#### GeoSMEFT and an application

Tyler Corbett Niels Bohr Institute

Based on: TC, A. Helset, A. Martin, M. Trott, arXiv:2102.02819



## SMEFT

In studying NP at  $\Lambda_{\rm NP} \gg v,$  we employ the Standard Model EFT





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## The SMEFT at dimension-six

#### D6 operators from SM field content $\Rightarrow$ SMEFT @ D6

Type I: X <sup>3</sup>		Type II, III: $H^6$ , $H^4D^2$		Type V: $\Psi^2 H^3 + h.c.$	
Q <sub>G</sub>	$f^{ABC} G^{A u}_{\mu} G^{B ho}_{ u} G^{C\mu}_{ ho}$	Q <sub>H</sub>	$(H^{\dagger}H)^3$	Q <sub>eH</sub>	$(H^{\dagger}H)(\overline{L}eH)$
Q <sub>Ĝ</sub>	$f^{ABC} \tilde{G}^{A u}_{\mu} G^{B ho}_{ u} G^{C\mu}_{ ho}$	$Q_{H\square}$	$(H^{\dagger}H)\Box(H^{\dagger}H)$	$Q_{uH}$	$(H^{\dagger}H)(\bar{Q}u\tilde{H})$
$Q_W$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$	Q <sub>HD</sub>	$(H^{\dagger}D^{\mu}H)^{*}(H^{\dagger}D^{\mu}H)$	$Q_{dH}$ $(H^{\dagger}H)(\bar{Q}dH)$	
$Q_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$				
Type IV: $X^2 \Phi^2$		Type VI: $\Psi^2 H^3$		Type VII: $\Psi^2 H^2 D$	
Q <sub>HG</sub>	$(H^{\dagger}H)G^{A}_{\mu\nu}G^{A\mu\nu}$	Q <sub>eW</sub>	$(\bar{L}\sigma^{\mu\nu}e)\tau^{I}HW^{I}_{\mu\nu}$	$Q_{HL}^{(1)}$	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{L}\gamma^{\mu}L)$
Q <sub>HĜ</sub>	$(H^{\dagger}H)\tilde{G}^{A}_{\mu u}G^{A\mu u}$	Q <sub>eW</sub>	$(\bar{L}\sigma^{\mu\nu}e)\tau^{I}HB_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^{\dagger}i\vec{D}^{I}_{\mu}H)(\bar{L}\tau^{I}\gamma^{\mu}L)$
Q <sub>HW</sub>	$(H^{\dagger}H)W^{I}_{\mu\nu}W^{I\mu\nu}$	Q <sub>uG</sub>	$(\bar{Q}\sigma^{\mu\nu}T^{A}u)\tilde{H}G^{A}_{\mu\nu}$	Q <sub>He</sub>	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{e}\gamma^{\mu}e)$
Q <sub>HŴ</sub>	$(H^{\dagger}H)\tilde{W}^{I}_{\mu\nu}W^{I\mu\nu}$	Q <sub>uW</sub>	$(\bar{Q}\sigma^{\mu\nu}u)\tau^{I}\tilde{H}W^{I}_{\mu\nu}$	$Q_{HQ}^{(1)}$	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{q}\gamma^{\mu}q)$
Q <sub>HB</sub>	$(H^{\dagger}H)B_{\mu\nu}B^{\mu\nu}$	Q <sub>uB</sub>	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{H}B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^{\dagger}i\vec{D}^{I}_{\mu}H)(\bar{q}\tau^{I}\gamma^{\mu}q)$
Q <sub>HĨ</sub>	$(H^{\dagger}H)\tilde{B}_{\mu u}B^{\mu u}$	Q <sub>dG</sub>	$(\bar{Q}\sigma^{\mu\nu}T^{A}d)HG^{A}_{\mu\nu}$	Q <sub>Hu</sub>	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{u}\gamma^{\mu}u)$
Q <sub>HWB</sub>	$(H^{\dagger}\tau^{I}H)W^{I}_{\mu\nu}B^{\mu\nu}$	Q <sub>dW</sub>	$(\bar{Q}\sigma^{\mu\nu}d)\tau^{I}HW^{I}_{\mu\nu}$	Q <sub>Hd</sub>	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{d}\gamma^{\mu}d)$
Q <sub>HŴB</sub>	$(H^{\dagger}\tau^{I}H)\tilde{W}^{I}_{\mu\nu}B^{\mu\nu}$	Q <sub>dB</sub>	$(\bar{Q}\sigma^{\mu u}d)\tilde{H}B_{\mu u}$	Q <sub>Hud</sub>	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{u}\gamma^{\mu}d)$

