Top-quark production at approximate N^{3}LO

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- **Higher-order soft-gluon corrections**
- **Three-loop soft anomalous dimensions**
- **Top-pair and tW production**
- **tqH production**

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Soft-gluon corrections

partonic processes (in general $2 \to n$)

$$f_1(p_1) + f_2(p_2) \to t(p_t) + X$$

define $s = (p_1 + p_2)^2$, $t = (p_1 - p_t)^2$, $u = (p_2 - p_t)^2$ **and** $s_4 = s + t + u - p_t^2 - p_X^2$

At partonic threshold $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \leq 2n - 1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

At NNLL accuracy we need two-loop soft anomalous dimensions

At N^3LL accuracy we need three-loop soft anomalous dimensions

Finite-order expansions-no prescription needed

Approximate NNLO (aNNLO) and N^3LO (aN^3LO) predictions

for cross sections and differential distributions (single and double)

Soft-gluon Resummation

moments of the differential partonic cross section with moment variable N:
$d\hat{\sigma}(N) = \int (ds_4/s)\, e^{-N s_4/s}\, d\hat{\sigma}(s_4)$

factorized expression for the cross section in $4-\epsilon$ dimensions

$$d\sigma^{f_1 f_2 \to tX}(N,\epsilon) = H_{IL}^{f_1 f_2 \to tX}\left(\alpha_s(\mu_R)\right)\, S_{LI}^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F}, \alpha_s(\mu_R)\right)$$
$$\times\, \psi_1\left(N_1, \mu_F, \epsilon\right)\, \psi_2\left(N_2, \mu_F, \epsilon\right) \prod J\left(N, \mu_F, \epsilon\right)$$

$H_{IL}^{f_1 f_2 \to tX}$ **is hard function and** $S_{LI}^{f_1 f_2 \to tX}$ **is soft function**

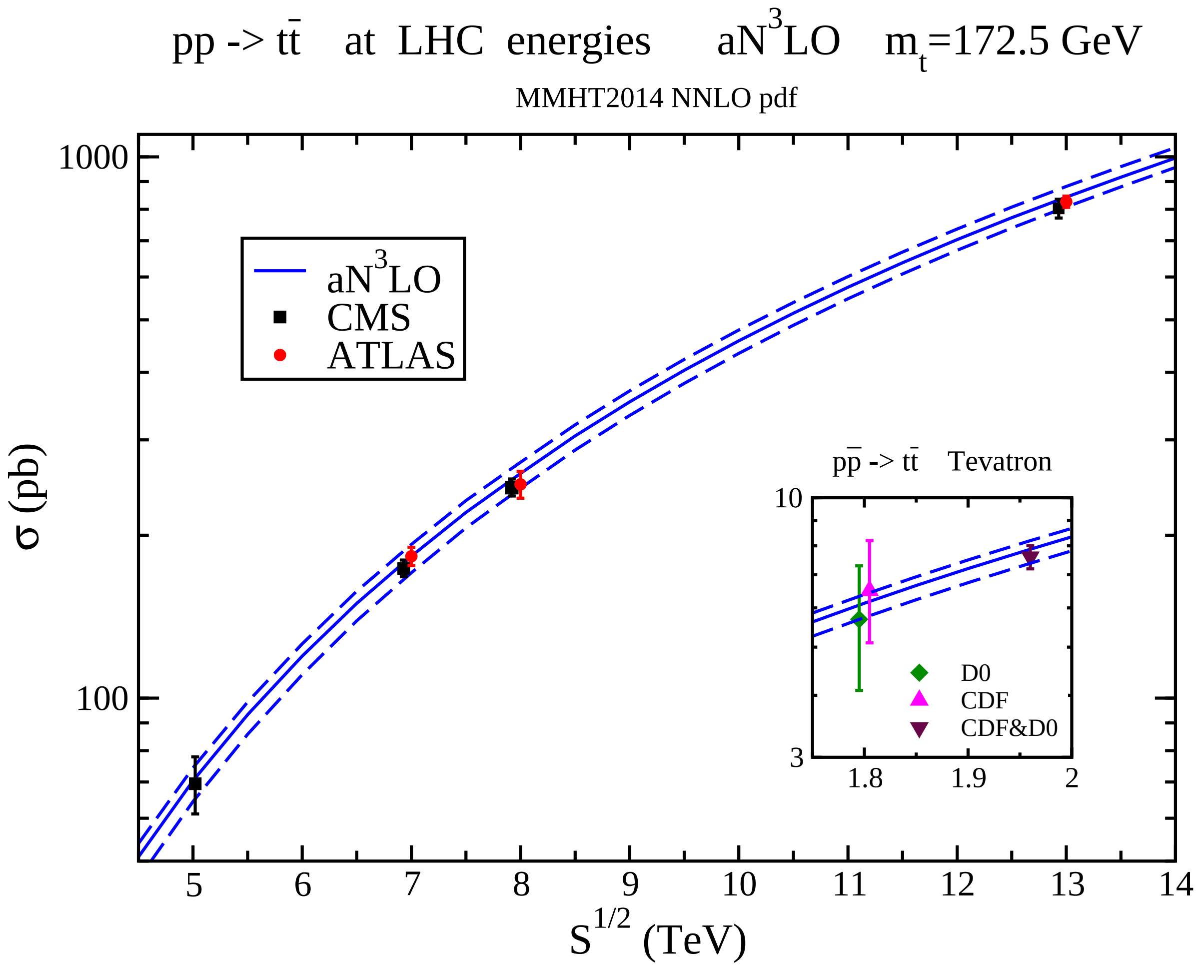
$S_{LI}^{f_1 f_2 \to tX}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI}^{f_1 f_2 \to tX} = -(\Gamma_S^{\dagger})_{LK}^{f_1 f_2 \to tX}\, S_{KI}^{f_1 f_2 \to tX} - S_{LK}^{f_1 f_2 \to tX}\, (\Gamma_S)_{KI}^{f_1 f_2 \to tX}$$

