

The Singly-Charged Scalar Singlet as the Origin of Neutrino Masses

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Singly-Charged Scalar Singlet

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} - h^* (D^\mu D_\mu + M_h) h - (y_h^{ij} L_i L_j h + \text{h.c.})$$

Lepton number: Continuous global $U(1)$ symmetry. $L_i \sim 1$, $\bar{e}_i \sim -1$, $h \sim -2$

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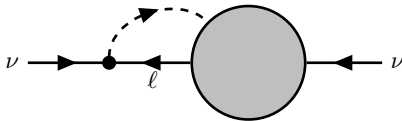
Antisymmetric coupling matrix:

$$y_h = \begin{pmatrix} 0 & y_h^{e\mu} & y_h^{e\tau} \\ -y_h^{e\mu} & 0 & y_h^{\mu\tau} \\ -y_h^{e\tau} & -y_h^{\mu\tau} & 0 \end{pmatrix}$$

→ Eigenvector $v_h = (y_h^{\mu\tau}, -y_h^{e\tau}, y_h^{e\mu})^T$ with eigenvalue zero, $y_h v_h = 0$.

Neutrino Mass Matrix: Linear Case

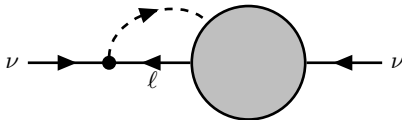
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Most general form of neutrino mass matrix: $M_\nu = U^* m_{\text{diag}} U^\dagger = X y_h - y_h X^T$

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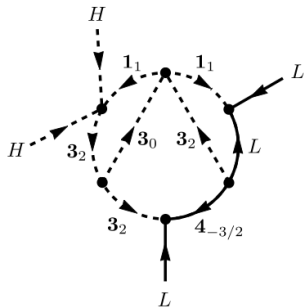
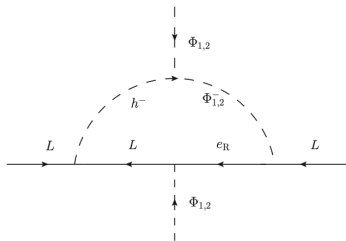


Most general form of neutrino mass matrix: $M_\nu = U^* m_{\text{diag}} U^\dagger = X y_h - y_h X^T$
→ All three neutrinos in general massive.

$$\Rightarrow \mathbf{v}_h^T \mathbf{U}^* m_{\text{diag}} \mathbf{U}^\dagger \mathbf{v}_h = 0$$

Neutrino Mass Matrix: Linear Case

Example: Effective dim-5 operator $\frac{c^{ij}}{\Lambda} h^* \bar{e}_i L_j H + \text{h.c.}$ which violates LN
 $\Rightarrow M_\nu \approx \frac{v^2}{(4\pi)^2 \Lambda} (c y_e y_h - y_h y_e c^T)$



Zee, Phys. Lett. B93 389 (1980)

Cheng, Li, Phys. Rev. D22 2860 (1980)

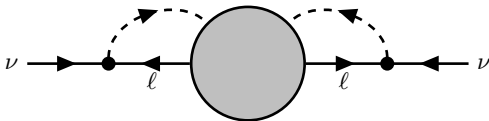
Wolfenstein, Nucl. Phys. B175 93 (1980)

HG, Ohlsson, Riad, Wiren, JHEP 04, 130 (2017)

Cepedello, Hirsch, Helo, JHEP 07, 079 (2017)

Neutrino Mass Matrix: Quadratic Case

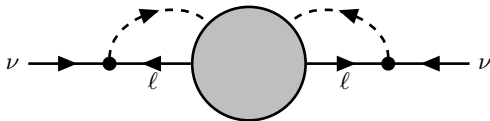
Eigenvector $v_h = (y_h^{\mu\tau}, -y_h^{e\tau}, y_h^{e\mu})^T$ with eigenvalue zero, $y_h v_h = 0$.



Most general form of neutrino mass matrix: $M_\nu = U^* m_{\text{diag}} U^\dagger = y_h S y_h$

Neutrino Mass Matrix: Quadratic Case

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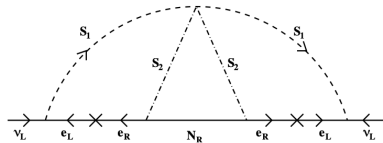
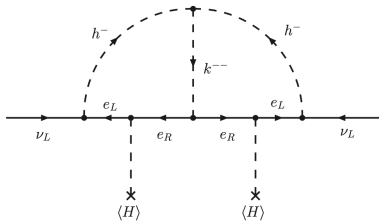


Most general form of neutrino mass matrix: $M_\nu = U^* m_{\text{diag}} U^\dagger = y_h S y_h$
→ One neutrino remains massless.

$$\Rightarrow m_{\text{diag}} \mathbf{U}^\dagger \mathbf{v}_h = \mathbf{0}$$

Neutrino Mass Matrix: Quadratic Case

Example: Effective dim-5 operator $\frac{d^{ij}}{\Lambda} (h^*)^2 \bar{e}_i \bar{e}_j + \text{h.c.}$ which violates LN
 $\Rightarrow M_\nu \approx \frac{v^2}{(4\pi)^4 \Lambda} y_h y_e d y_e y_h$



Zee, Phys. Lett. B161 141 (1985)

Zee, Nucl. Phys. B264 99 (1986)

Babu, Phys. Lett. B203 132 (1988)

Nebot, Oliver, Palao, Santamaria, Phys. Rev. D77,
093013 (2008)

Krauss, Nasri, Trodden, Phys. Rev. D67, 085002
(2003)

Neutrino-Mass Constraints

Eigenvector $v_h = (y_h^{\mu\tau}, -y_h^{e\tau}, y_h^{e\mu})^T$ with eigenvalue zero, $y_h v_h = 0$.

