Constraining transport properties of quark-gluon plasma using non-linear hydrodynamic response

Zuzana Moravcová, You Zhou

zuzana.moravcova@cern.ch
you.zhou@cern.ch

Niels Bohr Institute, University of Copenhagen

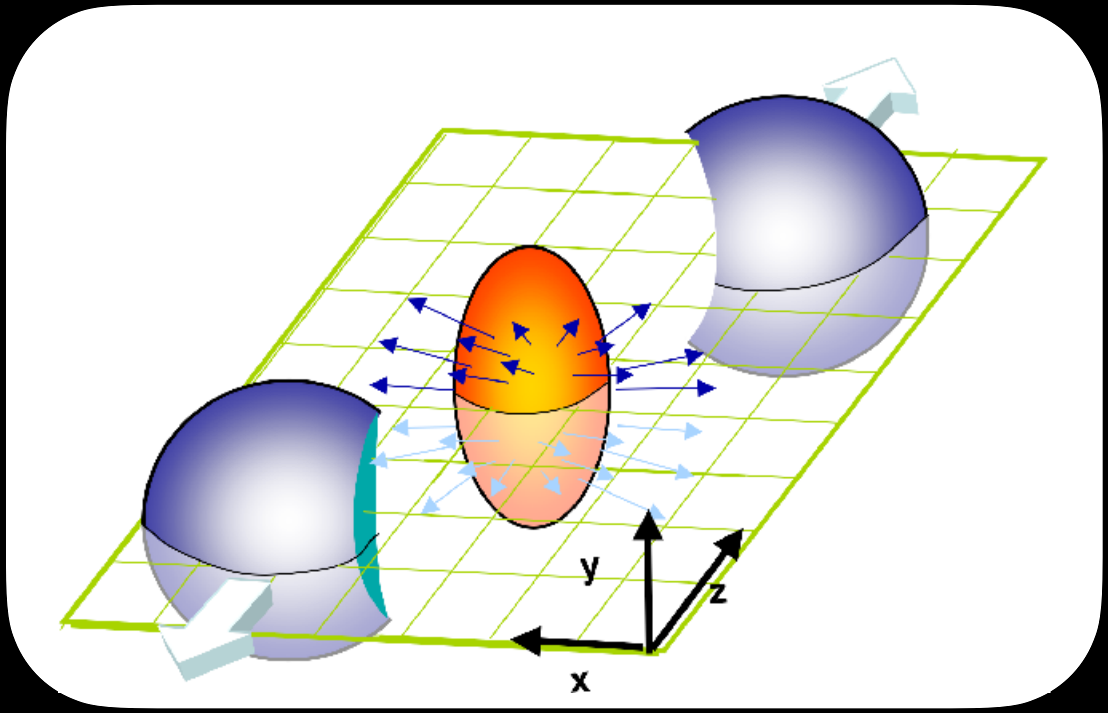
THE VELUX FOUNDATIONS
VILLUM FONDEN VELUX FONDEN

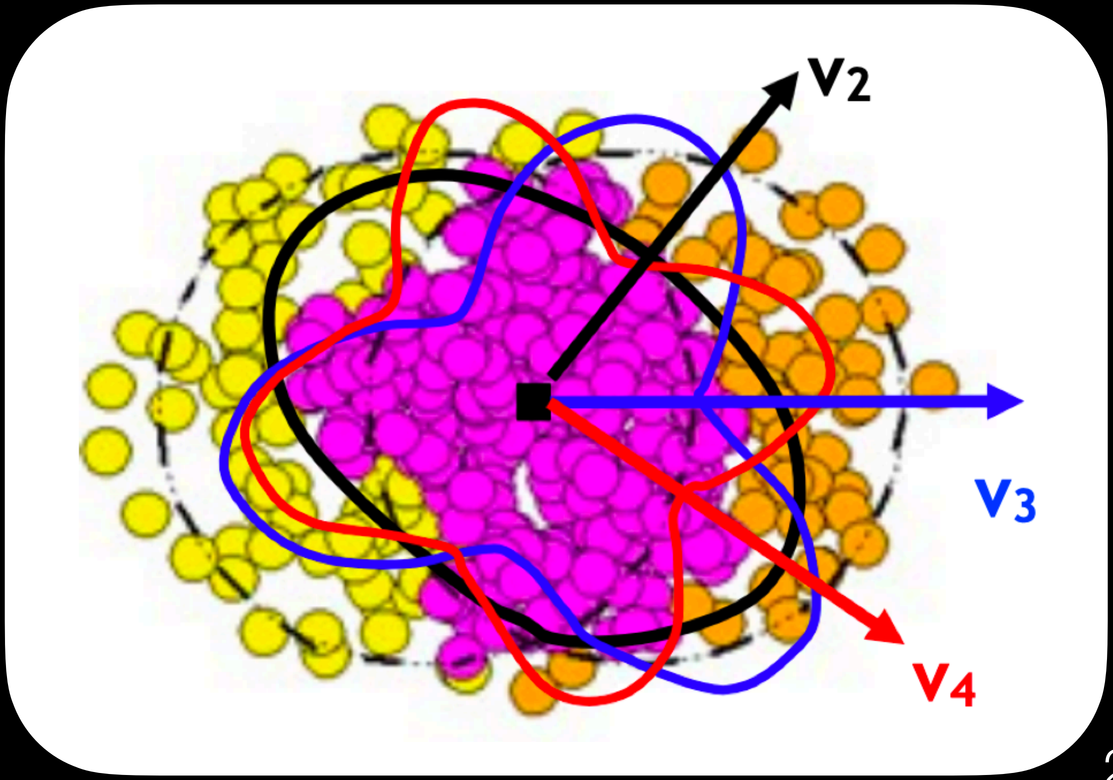
Anisotropic flow

- Partial overlap ⇨ Spatial anisotropy ⇨
  ⇨ Different pressure gradients ⇨
  ⇨ Particles "flow"
- Fourier series decomposition of azimuthal distribution of emitted particles:

