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First steps towards the quantum simulation of jet quenching

arXiv: 2104.04661

26th July 2021, EPS-HEP Conference 2021

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Why Quantum computing might be useful to jet quenching

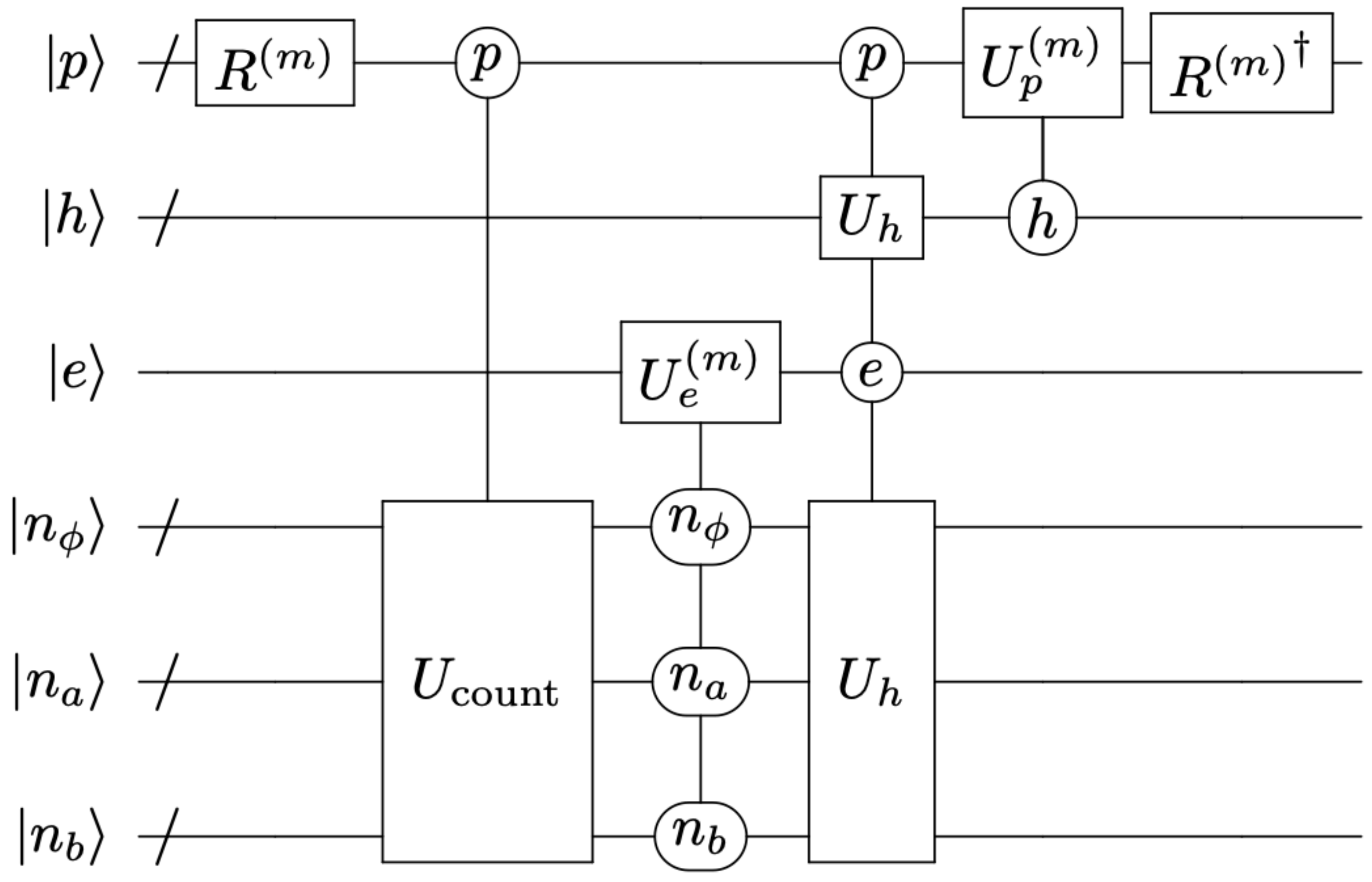
One hopes to be able to simulate QFTs in a Quantum Computer (QC)

- QCs naturally describe highly entangled states in large Hilbert spaces
- Current devices still too small and noisy to study dynamical processes in QFT

e.g. high energy scattering

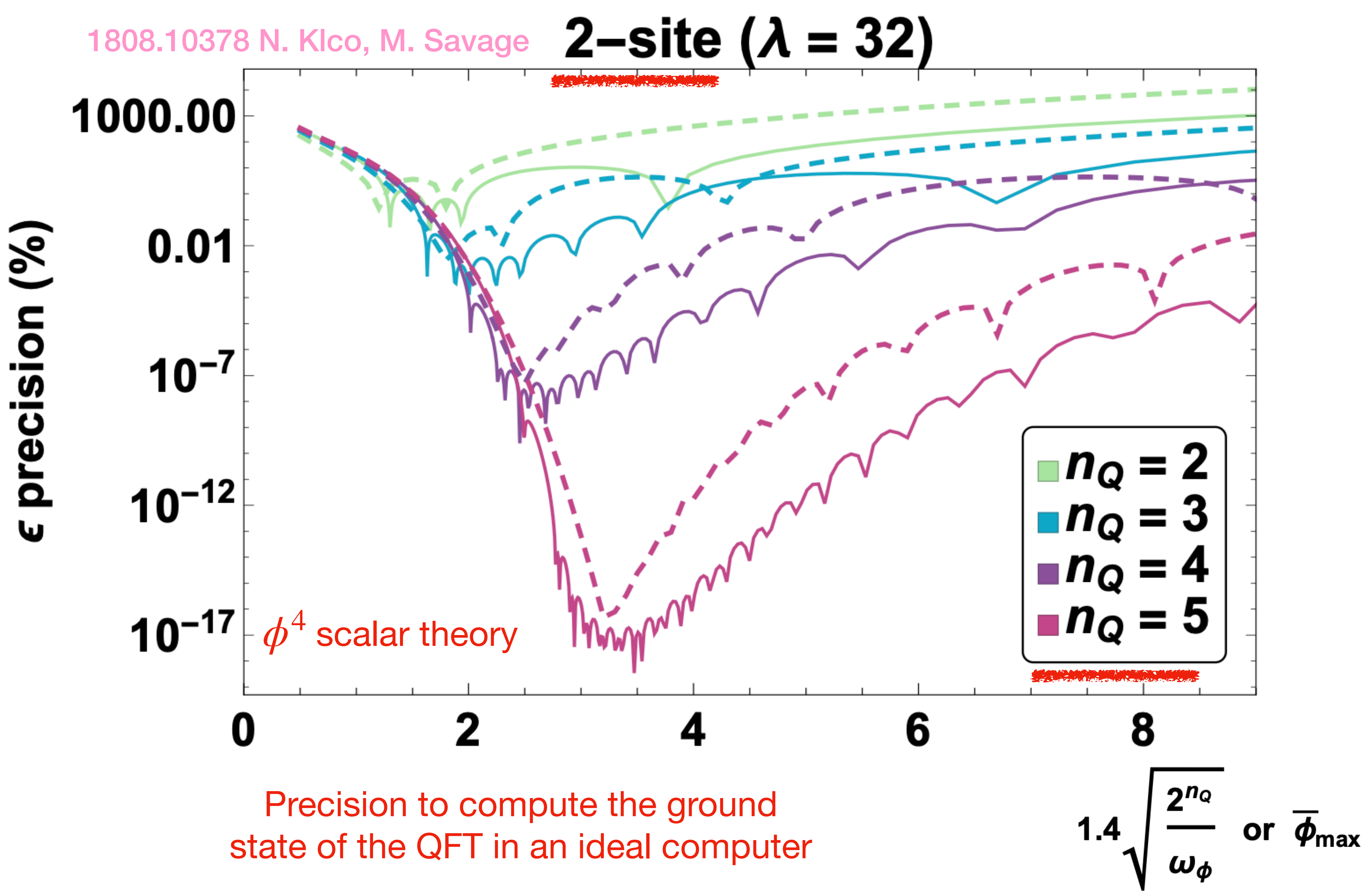
However, QCs can be used to study simpler problems

$$\mathcal{L} = \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2$$
$$+ g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\underline{\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]}\phi$$