Derived constraints:

$$\mathbf{v}_h^T \mathbf{U}^* \mathbf{m}_{\text{diag}} \mathbf{U}^\dagger \mathbf{v}_h = 0, \quad \text{Linear case.}$$

$$\mathbf{m}_{\text{diag}} \mathbf{U}^\dagger \mathbf{v}_h = 0, \quad \text{Quadratic case.}$$

They ...

- ... directly relate the couplings y_h^{ij} to measured neutrino data.
- ... provide a *necessary condition* for the correct description of neutrino masses in the SM extended by h .
- ... are independent of the mechanism of lepton-number breaking.

How to solve the constraint? Example: Quadratic case

Constraints yield four real conditions for both neutrino-mass orderings:

Normal Ordering

$$\frac{y_h^{e\tau}}{y_h^{\mu\tau}} = \tan(\theta_{12}) \frac{\cos(\theta_{23})}{\cos(\theta_{13})} + \tan(\theta_{13}) \sin(\theta_{23}) e^{i\delta}$$
$$\frac{y_h^{e\mu}}{y_h^{\mu\tau}} = \tan(\theta_{12}) \frac{\sin(\theta_{23})}{\cos(\theta_{13})} - \tan(\theta_{13}) \cos(\theta_{23}) e^{i\delta}$$

Inverted Ordering

$$\frac{y_h^{e\tau}}{y_h^{\mu\tau}} = -\frac{\sin(\theta_{23})}{\tan(\theta_{13})} e^{i\delta}$$
$$\frac{y_h^{e\mu}}{y_h^{\mu\tau}} = \frac{\cos(\theta_{23})}{\tan(\theta_{13})} e^{i\delta}$$

- RHS: Assign pseudo-random variates from normal distributions. (NuFIT)
Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP 09, 178 (2020)
- LHS: Assign random values to one magnitude and one phase
→ Rest fixed by constraint.
- Mass $M_h \in [350 \text{ GeV}, 100 \text{ TeV}]$

Linear case: One very lengthy complex expression, see next slide ...

Constraint in the Linear Case

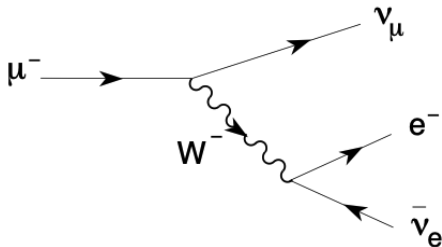
$$\begin{aligned}
 & \left(\left(c_{13}^2 m_3 c_{23}^2 + e^{-2i\eta_2} m_2 \left(e^{-i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right)^2 + e^{-2i\eta_1} m_1 \left(e^{-i\delta} c_{12} c_{23} s_{13} - s_{12} s_{23} \right)^2 \right) y_h^{e\mu} \right. \\
 & - \left(c_{23} m_3 s_{23} c_{13}^2 + e^{-2i\eta_1} m_1 \left(e^{-i\delta} c_{12} c_{23} s_{13} - s_{12} s_{23} \right) \left(c_{23} s_{12} + e^{-i\delta} c_{12} s_{13} s_{23} \right) \right. \\
 & - e^{-2i\eta_2} m_2 \left(e^{-i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right) \left(c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23} \right) \left. \right) y_h^{e\tau} \\
 & + c_{13} \left(e^{i\delta} c_{23} m_3 s_{13} - e^{-2i\eta_2} m_2 s_{12} \left(e^{-i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right) + e^{-2i\eta_1} c_{12} m_1 \left(s_{12} s_{23} - e^{-i\delta} c_{12} c_{23} s_{13} \right) \right) y_h^{\mu\tau} \left. \right) y_h^{e\mu} \\
 & - \left(\left(c_{23} m_3 s_{23} c_{13}^2 + e^{-2i\eta_1} m_1 \left(e^{-i\delta} c_{12} c_{23} s_{13} - s_{12} s_{23} \right) \left(c_{23} s_{12} + e^{-i\delta} c_{12} s_{13} s_{23} \right) \right. \right. \\
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 & - \left(c_{13}^2 m_3 s_{23}^2 + e^{-2i\eta_1} m_1 \left(c_{23} s_{12} + e^{-i\delta} c_{12} s_{13} s_{23} \right)^2 + e^{-2i\eta_2} m_2 \left(c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23} \right)^2 \right) y_h^{e\tau} \\
 & - e^{i\delta} c_{13} \left(e^{-2i(\delta+\eta_1)} m_1 s_{13} s_{23} c_{12}^2 + e^{-i\delta} c_{23} \left(e^{-2i\eta_1} m_1 - e^{-2i\eta_2} m_2 \right) s_{12} c_{12} \right. \\
 & - \left. \left(m_3 - e^{-2i(\delta+\eta_2)} m_2 s_{12}^2 \right) s_{13} s_{23} \right) y_h^{\mu\tau} \left. \right) y_h^{e\tau} \\
 & + \left(c_{13} \left(e^{i\delta} c_{23} m_3 s_{13} - e^{-2i\eta_2} m_2 s_{12} \left(e^{-i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right) + e^{-2i\eta_1} c_{12} m_1 \left(s_{12} s_{23} - e^{-i\delta} c_{12} c_{23} s_{13} \right) \right) \right) y_h^{e\mu} \\
 & + e^{i\delta} c_{13} \left(e^{-2i(\delta+\eta_1)} m_1 s_{13} s_{23} c_{12}^2 + e^{-i\delta} c_{23} \left(e^{-2i\eta_1} m_1 - e^{-2i\eta_2} m_2 \right) s_{12} c_{12} \right. \\
 & - \left. \left(m_3 - e^{-2i(\delta+\eta_2)} m_2 s_{12}^2 \right) s_{13} s_{23} \right) y_h^{e\tau} + \left(e^{-2i\eta_2} m_2 s_{12}^2 c_{13}^2 + e^{-2i\eta_1} c_{12}^2 m_1 c_{13}^2 + e^{2i\delta} m_3 s_{13}^2 \right) y_h^{\mu\tau} \left. \right) y_h^{\mu\tau} = 0
 \end{aligned}$$