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos n(\varphi - \Psi_n)$$

- Ψ_n — flow symmetry plane (defined by xz)
- v_n — flow coefficients
  - Nowadays typically calculated using m-particle correlations, $v_n\{m\}$
- Together they make flow vector $\overrightarrow{V}_n = v_n e^{in\Psi_n}$

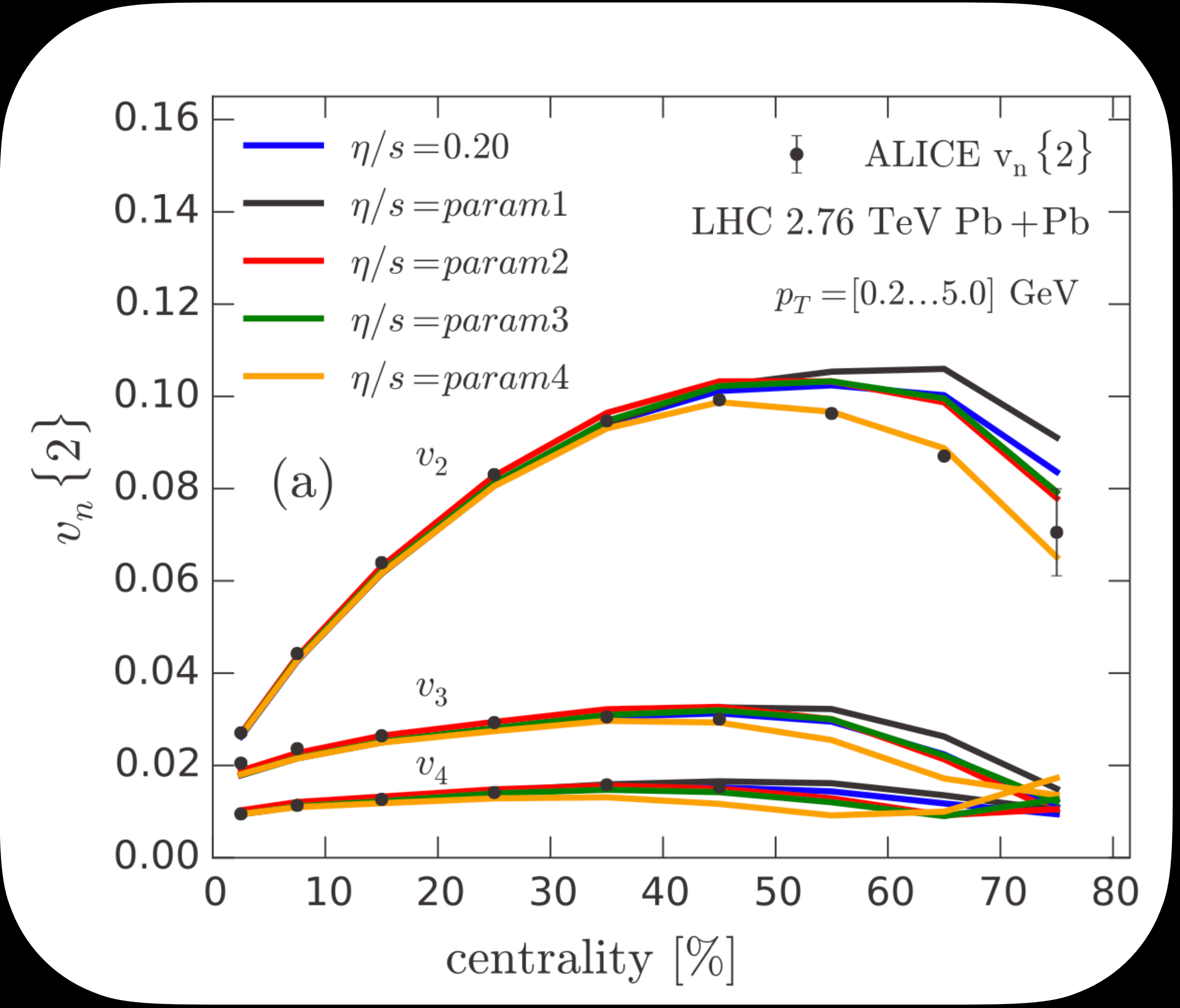


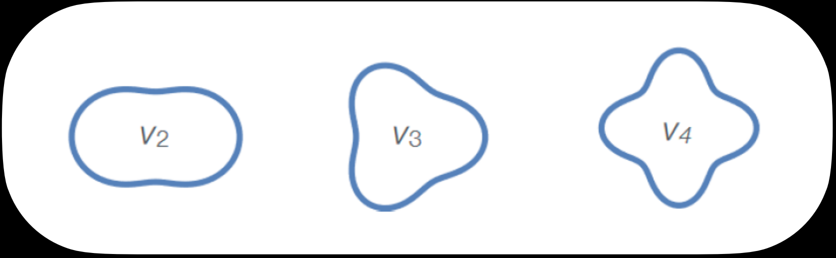


Relation to QGP properties

H. Niemi et al., Phys. Rev. C 93, 024907 (2016)

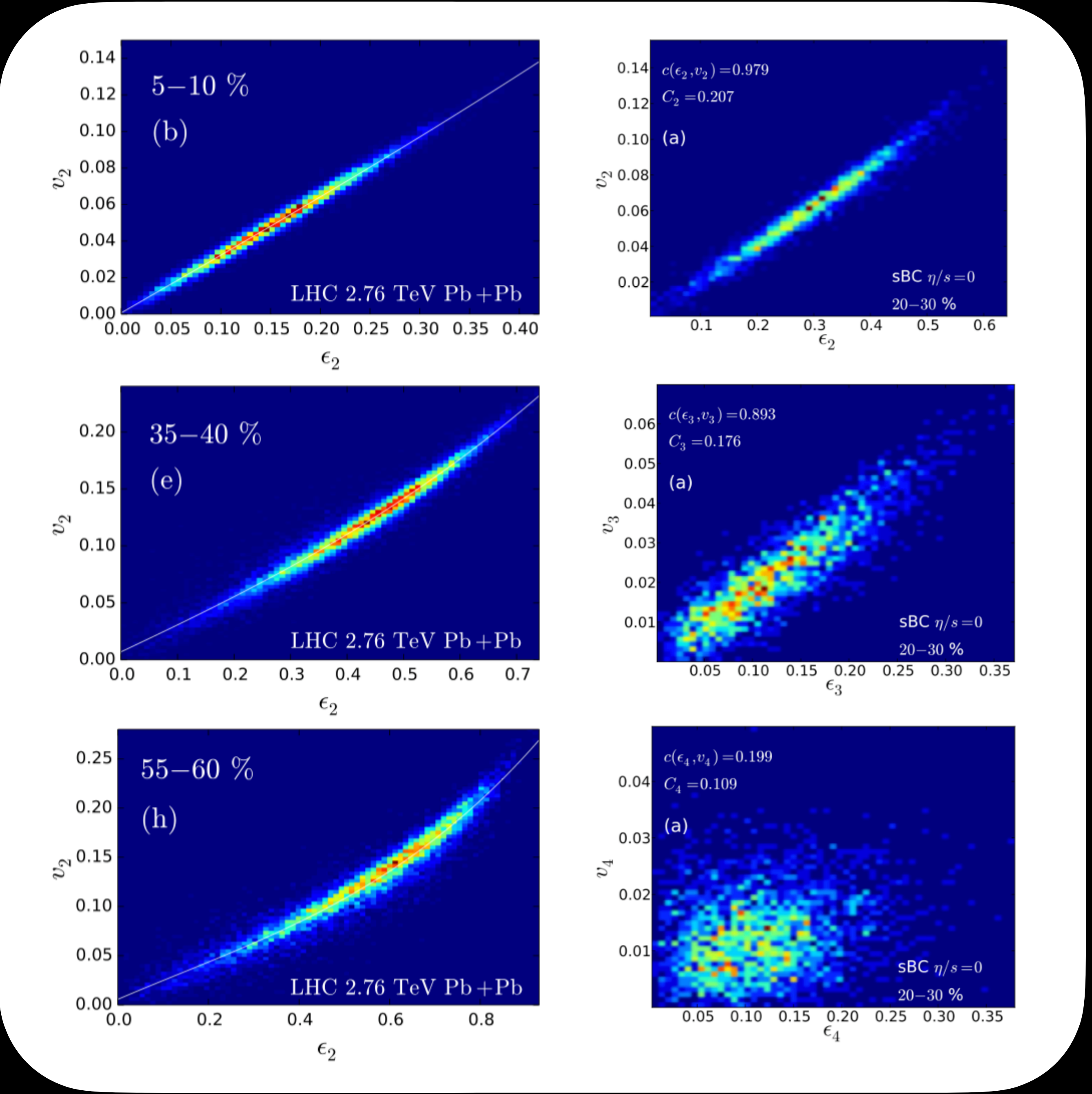
- What information can measuring of flow vector provide?
  - Initial-state conditions (initial geometry)
  - QGP properties, e.g. transport coefficients (shear viscosity η/s, bulk viscosity ξ/s) as a function of temperature
- Measured flow coefficients v_n alone do not make it possible to distinguish between the different η/s temperature dependencies