Type VIII:  $5 \times (\bar{L}L)(\bar{L}L) + 7 \times (\bar{R}R)(\bar{R}R) + 8 \times (\bar{L}L)(\bar{R}R) + (\bar{L}R)(\bar{R}L) + 4[(\bar{L}R)(\bar{L}R) + h.c.] = 25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$ 

## The SMEFT at dimension-six

#### D6 operators from SM field content $\Rightarrow$ SMEFT @ D6

Type I: X <sup>3</sup>		Type II, III: <i>H</i> <sup>6</sup> , <i>H</i> <sup>4</sup> <i>D</i> <sup>2</sup>		Тур	be V: $\Psi^2 H^3 + h.c.$
QG	$f^{ABC} G^{A\nu}_{,\nu} G^{B\rho}_{,\nu} G^{C\mu}_{,\nu}$	$^{ABC}G^{A\nu}_{,,\nu}G^{B\rho}_{,,\nu}G^{C\mu}_{,\nu} \qquad Q_H$		Q <sub>eH</sub>	$(H^{\dagger}H)(\overline{L}eH)$
Q <sub>Ĝ</sub>					$(H^{\dagger}H)(\bar{Q}u\tilde{H})$
$Q_W$	<ol> <li>Take Field Cor</li> </ol>	$(H^{\dagger}H)(\bar{Q}dH)$			
$Q_{\tilde{W}}$	X – Field Stre				
	$\Psi$ – Fermions	VII: $\Psi^2 H^2 D$			
QHG	H – Higgs dou	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{L}\gamma^{\mu}L)$			
Quã	D – Covariant	$I^{\dagger} i \vec{D}_{\mu}^{I} H) (\bar{L} \tau^{I} \gamma^{\mu} L)$			
Q <sub>HW</sub>	2 Form D6 operation	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{e}\gamma^{\mu}e)$			
Q <sub>HŴ</sub>	$X^3, H^6, H^4 D^2$	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{q}\gamma^{\mu}q)$			
Q <sub>HB</sub>	Remove, e.g.,	$M = I^{\dagger} i \vec{D}^{I}_{\mu} H (\bar{q} \tau^{I} \gamma^{\mu} q)$			
Q <sub>HB</sub>				, _0	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{u}\gamma^{\mu}u)$
Q <sub>HWE</sub>	β (Π'Τ'Π)W <sup>+</sup> <sub>μν</sub> Β <sup></sup>	<i>\</i> <i>\dW</i>	$(\varphi \sigma r^{-} a) \tau m v \mu \nu$	\	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{d}\gamma^{\mu}d)$
Q <sub>HŴE</sub>	$(H^{\dagger}\tau^{I}H)\tilde{W}^{I}_{\mu\nu}B^{\mu\nu}$	Q <sub>dB</sub>	$(\bar{Q}\sigma^{\mu u}d)\tilde{H}B_{\mu u}$	Q <sub>Hud</sub>	$(H^{\dagger}i\vec{D}_{\mu}H)(\bar{u}\gamma^{\mu}d)$