Lepton-Flavour Non-Universality

Define the ratios of effective leptonic gauge couplings inferred from $l_i \rightarrow l_j \bar{\nu} \nu$:

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\tau \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\tau e}} = \frac{g_\mu}{g_e}, \quad \sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\mu e}} = \frac{g_\tau}{g_e}, \quad \sqrt[4]{\frac{\Gamma_{\tau \rightarrow e}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau e}}{G_{\mu e}} = \frac{g_\tau}{g_\mu}$$

They are all equal to one in the SM (LFU).



Pich, Prog.Part.Nucl.Phys. 75 (2014) 41-85

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They are all equal to one in the SM (LFU). Current best-fit values:

$$\left| \frac{g_\mu}{g_e} \right| = 1.0018 \pm 0.0032, \quad \left| \frac{g_\tau}{g_e} \right| = 1.0030 \pm 0.0030, \quad \left| \frac{g_\tau}{g_\mu} \right| = 1.0011 \pm 0.0030$$

Gersabeck, Pich, Comptes Rendus Physique 21 (2020) 75

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Gersabeck, Pich, Comptes Rendus Physique 21 (2020) 75

- h contributes to g_i/g_j at tree level
- 1σ ranges of $|y_h^{e\mu}|/M_h$, $|y_h^{\mu\tau}|/M_h$ determined in 2012.09845
- Effect in $\mu \rightarrow e$ explains CAA (indirectly: $G_F \rightarrow |V_{us}^{CKM}|$)

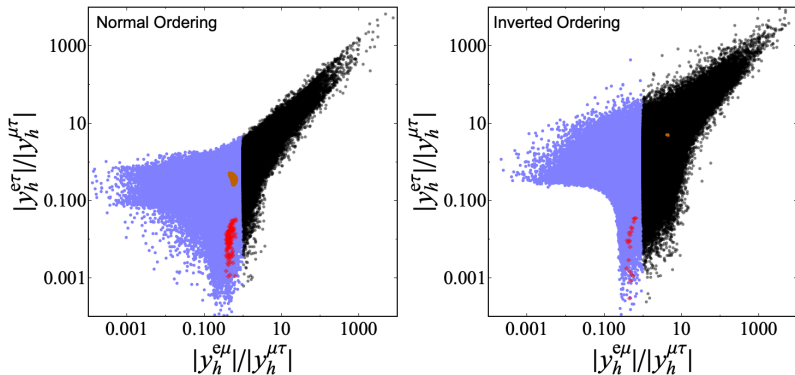
Crivellin, Kirk, Manzari, Panizzi,
Phys. Rev. D. 103, 073002 (2021)

$$\frac{g_\mu}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\tau}|^2}{M_h^2}$$

$$\frac{g_\tau}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2}$$

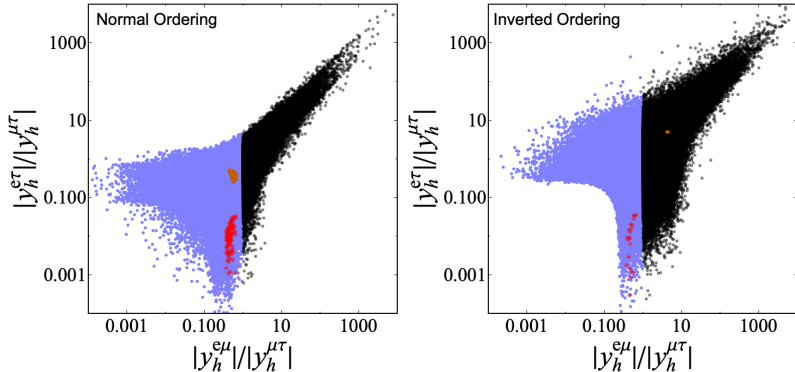
$$\frac{g_\tau}{g_\mu} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{e\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2}$$

Solution to the Neutrino-Mass Constraints



Neutrino-mass constraints shape available parameter space for y_h^{ij} non-trivially.

