

Charmless b-meson and b-baryon decays at LHCb

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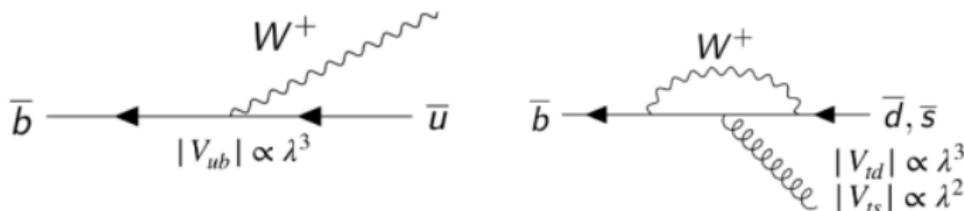
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Charmless b-meson decays

- Involve both **Tree** $b \rightarrow u$ and **Penguin** $b \rightarrow s, d$ transitions



- Dominant tree-level and penguin diagrams contribute in the same order of magnitude
- Sensitive to CP violation studies
- Multi-body decays possess rich resonant structures
 - Large CP violation signatures found in regions of the phase space
- Interesting to search for new sources of CP violation

- **Measurement of the relative branching fractions of $B^+ \rightarrow h^+ h'^+ h'^-$ decays**
 - Published as [Phys. Rev. D102\(2020\) 112010](#) ([arXiv:2010.11802](#))
- **Search for CP and observation of P violation in $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$**
 - Published as [Phys. Rev. D102\(2020\) 051101](#) ([arXiv:1912.10741](#))
- **Search for CP violation in $\Xi_b^- \rightarrow pK^-K^-$**
 - Submitted to Phys. Rev. D ([arXiv:2104.15074](#))

Measurement of the relative branching fractions of

$$B^+ \rightarrow h^+ h'^+ h'^- \text{ decays}$$

$$(B^+ \rightarrow K^+ K^+ K^-, B^+ \rightarrow \pi^+ K^+ K^-, \\ B^+ \rightarrow K^+ \pi^+ \pi^-, B^+ \rightarrow \pi^+ \pi^+ \pi^-)$$

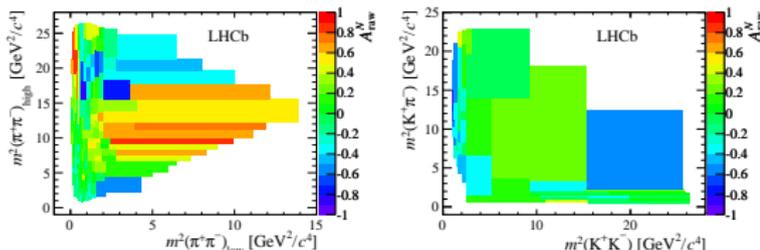
Phys. Rev. D102(2020) 112010
(arXiv:2010.11802)

Motivation

- Large integrated CP asymmetries and CP signatures in localized regions of the phase space
- Confirmed with recent amplitude analyses:
 - $B^+ \rightarrow \pi^+ \pi^+ \pi^-$
(PRL 124(2020)031801)
(PRD 101(2020)012006)
 - $B^+ \rightarrow \pi^+ K^+ K^-$
(PRL 123(2019)231802)
- Fit fractions from amplitude analysis are converted into quasi-two-body branching fractions
 - Precise knowledge of three-body branching fraction is needed
 - Current knowledge of branching fraction not sufficient given the sensitivity of the Dalitz plot analyses

$$B^+ \rightarrow \pi^+ \pi^+ \pi^- \text{ and } B^+ \rightarrow \pi^+ K^+ K^-$$

(PRD 90(2014)112004)



Current knowledge BF $B^+ \rightarrow h^+ h'^+ h'^-$ (PDG)

Decay	PDG average (10^{-6})
$B^+ \rightarrow K^+ K^+ K^-$	34.0 ± 1.4
$B^+ \rightarrow \pi^+ K^+ K^-$	5.2 ± 0.4
$B^+ \rightarrow K^+ \pi^+ \pi^-$	51.0 ± 2.9
$B^+ \rightarrow \pi^+ \pi^+ \pi^-$	15.2 ± 1.4