 $\begin{array}{l} \text{Type VIII: } 5 \times (\bar{L}L)(\bar{L}L) + 7 \times (\bar{R}R)(\bar{R}R) + 8 \times (\bar{L}L)(\bar{R}R) \\ + (\bar{L}R)(\bar{R}L) + 4[(\bar{L}R)(\bar{L}R) + \text{h.c.}] = 25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) \end{array}$ 

# The geoSMEFT, or SMEFT to all orders $\frac{v}{\Lambda}$

(the very simplified version)

- **1** Take SM field content, X,  $\Psi$ , H, D
- 2 Form Lorentz invariant, but gauge variant combos of  $\geq$  3 fields
- Stick towers of H<sup>†</sup>H and/or H<sup>†</sup>τ<sup>A</sup>H → gauge invariant use Hilbert Series to confirm all operators included/no redundancy (see e.g. Lehman & Martin 2015, Henning et al. 2015)

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For example:

$$\begin{aligned} \mathcal{L}_{\text{class3}} &= (DH)^{\dagger}(DH) + \frac{c_{H\square}}{\Lambda^2} |H|^2 \square |H|^2 + \frac{c_{HD}}{\Lambda^2} |H^{\dagger}DH|^2 \\ &+ \frac{c_{HD}^{(8)}}{\Lambda^4} |H|^4 |DH|^2 + \frac{c_{HD,2}^{(8)}}{\Lambda^4} |H|^2 (H^{\dagger}\tau^A H) (DH)^{\dagger}\tau^A (DH) + \mathcal{O}\left(\frac{1}{\Lambda^6}\right) \end{aligned}$$



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$$\begin{split} h_{IJ} &\equiv \left[ 1 + \phi^2 \frac{c_{H\Box}}{\Lambda^2} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+2} \left( \frac{c_{HD}^{(8+2n)} - c_{HD,2}^{(8+2n)}}{\Lambda^{2+2n}} \right) \right] \delta_{IJ} \\ &+ \frac{\Gamma_{A,J}^{I} \phi_K \Gamma_{A,L}^{K} \phi^L}{2} \left( \frac{c_{HD}^{(6)}}{2\Lambda^2} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+1} \frac{c_{H,D2}^{(8+2n)}}{\Lambda^{2+2n}} \right) \end{split}$$

#### three-point functions from geoSMEFT

operator form	shifts:
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3-point functions + Masses
$g_{AB}W^A_{\mu u}W^{B,\mu u}$	SM triple gauge couplings + $h(\partial V)^2$ + mixing angles
$Y^{\psi} \bar{\Psi}_L \psi_R + h.c.$	SM Yukawas $+\psi$ masses
$L^\psi_J(D^\mu\phi)^J(ar\psi\Gamma_\mu\psi)$	SM gauge-fermion couplings
$d^\psi_A W^{A,\mu u} (ar\psi \sigma_{\mu u} \psi)$	Dipoles
$f_{ABC}W^{A,\mu u}W^B_{ u ho}W^{C, ho}_{\mu}$	new TGCs $(\partial V)^3$
$\kappa^A_{IJ}(D_\mu\phi)^I(D_ u\phi)^J W^A_{\mu u}$	new TGCs $(\partial h)^2 (\partial V)$ , removed from D6 in Warsaw

## geoSMEFT on the *Z*-pole

operator form	shifts:
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3-point functions + Masses
$g_{AB}W^A_{\mu u}W^{B,\mu u}$	SM triple gauge couplings + $h(\partial V)^2$ + mixing angles
$L^\psi_J (D^\mu \phi)^J (ar \psi \Gamma_\mu \psi)$	SM gauge-fermion couplings

$$\Gamma_{Z \to \bar{\psi}\psi} = \frac{N_c^{\psi}}{24\pi} \underline{\bar{m}}_Z |g_{\text{eff}}^{Z,\psi}|^2 \left(1 - \frac{4\bar{m}_{\psi}^2}{\underline{\bar{m}}_Z^2}\right)^{3/2}$$

$$g_{\text{eff}}^{Z,\psi} = \frac{\bar{g}_Z}{2} \left[ (2 \frac{s_{\theta_Z}^2}{Q_{\psi} - \sigma_3}) + \bar{v}_T \left\langle L_{3,4}^{\psi} \right\rangle + \sigma_3 \bar{v}_T \left\langle L_{3,3}^{\psi} \right\rangle \right]$$

$$\bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4} \langle h_{33} \rangle \bar{v}_T^2 \qquad s_{\theta_Z}^2 = f(\langle g_{AB} \rangle, g_1, g_2)$$