  - Differences only in peripheral collisions





Initial condition dependence

- Assumption $v_n \propto \epsilon_n$
  - ϵ_n is n-th eccentricity
    - $\epsilon_n e^{in\Phi_n} = -\frac{\int r dr d\varphi r^n e^{in\varphi} \varepsilon(r,\varphi)}{\int r dr d\varphi r^n \varepsilon(r,\varphi)}$
    - $\varepsilon(r,\varphi)$ is the initial energy density in the transverse plane
- Stronger linear correlations between v_n and ϵ_n in central collisions
- Approximately linear response for v_2 and v_3 harmonics
- Stronger linear correlations for v_2 than v_3
- Basically no linear correlation between v_4 and ϵ_4



H. Niemi et al., Phys. Rev. C 87, 054901 (2013)

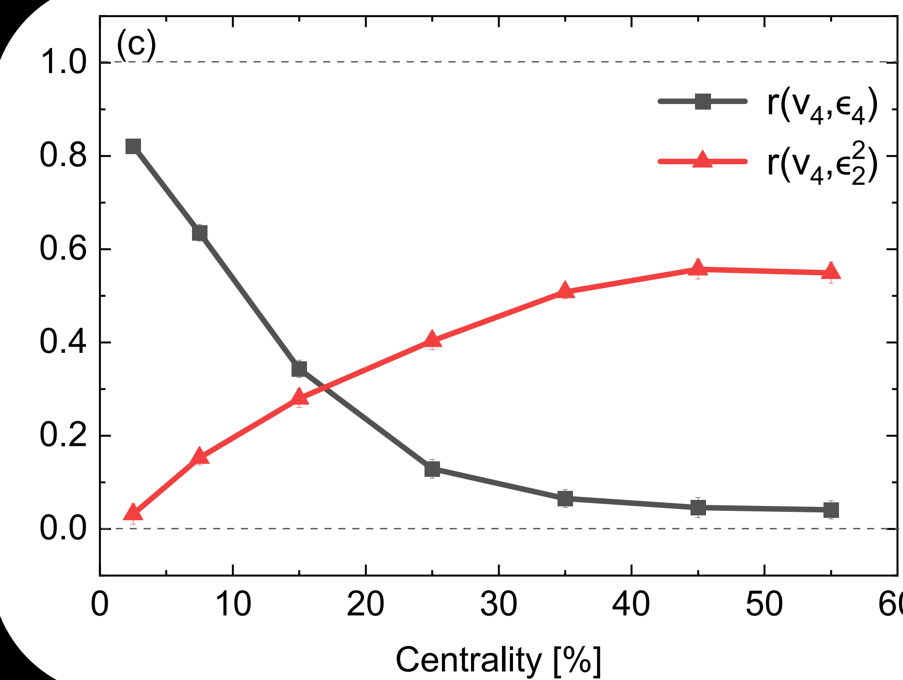
Non-linear hydrodynamic response

- Non-negligible non-linear response for v_n with $n \geq 4$
- Pearson correlation coefficient:

