Global properties of SMEFT and HEFT

based on 2008.08597 with T. Cohen, N. Craig and X. Lu, 2001.00017 with N. Craig, M. Jiang, Y-y. Li

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Consider the (simplified) EFT of the SM scalar sector

We observe four scalar degrees of freedom in high energy collisions: the Higgs boson and the three longitudinal components of the W^+, W^- and Z. What field theory should we use to parameterise their interactions?

On first principles, what is the difference between scalar sectors of:

- **SMEFT**: built about the electroweak preserving vacuum, out of fields $\vec{\phi}$ that linearly realise electroweak symmetry, and
- **HEFT**: built about our low energy vacuum, out of fields h, $\vec{\pi}$ that don't?

Are **SMEFT** and **HEFT** just field redefinitions of each other?

Use (field redefinition invariant) geometric properties

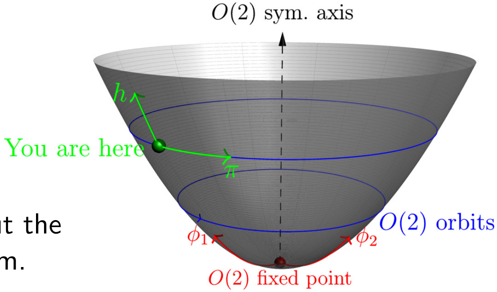
(Alonso, Jenkins, and Manohar 2016)

Dynamics encoded by a metric and potential.

$$\mathcal{L} = \frac{1}{2} g_{\alpha\beta}(\phi)\partial^\mu \phi^\alpha \partial_\mu \phi^\beta - V(\phi)$$

HEFT is an expansion about our vacuum.

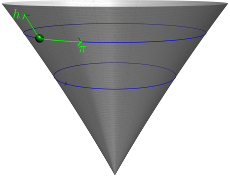
SMEFT is an expansion about the electroweak preserving vacuum.



A HEFT is poorly described by SMEFT when sufficient violence is done to the manifold between us and the EW preserving vacuum.

When is a HEFT not a SMEFT?

1) When turning off the Higgs vev gives massless BSM particles (Falkowski and Rattazzi 2019)



Extend the scalar sector with an EW singlet

$$\mathcal{L}_{\rm UV} = |\partial H|^2 + \frac{1}{2}(\partial S)^2 - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2}\left(m^2 + \kappa |H|^2\right) S^2 + \frac{1}{4}\lambda_S S^4\right)$$

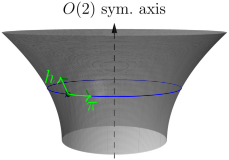
Match at tree-level: sub in the solution $S^{\rm c}$ to the EOM, assume $m^2, \kappa \leq 0$.

$$\frac{\delta S_{\rm UV}}{\delta S} = (\partial^2 + m^2 + \kappa |H|^2 + \lambda_S S^2) S = 0 \implies S^{\rm c} = \sqrt{-\frac{m^2 + \kappa |H|^2}{\lambda_S}} + \mathrm{O}(\partial^2)$$

$$\mathcal{L}_{\rm EFT} = |\partial H|^2 - \frac{\kappa^2 \left(\partial_\mu |H|^2\right)^2}{4\lambda_S \left(m^2 + \kappa |H|^2\right)} - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 - \frac{\left(m^2 + \kappa |H|^2\right)^2}{4\lambda_S}\right) + \mathrm{O}(\partial^4)$$

The lagrangian is non-analytic at $H = 0$ when $m^2 = 0$.

When is a HEFT not a SMEFT?



2) When there are extra sources of EWSB, e.g., a triplet

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \frac{1}{2}(\partial \Phi)^2$$
$$- \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2} m^2 \Phi^2 - \frac{1}{2} \mu H^\dagger \sigma^a H \Phi_a + \kappa |H|^2 \Phi^2 + \frac{1}{4} \lambda_\Phi \Phi^4 \right)$$