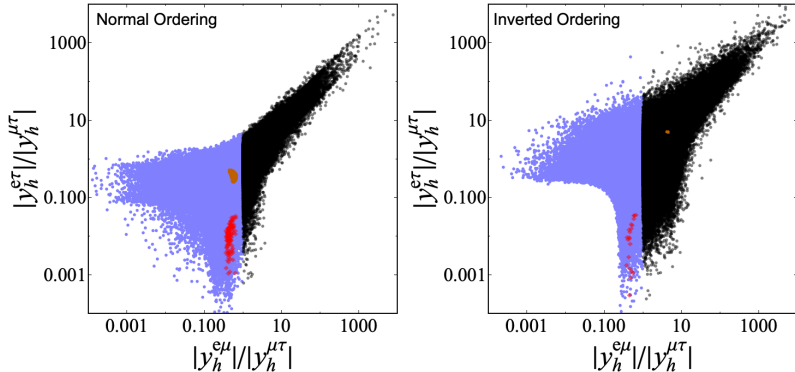
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- Quadratic case (brown): **Very predictive**, stringently constrained.

Solution to the Neutrino-Mass Constraints

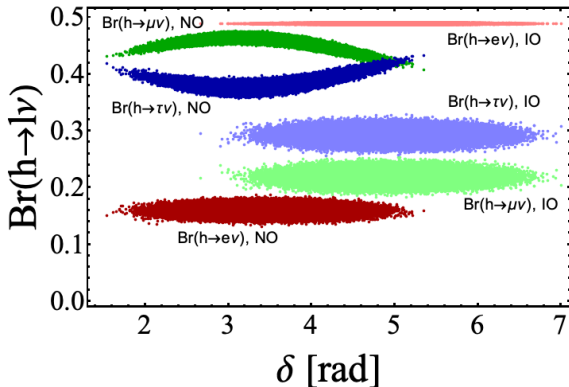


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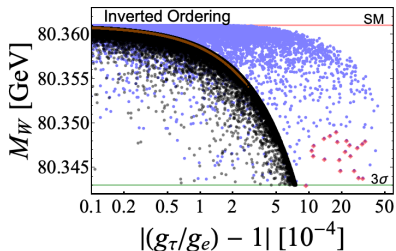
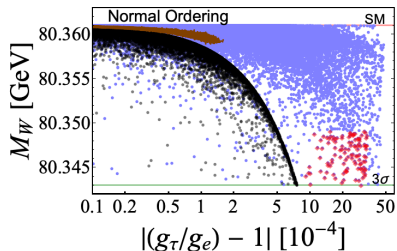
- Quadratic case (brown): **Very predictive**, stringently constrained.
- Linear case (blue, black): **Less predictive**, but **simultaneous explanation of LFU anomalies** (V_{us}^{CKM} , $l_i \rightarrow l_j \bar{\nu} \nu$; in red) at 1σ possible.

Quadratic Case: Branching Ratios

$$\Gamma(h \rightarrow \ell_a \nu_b) = \Gamma(h \rightarrow \ell_b \nu_a) = \frac{|y_h^{ab}|^2}{4\pi} M_h$$
$$\text{Br}(h \rightarrow \ell_a \nu) = \frac{\sum_{b \neq a} |y_h^{ab}|^2}{2(|y_h^{e\mu}|^2 + |y_h^{e\tau}|^2 + |y_h^{\mu\tau}|^2)}$$



$$\delta M_W^2 = -\frac{M_W^2}{\sqrt{2}G_F} \left| 1 - \frac{M_W M_Z}{2M_W^2 - M_Z^2} \right| \frac{|y_h^{e\mu}|^2}{M_h^2}$$

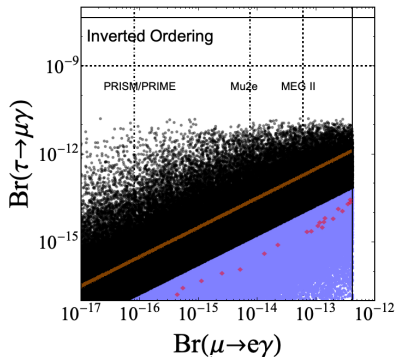
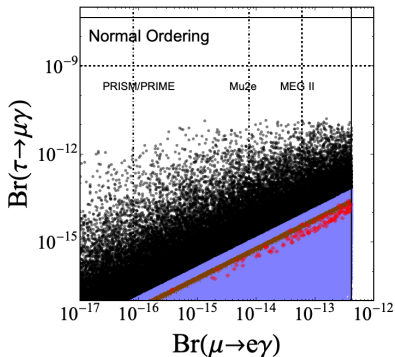