Soft anomalous dimension $\Gamma_S^{f_1 f_2 \to tX}$ controls the evolution of the soft function

which gives the exponentiation of logarithms of N

Top-antitop pair production



Top-antitop pair production

$\Gamma_S^{q\bar{q}\to t\bar{t}}$ is a 2×2 matrix while $\Gamma_S^{gg\to t\bar{t}}$ is a 3×3 matrix

At one loop for $q\bar{q}\to t\bar{t}$ (with s-channel singlet-octet color basis)

$$\Gamma_{S\,11}^{(1)q\bar{q}\to t\bar{t}} = \Gamma_{\rm cusp}^{(1)\,\beta}, \quad \Gamma_{12}^{(1)q\bar{q}\to t\bar{t}} = \frac{C_F}{C_A}\ln\left(\frac{t-m_t^2}{u-m_t^2}\right), \quad \Gamma_{21}^{(1)q\bar{q}\to t\bar{t}} = 2\ln\left(\frac{t-m_t^2}{u-m_t^2}\right)$$

$$\Gamma_{22}^{(1)q\bar{q}\to t\bar{t}} = \left(1-\frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)} + 4C_F\ln\left(\frac{t-m_t^2}{u-m_t^2}\right) - \frac{C_A}{2}\left[1+\ln\left(\frac{sm_t^2(t-m_t^2)^2}{(u-m_t^2)^4}\right)\right]$$

At two loops for $q\bar{q}\to t\bar{t}$

$$\Gamma_{S\,11}^{(2)q\bar{q}\to t\bar{t}} = \Gamma_{\rm cusp}^{(2)\,\beta}, \quad \Gamma_{12}^{(2)q\bar{q}\to t\bar{t}} = \left(K_2 - C_A\,N_2^\beta\right)\Gamma_{12}^{(1)q\bar{q}\to t\bar{t}}, \quad \Gamma_{21}^{(2)q\bar{q}\to t\bar{t}} = \left(K_2 + C_A\,N_2^\beta\right)\Gamma_{21}^{(1)q\bar{q}\to t\bar{t}}$$

$$\Gamma_{22}^{(2)q\bar{q}\to t\bar{t}} = K_2\Gamma_{22}^{(1)q\bar{q}\to t\bar{t}} + \left(1-\frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(2)\,\beta} - K_2\Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{C_A^2}{4}(1-\zeta_3)$$

where

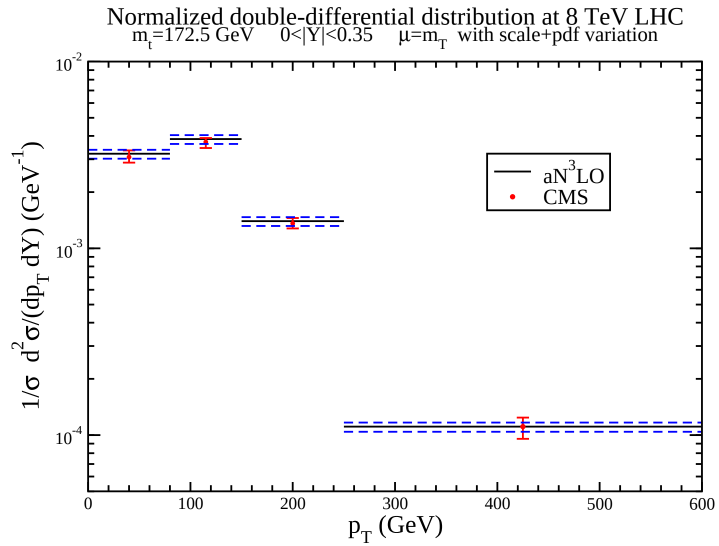
$$N_2^\beta = \frac{1}{4}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \frac{(1+\beta^2)}{8\beta}\left[\zeta_2 - \ln^2\left(\frac{1-\beta}{1+\beta}\right) - {\rm Li}_2\left(\frac{4\beta}{(1+\beta)^2}\right)\right]$$