1904.03196 B. Nachman, D. Provasoli, C. Bauer, W. de Jong

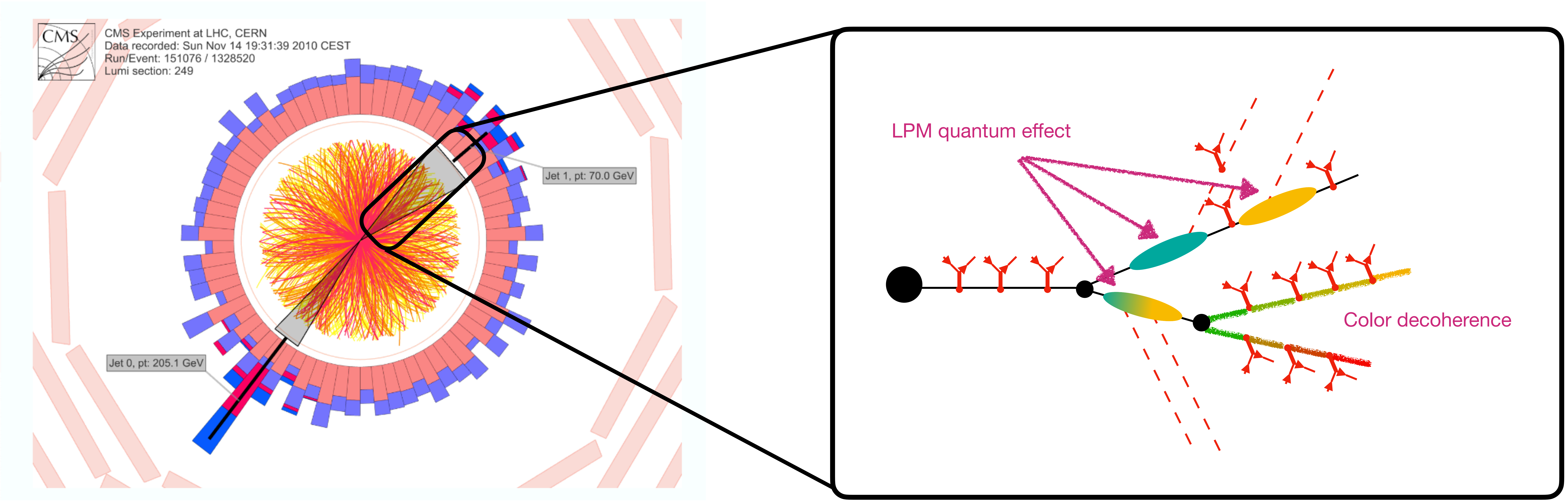
1808.10378 N. Klco, M. Savage



Precision to compute the ground state of the QFT in an ideal computer

Why Quantum computing might be useful to jet quenching

Detailed evolution of jets in-medium requires **understanding multi-parton interference**

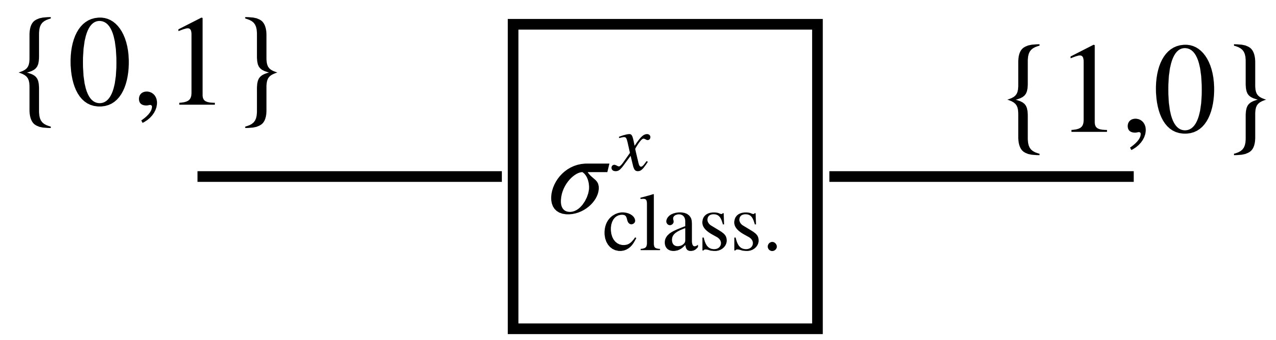


Such effects should be **naturally** tracked if one simulates events at the **amplitude level** and **not the squared amplitude** (as in M.C.)

Some aspects of QC: Qubits and Quantum gates

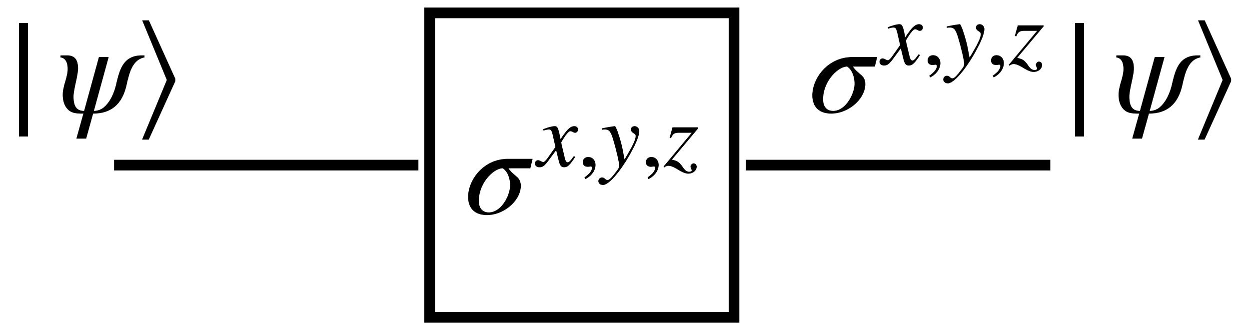
Consider 1 information “unit”

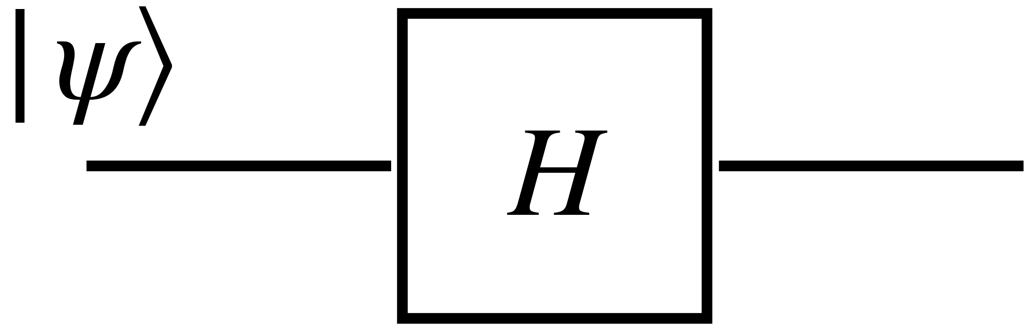
In the classical circuit model: $\psi = \{0,1\}$ (bit). Only two possible operations