Radiative Charged-Lepton Decays

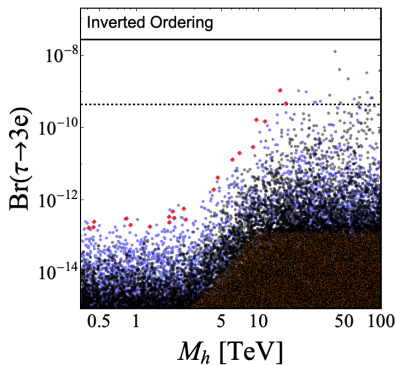
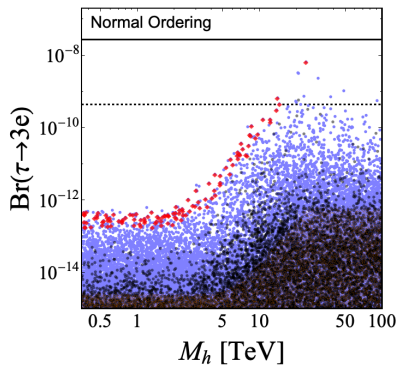
$$\text{Br}(\mu \rightarrow e\gamma) = \text{Br}(\mu \rightarrow e\nu\bar{\nu}) \frac{\alpha_{\text{EM}}}{48\pi G_F^2} \frac{|y_h^{e\tau} y_h^{\mu\tau}|^2}{M_h^4},$$

$$\text{Br}(\tau \rightarrow e\gamma) = \text{Br}(\tau \rightarrow e\nu\bar{\nu}) \frac{\alpha_{\text{EM}}}{48\pi G_F^2} \frac{|y_h^{e\mu} y_h^{\mu\tau}|^2}{M_h^4},$$

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Tri-Lepton Decays of Charged Leptons



Assumption: Neutrino masses generated via a singly-charged scalar singlet.
→ **Model-independent constraints** for couplings y_h^{ij} in terms of neutrino data.

Discussion of *two distinct structures* of the neutrino mass matrix:

- *Linear case*: **Zee Model** and variants, ...
- *Quadratic case*: **Zee-Babu Model**, **Krauss-Nasri-Trodden Model** and their variants, ...

Felkl, T., Herrero-García, J. & Schmidt, M. A.

The singly-charged scalar singlet as the origin of neutrino masses.

J. High Energ. Phys. 2021, 122 (2021). arxiv: e-Print 2102.09898

Thank you for watching the talk!

Back-Up

Conventions for the Neutrino Sector

Neutrino mass eigenstates ν_i and flavour eigenstates related ν_α via

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i. \quad (1)$$

PMNS matrix:

$$U = P U_{23} U_{13} U_{12} U_{\text{Maj}} \quad (2)$$

with

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \quad U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}, \quad (3)$$

$$U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

$U_{\text{Maj}} \equiv \text{diag}(e^{i\eta_1}, e^{i\eta_2}, 1)$ and $P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$.

Squared-mass differences $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$:

$$m_1 = m_0, \quad m_2 = \sqrt{\Delta m_{21}^2 + m_0^2}, \quad m_3 = \sqrt{\Delta m_{31}^2 + m_0^2} \quad (5)$$

in the case of Normal Ordering (NO) $m_1 < m_2 \ll m_3$, and

$$m_1 = \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2 + m_0^2}, \quad m_2 = \sqrt{|\Delta m_{32}^2| + m_0^2}, \quad m_3 = m_0 \quad (6)$$

in the case of Inverted Ordering (IO) $m_3 \ll m_1 < m_2$.

Input values

m_e [keV]	m_μ [MeV]	m_τ [GeV]	$G_F [\frac{1}{\text{GeV}^2}]$	α_{EM}^{-1}	M_Z [GeV]	
510.9989	105.6584	1.777	1.16638×10^{-5}	137.035999	91.1535	
	$\Delta m_{3l}^2 [10^{-3} \text{eV}^2]$	$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	δ [rad]			
NO	2.517 ± 0.026	7.42 ± 0.20	3.44 ± 0.42			
IO	-2.498 ± 0.028	7.42 ± 0.20	4.92 ± 0.45			
	$\sin^2(\theta_{12})$	$\sin^2(\theta_{13})$	$\sin^2(\theta_{23})$			
NO	0.304 ± 0.012	0.02219 ± 0.00062	0.573 ± 0.016			
IO	0.304 ± 0.012	0.02238 ± 0.00062	0.575 ± 0.016			
	$ y_h^{ij} $	$\arg(y_h^{ei})$	$\arg(y_h^{\mu\tau})$	m_0 [meV]	$\eta_{1,2}$ [rad]	M_h [GeV]
Prior	Log-Flat	Flat	Fixed	Log-Flat	Flat	Log-Flat
Range	$[10^{-4}, 2\pi]$	$[0, 2\pi]$	0	$[10^{-4}, 30]$ (NO) $[10^{-4}, 15]$ (IO)	$[0, \pi]$	$[350, 10^5]$

$\Delta m_{31}^2 > 0$ for NO, and $\Delta m_{32}^2 < 0$ for IO.