Reparameterise as radial (r, f) and angular modes (π^a, β^i)

$$H = \frac{1}{\sqrt{2}} r \exp\left(i \frac{\pi^a}{v} \sigma^a \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad \Phi_a = \frac{2f}{r^2} \exp \begin{pmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 \\ -\beta_1 & -\beta_2 & 0 \end{pmatrix} \begin{pmatrix} H^\dagger \sigma^1 H \\ H^\dagger \sigma^2 H \\ H^\dagger \sigma^3 H \end{pmatrix}$$

to integrate out at tree-level (sub. in EOM solutions of f and β)

$$\frac{\partial V}{\partial \beta^i} = 0 \implies \beta^i = 0; \frac{\partial V}{\partial f}\bigg|_{\beta^i = 0} = -\frac{1}{4} \mu r^2 + (m^2 + \kappa r^2) f + \lambda_\Phi f^3 = 0$$

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}\left[1 + (f_c')^2 + \frac{8 f_c^2}{r^2} \right] (\partial r)^2 + \frac{1}{2} \left[\frac{r^2 + 4 f_c^2}{v^2} \right] \left((\partial \pi_1)^2 + (\partial \pi_2)^2 \right) + \frac{1}{2} \left[\frac{r^2}{v^2} \right] (\partial \pi_3)^2$$
$$- V + \mathrm{O}(\partial^4, \pi^4)$$

Need $f_c \to 0$ as $r \to 0$ for SMEFT

SMEFT, like the SM, is a chiral theory

Can write the lagrangian in terms of fields in irreps of Lorentz group $\approx SU(2)_L \times SU(2)_R$

$$\phi, \psi_\alpha, \bar{\psi}_{\dot{\alpha}}, F_{\alpha\beta}, \bar{F}_{\dot{\alpha}\dot{\beta}}, D_{\alpha\dot{\alpha}}\,,$$

These excite massless particles of definite helicities

$$\begin{array}{ccccccc} & \phi & \psi^+ & \psi^- & V^+ & V^- & V^\pm \\ h = & 0 & \frac{1}{2} & -\frac{1}{2} & 1 & -1 & \pm 1 \end{array}$$

Leading to the familiar dimension 4 Lagrangian

$$\mathcal{L}_4 = -F^2 - \bar{F}^2 + i\bar{\psi}D\psi + (D\phi)^2 - \lambda\phi^4 - y\phi\psi\psi + \text{h.c.}$$

Dimension 4 tree-level amplitude map

(Cheung and Shen 2015)

In terms of the coordinates

1. n, number of external legs
2. $\sum h$, total helicity of external legs, with all particles' momenta outgoing