Measurement of the relative branching fractions of $B^+ \rightarrow h^+ h'^+ h'^-$

- Performed with Run 1 dataset: 3fb^{-1} from 2011+12
- Relative branching fraction ratios determined by

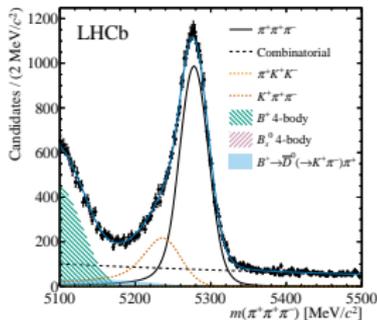
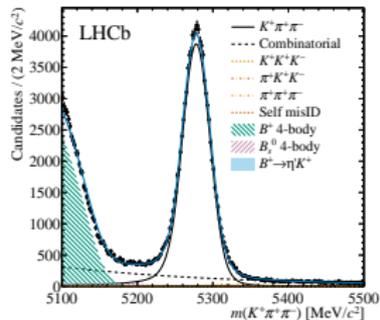
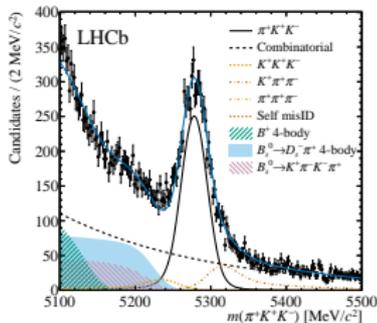
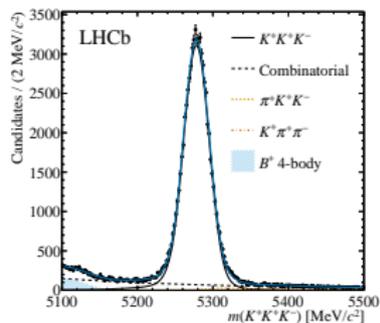
$$\frac{\mathcal{B}(B^+ \rightarrow h^+ h'^+ h'^-)}{\mathcal{B}(B^+ \rightarrow K^+ K^+ K^-)} = \frac{\mathcal{N}_{hh'h'}^{corr}}{\mathcal{N}_{KKK}^{corr}}$$

$\mathcal{N}^{corr} \rightarrow$ signal yield efficiency corrected

- Obtained as a function of Dalitz plot position

Signal Yields

Extraction of the signal and background yields from a simultaneous fit to all channels



Mass fit results:

Decay	Fit yield
$B^+ \rightarrow K^+K^+K^-$	$69\,310 \pm 280$
$B^+ \rightarrow \pi^+K^+K^-$	$5\,760 \pm 140$
$B^+ \rightarrow K^+\pi^+\pi^-$	$94\,950 \pm 430$
$B^+ \rightarrow \pi^+\pi^+\pi^-$	$25\,480 \pm 200$

Signal Yield correction:

$$\mathcal{N}^{corr} \propto \sum_{bins} \frac{W_{bin}}{\epsilon_{bin}}$$

$w_{bin} \rightarrow$ signal data background subtracted, $\epsilon_{bin} \rightarrow$ efficiency