#### **1** Calculate the following: $\Gamma_{e,\mu}, \Gamma_{\tau}, \Gamma_{\nu}, \Gamma_{c}, \Gamma_{b}, \Gamma_{Z},$ $R_{I}, R_{c}, R_{b}, A'_{FB}, A^{c}_{FB}, A^{b}_{FB}, \sigma^{0}_{Had}$

#### Supplement with SM predictions up to two-loops Awramik et al. hep-ph/0608099, Dubovyk et al. arXiv:1906.08815 Freitas arXiv: 1401.2447, Awramik et al. hep-ph/0311148

**③** Interpolate SM pred. to  $\{M_W, M_Z, G_F\}$  input scheme from  $\{\alpha, M_Z, G_F\}$  $\rightarrow$  allows for comparison of scheme dependence



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- **5** use  $\delta O_8 / O_{\rm SM}$  as a measure of impact of D8 operators on O
- 6 take  $\Lambda = 1$  TeV
- randomly sample values of c<sub>i</sub> between 1 ("tree"-) and .01 ("loop"-induced operator assumption)











**1** Calculate the following:  $\Gamma_{e,\mu}, \Gamma_{\tau}, \Gamma_{\nu}, \Gamma_{u}, \Gamma_{c}, \Gamma_{d,s}, \Gamma_{b}, \Gamma_{Z}, R_{l}, R_{c}, R_{b}, A_{FB}^{l}, A_{FB}^{c}, A_{FB}^{b}, \sigma_{Had}^{0}$ 

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- **(5)** Take triplet scalar model w  $Y_{\Phi} = 0$  and match to SMEFT to D8
- **(6)** Perform  $\chi^2$  analysis to Z-pole data gives constraints in the  $M_{\Phi}-\kappa$  plane  $(2\kappa H^{\dagger}\tau^a H \Phi^a, \eta |H|^2 \Phi^2 w/\eta = .1)$







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 $\kappa/v$ 

-4 -2





$$|\mathcal{M}|^2 = |\mathcal{M}_{\mathrm{SM}}|^2 + \frac{2}{\Lambda^2} \mathrm{Re}\left[\mathcal{M}^*_{\mathrm{SM}} \mathcal{M}_6\right] + \frac{1}{\Lambda^4}\left(|\mathcal{M}_6|^2 + \mathrm{Re}\left[\mathcal{M}^*_{\mathrm{SM}} \mathcal{M}_8\right]\right) + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$



$$|\mathcal{M}|^{2} = |\mathcal{M}_{\rm SM}|^{2} + \frac{2}{\Lambda^{2}} \operatorname{Re}\left[\mathcal{M}_{\rm SM}^{*} \mathcal{M}_{6}\right] + \frac{1}{\Lambda^{4}} \left(|\mathcal{M}_{6}|^{2} + \operatorname{Re}\left[\mathcal{M}_{\rm SM}^{*} \mathcal{M}_{8}\right]\right) + \mathcal{O}\left(\frac{1}{\Lambda^{6}}\right)$$
LO SMEFT contribution











