Entropy in the early universe

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Work in collaboration with Juan García-Bellido, see:
arXiv:2106.16012 and arXiv:2106.16014

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Outline

1. Motivation
2. Reversible cosmology
3. Entropic forces in General Relativity
4. Non-equilibrium cosmology
5. Cosmic acceleration
6. Conclusions

1. Motivation

- General Relativity is a time-reversible theory.
- Most of the universe expansion history is adiabatic, but there are a few non-equilibrium epochs.
- Irreversible phenomena are not included in General Relativity in a complete and systematic way.

2. Reversible cosmology

Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

Filled with a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

+ Einstein →

Friedmann equations

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad \text{(F1)}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \text{(F2)}$$

2. Reversible cosmology

What can we say about this perfect fluid? Energy conservation:

$$D_\mu T^{\mu\nu} = 0 \rightarrow \dot{\rho} + 3H(\rho + p) = 0$$

But this also arises from the second law of thermodynamics

$$T\frac{dS}{dt} = \frac{d}{dt}\left(\rho a^3\right) + p\frac{d}{dt}\left(a^3\right) = 0$$

Which is true only in equilibrium

Internal energy Change

Work

More generally $T\frac{dS}{dt} \geq 0$

2. Reversible cosmology

Should we go beyond adiabatic cosmology?

Continuity equation becomes $\dot{\rho} + 3H(\rho + p) = \frac{T\dot{S}}{a^3}$

+F1

→ Non-equilibrium F2 $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p - \frac{T\dot{S}}{a^3 H}\right)$$

3. Entropic forces in General Relativity

Can we make the previous statement more rigorous?

We will use the variational formulation of non-equilibrium thermodynamics (Gay-Balmaz & Yoshimura, 2017)

Stationary-action principle + Laws of thermodynamics

Stationary-action principle → Defines the (physical) Variational problem

Laws of thermodynamics → Imposes constraints on → Defines the (physical) Variational problem

3. Entropic forces in General Relativity

Let us apply this formalism to General Relativity

See LEP, J. García-Bellido
arXiv:2106.16012

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + \int d^4x\mathcal{L}_m(g_{\mu\nu}, s)$$

$$\delta\mathcal{S} = \int d^4x\left(\frac{1}{2\kappa}\frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} + \frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}\right)\delta g^{\mu\nu} + \int d^4x\frac{\delta\mathcal{L}_m}{\delta s}\delta s$$

Full treatment of time-evolution done in the ADM formalism

Add the constraint

$$\frac{\delta\mathcal{L}_m}{\delta s}\delta s = \frac{1}{2}\sqrt{-g}f_{\mu\nu}\delta g^{\mu\nu}$$

→ Non-equilibrium Einstein field equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa\left(T_{\mu\nu} - f_{\mu\nu}\right)$$

← Friction or Entropic force

3. Entropic forces in General Relativity

Consider a congruence of geodesics

$$\Theta_{\mu\nu} = D_\nu n_\mu = \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu} - a_\mu n_\nu$$

$$£_n\Theta = -\frac{1}{3}\Theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}n^\mu n^\nu + D_\mu a^\mu$$ → Raychauduri equation

If the strong energy condition $T_{\mu\nu}n^\mu n^\nu \geq -\frac{1}{2}T$ is satisfied and $a^\mu = 0$

$$£_n\Theta + \frac{1}{3}\Theta^2 \leq 8\pi G\left(f_{\mu\nu}n^\mu n^\nu + \frac{1}{2}f\right)$$ → Collapse can be avoided by entropic forces

3. Entropic forces in General Relativity

Temperature and entropy from the matter content

- Hydrodynamical matter

$$\mathcal{L} = -\sqrt{-g}\rho(g_{\mu\nu}, s)$$

- Gravity and horizons

$$\mathcal{S}_{GHY} = \frac{1}{8\pi G}\int_H d^3y\sqrt{h}K$$

$$T = -\frac{\partial L}{\partial S}$$

4. Non-equilibrium cosmology

Homogeneous and isotropic universe

$$ds^2 = -N(t)dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

Filled with a fluid with

$$\rho = \rho(a,S) \quad p = p(a,S)$$

Choose

$N(t) = 1$

Hamilton constraint

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

Hamilton equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\tilde{f}$$

with $\tilde{f} = h^{ij} f_{ij}$

4. Non-equilibrium cosmology

Consistency with the laws of thermodynamics means

Term of entropic origin

$$\tilde{f} = \frac{T\dot{S}}{a^2\dot{a}} \quad \Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\frac{T\dot{S}}{a^2\dot{a}}$$

Non-equilibrium F2

Entropy production
(P)reheating, phase transitions, black hole formation… → Cosmic acceleration

5. Cosmic acceleration

The entropy of the causal horizon in open inflation may explain cosmic acceleration