At $(n, \sum h) = (3, 1)$,

$$\mathcal{A}(\phi\psi^+\psi^+),\ \mathcal{A}(\psi^+\psi^- V^+)$$

$$\mathcal{A}(\phi\phi V^+),\ \mathcal{A}(V^+V^+V^-)$$

At $(n, \sum h) = (4, 2)$,

$$\mathcal{A}(V^+V^+V^+V^-)$$

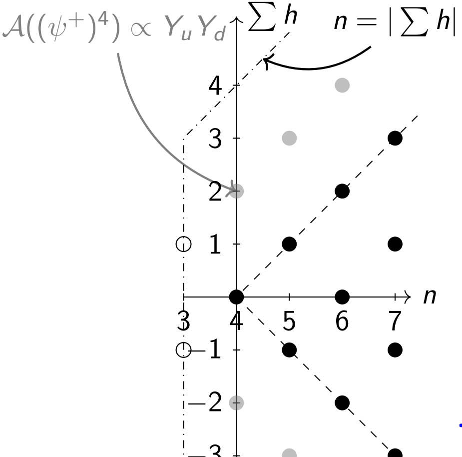
$$\mathcal{A}(V^+V^+\psi^+\psi^-)$$

$$\mathcal{A}(V^+V^+\phi\phi)$$

$$\mathcal{A}(V^+\psi^+\psi^+\phi)$$

vanish at dim 4, leaving just

$$\mathcal{A}(\psi^+\psi^+\psi^+\psi^+)$$

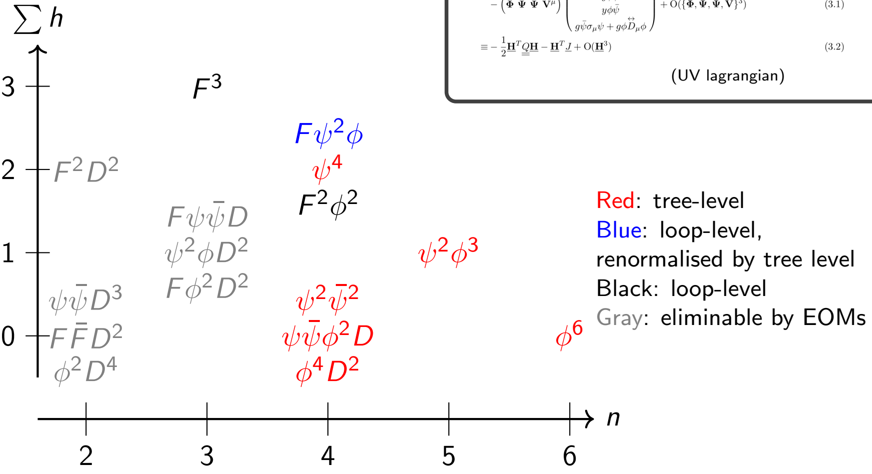


A map of dimension 6 operators

Classified by tree/loop-level generated

$$
\begin{aligned}
\mathcal{L}_{\mathrm{UV}} = &-\frac{1}{2}\begin{pmatrix}\mathbf{\Phi} & \mathbf{\Psi} & \bar{\mathbf{\Psi}} & \mathbf{V}^{\mu}\end{pmatrix}\begin{pmatrix} D^2+M^2+\lambda\phi^2 & y\psi & y\bar{\psi} & 0 \\ y\psi & M+y\phi & -i\not{D} & 0 \\ y\bar{\psi} & i\not{D} & M+y\phi & 0 \\ 0 & 0 & 0 & \begin{matrix} -g_{\mu\nu}(D^2+M^2+g\phi^2)+ \\ D_\nu D_\mu - [D_\mu, D_\nu] \end{matrix} \end{pmatrix}\begin{pmatrix}\mathbf{\Phi} \\ \mathbf{\Psi} \\ \bar{\mathbf{\Psi}} \\ \mathbf{V}^{\nu}\end{pmatrix} \\
&-\begin{pmatrix}\mathbf{\Phi} & \mathbf{\Psi} & \bar{\mathbf{\Psi}} & \mathbf{V}^{\mu}\end{pmatrix}\begin{pmatrix} y\psi\psi + y\bar{\psi}\bar{\psi} + \lambda\phi^3 \\ y\phi\psi \\ y\phi\bar{\psi} \\ g\bar{\psi}\sigma_\mu\psi + g\phi\overleftrightarrow{D}_\mu\phi \end{pmatrix} + \mathrm{O}(\{\mathbf{\Phi}, \mathbf{\Psi}, \bar{\mathbf{\Psi}}, \mathbf{V}\}^3) \qquad (3.1) \\
\equiv &-\frac{1}{2}\underline{\mathbf{H}}^T \underline{\underline{Q}}\,\underline{\mathbf{H}} - \underline{\mathbf{H}}^T \underline{J} + \mathrm{O}(\underline{\mathbf{H}}^3) \qquad (3.2)
\end{aligned}
$$

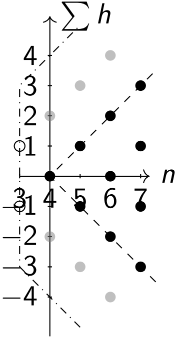
(UV lagrangian)



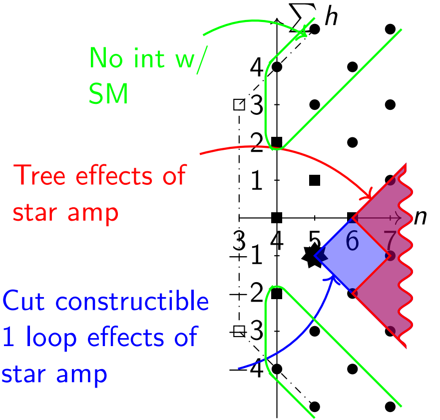
...plotted in terms of the number of legs n, and total helicity $\sum h$, of the associated contact interactions.

A map of dim 6 processes

(Dim. 4)



(Dim. 6)



(Azatov, Contino, Machado, and Riva 2017): Many dim 6 operators, at tree level, do not interfere with SM, e.g. $\mathcal{A}(V^+\psi^+\psi^+\phi)$.

(Cheung and Shen 2015): Many operators do not renormalise others at one loop, e.g., $\phi^4 D^2 \not\to F^2\phi^2$.