Zyla et al. (PDG), PTEP 2020, 083C01 (2020)
Freitas, 2020

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, 2020

Observable	Experiment		Scan: Max.	
	Current Bound	Future Sensitivity	NO	IO
$\text{Br}(\mu \rightarrow e\gamma)$	4.2×10^{-13} (90% CL)	6×10^{-14}	4.2×10^{-13}	4.2×10^{-13}
$\text{Br}(\tau \rightarrow e\gamma)$	3.3×10^{-8} (90% CL)	3×10^{-9}	6.4×10^{-11}	4.9×10^{-11}
$\text{Br}(\tau \rightarrow \mu\gamma)$	4.4×10^{-8} (90% CL)	10^{-9}	1.6×10^{-11}	1.6×10^{-11}
$\text{Br}(\mu \rightarrow 3e)$	10^{-12} (90% CL)	10^{-16}	10^{-12}	10^{-12}
$\text{Br}(\tau \rightarrow 3e)$	2.7×10^{-8} (90% CL)	4.3×10^{-10}	6.6×10^{-9}	1.3×10^{-8}
$\text{Br}(\tau \rightarrow 3\mu)$	2.1×10^{-8} (90% CL)	3.3×10^{-10}	3.0×10^{-9}	1.2×10^{-8}
$ g_{\mu}/g_e $	[0.9986, 1.0050] (2σ)		1.0050	1.0047
$ g_{\tau}/g_{\mu} $	[0.9981, 1.0041] (2σ)		1.0009	1.0014
$ g_{\tau}/g_e $	[1.0000, 1.0060] (2σ)		1.0048	1.0043
	[0.9985, 1.0075] (3σ)			
$ \delta M_W [\text{GeV}]$	0.018 (3σ)		0.018	0.018

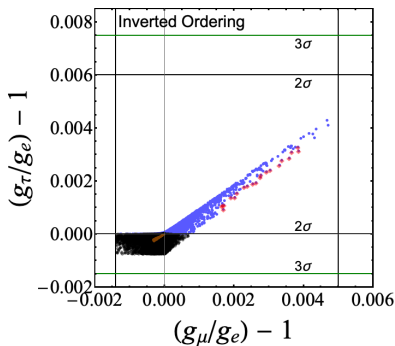
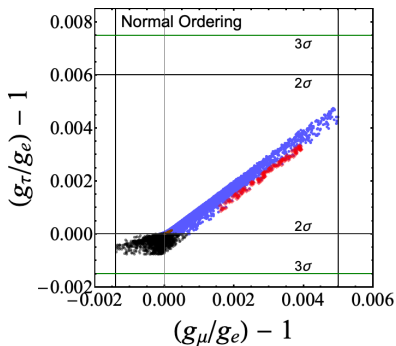
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Universality of Leptonic Gauge Couplings

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\tau \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\tau e}} = \frac{g_\mu}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\tau}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\tau\tau} - \epsilon_{\mu\mu}^{\tau\tau}, \quad (7)$$

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\mu e}} = \frac{g_\tau}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\mu\mu} - \epsilon_{\mu\mu}^{\tau\tau}, \quad (8)$$

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow e}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau e}}{G_{\mu e}} = \frac{g_\tau}{g_\mu} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{e\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\mu\mu} - \epsilon_{ee}^{\tau\tau} \quad (9)$$

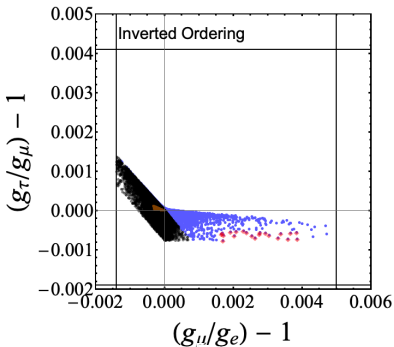
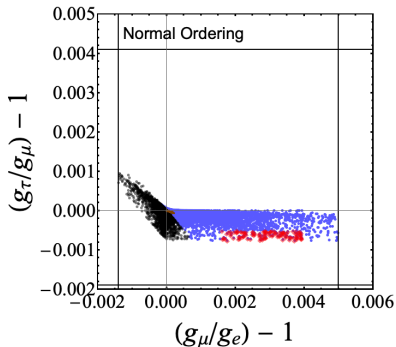


Universality of Leptonic Gauge Couplings

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\tau \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\tau e}} = \frac{g_{\mu}}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\tau}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\tau\tau} - \epsilon_{\mu\mu}^{\tau\tau}, \quad (10)$$

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow \mu}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau\mu}}{G_{\mu e}} = \frac{g_{\tau}}{g_e} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{\mu\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\mu\mu} - \epsilon_{\mu\mu}^{\tau\tau}, \quad (11)$$

$$\sqrt[4]{\frac{\Gamma_{\tau \rightarrow e}}{\Gamma_{\mu \rightarrow e}}} \propto \frac{G_{\tau e}}{G_{\mu e}} = \frac{g_{\tau}}{g_{\mu}} \approx 1 + \frac{1}{\sqrt{2}G_F} \frac{|y_h^{e\tau}|^2 - |y_h^{e\mu}|^2}{M_h^2} \equiv 1 + \epsilon_{ee}^{\mu\mu} - \epsilon_{ee}^{\tau\tau} \quad (12)$$