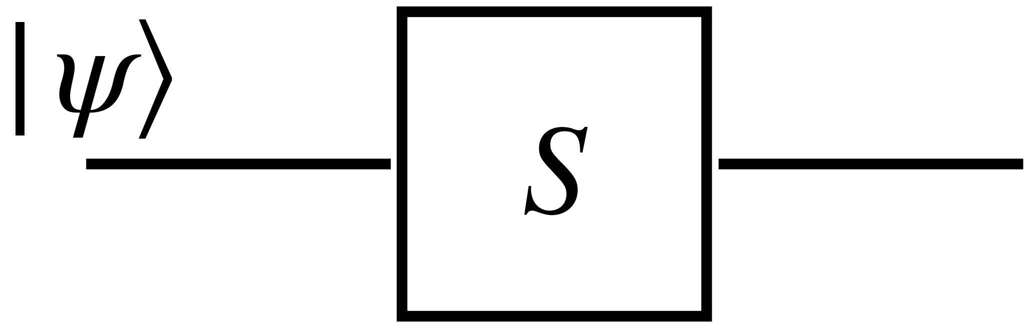
In the quantum circuit model: $|\psi\rangle = a|0\rangle + b|1\rangle \equiv a|\uparrow\rangle + b|\downarrow\rangle$ (qubit)

Infinite set of operations: **Pauli’s + identity**





$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}|\psi\rangle$$

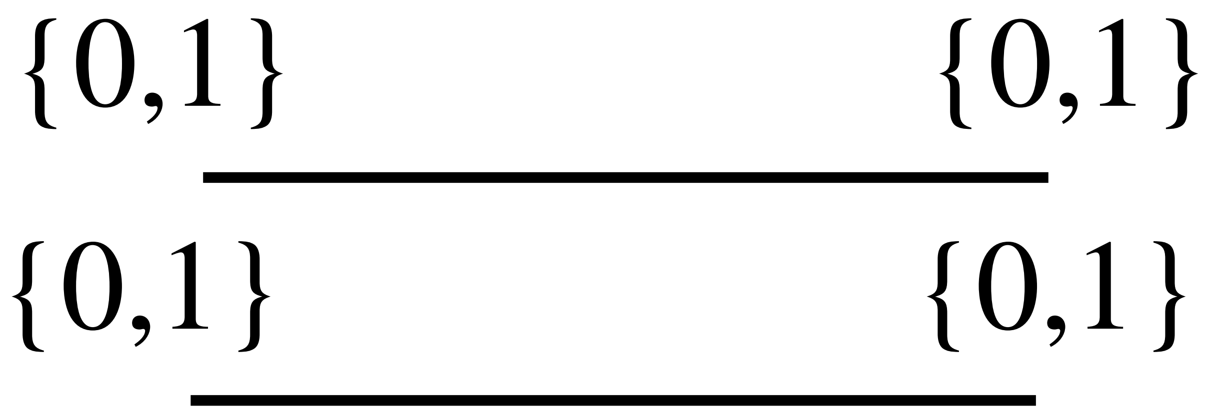


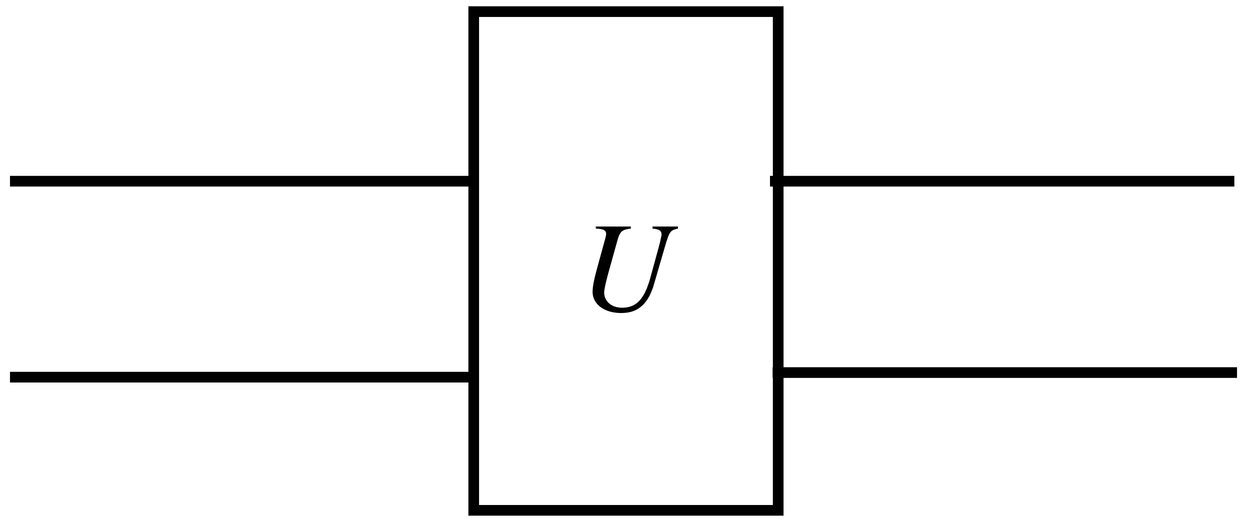
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}|\psi\rangle$$

Some aspects of QC: Qubits and Quantum gates

Consider 2 information "units"

In the **classical** circuit model: $\psi = \{0,1,2,3\}$ (in binary).

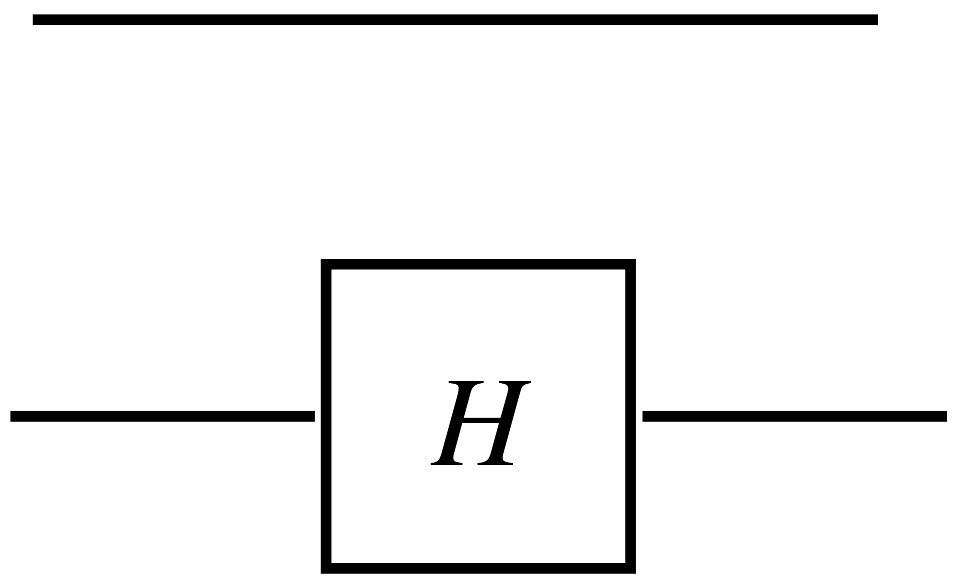




In the **quantum** circuit model: $|\psi\rangle = \sum_{i,j} c_{ij} |x_i, x_j\rangle$

