Influence of scattering versus coherent parton branching on the k_T broadening of QCD cascades in a medium

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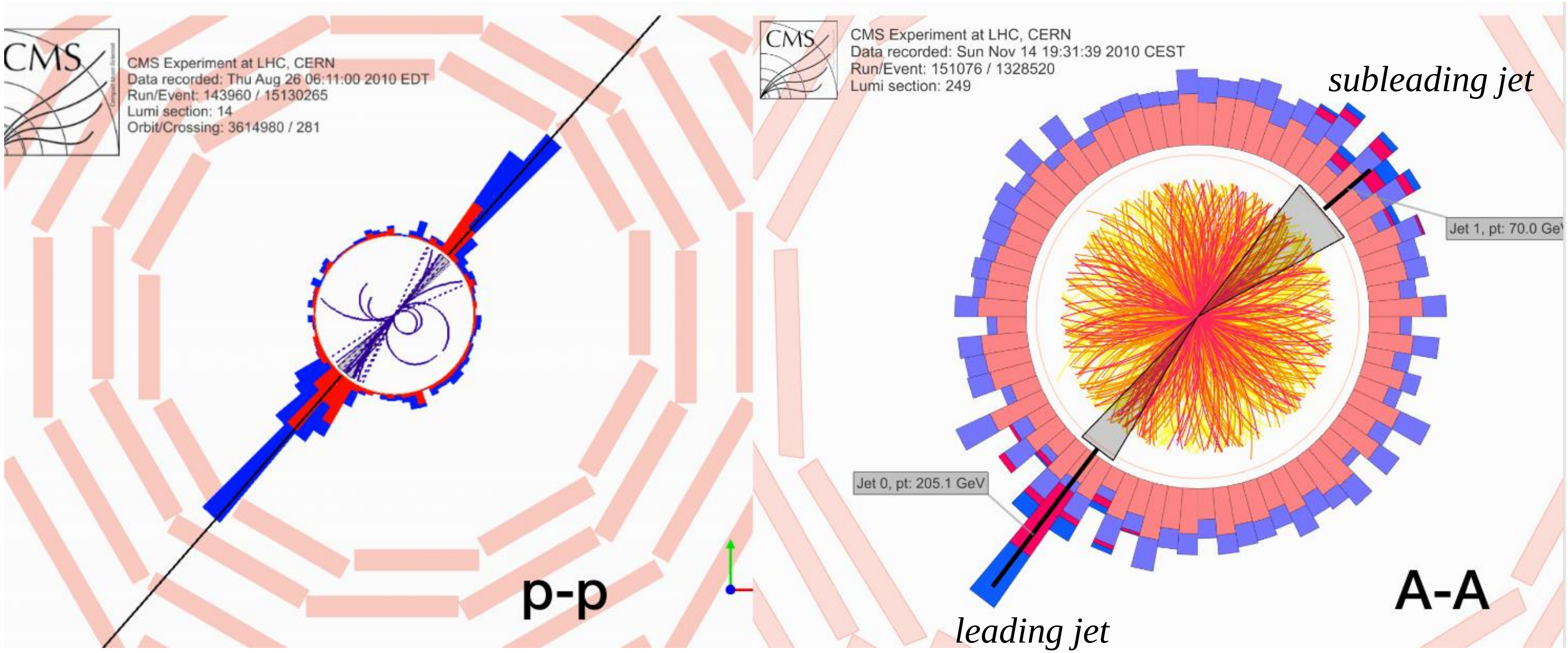
based on:

[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014] (k_T broadening/entropy in jets)

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (Monte Carlo for gluon fragmentation functions)

Jet Quenching



Jets interact with medium → Jet Quenching!