Cabibbo Angle Anomaly

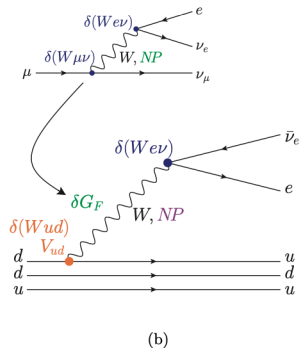
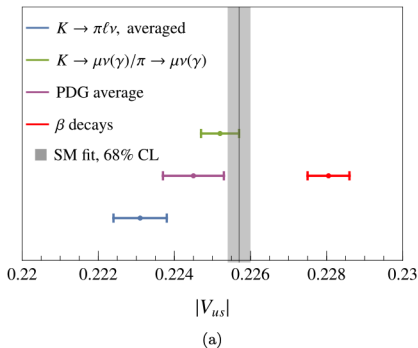
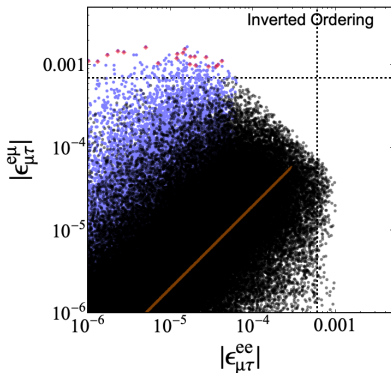
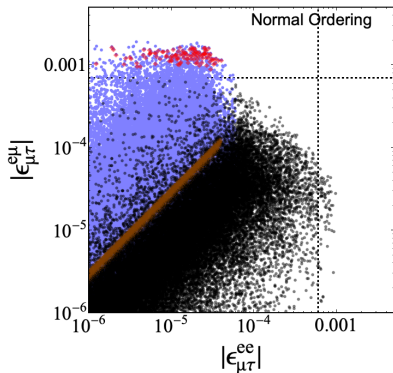


Figure 1: (a) Determinations of $|V_{us}|$ from different sources.^{7,8,6} $|V_{us}| = 0.22805(64) \equiv |V_{us}^\beta|$, which was determined, using CKM unitarity from $|V_{ud}| = 0.97365(15)$,⁹ is shown in red. (b) Possible NP interpretations leading to effects in β -decays (and $\mu \rightarrow e \bar{\nu} \nu$). Note that the contribution of $\delta(W_{e\nu})$ to $\mu \rightarrow e \bar{\nu} \nu$ cancels against the direct contribution of $\delta(W_{e\nu})$ to the β -decay. $\delta(W_{\mu\nu})$, however, leads to an effect in β -decays.

Leptonic Non-Standard Interactions

$$\epsilon_{\tau e}^{e\mu} \equiv \frac{(y_h^{e\tau})^* y_h^{e\mu}}{\sqrt{2} G_F M_h^2} = -(\epsilon_{\mu\tau}^{ee})^*, \quad \epsilon_{\mu\tau}^{e\mu} \equiv -\frac{(y_h^{e\mu})^* y_h^{\mu\tau}}{\sqrt{2} G_F M_h^2} \quad (13)$$

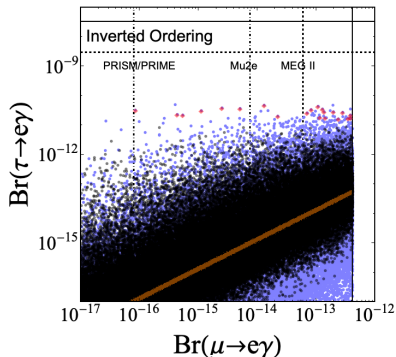
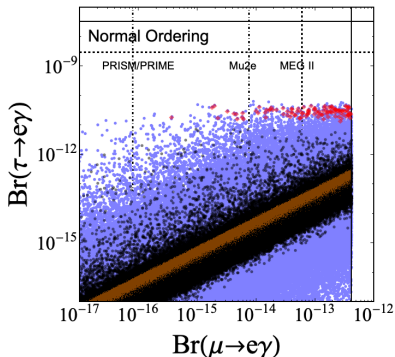


Radiative Charged-Lepton Decays

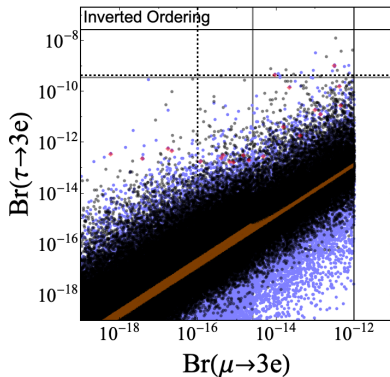
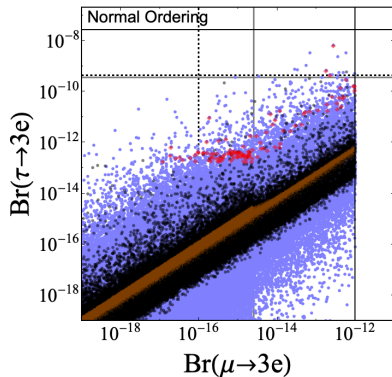
$$\text{Br}(\mu \rightarrow e\gamma) = \text{Br}(\mu \rightarrow e\nu\bar{\nu}) \frac{\alpha_{\text{EM}}}{48\pi G_F^2} \frac{|y_h^{e\tau} y_h^{\mu\tau}|^2}{M_h^4}, \quad (14)$$

$$\text{Br}(\tau \rightarrow e\gamma) = \text{Br}(\tau \rightarrow e\nu\bar{\nu}) \frac{\alpha_{\text{EM}}}{48\pi G_F^2} \frac{|y_h^{e\mu} y_h^{\mu\tau}|^2}{M_h^4}, \quad (15)$$