Single qubit operations generalize in a simple way, for example

$$1 \otimes H|\psi\rangle \quad =$$



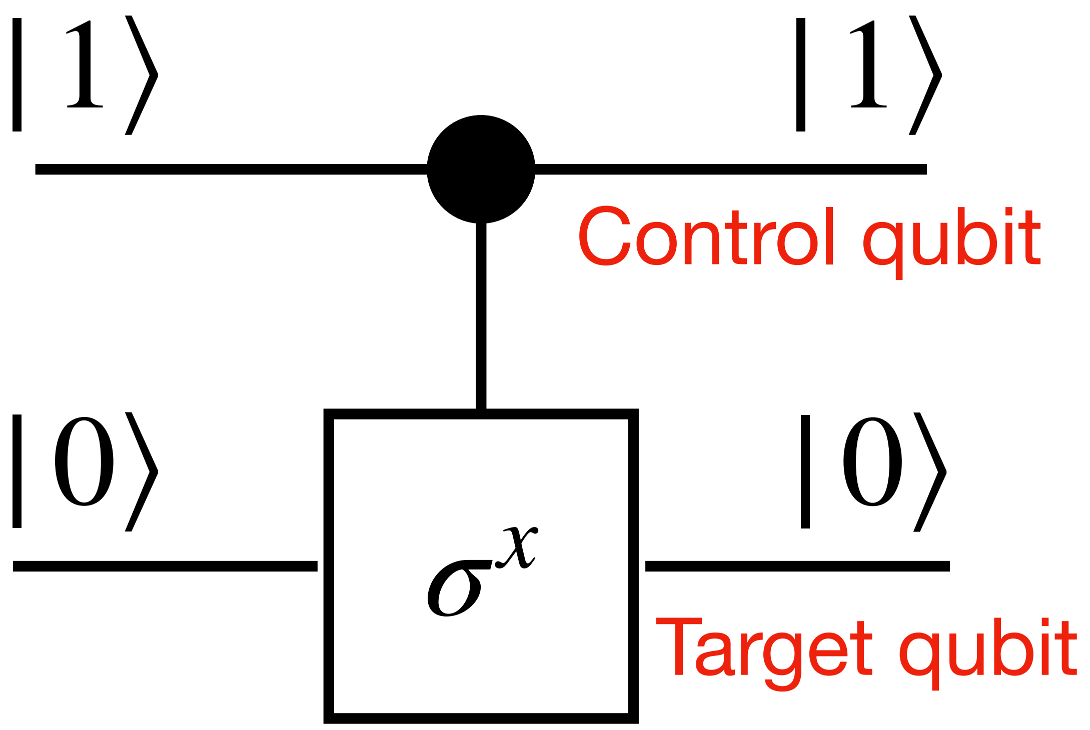
Some aspects of QC: Qubits and Quantum gates

Consider 2 information “units”

In the **quantum** circuit model: $|\psi\rangle = \sum_{i,j} c_{ij} |x_i, x_j\rangle$

Beyond the 1 qubit case, there are **non-Pauli** gates, for example is the CNOT (C=controlled)

$$\mathrm{CNOT}\,|\psi\rangle = \mathrm{C}\sigma^x |\psi\rangle =$$



$$= \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}$$

Unity Negate

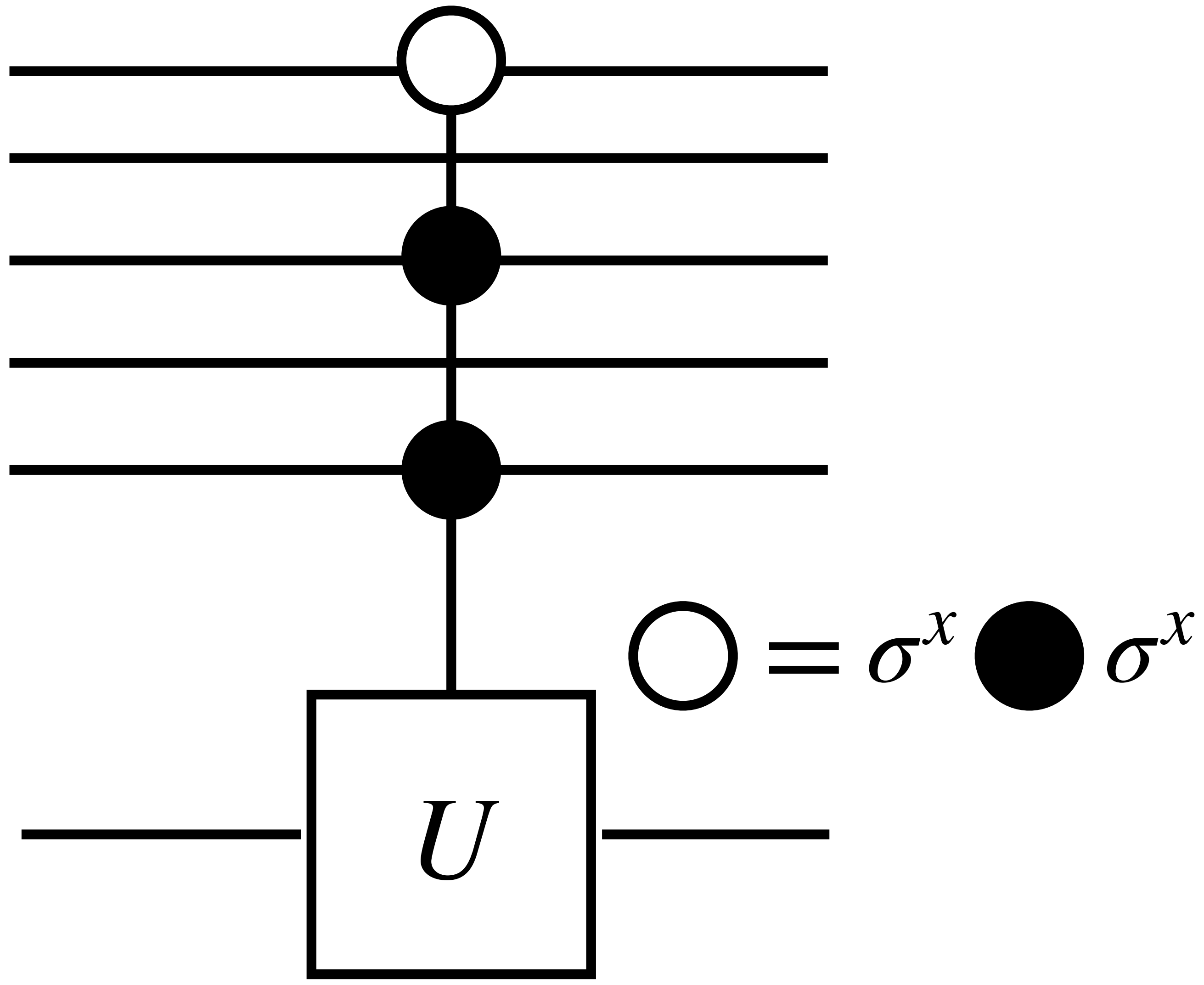
A simple example:

$$|\downarrow\downarrow\rangle \xrightarrow{H\otimes 1} \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\downarrow\rangle) \xrightarrow{\mathrm{CNOT}} \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle) \quad \text{Bell state}$$

Some aspects of QC: Qubits and Quantum gates

Consider n_{qubits} information "units"

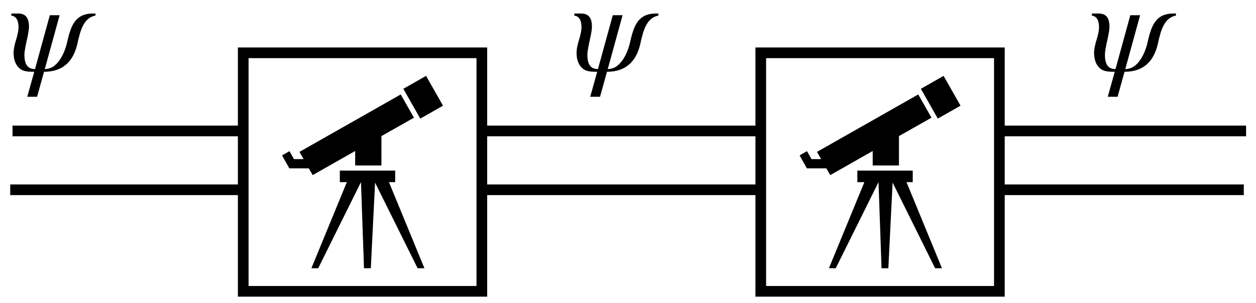
In the quantum circuit model: $|\psi\rangle = \sum_{x=0}^{2^{n_{\text{qubits}}}-1} c_x |x\rangle$



