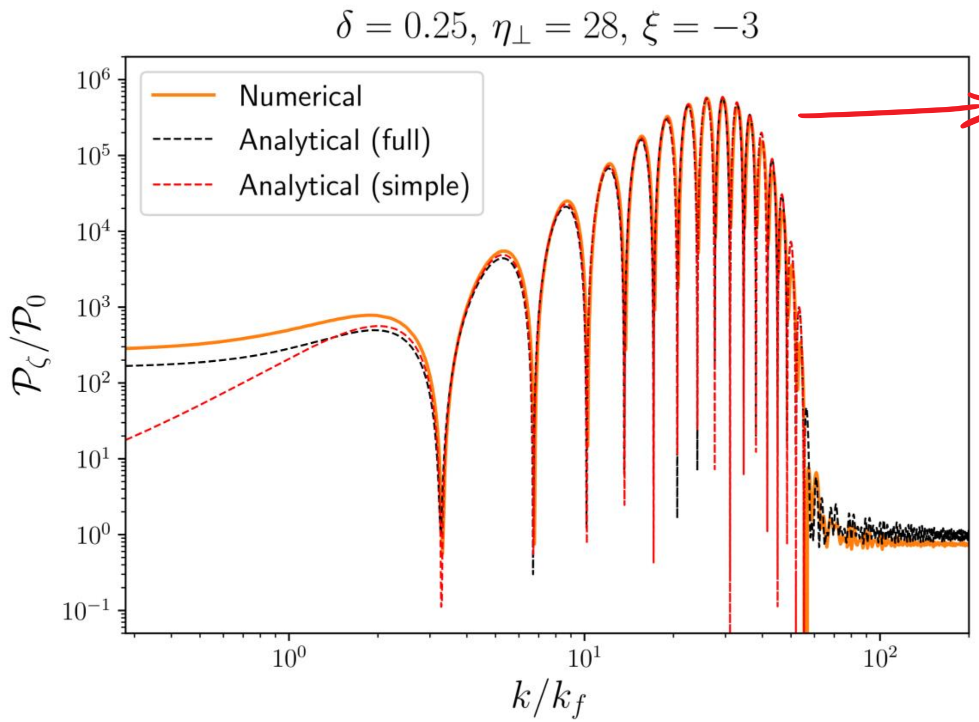
$\eta_\perp$ RATE OF THE TURN

δ DURATION OF THE TURN

$k_{\rm f}$ SCALE LEAVING HORIZON AT THE TURN

EXAMPLE: INFLATION IN NON-GEODESIC MOTION

- $\Delta t \ll H^{-1}$



OSCILLATORY PATTER IN $\mathcal{P}_\zeta(k)$

$\eta_\perp$ RATE OF THE TURN

δ DURATION OF THE TURN

k_f SCALE LEAVING HORIZON AT THE TURN

$$\frac{\mathcal{P}_\zeta(k)}{\mathcal{P}_0} = \frac{e^{2\eta_\perp \delta\, S}}{2S^2\left(1+X+\sqrt{X(1+X)}\right)} \times \left(1-\cos\left(\frac{2k}{k_\mathrm{f}} + \arctan\left(\frac{k}{\sqrt{(2\eta_\perp k_\mathrm{f} - k)k}}\right)\right)\right)$$