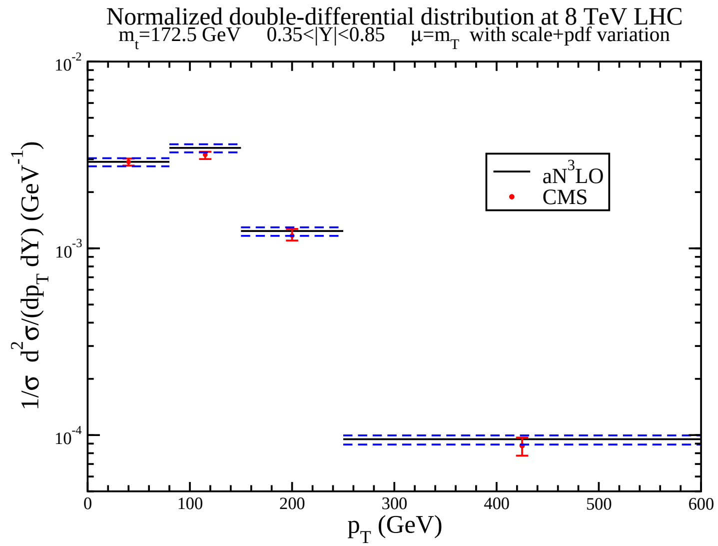
At three loops for $q\bar{q}\to t\bar{t}$

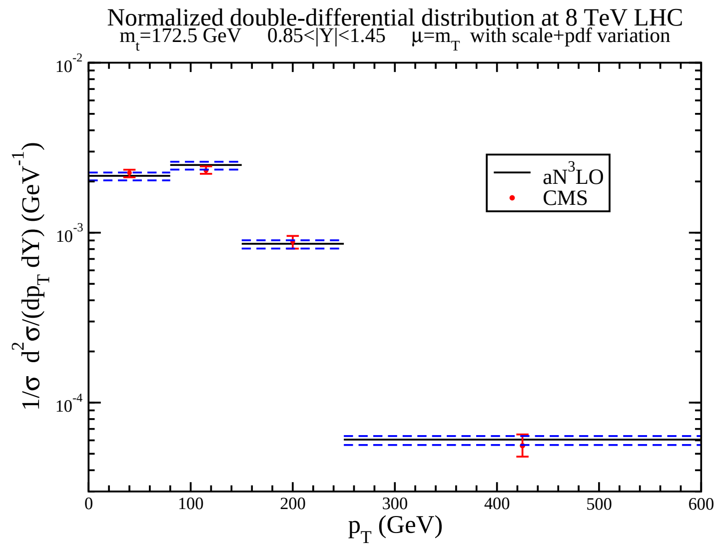
$$\Gamma_{S\,22}^{(3)q\bar{q}\to t\bar{t}} = K_3\,\Gamma_{S\,22}^{(1)q\bar{q}\to t\bar{t}} + \left(1-\frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(3)\,\beta} - K_3\Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{K_2}{2}C_A^2(1-\zeta_3)$$
$$+ C_A^3\left(-\frac{1}{4}+\frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)q\bar{q}\to t\bar{t}}$$

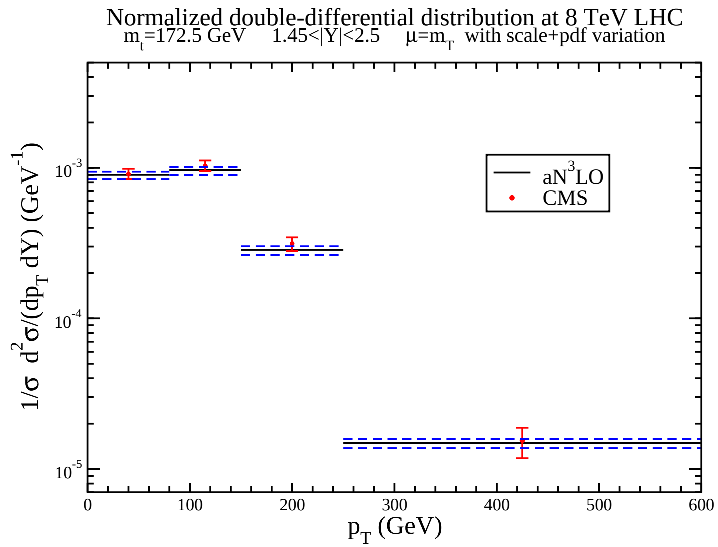
where $X_{S\,22}^{(3)q\bar{q}\to t\bar{t}}$ denotes unknown three-loop contributions from four-parton correlations