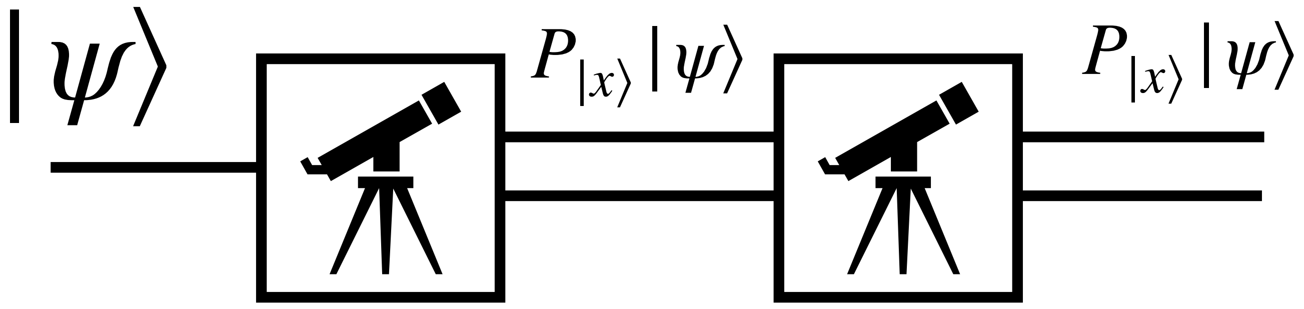
Straightforward combination of all the previous topologies

Finally, character of measurement follows from QM postulates

Classical case:



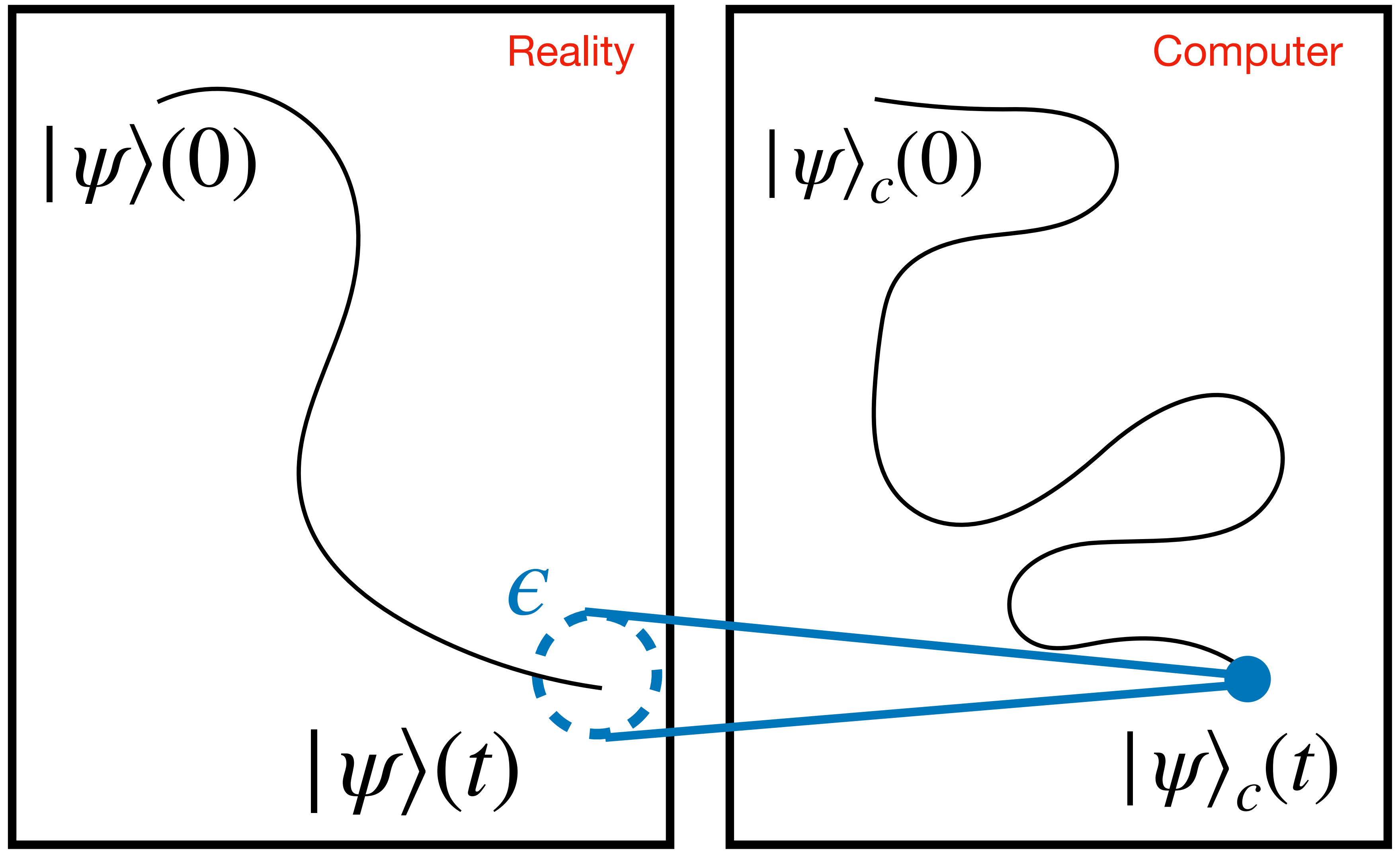
Quantum case:



The Quantum Simulation Algorithm

QC allows to efficiently simulate quantum systems:

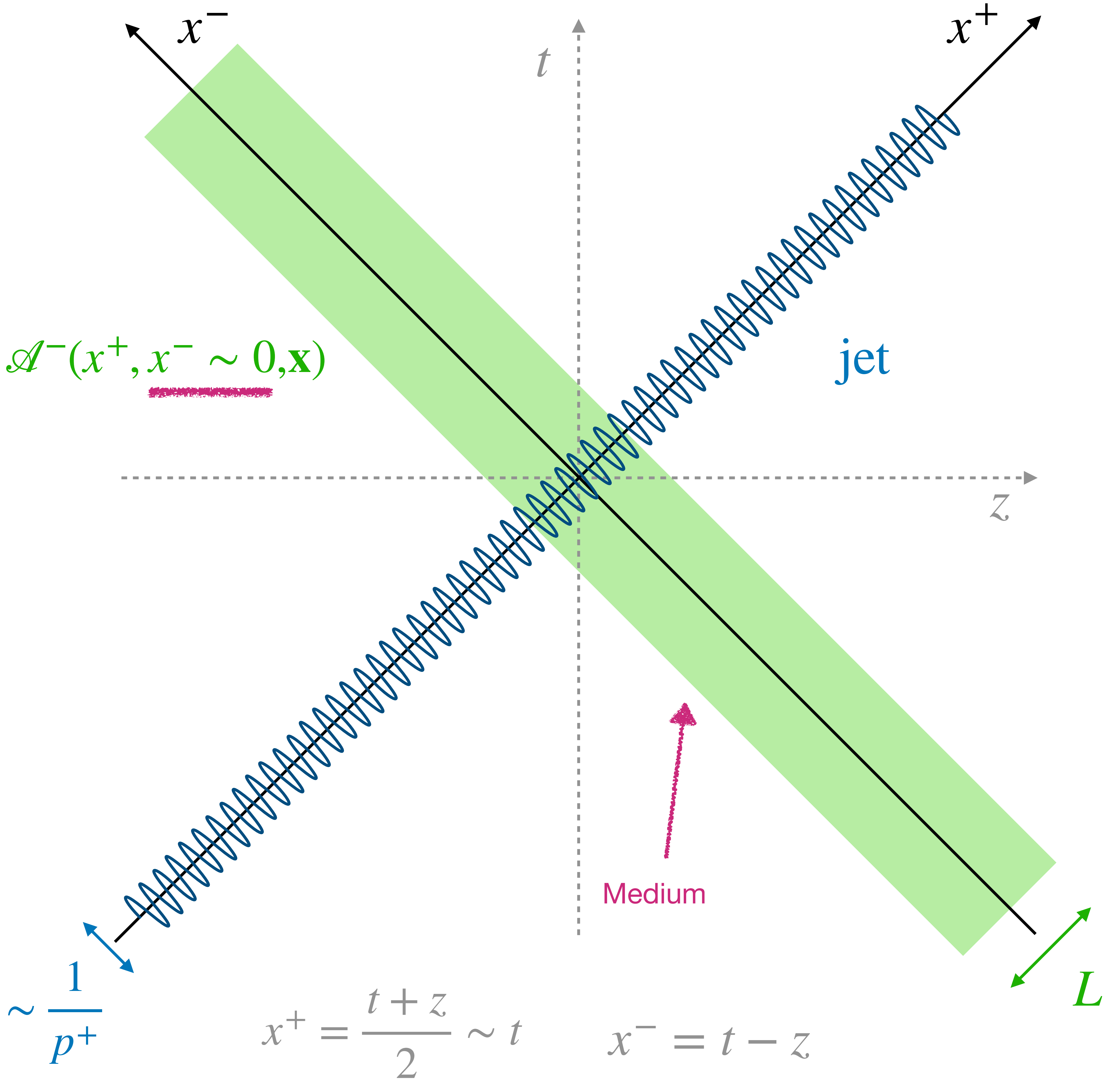
$$|\psi\rangle(t) = \exp(-iHt)\,|\psi\rangle(0)$$