↓

probe of the medium

Coherent emission

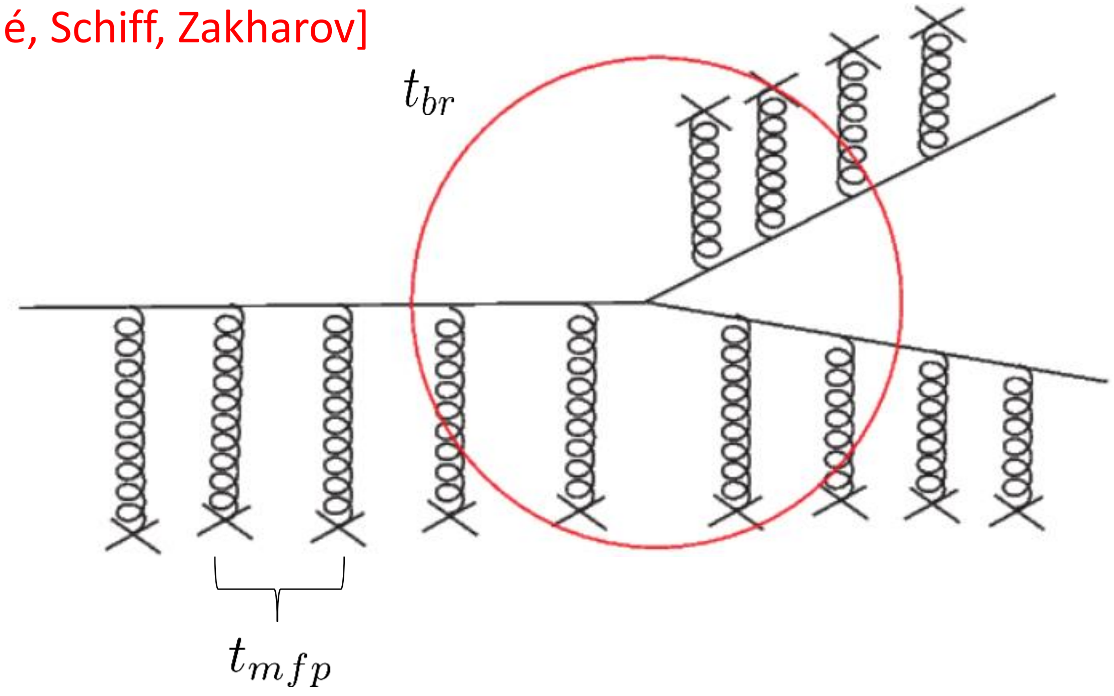
…à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$t_{br} \sim t_{mfp}$: one scattering + radiation
…Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$



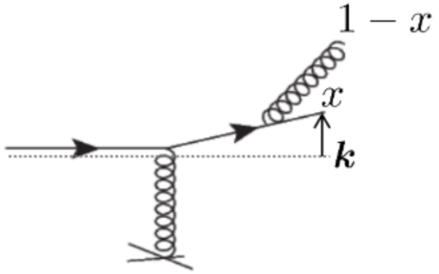
Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]



Momentum distribution:
$p \to xp$
Momentum transfer:
$p \to p + \mathbf{k}$

- Generalizes BDMPS-Z approach
- Includes transverse momentum broadening

For gluon-jets:

$$\frac{\partial}{\partial t} D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] + \int \frac{d^2\mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t).$$

Induced Radiation:

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{2}{p_0^+} \frac{P_{gg}(z)}{z(1-z)} \sin\left[\frac{\mathbf{Q}^2}{2k_{br}^2}\right] \exp\left[-\frac{\mathbf{Q}^2}{2k_{br}^2}\right]$$

$$\omega = x p_0^+, \;\; k_{br}^2 = \sqrt{\omega_0 \hat{q}_0}, \;\; \mathbf{Q} = \mathbf{k} - z\mathbf{q}, \;\; \omega_0 = z(1-z) p_0^+$$

$$\hat{q}_0 = \hat{q} f(z), \; f(z) = 1 - z(1-z), \; P_{gg}(z) = N_c \frac{[1 - z(1-z)]^2}{z(1-z)}$$

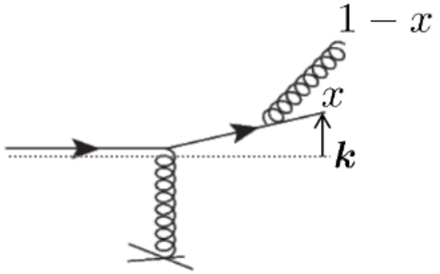
Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2\boldsymbol{q}' w(\boldsymbol{q}')$$

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Generalizes BDMPS-Z approach



Momentum distribution:
$p \to xp$
Momentum transfer:
$p \to p + \mathbf{k}$

k_T averaged Kernel:

$$\int_0^\infty d^2\mathbf{Q}\mathcal{K}(z,\mathbf{Q},p_0^+) = 2\pi\sqrt{\frac{\hat{q}}{p_0^+}}N_c\mathcal{K}(z) \longrightarrow \mathcal{K}(z) = \frac{(1-z+z^2)^{\frac{5}{2}}}{[z(1-z)]^{\frac{3}{2}}}$$

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$

Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

Splitting:

$$\mathcal{K}(z) = \frac{(1-z+z^2)^{\frac{5}{2}}}{[z(1-z)]^{\frac{3}{2}}}$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\sqrt{x}t^* \propto t_{\text{br}}$$