Top double-differential distributions in $t\bar{t}$ production

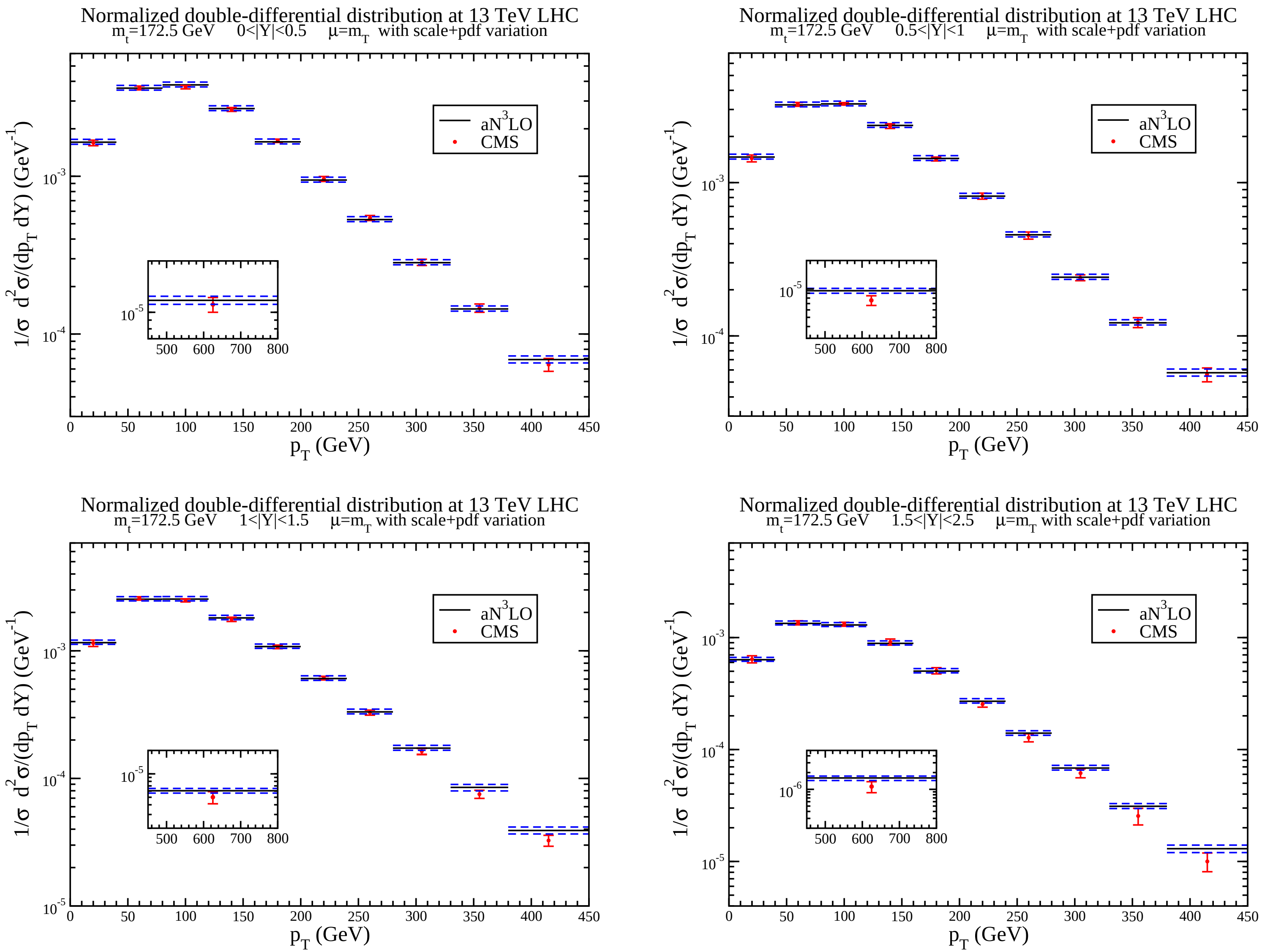








Top double-differential distributions in $t\bar{t}$ production



tW production

At one loop

$$\Gamma_S^{(1)bg\to tW} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{u - m_t^2}{t - m_t^2}\right)$$