The 5 main steps of the Quantum Simulation Algorithm:

1. Provide $H = \sum_k H_k$ and $\psi(0)$
2. Encode the physical d.o.f's in terms of qubits and decompose H_k in terms of gates
3. Prepare the initial wave function from a fiducial state ($|0\rangle^{\otimes n_{\text{qubits}}}$)
4. Time evolve according to $\exp(-iHt)$
5. Implement a measurement protocol

Parton propagation in a stochastic background



Integrating out x^- the **quark propagator** satisfies

$$\left(i\partial_t + \frac{\partial_{\boldsymbol{x}}^2}{2\omega} + g\mathcal{A}^-(t,\boldsymbol{x})\cdot T\right) G(t,\boldsymbol{x};0,\boldsymbol{y}) = i\delta(t)\delta(\boldsymbol{x}-\boldsymbol{y})$$

Parton evolution is equivalent to **2+1d non-rel. QM**

$$\mathcal{H}(t) = \underbrace{\frac{\boldsymbol{p}^2}{2\omega}}_{\text{p-space}} + \underbrace{g\mathcal{A}^-(t,\boldsymbol{x})\cdot T}_{\text{x-space}} = \mathcal{H}_K + \mathcal{H}_{\mathcal{A}}(t)$$

+vertices

Consider the **simplest** case:

1. $|q\rangle$ Fock space only
2. $T = 1$
3. Stochastic background (hybrid approach)

Setting up the QS algorithm

1. Provide $\mathscr{H} = \mathscr{H}_K + \mathscr{H}_A(t)$ ✓ and $\psi(0) = \psi(\mathbf{p} = 0)$ ✓ + ensemble of $\{\mathscr{A}, p_A\}$ ✓

2. Encode the physical d.o.f's in terms of qubits and write $\mathscr{H}$ in terms of gates

Introduce 2d spatial lattice with $N_s = 2^{n_Q}$ sites per dimension

a_s
Lattice spacing

$a_d = \dfrac{a_s}{2\pi N_s}$
Momentum Lattice spacing

$$\text{e.g.}\,|\mathbf{q}\rangle = |q_1, q_2\rangle = a_d|n_{q_1}, n_{q_2}\rangle$$

such that

$$H = \frac{\boldsymbol{P}^2}{2E} + gA(t, \boldsymbol{X}) \cdot T = H_K + H_A(t)$$

$H = a_s\mathscr{H}$
Dimensionless Hamiltonian

$A = a_s\mathscr{A}$
Dimensionless Field

$E \propto a_s\omega$
Dimensionless Energy

where

$$\hat{P}|p\rangle = p|p\rangle \qquad \hat{X}|x\rangle = x|x\rangle \qquad x, n \in \mathbb{Z}$$

3. Prepare the initial wave function from a fiducial state $(|0\rangle^{\otimes n_{\text{qubits}}})$ ✓

Setting up the QS algorithm

4. Time evolve according to $\exp(-iHt)$

$$H = \frac{\boldsymbol{P}^2}{2E} + gA(t, \boldsymbol{X}) \cdot T = H_K + H_A(t)$$

Time dependent evolution a bit more tricky. Simplest product formula

$$U(L', 0) \approx \prod_{k_t=1}^{N_t} \left\{ \exp\left[-iH_K \frac{L'}{N_t}\right] \exp\left[-iH_A\left(k_t \cdot \frac{L'}{N_t}\right) \frac{L'}{N_t}\right]\right\} \equiv \prod_{k_t=1}^{N_t} \{U_K(\varepsilon_t) U_A(k_t \cdot \varepsilon_t, \varepsilon_t)\}$$

valid for very smooth H_A

$$L' = \frac{L}{a_s}$$

Dimensionless medium size

Implement operators with a Fourier Transform in between

$$U_K(\varepsilon_t) \, |\boldsymbol{p}\rangle = \exp\left(-i \frac{\varepsilon_t}{2E} \boldsymbol{p}^2\right) |\boldsymbol{p}\rangle$$

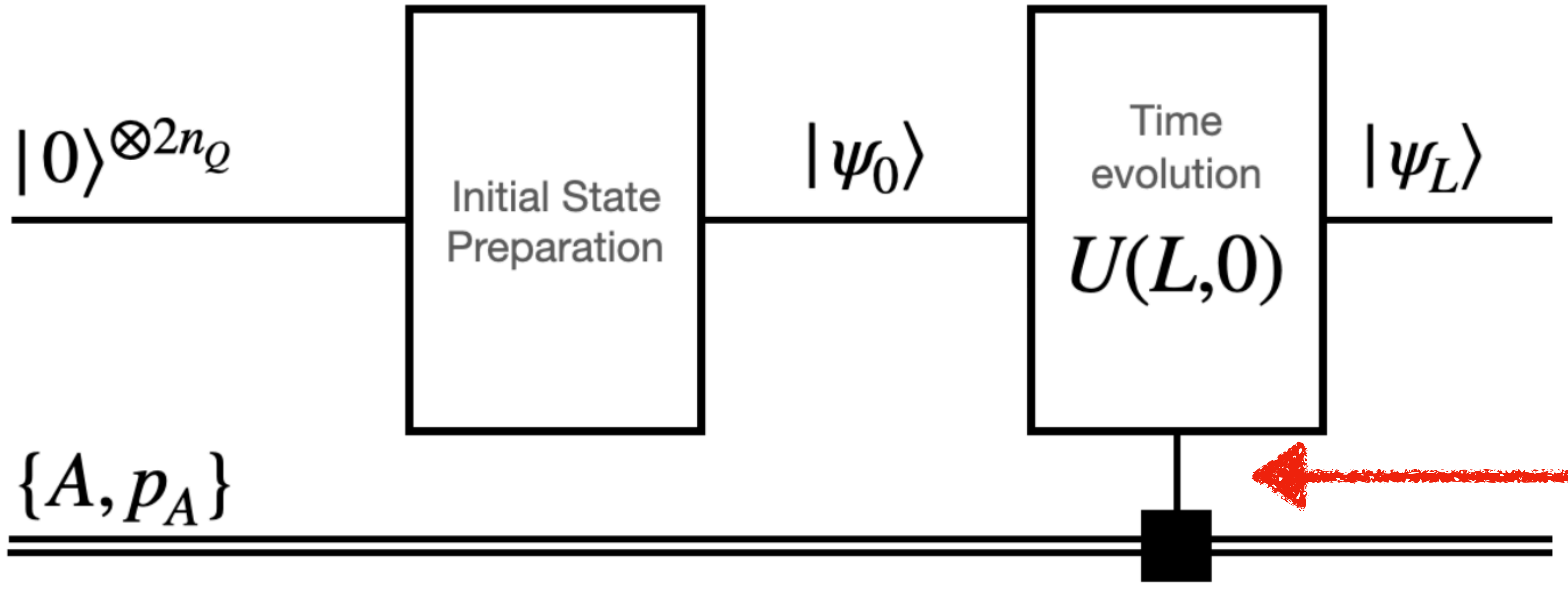
$$|\mathbf{p}\rangle \xrightarrow[\text{qFT}]{} |\mathbf{x}\rangle$$

$$U_A(k_t \cdot \varepsilon_t, \varepsilon_t) \, |\boldsymbol{x}\rangle = \exp(-ig\varepsilon_t A(k_t \cdot \varepsilon_t, \boldsymbol{x})) \, |\boldsymbol{x}\rangle$$

