Top quark contribution to two-loop helicity amplitudes for W/Z boson pair production in gluon fusion

Christian Brønnum-Hansen and Chen-Yu Wang

Karlsruhe Institute of Technology

We compute the top quark contribution to the two-loop amplitude for on-shell W/Z boson pair production in gluon fusion. Exact dependence on the top quark mass is retained. For each phase space point the integral reduction is performed numerically and the master integrals are evaluated using the auxiliary mass flow method, allowing fast computation of the amplitude with very high precision.

1 Motivation

Experiments at the Large Hadron Collider (LHC) continue to decrease statistical and systematic uncertainties and this calls for more accurate theoretical predictions. In the context of perturbative quantum chromodynamics this can be achieved by calculating processes to higher loop order. Furthermore, as the collision energy is increased, internal massive particles become increasingly important. On this poster **we present calculations at two-loop order for electroweak diboson production in gluon fusion**,

$$g(p_1)+g(p_2)\rightarrow V(p_3)+V(p_4) \tag{1}$$

where $V=W,Z$ and we retain **full dependence on the top quark mass**, m_t. All other quarks are considered massless. Despite being loop-induced, this production channel becomes **significant due to the high gluon flux and event selection**. These processes serve as **important backgrounds to Higgs production** and a precise prediction can be used to constrain the Higgs width as well as anomalous gauge couplings. At leading order inclusion of top mass effects increases the gluon fusion production channel by about 10 %, see figure 1 below. At high p_T it even becomes the dominant contribution, see figure 2.

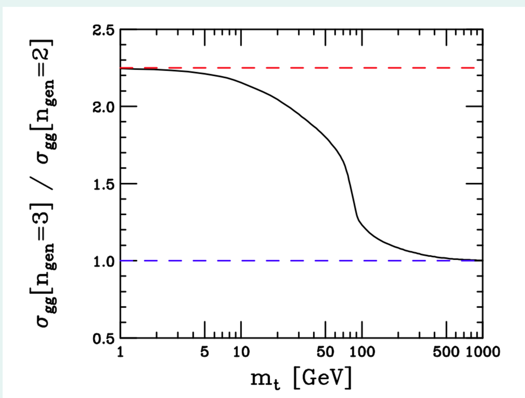

Figure 1. Effect on total cross section of third quark generation as function of m_t. Campbell et al. 2011.

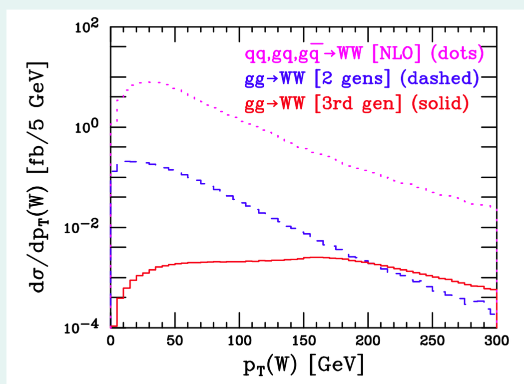

Figure 2. Distribution in transverse momentum, p_T, split in massless and massive quark generations. Campbell et al. 2011.

2 Amplitude calculation

There are 136 and 138 diagrams for $gg\rightarrow WW$ and $gg\rightarrow ZZ$ respectively. In both cases we ignore diagrams with an intermediate Higgs or Z boson and decompose the amplitude according to colour and Lorentz structure. In both cases there are 20 parity-even Lorentz tensor structures. For the final state ZZ there is no parity-odd contribution due to charge-parity conservation, while for WW this contribution can be projected onto another 18 tensor structures (Binoth et al. 2006).
For ZZ the dependence on γ_5 can for all but the factorisable two-loop diagrams be treated naively with anti-commutativity. For the factorisable diagrams and the WW process we employ the **Larin scheme** and replace

$$\gamma_5\gamma^\mu \rightarrow \frac{i}{3!}\epsilon^{\mu\nu\sigma\rho}\gamma_\nu\gamma_\sigma\gamma_\rho. \tag{2}$$

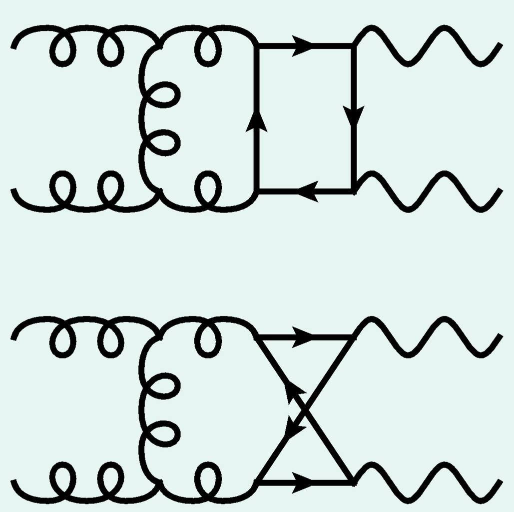

Figure 3. Representative diagrams for $gg\rightarrow VV$. The internal quark loop contains top quarks as well as bottom quarks for the WW final state.

We apply symmetries between the diagrams and perform integral reduction via integration-by-parts identities. The reduction can be performed numerically for each phase space point setting the masses to integer values close to their physical values

$$m_t = 173~\mathrm{GeV}, \qquad m_W = 80~\mathrm{GeV}, \qquad m_Z = 91~\mathrm{GeV}, \tag{3}$$

and using rational numbers for the Mandelstam variables s and t. A **numerical reduction can be completed in a few hours** with `KIRA` (Klappert et al. 2020) on a single core using reasonable memory resources.