Results

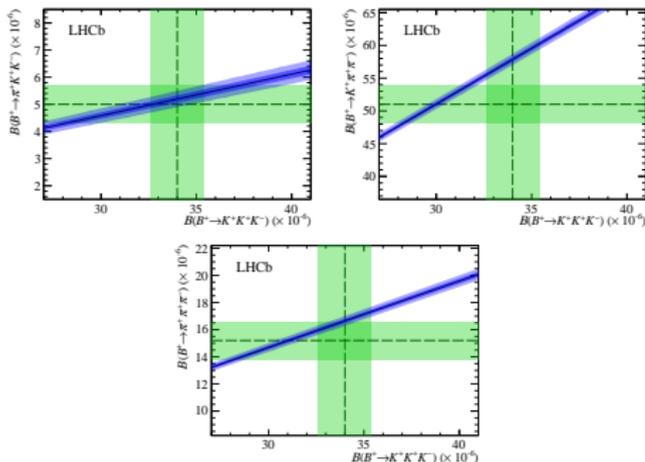
Measured relative branching fractions

\mathcal{B} ratio	Value
$\mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow K^+ K^+ K^-)$	$0.151 \pm 0.004 \pm 0.008$
$\mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow K^+ K^+ K^-)$	$1.703 \pm 0.011 \pm 0.022$
$\mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow K^+ K^+ K^-)$	$0.488 \pm 0.005 \pm 0.009$
$\mathcal{B}(B^+ \rightarrow K^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-)$	$6.61 \pm 0.17 \pm 0.33$
$\mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-)$	$11.27 \pm 0.29 \pm 0.54$
$\mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-)$	$3.23 \pm 0.09 \pm 0.19$
$\mathcal{B}(B^+ \rightarrow K^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-)$	$0.587 \pm 0.004 \pm 0.008$
$\mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-)$	$0.0888 \pm 0.0023 \pm 0.0047$
$\mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-)$	$0.2867 \pm 0.0029 \pm 0.0045$
$\mathcal{B}(B^+ \rightarrow K^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-)$	$2.048 \pm 0.020 \pm 0.040$
$\mathcal{B}(B^+ \rightarrow \pi^+ K^+ K^-) / \mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-)$	$0.310 \pm 0.008 \pm 0.020$
$\mathcal{B}(B^+ \rightarrow K^+ \pi^+ \pi^-) / \mathcal{B}(B^+ \rightarrow \pi^+ \pi^+ \pi^-)$	$3.488 \pm 0.035 \pm 0.053$

- Large systematic uncertainties:
 - Dominant source from background modelling

Results

Comparison with the current world averages



Branching fractions ratios obtained (violet) compared to the PDG (green)

- All measurements in good agreement
- Significant improvement in the precision of all measured ratios

Results applied to quasi-two-body
BFs from $B^{\pm} \rightarrow \pi^{\pm} \pi^{\pm} \pi^{\mp}$
amplitude analysis

$\mathcal{B}(B^{\pm} \rightarrow \rho^0(770)\pi^{\pm})$ improves
relative error from 16% to 6%

Current world average
(arXiv:1612.07233):

- $\mathcal{B}(B^{\pm} \rightarrow \rho^0(770)\pi^{\pm}) = (8.3^{+1.2}_{-1.3}) \times 10^{-6}$

Improved measurement:

- $\mathcal{B}(B^{\pm} \rightarrow \rho^0(770)\pi^{\pm}) = (9.5 \pm 0.6) \times 10^{-6}$

Charmless b-baryon decays

Introduction

- Theory predicts large CP violation asymmetries in some charmless b-baryon decays (Phys. Rev. D91,116007(2015))

Theory prediction

- $A_{CP}(\Lambda_b \rightarrow pK^-) = (5.8 \pm 0.2)\%$
 - $A_{CP}(\Lambda_b \rightarrow p\pi^-) = (-3.9 \pm 0.2)\%$
 - $A_{CP}(\Lambda_b \rightarrow pK^{*-}) = (19.6 \pm 1.3)\%$
 - $A_{CP}(\Lambda_b \rightarrow p\rho^-) = (-3.7 \pm 0.3)\%$
-
- CP violation not yet observed in any b-baryon decay
 - Abundant production of b-baryons in pp collisions at the LHC

Search for CP and observation of P violation in

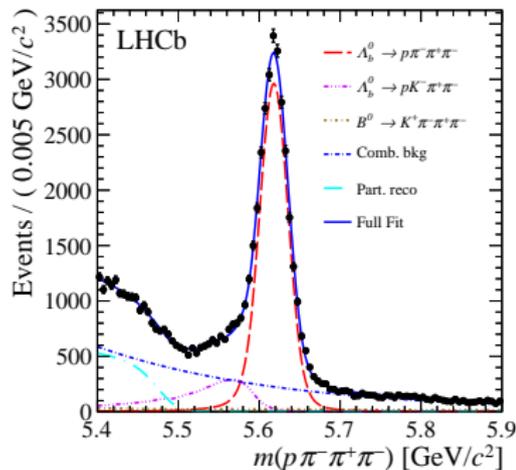
$$\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^- \text{ decays}$$