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q})\int d^2\boldsymbol{q}'w(\boldsymbol{q}')$$

$$w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$$

Different models

- Broadening in branching: $$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2}\left[2\mathcal{K}(\mathbf{Q},z,\frac{x}{z}p_0^+)D\left(\frac{x}{z},\mathbf{q},t\right) - \mathcal{K}(\mathbf{q},z,xp_0^+)D(x,\mathbf{k},t)\right] + \int \frac{d^2\mathbf{l}}{(2\pi)^2}C(\mathbf{l})D(x,\mathbf{k}-\mathbf{l},t).$$
    - No scattering
    - Scattering: $w(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
    - Scattering: $w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$

All models yield the same k_T averaged splitting kernel $\mathcal{K}(z)$!

- No broadening in branching: $$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$
    - Scattering: $w(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
    - Scattering: $w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$

Numerical simulations by **MINCAS** algorithm.

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

- Gaussian broadening:

  x given by $\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,t)\right]$

  $\boldsymbol{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

Departure from Gaussian broadening

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t)=\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$
$$+\frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)$$
$$\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x)-\frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right]$$

always same distribution for changes $p\to p+\boldsymbol{q}$
→central limit theorem

Splitting à la $p\to zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:
$$p\to z_1p\to z_1p+\boldsymbol{q}_1$$
$$\to z_1p+\boldsymbol{q}_1+\boldsymbol{q}_2$$
$$\to z_2(z_1+\boldsymbol{q}_1+\boldsymbol{q}_2)\to\dots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q})=\frac{16\pi^2\alpha_s^2N_cn}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$