At two loops

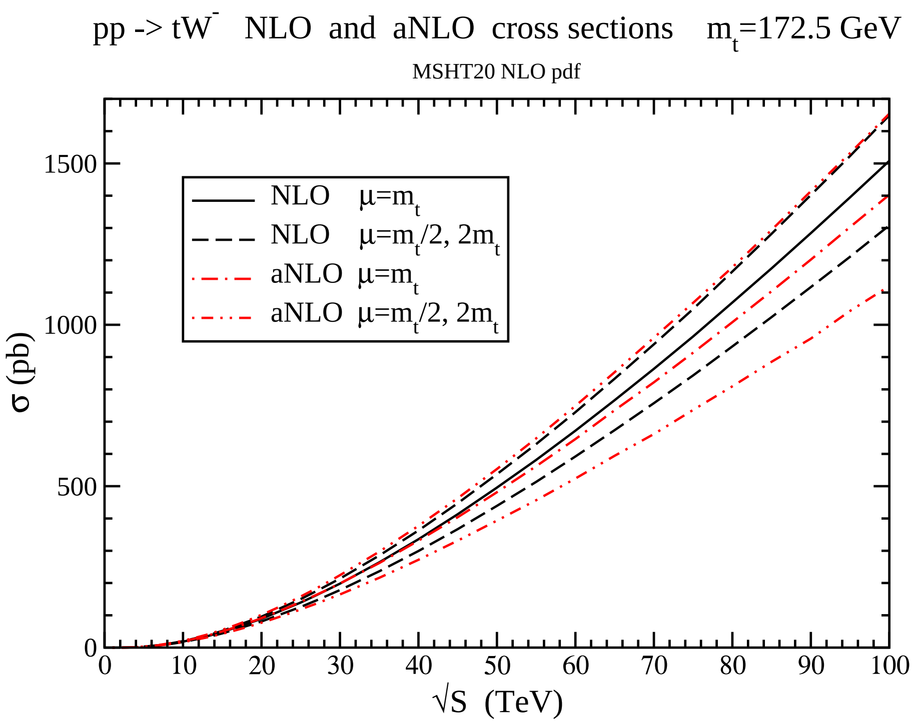
$$\Gamma_S^{(2)bg\to tW} = K_2\,\Gamma_S^{(1)bg\to tW} + \frac{1}{4}C_F C_A(1-\zeta_3)$$

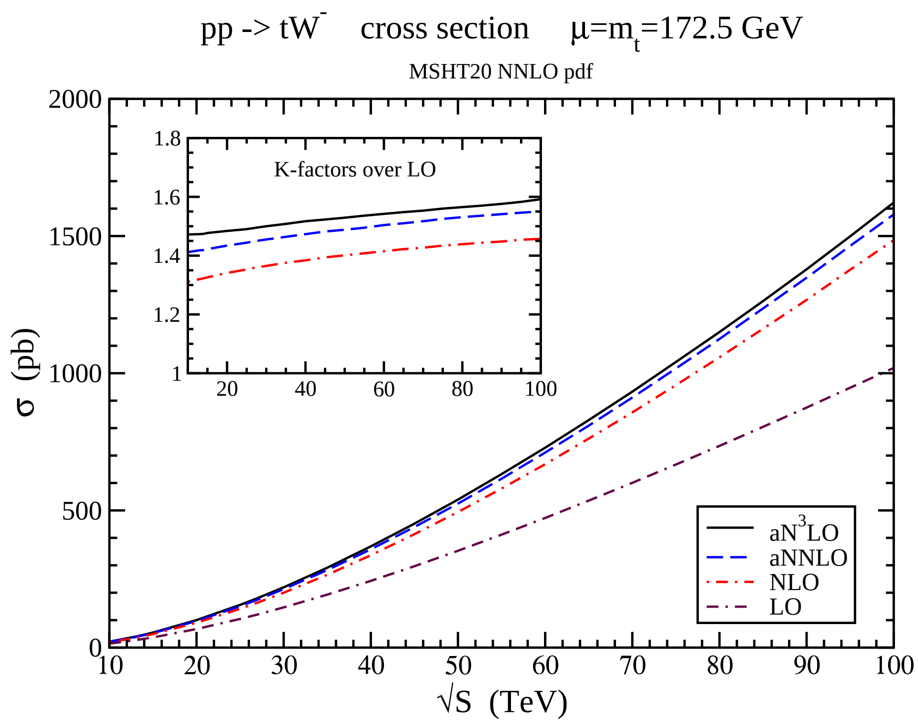
At three loops

$$\Gamma_S^{(3)bg\to tW} = K_3\,\Gamma_S^{(1)bg\to tW} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3) + C_F C_A^2\left(-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right)$$

tW production at high-energy colliders

(with Nodoka Yamanaka, arXiv:2102.11300)