EXAMPLE: INFLATION IN NON-GEODESIC MOTION

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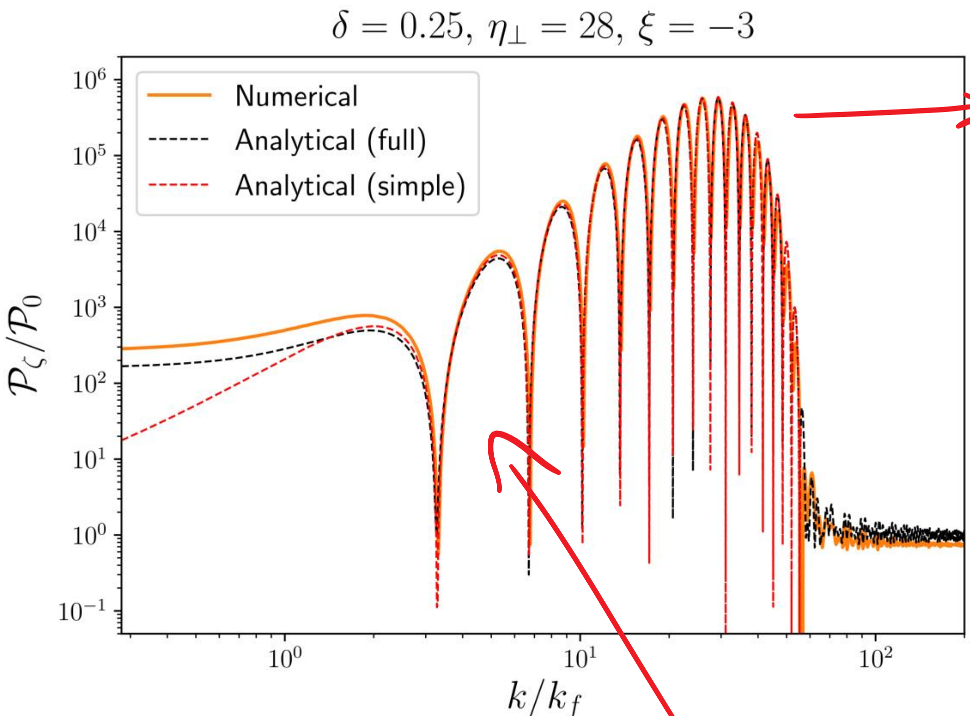


OSCILLATORY PATTER IN $\mathcal{P}_\zeta(k)$

$\eta_\perp$ RATE OF THE TURN

δ DURATION OF THE TURN

k_f SCALE LEAVING HORIZON AT THE TURN

CONSTANT FREQUENCY

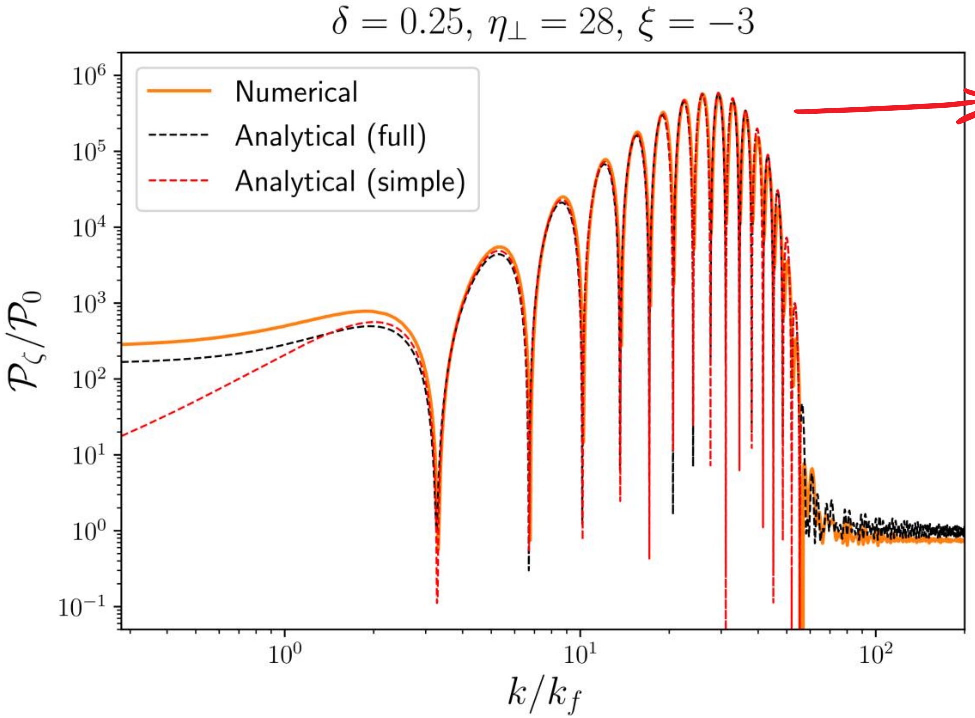
ENVELOPE DEPENDS ON m_s^2

$$\frac{\mathcal{P}_\zeta(k)}{\mathcal{P}_0} = \frac{e^{2\eta_\perp \delta\, S}}{2S^2\left(1 + X + \sqrt{X(1+X)}\right)} \times \left(1 - \cos\left(\frac{2k}{k_\mathrm{f}} + \arctan\left(\frac{k}{\sqrt{(2\eta_\perp k_\mathrm{f} - k)k}}\right)\right)\right)$$