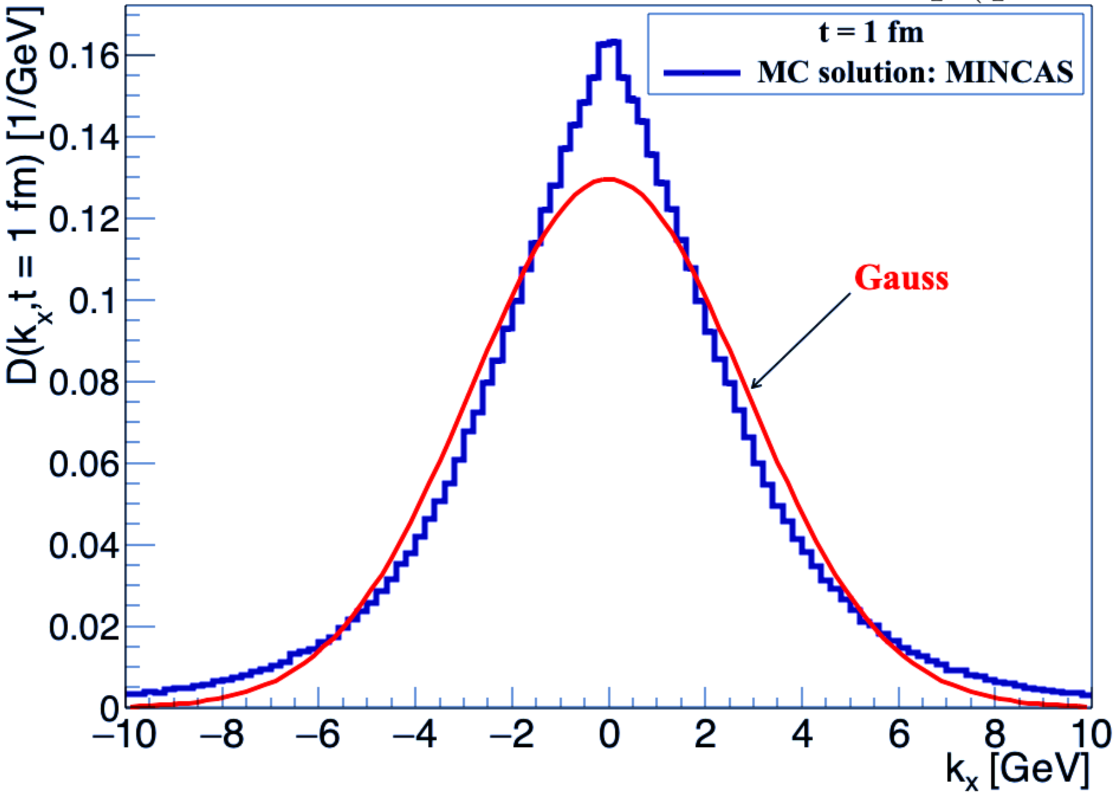
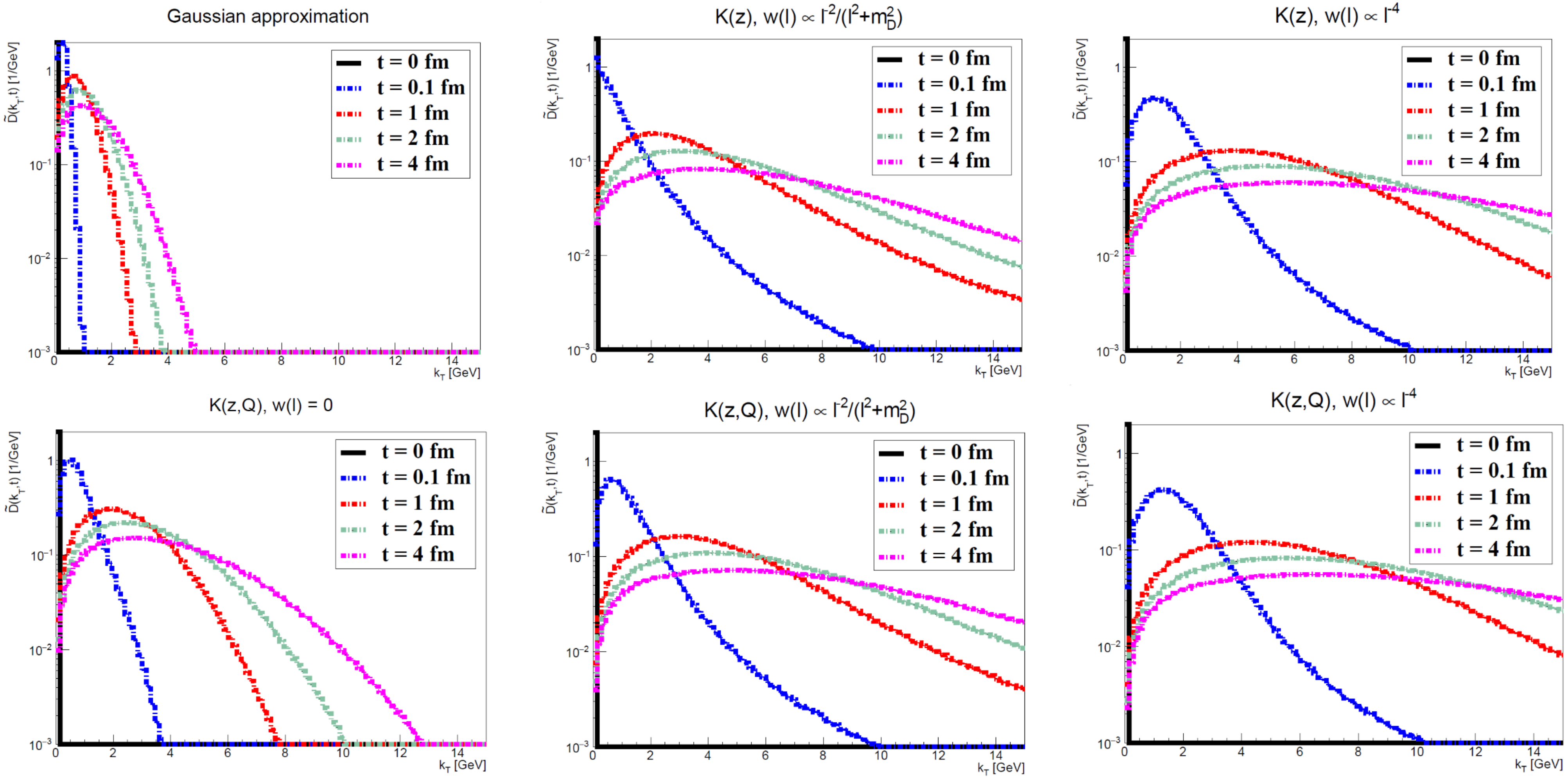


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Broadening (1/3)