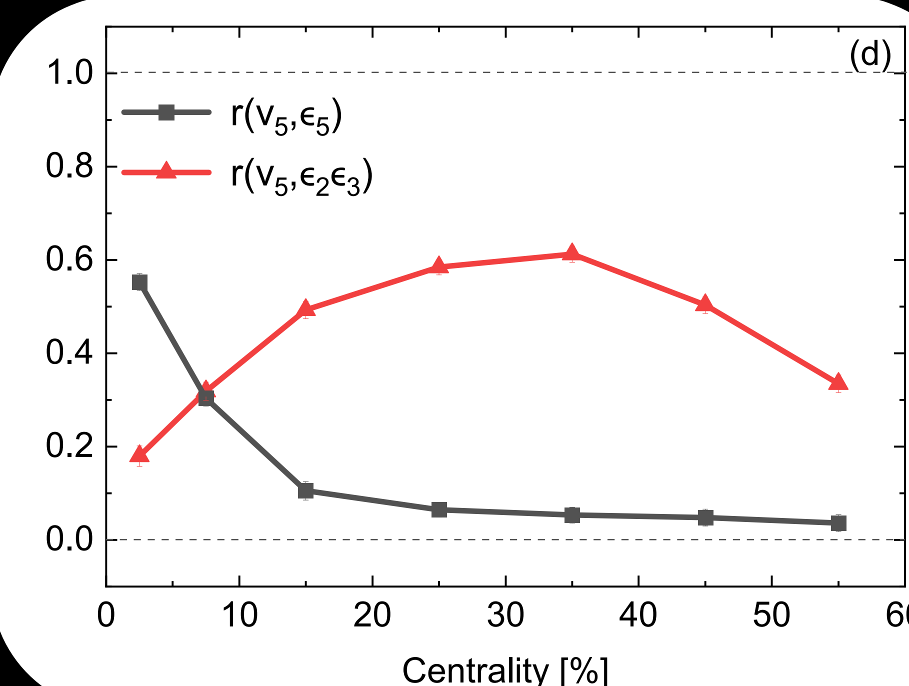
$$r(X,Y) = \frac{\langle XY \rangle - \langle X \rangle \langle Y \rangle}{\sqrt{Var[X]}\sqrt{Var[Y]}}$$

$r(X,Y) = 1$ — linearly correlated
$r(X,Y) = 0$ —uncorrelated

- Linear response:
  - v_4 almost linearly correlated with ϵ_4 in central collisions, rapid decrease towards peripheral collisions
  - Similar case for v_5 and ϵ_5
- Non-linear response:
  - Correlation between v_4 and ϵ_2^2 gets more important from semi-central collisions, dominant in peripheral collisions
  - Non-monotonic centrality dependence of correlation between v_5 and $\epsilon_2\epsilon_3$
- Not possible to do such study experimentally



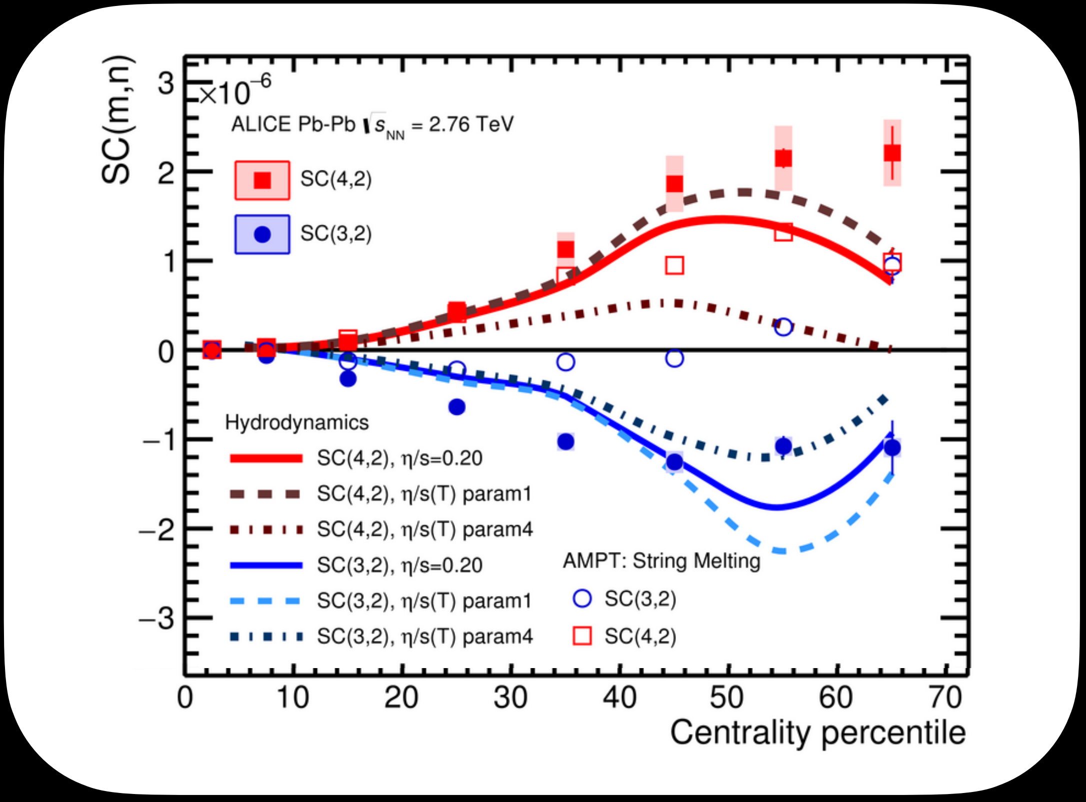
M. Li et al., arXiv:2104.10422 (accepted by PRC)