Phys. Rev. D102(2020) 051101 (arXiv:1912.10741)

Search for CP and P violation in $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$ decays

- $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$ has contributions of many resonances
 - Possibility of CP asymmetries in regions of the phase space
- First evidence of CPV in baryons found in this decay with 3.3σ using Run 1 data (Nature Phys. 13 (2017) 391)
- Updated result using 6.6 fb^{-1} from Run 1+Run 2 (2011+12+15+16+17)
- Search for CPV with 2 methods:
 - Triple Product Asymmetries (TPA)
 - Unbinned energy test
- Published as PRD 102(2020)051101

Run 1+Run 2 (PRD 102(2020)051101)



• $N_{\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-}^{\text{Run1+Run2}} = 27600 \pm 200$

Triple Product Asymmetries and Energy Test method

Triple Product Asymmetries $A_{\hat{\tau}}$

Based on scalar triple products $C_{\hat{\tau}} \equiv \vec{p}_p \cdot (\vec{p}_{\pi_{fast}} \times \vec{p}_{\pi_+})$

CP and P violating asymmetries observables:

$$a_{CP}^{\hat{\tau}-odd} = \frac{1}{2}(A_{\hat{\tau}} - \bar{A}_{\hat{\tau}}) \qquad a_P^{\hat{\tau}-odd} = \frac{1}{2}(A_{\hat{\tau}} + \bar{A}_{\hat{\tau}})$$

Measurements in bins and integrated over the phase space

Energy Test

- Sensitive to local asymmetries
- Relies on a test statistic to calculate the differences between two samples

Measurements in regions of the phase space

Results

Integrated measurements (TPA)

$$a_{CP}^{\hat{T}^{-odd}} = (-0.7 \pm 0.7 \pm 0.2)\%$$

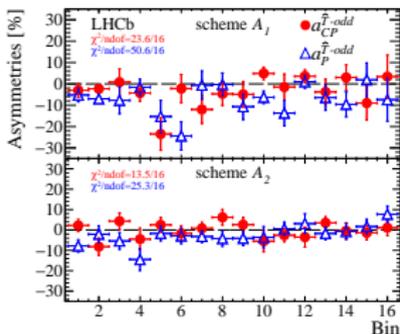
No evidence for CPV

$$a_P^{\hat{T}^{-odd}} = (-4.0 \pm 0.7 \pm 0.2)\%$$

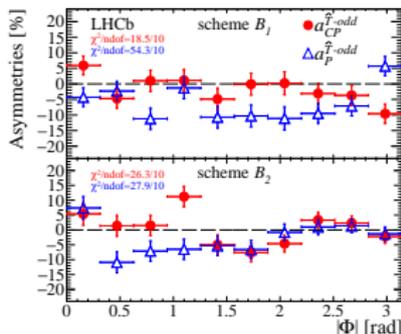
P violation observed with 5.5σ

Phase space bins measurements (TPA)

16 bins on helicity angles



10 bins on $|\Phi|$ angle



- No evidence for CPV in regions of the phase space
- Local P violation at 5.1σ

Energy Test

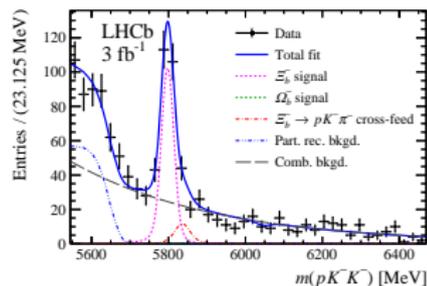
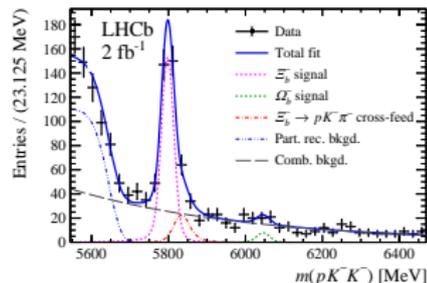
- No evidence for local CPV
- Confirms local P violation observed at 5.3σ