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

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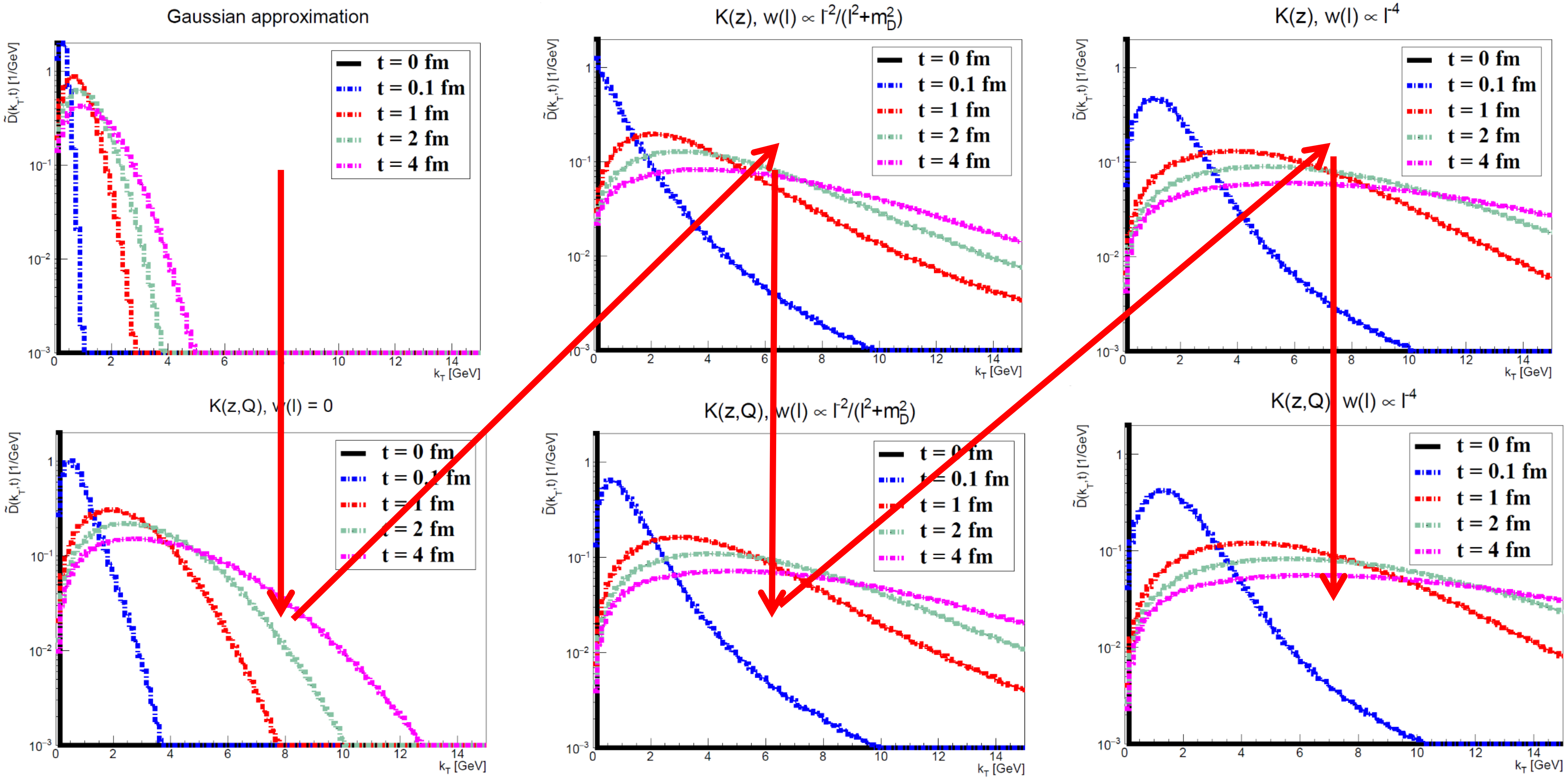


[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

Entropy for leading particles

$$S_\Delta(t) = -\sum_{i=1}^{N} p_i(x,k_T,t)\ln p_i(x,k_T,t) + P(t)\ln[\Delta x \Delta k_T]$$