Setting up the QS algorithm

4. Time evolve according to $\exp(-iHt)$ $\quad U_A(k_t \cdot \varepsilon_t, \varepsilon_t)\,|\boldsymbol{x}\rangle = \exp(-ig\varepsilon_t A(k_t \cdot \varepsilon_t, \boldsymbol{x}))\,|\boldsymbol{x}\rangle$

Field insertions require probing the field value. This is done classically



Classical channel

Classical memory

2 problems:

1. Requires $\mathcal{O}(N_t \times N_s^2)$ field evaluations; **Ok for small systems**

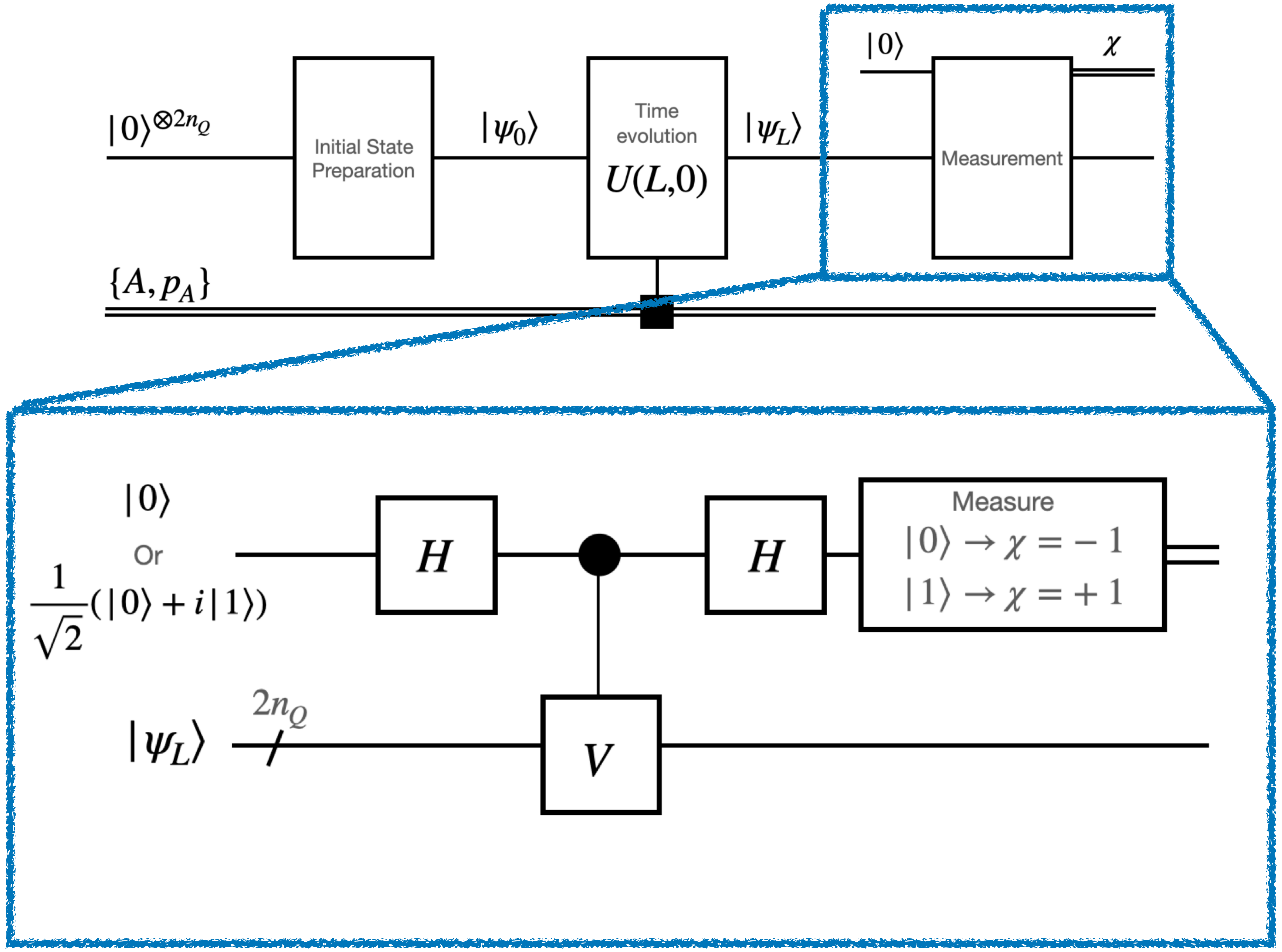
Major weakness/limitation of the approach due to classical treatment of medium

2. One needs to diagonalize a $\mathcal{O}(N_s \times N_s)$ matrix; highly sparse

Setting up the QS algorithm

5. Implement a measurement protocol

We set up a simple interference experiment



$|\psi_L\rangle = \sum_{\boldsymbol{q}} \psi_L^{\boldsymbol{q}} |\boldsymbol{q}\rangle$

Generic final state

$\chi = \{\pm 1\}$

Classical random variable

→ If ancilla is in $|0\rangle$

$$\langle\chi\rangle_{\mathrm{QM}} \equiv \langle\psi_L| V + V^\dagger |\psi_L\rangle = \Re \langle\psi_L| V |\psi_L\rangle$$

→ If ancilla is in $|0\rangle + i|1\rangle$

$$\langle\chi\rangle_{\mathrm{QM}} = \Im \langle\psi_L| V |\psi_L\rangle$$

Thus choose V such that

$$\langle \underset{\overset{\uparrow}{V}}{e^{i\alpha \boldsymbol{P}^2}} \rangle_{\mathrm{QM}} \approx 1 + i\frac{\alpha}{a_d^2}\hat{q}L \rightarrow \langle \sin(\alpha \boldsymbol{P}^2) \rangle_{\mathrm{QM}} \approx \frac{\alpha}{a_d^2}\hat{q}L$$

At the end average over background

$$\langle\chi\rangle_{\mathrm{M}} = \frac{1}{\sum_{i=1}^{m} p_{A^{(i)}}} \sum_{i=1}^{m} p_{A^{(i)}} \langle\chi\rangle_{\mathrm{QM}}^{(i)}$$