Search for CP violation in $\Xi_b^- \rightarrow pK^- K^-$
Submitted to Phys. Rev. D (arXiv:2104.15074)

Search for CP violation in $\Xi_b^- \rightarrow pK^- K^-$

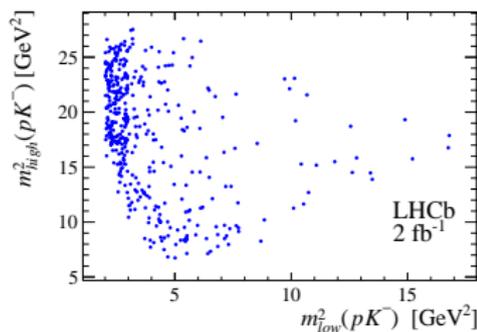
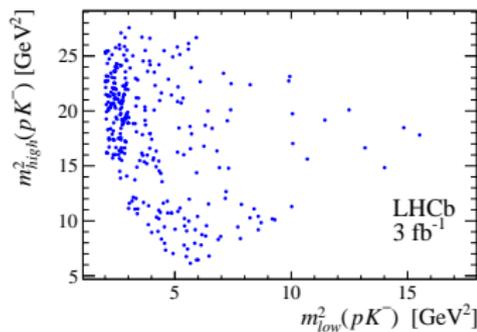
- Large local CPV observed in $B^\pm \rightarrow 3h$.
Can similar behaviour be observed in baryonic modes?
- $\Xi_b^- \rightarrow pK^- K^-$ was first observed by LHCb in 3 fb^{-1} from 2011+12 (PRL 118(2017)112004)
- First amplitude analysis of $\Xi_b^- \rightarrow pK^- K^-$
 - Run 1: 3 fb^{-1} from 2011+12
 - Run 2: 2 fb^{-1} from 2015+16
 - First amplitude analysis of any baryon decay accounting for CP violation
- Search for $\Omega_b^- \rightarrow pK^- K^-$ decays
 - New upper limit set on

$$\mathcal{R} = \frac{f_{\Omega_b}}{f_{\Xi_b}} \times \frac{BF(\Omega_b^- \rightarrow pK^- K^-)}{BF(\Xi_b^- \rightarrow pK^- K^-)}$$

Run 1 $N_{\Xi_b^-} = 193 \pm 21$ Run 2 $N_{\Xi_b^-} = 297 \pm 23$ 

Amplitude Analysis of $\Xi_b^- \rightarrow pK^- K^-$

- Model-dependent amplitude analysis
- Signal region: $m(\Xi_b^-) \pm 40$ MeV
 - Signal purity: 63%(Run 1) 70%(Run 2)
- Phase space of $\Xi_b^- \rightarrow pK^- K^-$ has 5 degrees of freedom
 - Assumption: Ξ_b^- produced unpolarised, decay characterised by 2 variables
- Resonant structures observed in $m_{low}^2(pK^-)$
 - Λ^* and Σ^* resonances expected



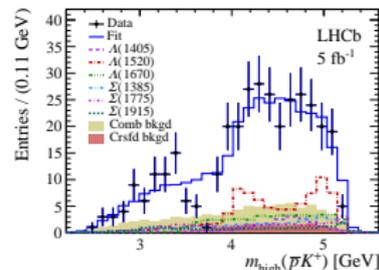
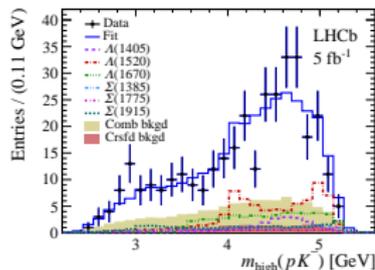
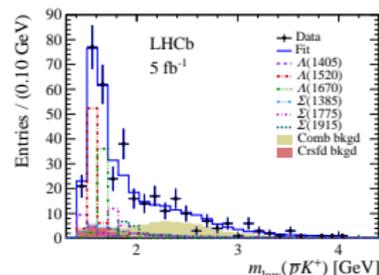
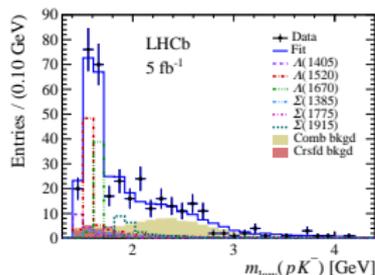
Fit to data