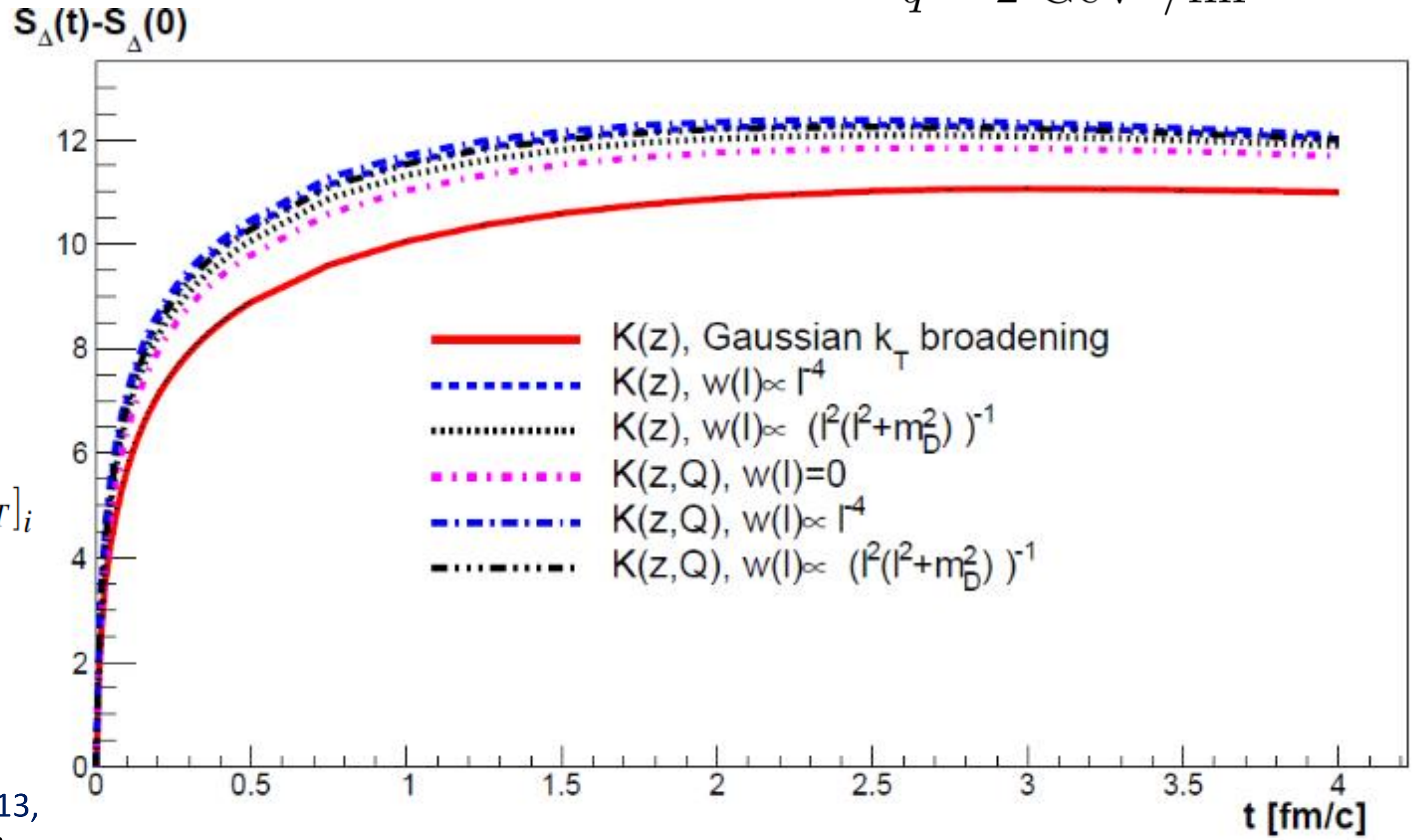
$$\Delta x \to 0,\ \Delta k_T \to 0$$

$$S_\Delta(t) \to -\int dx dk_T \tilde{D}(x,k_T,t)\ln\left(\tilde{D}(x,k_T,t)\right)$$

$$p_i(x,k_T,t) = \left[\tilde{D}(x,k_T,t)\Delta x \Delta k_T\right]_i = [2\pi k_T D(x,k_T,t)\Delta x\Delta k_T]_i$$