ANGLE SWEPT IN FIELD-SPACE

EXAMPLE: INFLATION IN NON-GEODESIC MOTION

- $\Delta t \ll H^{-1}$



OSCILLATORY PATTER IN $\mathcal{P}_\zeta(k)$

$\eta_\perp$ RATE OF THE TURN

δ DURATION OF THE TURN

k_f SCALE LEAVING HORIZON AT THE TURN

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_\mathrm{lin}\cos\left(\omega_\mathrm{lin}k + \phi_\mathrm{lin}\right)\Big)$$

RESONANT FEATURE - TEMPLATE

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\left(1 + A_{\log}\cos\left(\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log}\right)\right)$$

$$\Omega_{\rm GW} \sim \iint \mathcal{P}_\zeta^2 \qquad \Rightarrow \qquad \Omega_{\rm GW}(k) = \Omega_{\rm GW,0}(k) + A\,\Omega_{\rm GW,1}(k) + A^2\,\Omega_{\rm GW,2}(k)$$

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RESONANT FEATURE - TEMPLATE

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$$\Omega_{\rm GW,1}(k) = \overline{\mathcal{P}}_\zeta^2\left(A_1(\omega_{\log})^2 + B_1(\omega_{\log})^2\right)^{1/2}\cos\left[\omega_{\log}\log\left(\frac{\sqrt{3}k}{2k_{\rm ref}}\right) + \theta_1(\omega_{\log})\right]$$

$$\Omega_{\rm GW,2}(k) = \overline{\mathcal{P}}_\zeta^2\left\{\left(A_2(\omega_{\log})^2 + B_2(\omega_{\log})^2\right)^{1/2}\cos\left[2\omega_{\log}\log\left(\frac{\sqrt{3}k}{2k_{\rm ref}}\right) + \theta_2(\omega_{\log})\right] + C_2(\omega_{\log})\right\}$$

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RESONANT FEATURE - TEMPLATE

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\Big(1 + A_{\log}\cos\left(\omega_{\log}\log(k/k_{\rm ref}) + \phi_{\log}\right)\Big)$$

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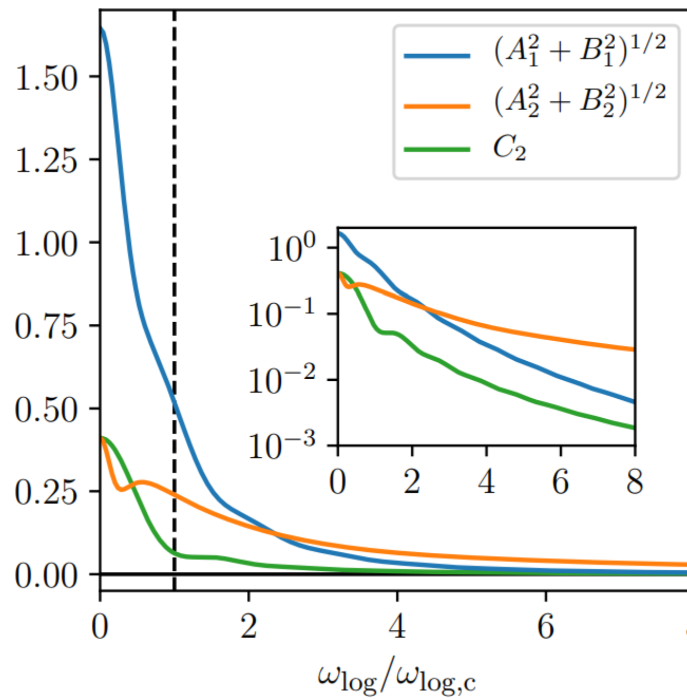
$$\Omega_{\rm GW,1}(k) = \overline{\mathcal{P}}_\zeta^2\left(A_1(\omega_{\log})^2 + B_1(\omega_{\log})^2\right)^{1/2}\cos$$