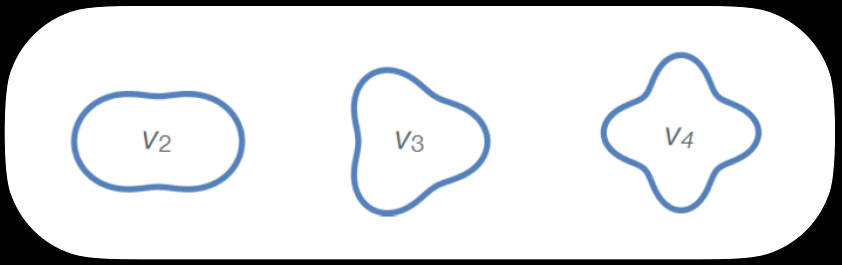


iEBE-VISH2+1 with AMPT initial conditions and $\eta/s = 0.08$

Experimental studies

ALICE Collaboration, Phys. Rev. Lett. 117, 182301 (2016)





- Symmetric cumulants
  - Correlations between two flow coefficients with their second moments
  - $SC(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$
  - More sensitive to both initial state information and medium properties than individual flow harmonics, e.g. strongly dependent on η/s of the QGP

Higher orders of multi-particle cumulants

- Multi-particle cumulant = a genuine multi-particle correlation after the subtraction of lower orders

- Can be used for the construction of the probability density function of a single harmonics, $P(v_n)$
  - Initial geometry fluctuations result in fluctuations in the final state
  - $\langle v_2^k \rangle \neq \langle v_2 \rangle^k$

- Correlations between different harmonics help further understand initial conditions and dynamic evolution of the QGP
  - Nearly insensitive to non-flow
  - High discriminating power between different initial-state and transport models

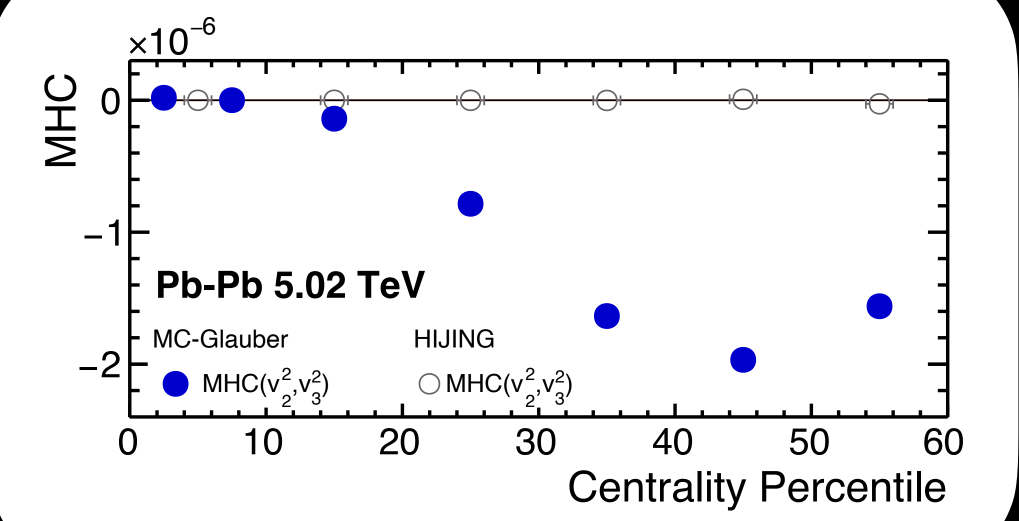
Moments of *p.d.f.*

$$\langle v_n \rangle \approx \left(\left(v_n\{2\}^2 + v_n\{4\}^2 \right)/2 \right)^{1/2}$$

$$\sigma_{v_n} \approx \left(\left(v_n\{2\}^2 - v_n\{4\}^2 \right)/2 \right)^{1/2}$$

$$\textit{Skewness} \simeq -2^{3/2} \frac{v_2\{4\}^3 - v_2\{6\}^3}{\left(v_2\{2\}^2 - v_2\{4\}^2 \right)^{3/2}}$$

$$\textit{Kurtosis} \simeq -\frac{3}{2} \frac{v_2\{4\}^4 - 12 v_2\{6\}^4 + 16 v_2\{8\}^4}{\left(v_2\{2\}^2 - v_2\{4\}^2 \right)^2}$$



ZM, YZ, et al., Physical Review C 103, 024913 (2021)

Multi-particle cumulants

- Number of terms of multi-particle cumulant follows Bell numbers
  - 6-particle cumulant: **203** terms
  - 8-particle cumulant: **4140** terms
- Many terms contain average over single particle correlation which ***must be zero***
  - Thanks to the isotropic azimuthal distribution over many events

- How to obtain formula for higher orders **without** calculating thousands of terms?

General algorithm for multi-particle cumulants of arbitrary order

- Possible to calculate easily and efficiently any order of multi-particle cumulants of single harmonic
  - $v_2\{6\}, v_2\{8\}, v_2\{10\}, v_2\{12\}, v_2\{14\}, \ldots$
- Introducing *mixed harmonic cumulants*
  - Correlations of any moments of different flow coefficients, e.g. $MHC(v_2^2, v_3^2), MHC(v_2^4, v_3^2), MHC(v_2^6, v_3^2), MHC(v_2^2, v_3^2, v_4^2)$
  - $SC(m, n) = MHC(v_m^2, v_n^2)$
- Physical Review C 103, 024913 (2021)

PHYSICAL REVIEW C

covering nuclear physics

Highlights Recent Accepted Collections Authors Referees Search Press

Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

Zuzana Moravcova, Kristjan Gulbrandsen, and You Zhou

Phys. Rev. C **103**, 024913 – Published 26 February 2021

```
complex Cumulant(int* harmonic, int n, bool remove_zeros=true, int negsplit=-1,
   int mult = 1, int skip = 0)
 {
   bool remove_term = false;
   if (remove_zeros)
   {
     int har_sum = 0;
     for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
     if (har_sum != 0) remove_term = true;
   }
   complex c = 0;
   if (!remove_term)
   {
     c = Corr(harmonic+(n-1), mult);
     if (n == 1) return c;
     c *= negsplit*Cumulant(harmonic, n-1, remove_zeros, negsplit-1);
   }

   int h_hold = harmonic[n-2];
   for (int counter = 0; counter <= n-2-skip; ++counter)
   {
     harmonic[n-2] = harmonic[counter];
     harmonic[counter] = h_hold;
     c += Cumulant(harmonic, n-1, remove_zeros, negsplit, mult+1, n-2-counter);
     harmonic[counter] = harmonic[n-2];
   }
   harmonic[n-2] = h_hold;
   return c;
 }
```