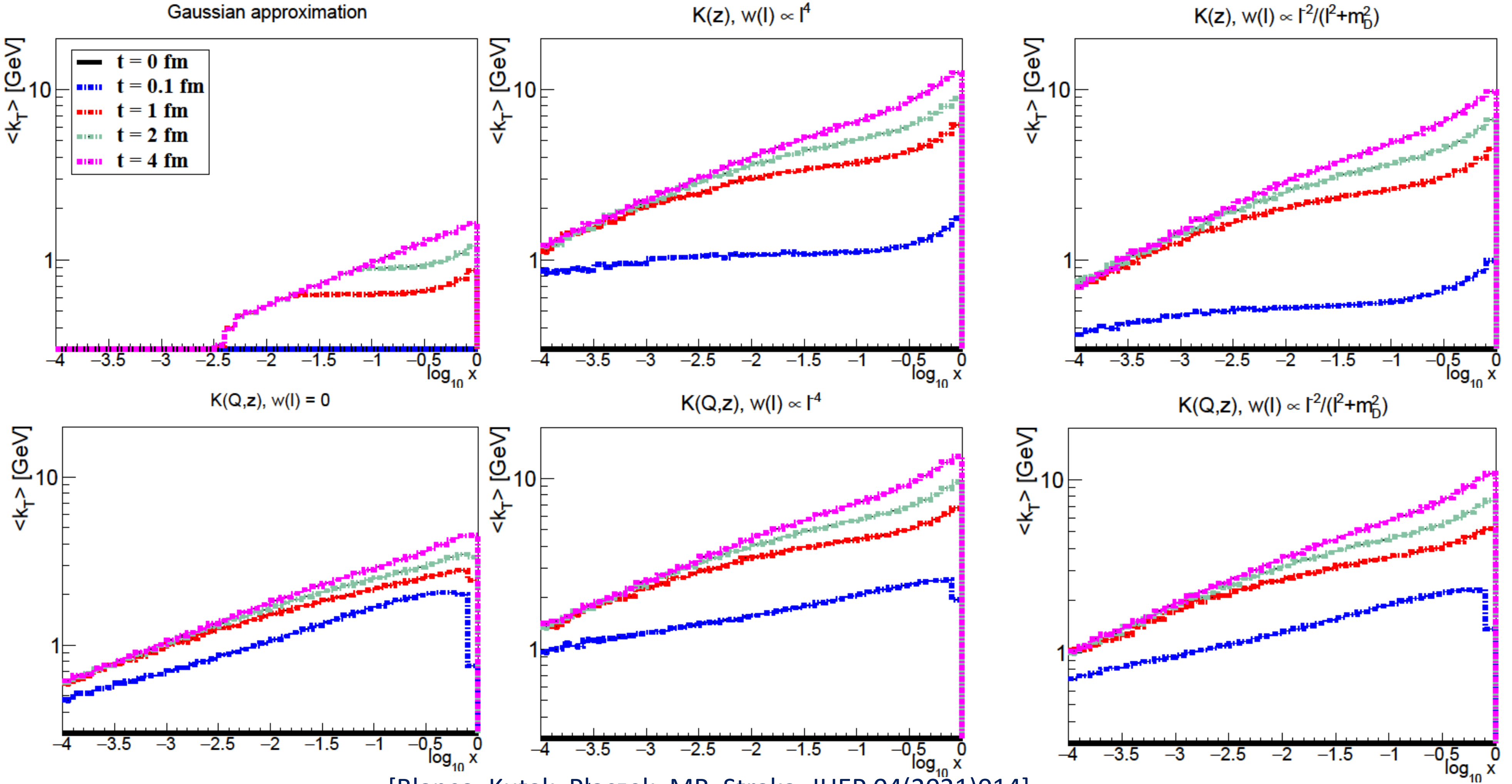
$$P(t) = \sum_{i=1}^{N} p_i(x,k_T,t)$$

$\hat{q} = 2\ \mathrm{GeV}^2/\mathrm{fm}$



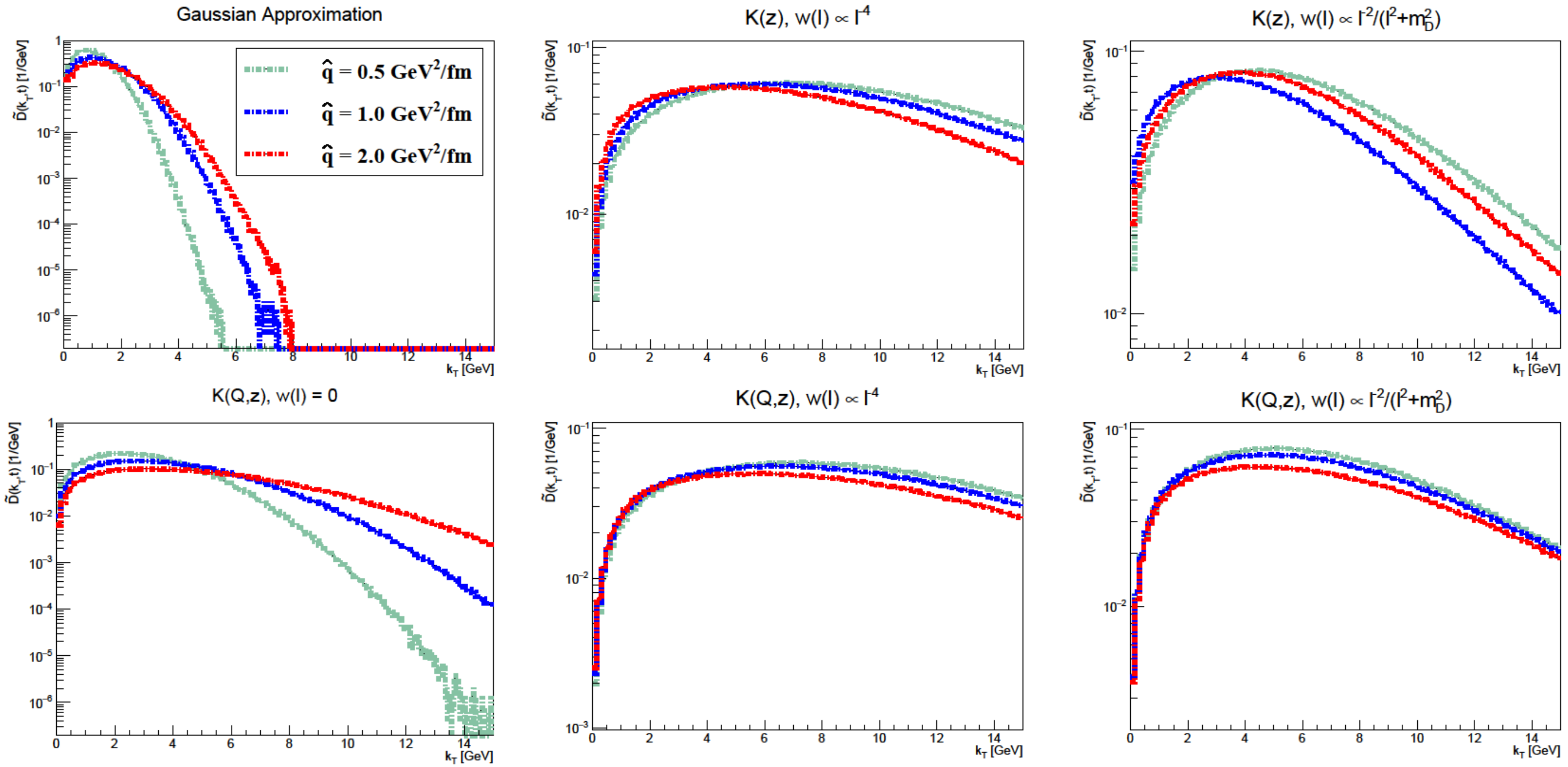
[B. Chen, Y. Zhu, J. Hu and J. C. Principe,System Parameter Identification. InformationCriteria and Algorithms. Elsevier, 2013, https://doi.org/10.1016/C2012-0-01233-1] (Delta-Entropy)

k_T Broadening (2/3)



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

k_T Broadening (3/3)



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

Summary

- **MINCAS**: jet evolution based on coherent emission and scattering
- Transverse momentum broadening differs from Gaussian distribution
- Gaussian distribution: smallest k_T broadening
- Clear ordering of broadening effects

Outlook

- to account for quarks
- to study more forward processes
- and vacuum-like emissions

Thank you for your attention!

BDIM Equation as Integral Equation

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

↓

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)}D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy\delta(x-zy)\sqrt{\frac{z}{x}}z\mathcal{K}(z)e^{-\phi(x)(\tau-\tau')}D(y,\tau')$$

$$\tau = \frac{t}{t^*} \qquad \phi(x) = \frac{1}{\sqrt{x}}\int_0^{1-\epsilon} dzz\mathcal{K}(z)$$

Monte-Carlo algorithm