Baseline model:

- Contributions from Λ^* and Σ^*
 - $\Lambda(1520)$ used as reference
 - modelled as Breit-Wigner
- Nonresonant components
 - modelled as exponential

Fit to data:

- Simultaneous fit to Run1 & Run2
- No significant difference between Ξ_b^- and Ξ_b^+



Results: Ratio \mathcal{R} of Ω_b^- and Ξ_b^- branching fractions

Combined Run1+Run2 results

Branching Fraction Limit

$$\mathcal{R} \equiv \frac{f_{\Omega_b^-}}{f_{\Xi_b^-}} \times \frac{\mathcal{B}(\Omega_b^- \rightarrow pK^- K^-)}{\mathcal{B}(\Xi_b^- \rightarrow pK^- K^-)} = (24 \pm 21(\text{stat}) \pm 14(\text{sys})) \times 10^{-3}$$

Consistent and more precise than the previous LHCb measurement performed on Run 1 data (PRL 118(2017) 071801)

No observation of $\Omega_b^- \rightarrow pK^- K^-$ decay mode

Upper limit on the ratio of fragmentation and branching fractions:

$$\mathcal{R} \equiv \frac{f_{\Omega_b^-}}{f_{\Xi_b^-}} \times \frac{\mathcal{B}(\Omega_b^- \rightarrow pK^- K^-)}{\mathcal{B}(\Xi_b^- \rightarrow pK^- K^-)} < 62(71) \times 10^{-3}$$

Results: Amplitude Analysis

CP-asymmetry

Component	$A^{CP} (10^{-2})$
$\Sigma(1385)$	-27 ± 34 (stat) ± 73 (syst)
$\Lambda(1405)$	-1 ± 24 (stat) ± 32 (syst)
$\Lambda(1520)$	-5 ± 9 (stat) ± 8 (syst)
$\Lambda(1670)$	3 ± 14 (stat) ± 10 (syst)
$\Sigma(1775)$	-47 ± 26 (stat) ± 14 (syst)
$\Sigma(1915)$	11 ± 26 (stat) ± 22 (syst)

No significant A^{CP}
is observed

Quasi 2-body Branching Fraction

$$\begin{aligned} \mathcal{B}(\Xi_b^- \rightarrow \Sigma(1385)K^-) &= (0.26 \pm 0.11 \pm 0.17 \pm 0.10) \times 10^{-6}, \\ \mathcal{B}(\Xi_b^- \rightarrow \Lambda(1405)K^-) &= (0.19 \pm 0.06 \pm 0.07 \pm 0.07) \times 10^{-6}, \\ \mathcal{B}(\Xi_b^- \rightarrow \Lambda(1520)K^-) &= (0.76 \pm 0.09 \pm 0.08 \pm 0.30) \times 10^{-6}, \\ \mathcal{B}(\Xi_b^- \rightarrow \Lambda(1670)K^-) &= (0.45 \pm 0.07 \pm 0.13 \pm 0.18) \times 10^{-6}, \\ \mathcal{B}(\Xi_b^- \rightarrow \Sigma(1775)K^-) &= (0.22 \pm 0.08 \pm 0.09 \pm 0.09) \times 10^{-6}, \\ \mathcal{B}(\Xi_b^- \rightarrow \Sigma(1915)K^-) &= (0.26 \pm 0.09 \pm 0.21 \pm 0.10) \times 10^{-6}, \end{aligned}$$

$$\mathcal{B}(\Xi_b^- \rightarrow R^-) = \mathcal{B}(\Xi_b^- \rightarrow pK^- K^-) \times \mathcal{F}_i$$

$$\mathcal{B}(\Xi_b^- \rightarrow pK^- K^-) = (2.3 \pm 0.9) \times 10^{-6}$$

Uncertainties

- statistical
- systematic
- knowledge of $\mathcal{B}(\Xi_b^- \rightarrow pK^- K^-)$