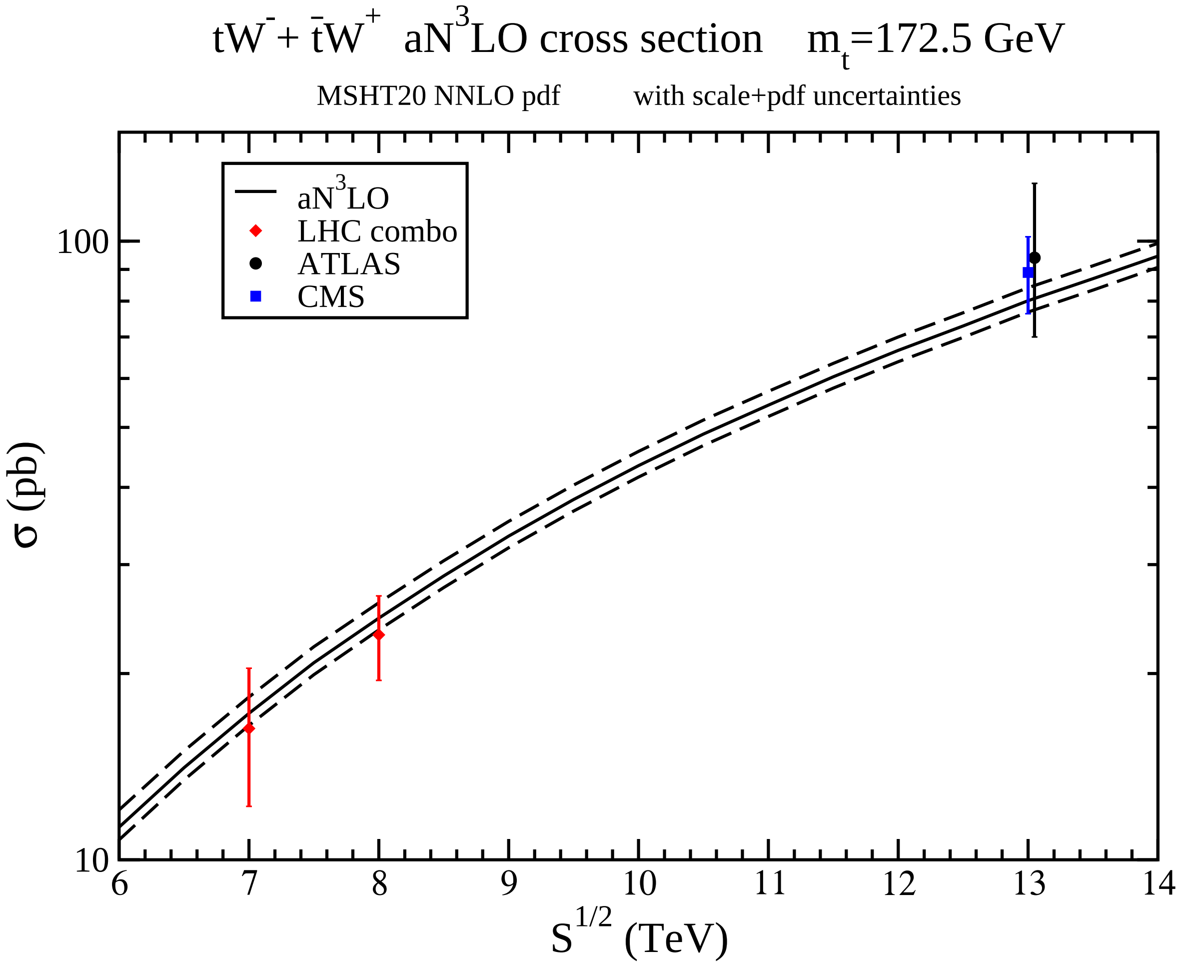




The aNLO cross section is a very good approximation to the complete NLO result for all forseeable collider energies

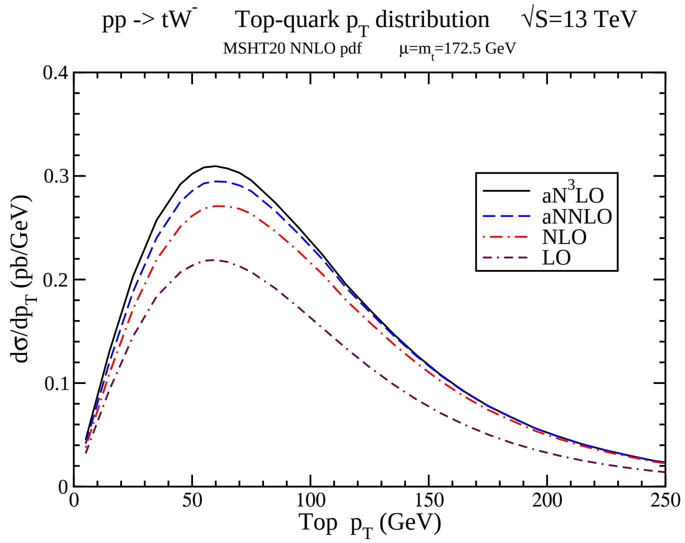
→ the soft-gluon corrections are dominant

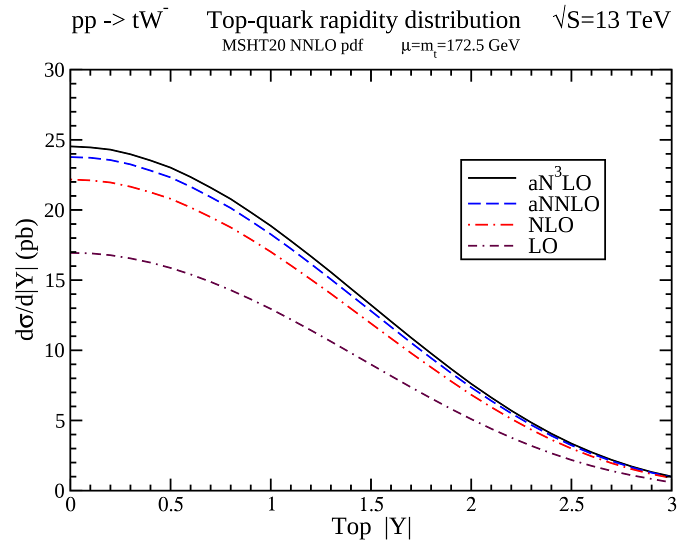
The aNNLO and aN^{3}LO corrections (at NNLL) are also significant