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)} D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy \delta(x-zy) \sqrt{\frac{z}{x}} z\mathcal{K}(z) e^{-\phi(x)(\tau-\tau')} D(y,\tau')$$

Set/Select x_0 at τ_0

1. Select τ_{i+1}:
   Probability density:
   $$\phi(x_i) e^{-\phi(x_i)(\tau_i - \tau_{i+1})}$$
2. Select $z = \frac{x_{i+1}}{x_i}$:
   Probability density:
   $$\frac{z\mathcal{K}(z)}{\int_0^{1-\epsilon} dz' z' \mathcal{K}(z')}$$

Repeat for next step in τ and x

Stop once $\tau > \tau_L$

Analogous for the k_T dependent equation in x, k_T, and, τ!

M I N C A S

Diffusion approximation (1/2)

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \, \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} \, D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t \right) \theta(z - x) - \frac{z}{\sqrt{x}} \, D(x, \mathbf{k}, t) \right]$$
$$+ \int \frac{d^2\mathbf{q}}{(2\pi)^2} \, C(\mathbf{q}) \, D(x, \mathbf{k} - \mathbf{q}, t),$$

$$\downarrow$$

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \, \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} \, D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t \right) \theta(z - x) - \frac{z}{\sqrt{x}} \, D(x, \mathbf{k}, t) \right]$$
$$+ \frac{1}{4} \hat{q} \nabla_k^2 \Big[D(x, \mathbf{k}, t) \Big].$$

Diffusion approximation (2/2)