Summary

Summary

$$B^+ \rightarrow h^+ h'^+ h'^-$$

- Measurements of all combinations of $B^+ \rightarrow h^+ h'^+ h'^-$ relative branching fractions
- All measurements in good agreement with their world-average results
- Significant improvement in the precision

$$\Lambda_b^0 \rightarrow p \pi^- \pi^+ \pi^-$$

- Two different methods to search for CP and P violation: TPA and Energy Test
- No evidence of CP violation
- Observation of P violation over 5σ locally and integrated over all phase space.

$$\Xi_b^- \rightarrow p K^- K^-$$

- First amplitude analysis of a baryon accounting for CP violation
- No evidence of CP violation
- Measurement of six quasi two-body branching fraction of $\Xi_b^- \rightarrow p K^- K^-$
- No significant signal of $\Omega_b^- \rightarrow p K^- K^-$
- Set a new upper limit on the ratio of fragmentation and branching fraction of $\Xi_b^- \rightarrow p K^- K^-$ and $\Omega_b^- \rightarrow p K^- K^-$

Thank you for your attention

Backup slides

$$\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$$

$\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: Triple Product Asymmetries

Triple products are \hat{T} -odd¹ observables built combining the momentum \vec{p} of three final states particles in the Λ_b^0 frame:

$$C_{\hat{T}} \equiv \vec{p}_p \cdot (\vec{p}_{\pi_{fast}} \times \vec{p}_{\pi_+})$$

Triple product asymmetries (TPA):

$$A_{\hat{T}} = \frac{N(C_{\hat{T}} > 0) - N(C_{\hat{T}} < 0)}{N(C_{\hat{T}} > 0) + N(C_{\hat{T}} < 0)}$$

$$\bar{A}_{\hat{T}} = \frac{\bar{N}(\bar{C}_{\hat{T}} > 0) - \bar{N}(\bar{C}_{\hat{T}} < 0)}{\bar{N}(\bar{C}_{\hat{T}} > 0) + \bar{N}(\bar{C}_{\hat{T}} < 0)}$$

$N, \bar{N} \rightarrow$ number of $\Lambda_b^0, \bar{\Lambda}_b^0$ decays

Clean CP and P violating asymmetries observables are defined as:

$$a_{CP}^{\hat{T}-odd} = \frac{1}{2}(A_{\hat{T}} - \bar{A}_{\hat{T}})$$

$$a_P^{\hat{T}-odd} = \frac{1}{2}(A_{\hat{T}} + \bar{A}_{\hat{T}})$$

Measurements in bins and integrated over the phase space

¹ \hat{T} is an operator called motion-reversal that reverts momentum and helicities.

$\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: Triple Product Asymmetries

Phase space bins measurements:

- Scheme A: asymmetries measured as function of the polar and azimuthal angles of $p(\Delta^{++})$ in the $\Delta^{++}(N^{*+})$ rest frame
- Scheme B: asymmetries measured as a function of the angle $|\Phi|$, between the decays planes $p\pi_{fast}^-$ and $\pi^+\pi_{slow}^-$
- $A_1, B_1 \rightarrow m(p\pi^+\pi_{slow}^-) > 2.8 \text{ GeV}/c^2$ dominated by a_1 resonance
- $A_2, B_2 \rightarrow m(p\pi^+\pi_{slow}^-) < 2.8 \text{ GeV}/c^2$ dominated by N^{*+} decay