Correlations of higher moments

- Change of signs for 4-, 6-, and 8-particle correlations
- In agreement with ALICE results
  - Phys. Lett. B (2021) 136354
  - See talk by ALICE Collaboration

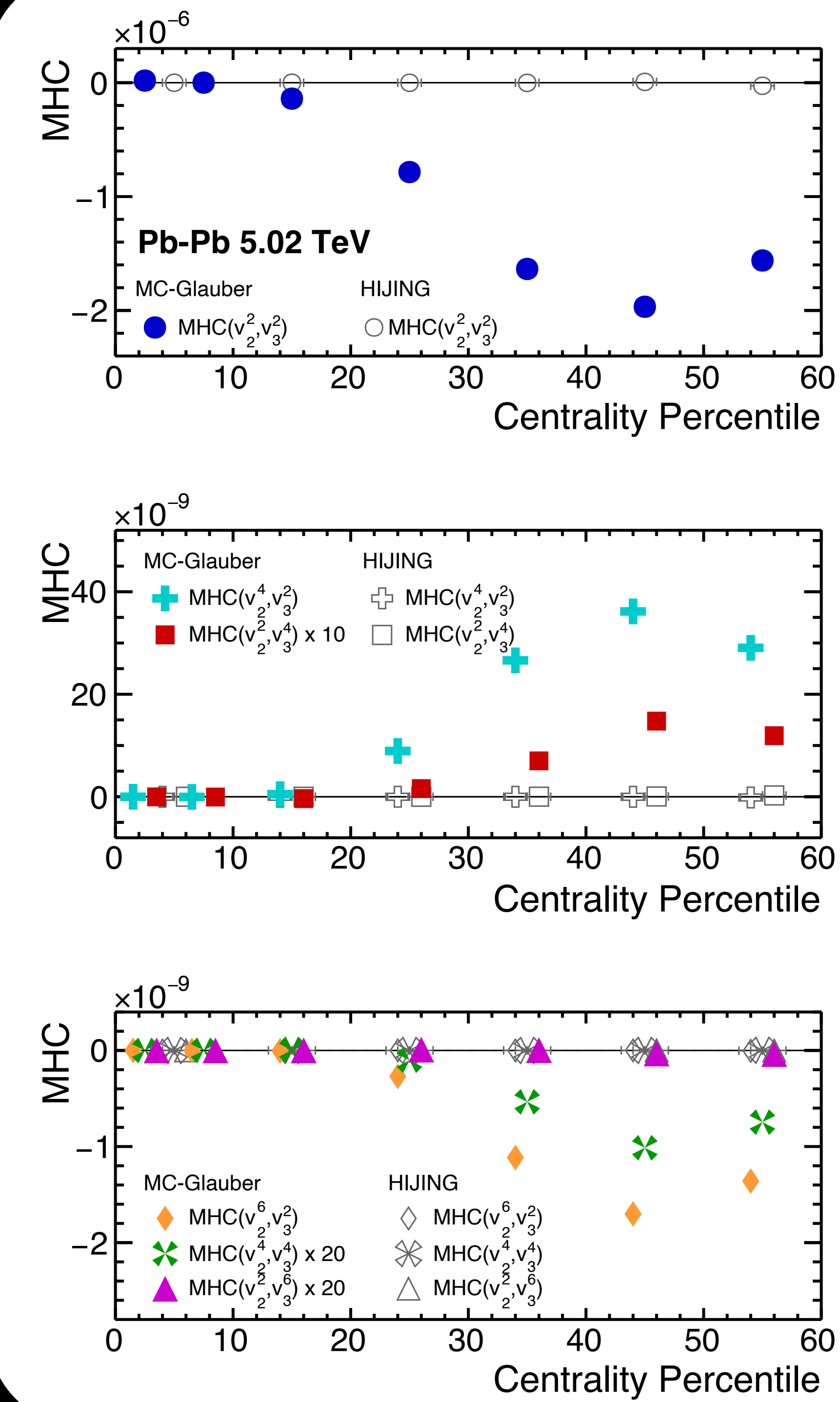
Measurements of mixed harmonic cumulants in Pb-Pb collisions at 5.02 TeV with ALICE