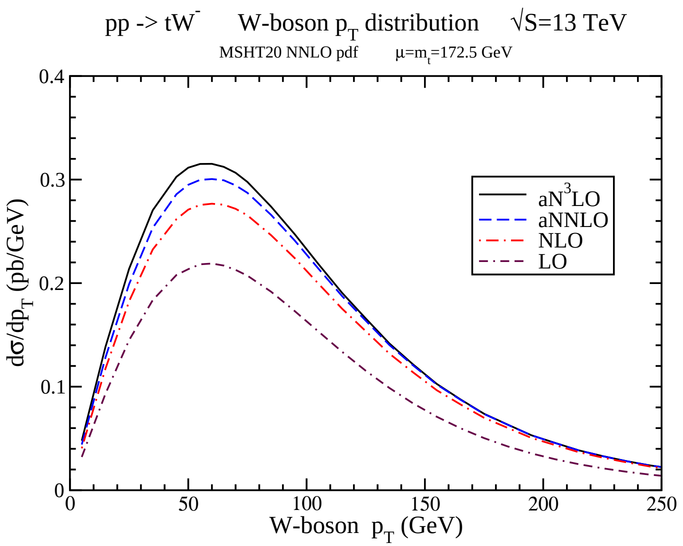


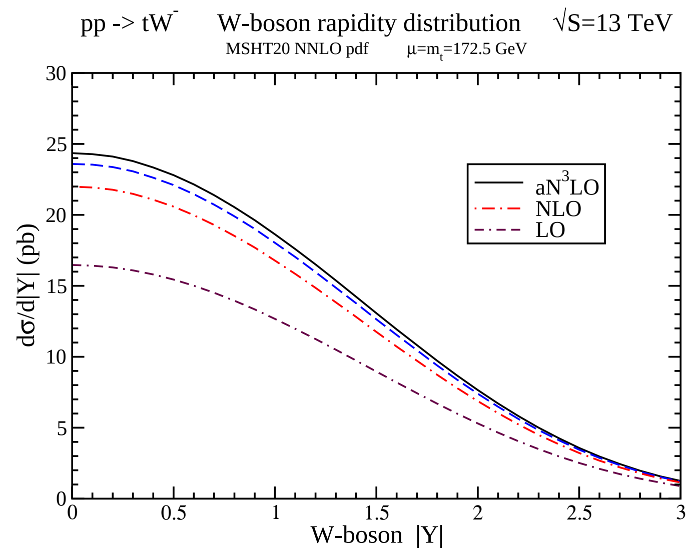
The aN^{3}LO cross section with scale and pdf (MSHT20) uncertainty is at 13 TeV: $79.5^{+1.9+2.0}_{-1.8-1.4}$ **pb** **at 14 TeV** $94.0^{+2.2+2.2}_{-2.1-1.6}$ **pb**

Top-quark and W-boson distributions in tW production









tqH, tqZ, $tq\gamma$, tqW production

we consider processes $bq \to tq'H$ as well as $bq \to tq'Z$, $bq \to tq'\gamma$, $bq \to tqW^-$, and use t-channel singlet-octet color basis

At one loop

$$\Gamma_{S\,11}^{(1)\,bq \to tq'H} = C_F \left[\ln\left(\frac{t'(t-m_t^2)}{m_t s^{3/2}}\right) - \frac{1}{2}\right]$$

$$\Gamma_{S\,12}^{(1)\,bq \to tq'H} = \frac{C_F}{2N_c} \ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right)$$

$$\Gamma_{S\,21}^{(1)\,bq \to tq'H} = \ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right)$$

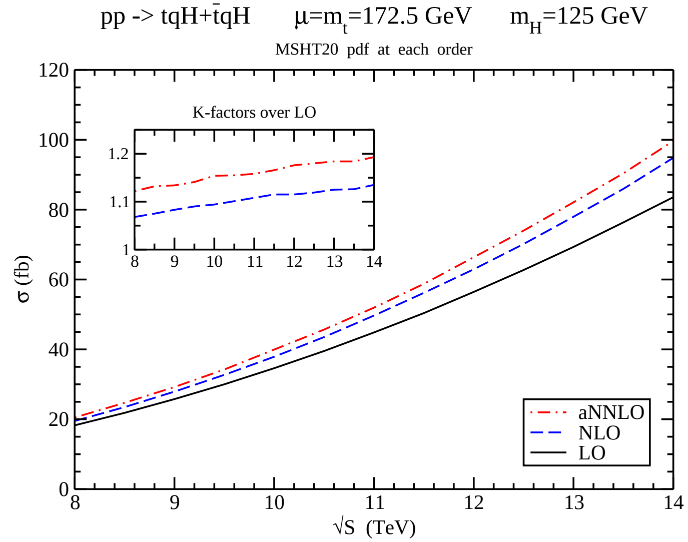
$$\Gamma_{S\,22}^{(1)\,bq \to tq'H} = C_F \left[\ln\left(\frac{t'(t-m_t^2)}{m_t s^{3/2}}\right) - \frac{1}{2}\right] - \frac{1}{N_c} \ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right) + \frac{N_c}{2} \ln\left(\frac{u'(u-m_t^2)}{t'(t-m_t^2)}\right)$$