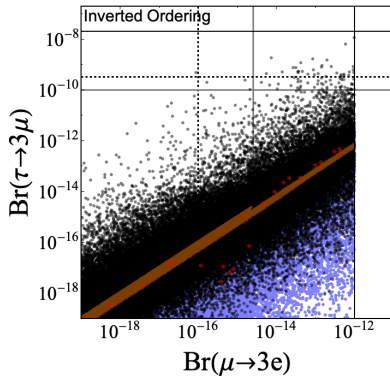
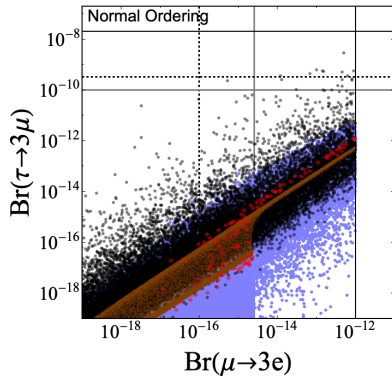
$$\text{Br}(\tau \rightarrow \mu\gamma) = \text{Br}(\tau \rightarrow \mu\nu\bar{\nu}) \frac{\alpha_{\text{EM}}}{48\pi G_F^2} \frac{|y_h^{e\mu} y_h^{e\tau}|^2}{M_h^4} \quad (16)$$



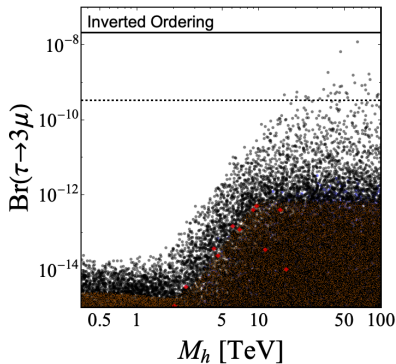
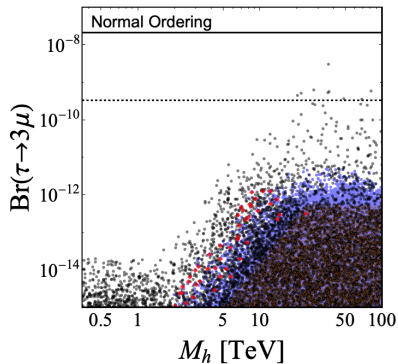
Tri-Lepton Decays of Charged Leptons



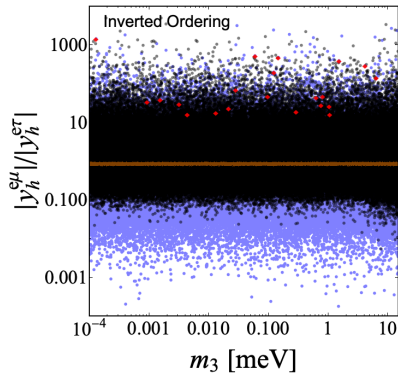
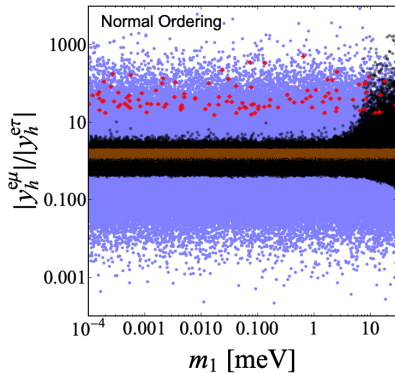
Tri-Lepton Decays of Charged Leptons



Tri-Lepton Decays of Charged Leptons



Coupling Magnitudes



Coupling Magnitudes

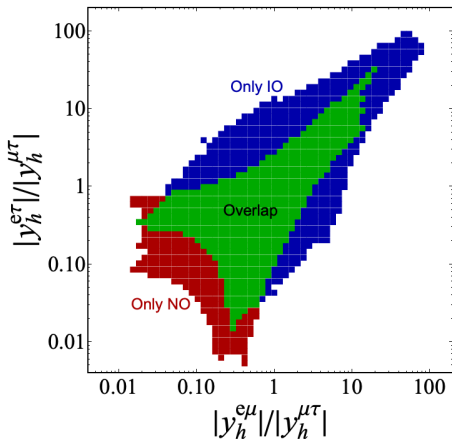


Figure: Plot of the coupling ratios $|y_h^{e\mu}|/|y_h^{\mu\tau}|$ and $|y_h^{e\tau}|/|y_h^{\mu\tau}|$ as obtained in the numerical scan if approximately 95.45 % of the overall number of 547 991 (542 287) sample points generated for NO (IO) are taken into account. Each square shown to be compatible with NO (IO) contains at least 97 (74) sample points.

Branching Ratios

$$\Gamma(h \rightarrow \ell_a \nu_b) = \Gamma(h \rightarrow \ell_b \nu_a) = \frac{|y_h^{ab}|^2}{4\pi} M_h \quad (17)$$

$$\text{Br}(h \rightarrow \ell_a \nu) = \frac{\sum_{b \neq a} |y_h^{ab}|^2}{2(|y_h^{e\mu}|^2 + |y_h^{e\tau}|^2 + |y_h^{\mu\tau}|^2)} \quad (18)$$