26 Jul 2021, 16:30
15m
Zoom

Parallel session talk | Heavy Ion Physics | T05: Heavy Ion Physics

Speaker

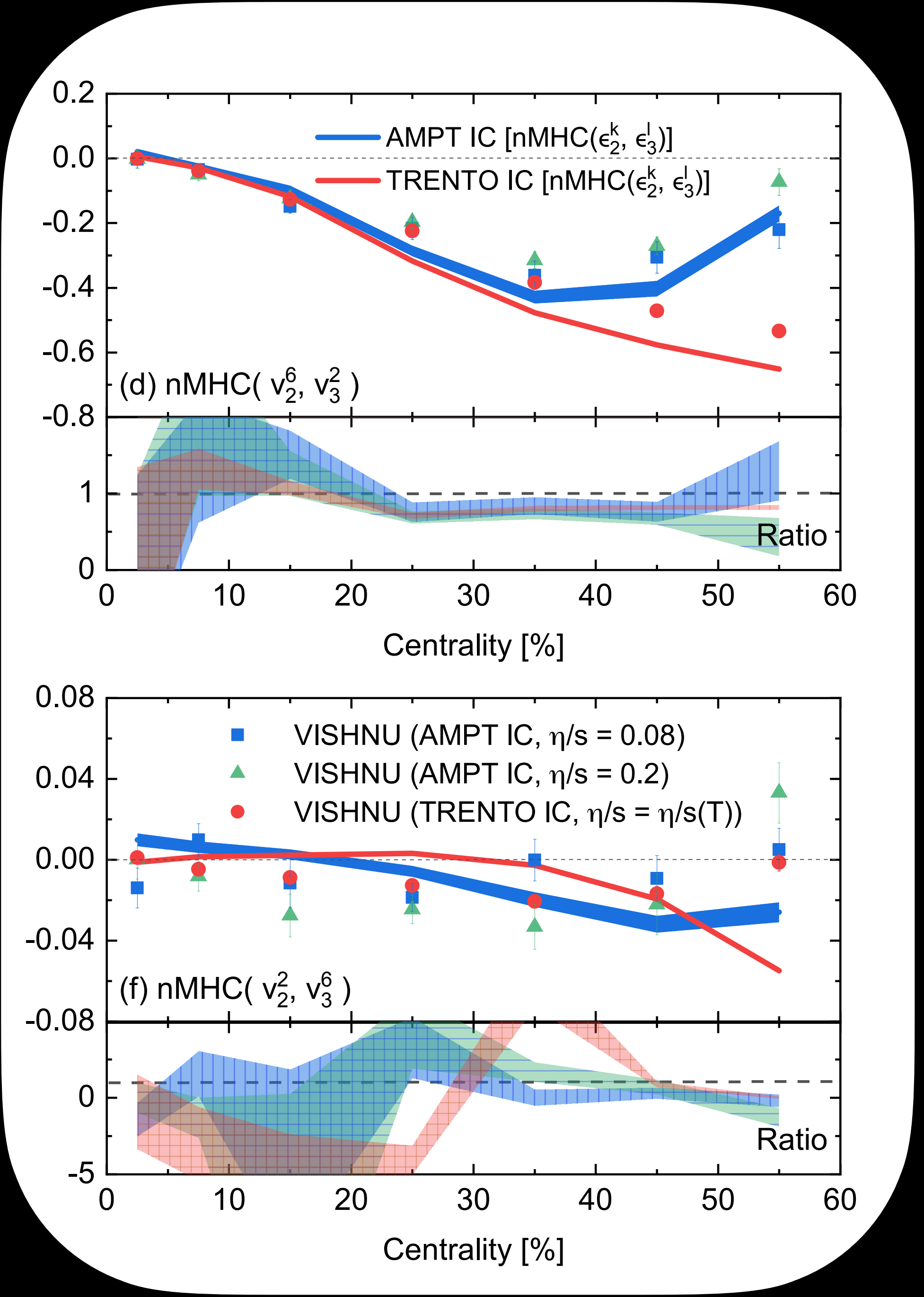
You Zhou (Niels Bohr Institute)



ZM, YZ, et al., Physical Review C 103, 024913 (2021)

Linear hydrodynamic response using $nMHC(v_2^k, v_3^l)$

- iEBE-VISHNU model with different initial conditions to study the sensitivity to η/s
- Ratio = $nMHC(v_2^k, v_3^l)/nMHC(\epsilon_2^k, \epsilon_3^l)$
- $nMHC(v_2^k, v_3^l)$ with v_2^k $(k > 2)$
  - Linear response holds only in central collisions
  - Sizeable difference for different initial conditions in peripheral collisions
- $nMHC(v_2^k, v_3^l)$ with v_3^l $(l > 2)$
  - Linear response totally breaks down



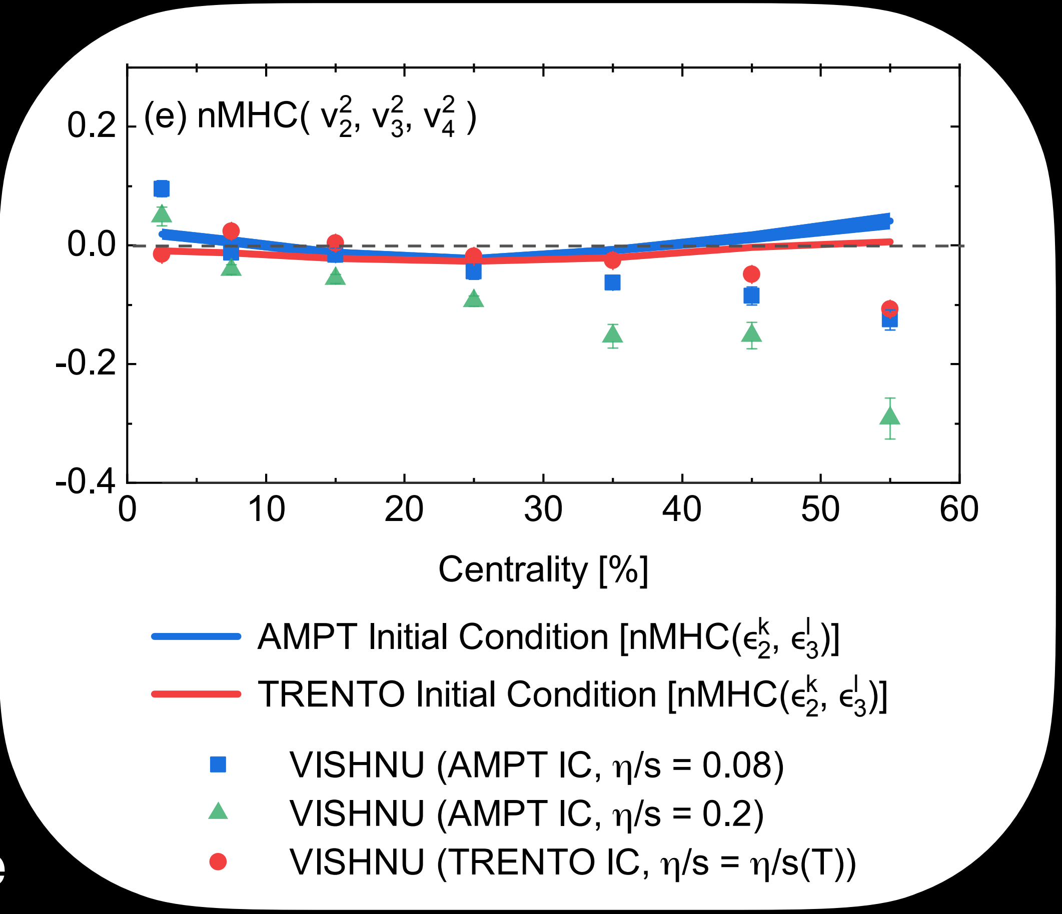
M. Li et al., arXiv:2104.10422 (accepted by PRC)