3 Auxiliary mass flow

To evaluate the master integrals we employ the **auxiliary mass flow method** (Liu et al. 2018, 2021). The master integrals are defined by

$$I \propto \int \prod_{i=1}^{2} \mathrm{d}^d l_i \prod_{a=1}^{9} \frac{1}{[q_a^2-(m_a^2-i0^+)]^{\nu_a}}. \tag{4}$$

where $D_a = q_a - m_a$ and q_a are linear combinations of loop and external momenta. The only non-zero masses come from top quark propagators. We add an **imaginary part to the massive propagators**

$$m_t^2 \rightarrow m_t^2 - i\eta \tag{5}$$

and solve a system of ordinary differential equations at each phase space point

$$\partial_x \boldsymbol{I} = M\boldsymbol{I}, \qquad m_t^2 - i\eta = m_t^2(1+x). \tag{6}$$

The **boundary condition is taken at infinity** $x\rightarrow -i\infty$ and the physical mass is at $x=0$. Several regions contribute in the boundary limit. The procedure is

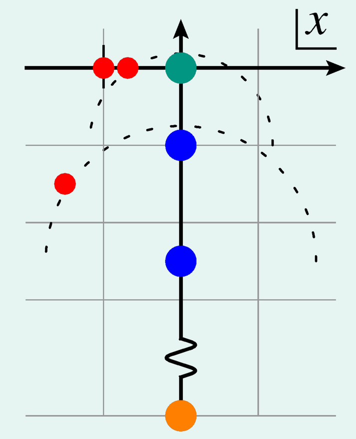

Figure 4. Transporting through complex mass space using a differential equation. Starting from the boundary at infinity, via regular points, to the physical mass. Step size is limited by singularities of the equation.

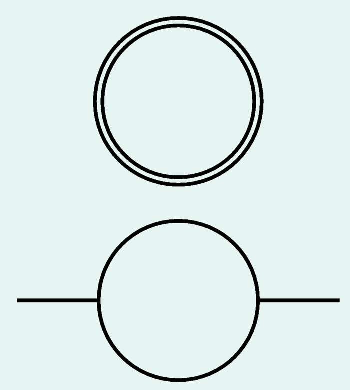

Figure 5. For $gg\rightarrow ZZ$ the only required integrals at the boundary are massive tadpole and massless bubbles.

- Expand $\boldsymbol{I}$ around the boundary in variable $y=x^{-1}=0$,

$$\boldsymbol{I}=\sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} y^k \ln^l y + \ldots \tag{7}$$

- Evaluate and expand around a regular point,

$$\boldsymbol{I}=\sum_j^M \epsilon^j \sum_{k=0}^N \boldsymbol{c}_{jk} x'^k + \ldots \tag{8}$$

- Repeat the previous step until reaching the physical point $x=0$.
- **Arbitrary precision** achieved with long expansions and/or short steps.

4 Results

We parametrise partonic phase space by relative velocity and scattering angle and plot the interference between the **two-loop finite remainder** and one-loop helicity amplitude

$$\frac{2\,\mathrm{Re}\left[(A^{(1)})^* F^{(2)}\right]}{|A^{(1)}|^2}. \tag{9}$$

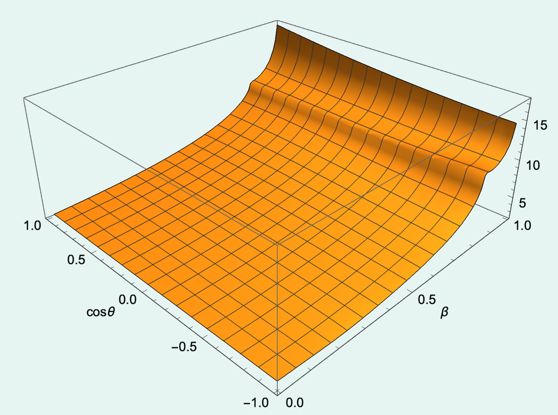

Figure 6. $gg\rightarrow ZZ$ for colour C_A and helicity $LLLL$.

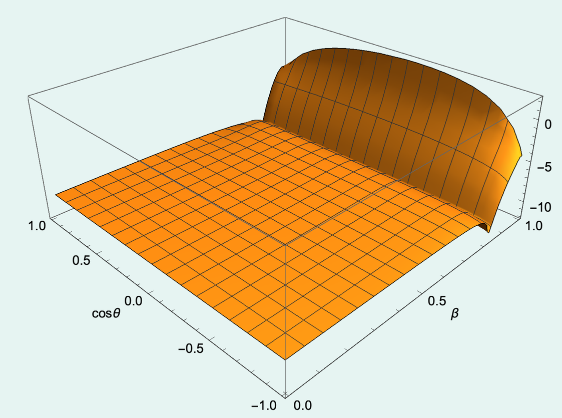

Figure 7. $gg\rightarrow ZZ$ for colour C_F and helicity $LLLR$.

The pole structure of the renormalised amplitudes has been checked against the universal infra-red pole structure (Catani 1998). In the bulk of phase space the amplitude is **accurate to more than 10 digits**. Only examples are shown here, the full results for $gg\rightarrow WW$ and $gg\rightarrow ZZ$ can be found in our publications (2009.03742 and 2101.12095 respectively).