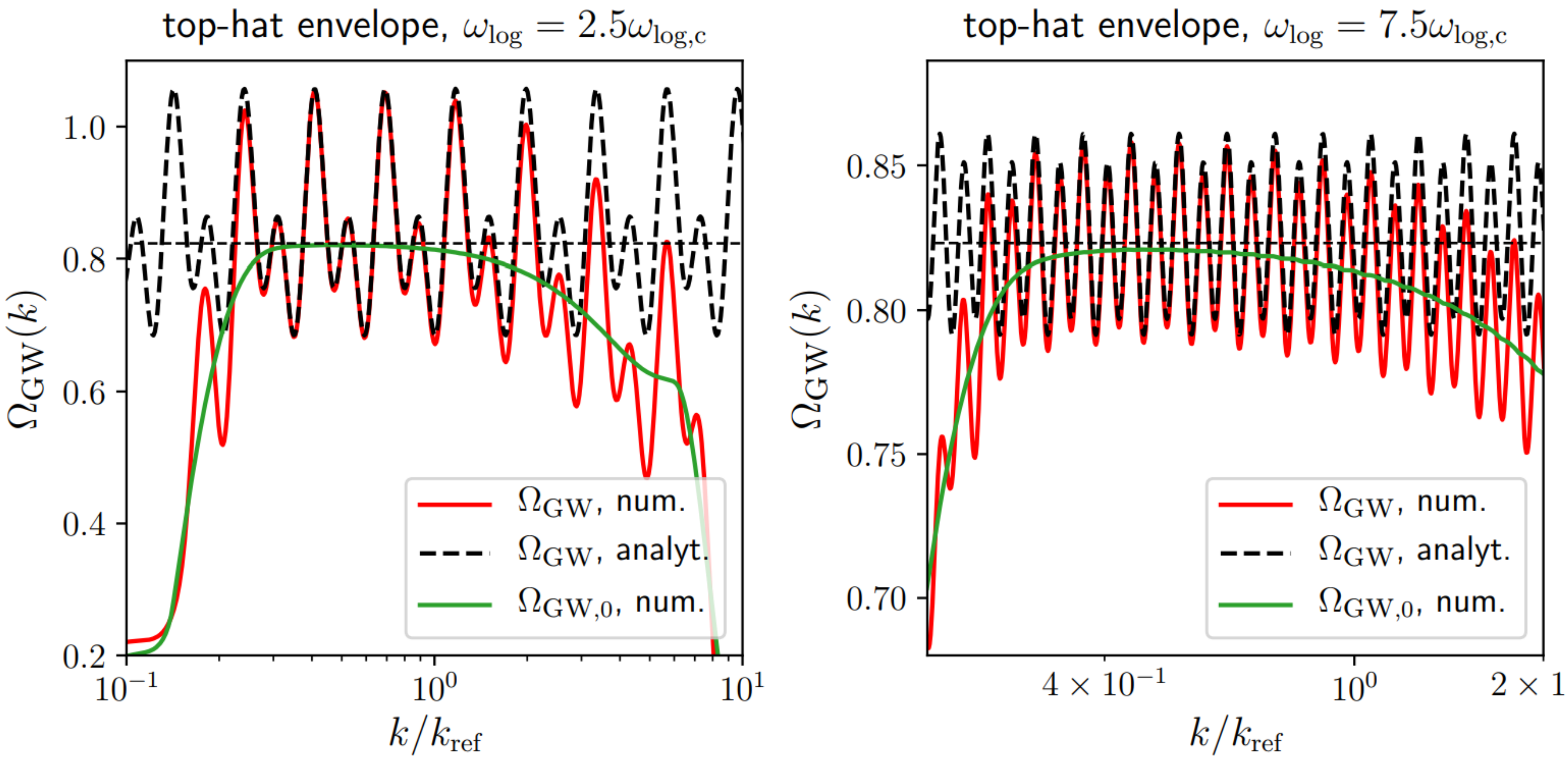
$$\Omega_{\rm GW,2}(k) = \overline{\mathcal{P}}_\zeta^2\Big\{\left(A_2(\omega_{\log})^2 + B_2(\omega_{\log})^2\right)^{1/2}\cos \quad \ldots \log)\Big\}$$



COMPUTED ONCE FOR ALL

J. F., S. Renaux-Petel, L. Witkowski; 2105.06481

RESONANT FEATURE - TEMPLATE



J. F., S. Renaux-Petel, L. Witkowski; 2105.02761