Correlations of three flow coefficients

- Together with studies of single v_n fluctuations and correlations between two flow coefficients, more information can be provided to the joint probability density function

- $nMHC(v_m^k, v_n^l, v_p^q)$ do not follow the centrality dependence of $nMHC(\epsilon_m^k, \epsilon_n^l, \epsilon_p^q)$
  - $nMHC(\epsilon_2^2, \epsilon_3^2, \epsilon_4^2)$ compatible with zero while $nMHC(v_2^2, v_3^2, v_4^2)$ has significantly non-zero value

- $nMHC(v_2^2, v_4^2)$ and $nMHC(v_3^2, v_5^2)$ have been studied as well (see back up), visible non-linear response



M. Li et al., arXiv:2104.10422 (accepted by PRC)

Summary

- Possible to calculate arbitrary order of single flow harmonic coefficient and any correlation between any moments of different harmonics using the general algorithm for multi-particle cumulants

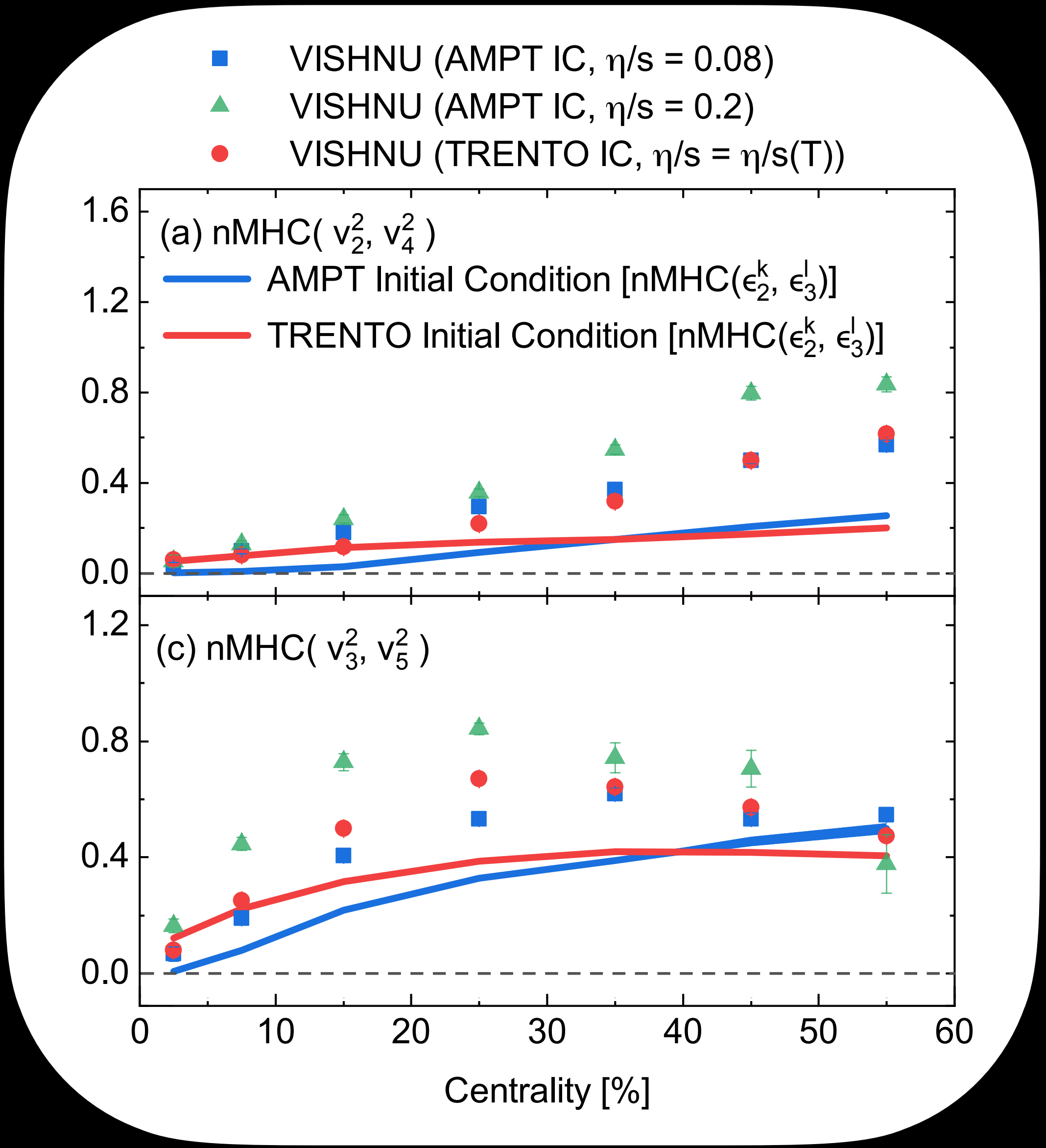
- Future investigations of mixed harmonic cumulants can be used to study non-linear hydrodynamic response — its understanding can help to better understand hydrodynamic evolution and to provide tighter constrains on initial models

Thank you for you attention!

BACK UP

Correlations with higher harmonics

- Sensitive to both η/s and initial conditions
- v_4 and v_5 have non-linear response to ϵ_2 and $\epsilon_2\epsilon_3$, respectively
  - Results in large deviation between $nMHC(v_2^2, v_4^2)$ and $nMHC(\epsilon_2^2, \epsilon_4^2)$ and between $nMHC(v_3^2, v_5^2)$ and $nMHC(\epsilon_3^2, \epsilon_5^2)$



M. Li et al., arXiv:2104.10422 (accepted by PRC)