The GHY term introduces an entropic fluid

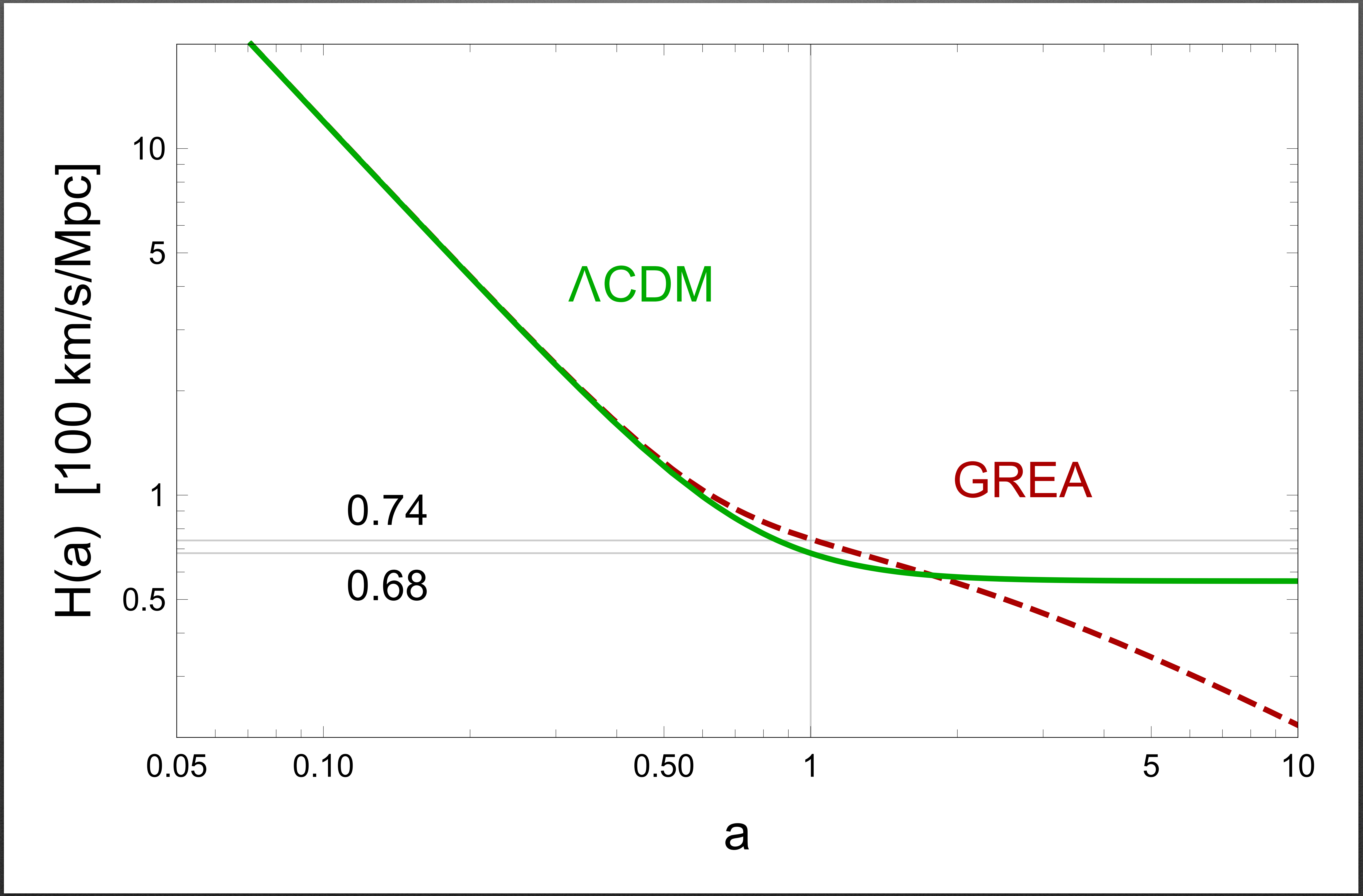
$$\rho_H\, a^2 = \frac{T_H S_H}{a} = \frac{x_0}{2G}\sinh(2a_0 H_0 \eta) \qquad x_0 \equiv \frac{1-\Omega_0}{\Omega_0} = e^{-2N}\left(\frac{T_{rh}}{T_{eq}}\right)^2 (1+z_{eq})$$

It satisfies the non-equilibrium continuity equation

$$\dot{\rho}_H + 3H(\rho_H + p_H) = \frac{T_H \dot{S}_H}{a^3}$$

See J. García-Bellido, LEP
arXiv:2106.16014

5. Cosmic acceleration



6. Conclusions

- The laws of thermodynamics modify the dynamics of any physical system and break time reversibility.
- The Lagrangian and Hamiltonian formulations of General Relativity can be modified to include the second law of thermodynamics.
- In Cosmology, this means the appearance of an additional term of entropic origin in the second Friedmann equation.
- The effect of non-adiabatic phenomena in the expansion history of the universe should be revisited.
- The entropy of the causal horizon can explain cosmic acceleration and the Hubble tension.