$\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: Energy Test

- Model independent unbinned test sensitive to local asymmetries
- Statistical test T used to calculate the differences between two samples:

$$T \equiv \frac{1}{2n(n-1)} \sum_{i \neq j}^n \psi_{ij} + \frac{1}{2\bar{n}(\bar{n}-1)} \sum_{i \neq j}^{\bar{n}} \psi_{ij} - \frac{1}{n\bar{n}} \sum_{i=1}^n \sum_{j=1}^{\bar{n}} \psi_{ij}$$

$n(\bar{n}) \rightarrow$ number of $\Lambda_b^0(\bar{\Lambda}_b^0)$ candidates and $\psi_{ij} = e^{-d_{ij}^2/2\delta^2} \rightarrow d_{ij}$ is the distance in phase space of each pair of candidates ij and δ is the distance scale (free parameter)

- 4 subsamples defined \rightarrow 3 tests performed (2 for CPV + 1 for P violation)

p-value results obtained for each test in 3 different δ

Distance scale δ	1.6 GeV ² /c ⁴	2.7 GeV ² /c ⁴	13 GeV ² /c ⁴
p-value (CP conservation, P even)	3.1×10^{-2}	2.7×10^{-3}	1.3×10^{-2}
p-value (CP conservation, P odd)	1.5×10^{-1}	6.9×10^{-2}	6.5×10^{-2}
p-value (P conservation)	1.3×10^{-7}	4.0×10^{-7}	1.6×10^{-1}

$$\Xi_b^- \rightarrow pK^- K^-$$

$\Xi_b^- \rightarrow pK^- K^-$: Total PDF of the phase space

$$\mathcal{P}_{tot}^Q(\Omega) = \frac{1}{N_{tot}} [N_{sig} P_{sig}^Q(\Omega) + N_{comb} \frac{(1 - QA_{comb})}{2} P_{comb}(\Omega) + \frac{N_{cf}}{2} P_{cf}(\Omega)]$$

$\Omega \rightarrow$ Phase space in terms of DP variables

Signal PDF

$$P_{sig}^Q(\Omega) = \frac{\epsilon^Q(\Omega)}{\Gamma} \frac{d\Gamma^Q}{d\Omega}$$

- $\frac{d\Gamma}{d\Omega}$ (differential decay density)
Helicity formalism to parametrise the decay dynamics and isobar formalism to coherently sum all intermediate states
- $\epsilon(\Omega)$: signal efficiency

Combinatorial Background PDF

- Modelled using neural networks
- Training in the right sideband region
- PDF is predicted by extrapolating the model at Ξ_b mass

Cross-Feed PDF

- $\Xi_b^- \rightarrow pK^- \pi^-$ simulated samples weighted according to a model consisting of Λ and N resonances

$\Xi_b^- \rightarrow pK^- K^-$: Outputs of the amplitude analysis

Decay density defined using helicity and isobar formalism:

$$\frac{d\Gamma^Q}{d\Omega} = \frac{1}{(8\pi m_{\Xi_b})^3} \sum_{M_{\Xi_b}, \lambda_p} \left| \sum_R A_{R, M_{\Xi_b}, \lambda_p}^Q \right|^2$$

- $A \rightarrow$ symmetrised decay amplitude

Dalitz Plot fit output:

Fit Fractions

$$\mathcal{F}_i = \frac{\int_{\Omega} (d\Gamma_i^+ / d\Omega + d\Gamma_i^- / d\Omega) d\Omega}{\int_{\Omega} (d\Gamma^+ / d\Omega + d\Gamma^- / d\Omega) d\Omega}$$

Interference Fit Fractions

$$\mathcal{I}_{ij} = \frac{\int_{\Omega} (d\Gamma_{i,j}^+ / d\Omega + d\Gamma_{i,j}^- / d\Omega) d\Omega}{\int_{\Omega} (d\Gamma^+ / d\Omega + d\Gamma^- / d\Omega) d\Omega}$$

CP violation

$$A_i^{CP} = \frac{\int_{\Omega} (d\Gamma_i^+ / d\Omega - d\Gamma_i^- / d\Omega) d\Omega}{\int_{\Omega} (d\Gamma^+ / d\Omega + d\Gamma^- / d\Omega) d\Omega}$$