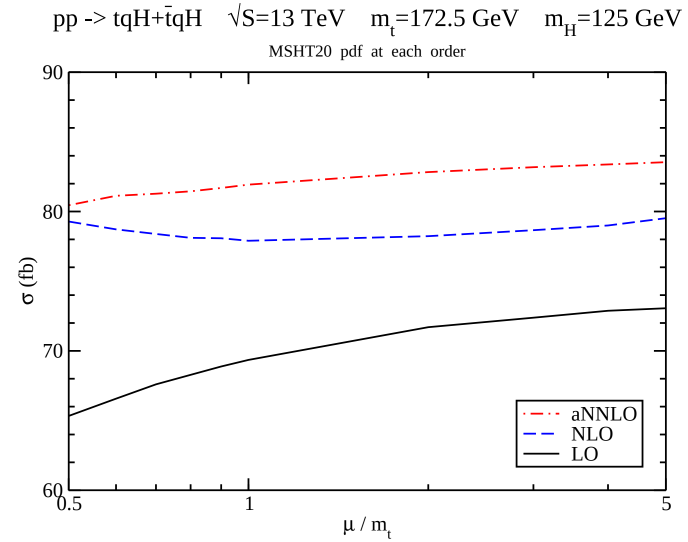
Two-loop and three-loop result structure as in t-channel single top

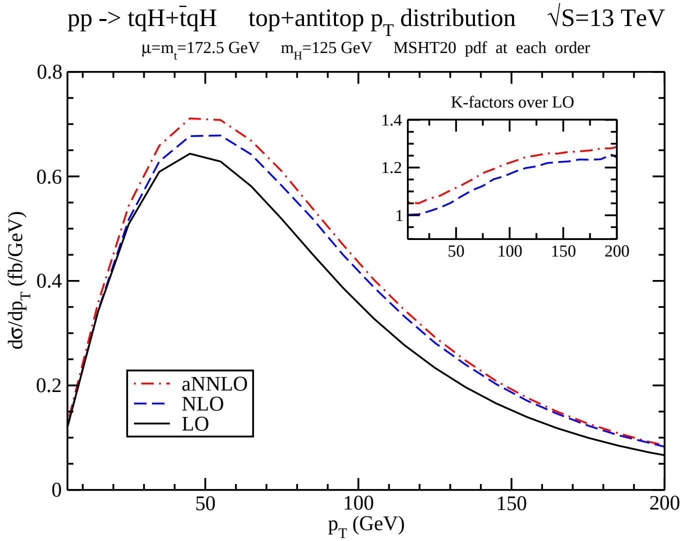
Results also known for s-channel processes

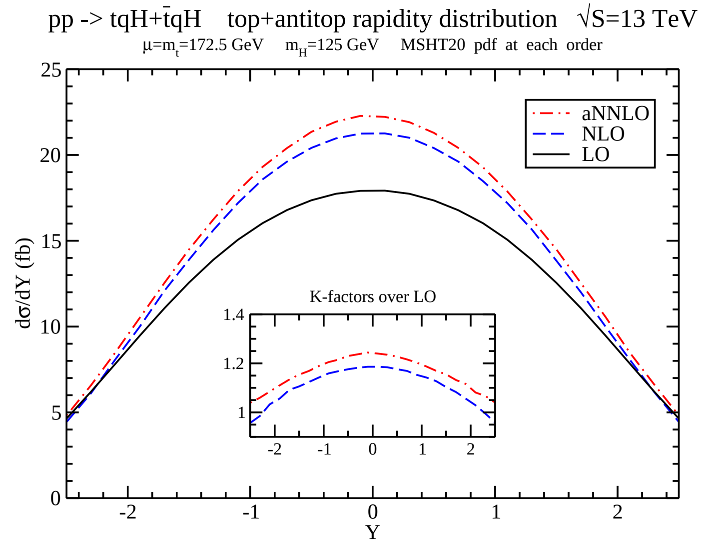
tqH production

(with Matthew Forslund, arXiv:2103.01228)









Summary

- **soft anomalous dimensions at three loops**
- **top-antitop pair production**
- **top-quark double-differential distributions in $t\bar{t}$ production**
- tW **cross sections and top-quark,** W**-boson distributions**
- tqH **cross sections and top-quark distributions**
- **soft-gluon corrections are dominant and they are significant through aN^{3}LO**