Extending in scope

1. Introducing dynamical evolution in color space

$$A \cdot T = A^a \frac{\lambda^a}{2}$$

Consider $a = 1$

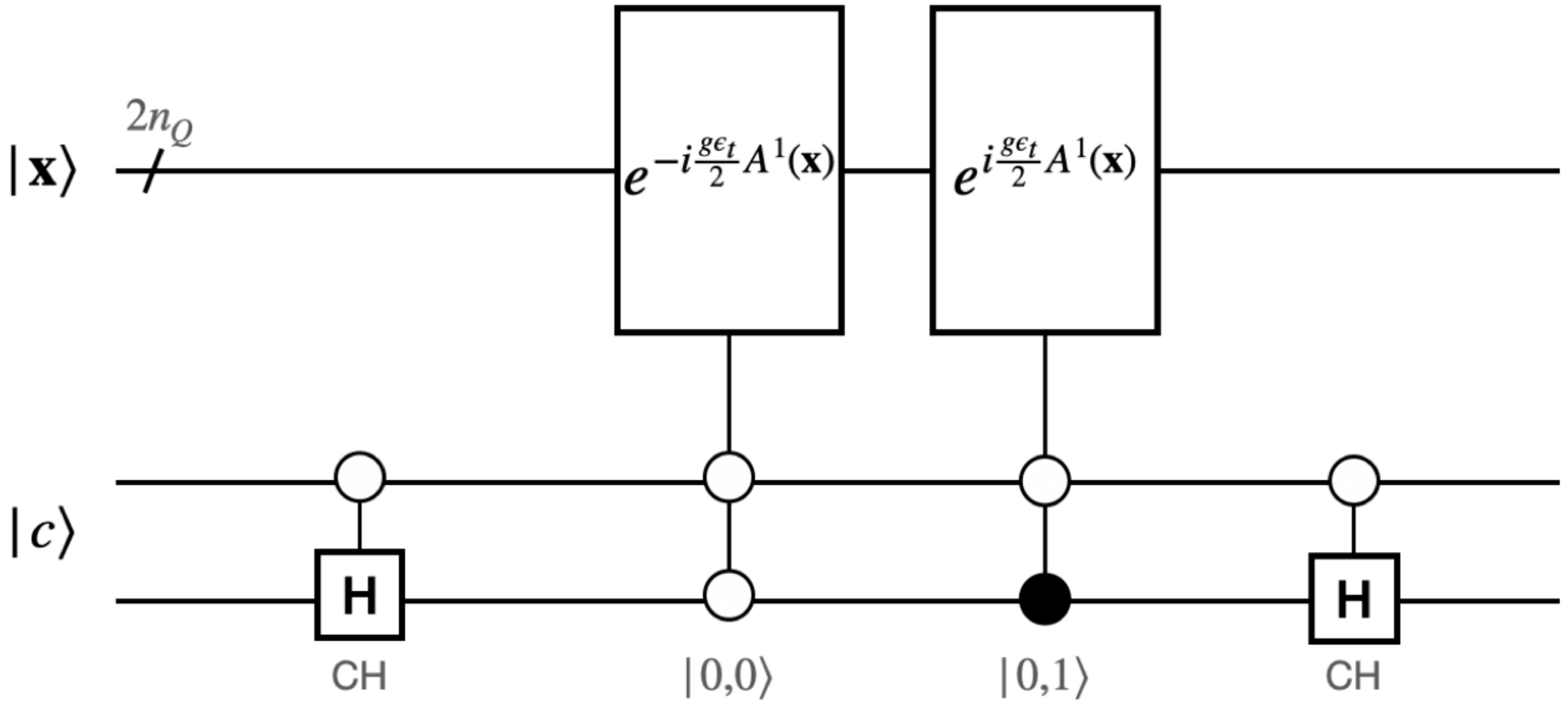
$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \tilde{\lambda}^1$$

Implementation follows usual tricks

→ $$e^{-\frac{ig\varepsilon_t}{2} A^1 \otimes \tilde{\lambda}^1} = (1 \otimes CH) e^{-\frac{ig\varepsilon_t}{2} A^1 \otimes \tilde{\sigma}^Z} (1 \otimes CH)$$

→ $$e^{-i\frac{g\varepsilon_t}{2} A^1 \otimes \tilde{\sigma}^Z} |\boldsymbol{x}\rangle \otimes |c\rangle = \sum_n \frac{(-ig\varepsilon_t)^n}{2^n n!} (A^1(\boldsymbol{X})\tilde{\sigma}^Z)^n |\boldsymbol{x}\rangle |c\rangle = |\boldsymbol{x}\rangle \sum_n \frac{(-ig\varepsilon_t A^1(\boldsymbol{x}))^n}{2^n n!} (\tilde{\sigma}^Z)^n |c\rangle$$

Circuit form



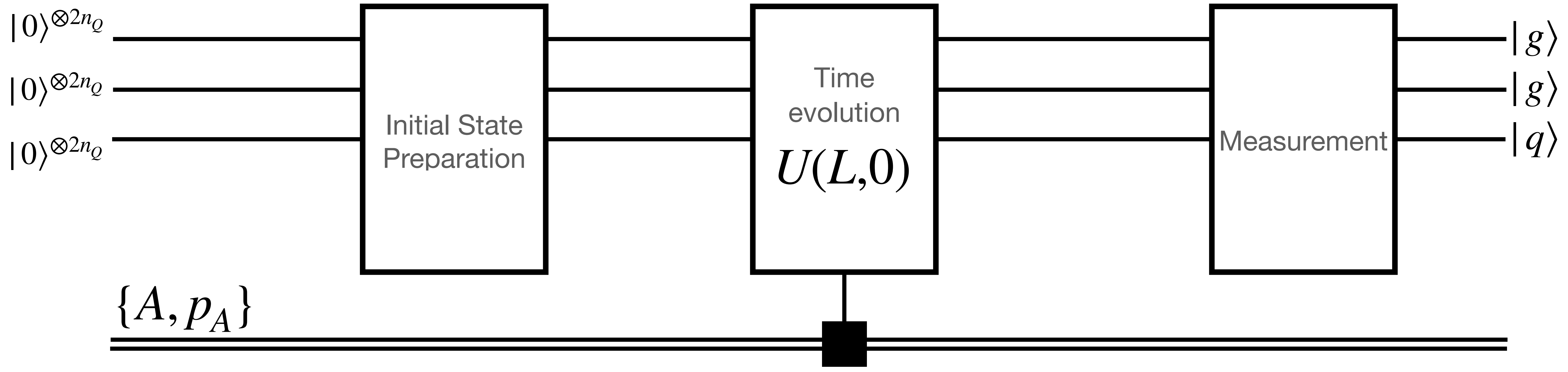
Extending in scope

2. Exploring higher multiplicity Fock states

One can integrate more single particle registers to accommodate more states

$$|\psi\rangle = |q\rangle + |qg\rangle + |qgg\rangle + \cdots$$

Quantum advantage crucial !

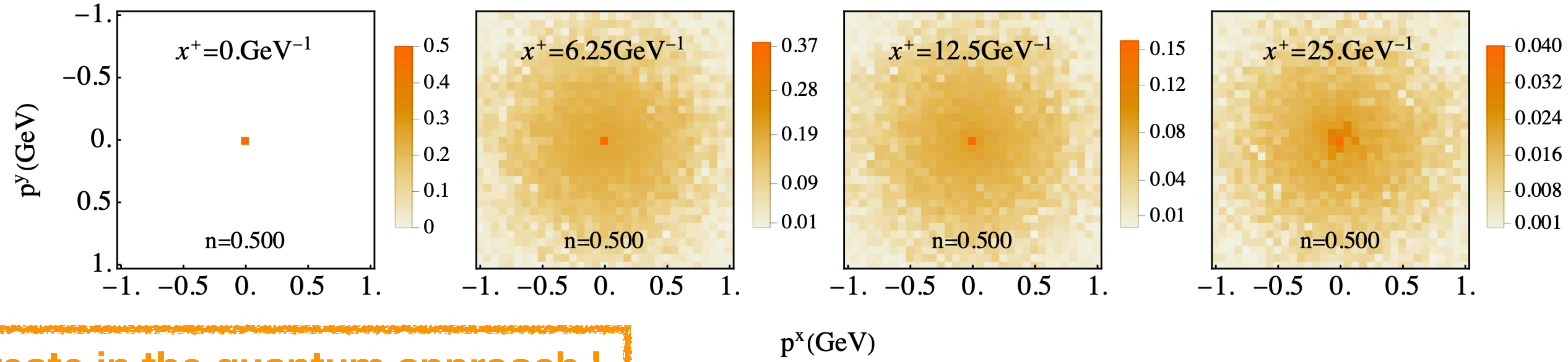


Initial State Preparation

Time evolution $U(L,0)$

Measurement

2107.02225 M. Li, T. Lappi, X. Zhao



(a) the quark in $|q\rangle$

Classical version

+

$|qg\rangle$ truncation

Not simple to recreate in the quantum approach !

Outlook

① Digital quantum simulation offers a **new way** to understand jet evolution

Some rough estimates on device requirements are not too “ambitious”

$$\mu < a_d < Q_s \sim \frac{\mu}{Q_s} < \frac{1}{N_s} < 1 \qquad\qquad 1 \geq \frac{\lambda}{L} \gg \frac{1}{\mu L}$$

$$1 < N_s < 100 \qquad\qquad 1 \leq N_t \ll \mathcal{O}(100)$$

② Next steps:

- 2.1 Implement momentum broadening circuit for $SU(2)$ plasma in an emulator
- 2.2 Workout efficient algorithm for including interactions
- 2.3 Improve treatment of the medium