Many one-loop dimension 6 amplitudes vanish exactly

Non-Abelian

		(4,0)				(4,2)	
		$V^+V^+V^-V^-$	$V^+V^-\psi^+\psi^-$	$V^+V^-\phi\phi$	$V^+\psi^-\psi^-\phi$	$V^+V^+V^+V^-$	$V^+V^+\psi^+\psi^-$
(4,0)	$\psi^2\bar\psi^2$	×	0	×	0*	×	R
	$\phi^4 D^2$	×	×	0	×	×	×
	$\phi^2\psi\bar\psi D$	×	0	0	0	×	R
(4,2)	$F\psi^2\phi$	×	R	R	R	×	0
	$F^2\phi^2$	R	0	R	R	0*	0*
	ψ^4	×	0	×	0	×	0
(4,−2)	$\bar F\bar\psi^2\phi$	×	R	R	R	×	0
	$\bar F^2\phi^2$	R	0	R	R	0	0
	$\bar\psi^4$	×	0	×	R	×	0

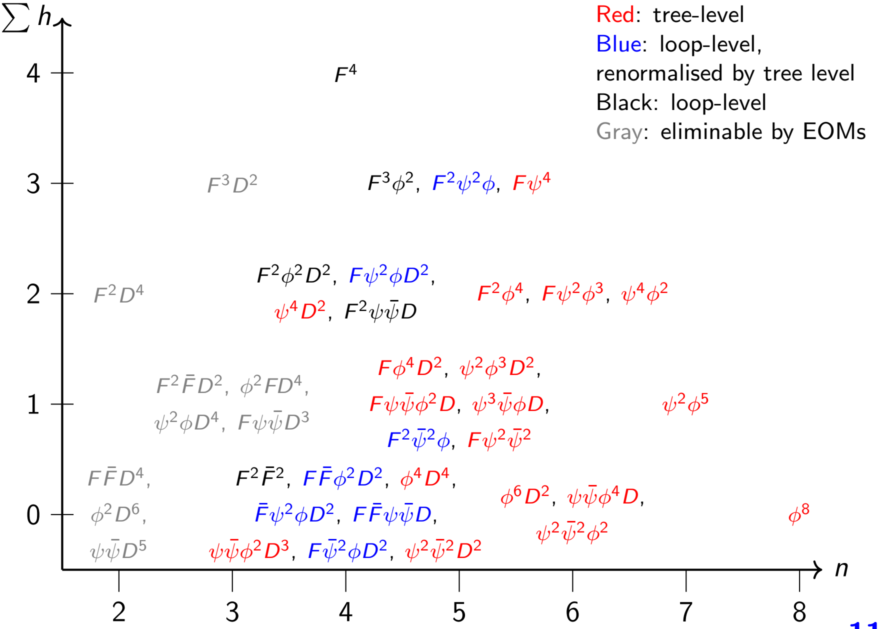
Abelian

		(4,0)				(4,2)	
		$V^+V^+V^-V^-$	$V^+V^-\psi^+\psi^-$	$V^+V^-\phi\phi$	$V^+\psi^-\psi^-\phi$	$V^+V^+V^+V^-$	$V^+V^+\psi^+\psi^-$
(4,0)	$\psi^2\bar\psi^2$	×	0	×	0*	×	0
	$\phi^4 D^2$	×	×	0	×	×	×
	$\phi^2\psi\bar\psi D$	×	0	0	0	×	0
(4,2)	$F\psi^2\phi$	×	R	R	R	×	0
	$F^2\phi^2$	R	0	R	R	0	0
	ψ^4	×	0	×	0	×	0
(4,−2)	$\bar F\bar\psi^2\phi$	×	R	R	R	×	0
	$\bar F^2\phi^2$	R	0	R	R	0	0
	$\bar\psi^4$	×	0	×	R	×	0

[Key: ×=No diagram, 0 = zero, R = rational (non-zero)];

See also (Jiang, Shu, Xiao, and Zheng 2021) for an explanation in terms of angular momentum selection rules.

A map of dimension 8 operators



Summary

2008.08597: Using a geometric picture, we argue that UV theories containing

1. particles getting most of their mass from electroweak symmetry breaking;
2. extra sources of electroweak symmetry breaking,

are in principle not amenable to a SMEFT description.

2001.00017: SMEFT operators have a particular pattern in helicity amplitudes (in the high energy limit). Many contributions vanish exactly at one-loop!

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