#### Conclusions

- the geoSMEFT allows for (some) all orders in v/A calculations in the SMEFT → most resonant processes doable, w/ some innovation can push to ~all
- 2 all Z-pole observables have been full calculated to all orders in  $v/\Lambda \rightarrow$  in two different input parameter schemes,  $M_W$  and  $\alpha$
- inclusion of D8 can have impact of ~ % in most cases
   → EWPD observables are (mostly) measured to per-mil accuracy
   → D8 is important for EWPD!
- ④ inclusion of D8 can have a large impact in other cases
   → HL-LHC precision goes to O(%) for Higgs
- Is geoSMEFT allows us to calculate beyond LO in SMEFT → a more reliable estimate of truncation error

Wednesday 10:30, Adam Martin talks more about geoSMEFT  $\Rightarrow$  HZ $\gamma$ , H $\gamma\gamma$ 

geoSMEFT in loops, TC arXiv:2106.10284

geoSMEFT D8 + Loops D6,  $gg \rightarrow H \rightarrow \gamma\gamma$ , TC, Martin, Trott, arXiv:2107.07470

#### Saturation of number of operators

Mass Dimension

(This information is contained in the Hilbert Series) (see e.g. Lehman & Martin 2015, Henning et al. 2015)

		Mass Dimension			
Operator form:		6	8	10	
	$h_{IJ}(D_{\mu}\phi)^{I}(D^{\mu}\phi)^{J}$	2	2	2	10 <sup>7</sup>
	$g_{AB}W^A_{\mu u}W^{B,\mu u}$	3	4	4	106
	$k_{IJA}(D^{\mu}\phi)^{I}(D^{\nu}\phi)^{J}W^{A}_{\mu u}$	0	3	4	105
	$f_{ABC} W^A_{\mu\nu} W^{B,\nu\rho} W^{C,\mu}_{\rho}$	1	2	2	104
	$Y_{ m pr}^{\psi} \overline{\Psi}_L \psi_R + h.c.$	$2N_f^2$	$2N_f^2$	$2N_{f}^{2}$	10 <sup>3</sup> 3,04
	$d_A^{\psi,pr} \bar{\Psi}_L \sigma_{\mu u} \psi_R W_A^{\mu u} + h.c.$	$4N_f^2$	$6N_f^2$	$6N_f^2$	10 <sup>2</sup>
	$L^{\psi_R}_{pr,J,A}(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	$N_f^2$	$N_f^2$	$N_f^2$	10
	$L_{pr,J,A}^{\Psi_L}(D^{\mu}\phi)^J(\bar{\Psi}_{p,L}\gamma_{\mu}\sigma_A\Psi_{r,L})$	$2N_f^2$	$4N_f^2$	$4N_f^2$	









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#### Backup: Inclusion of Cl 3 & 4 operators

Following geosmeft (Helset & Trott 1803.08001, + Martin 2001.01453) rewrite the doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix} \Rightarrow \phi_I = \{\phi_1, \phi_2, \phi_3, \phi_4\}$$

Then defining the field-space metric, *h*:

$$h_{IJ} \equiv \delta_{IJ} - 2c_{H\Box}\phi_I\phi_J + \frac{1}{4}c_{HD}\Gamma^A_{IJ}\Gamma^A_{KL}\phi_K\phi_L$$
 ( $\Gamma$  are matrices)

Then the Class 3 operators are written in simple form:

$$\mathcal{L}_{cl3} = h_{IJ}(D^{\mu}\phi)^{I}(D_{\mu}\phi)^{J} = (D^{\mu}H)^{\dagger}(D_{\mu}H) + c_{H\Box}(H^{\dagger}H)\Box(H^{\dagger}H) + c_{HD}(H^{\dagger}D^{\mu}H)^{*}(H^{\dagger}D_{\mu}H)$$



#### Backup: Inclusion of Cl 3 & 4 operators

Then the Class 3 and 4 operators are written in simple form:

$$\begin{array}{lll} \mathcal{L}_{\rm cl3} &=& h_{IJ} (D^{\mu} \phi)^{I} (D_{\mu} \phi)^{J} \\ \mathcal{L}_{\rm cl4} &=& -\frac{1}{4} g_{AB} W_{A\mu\nu} W_{B\mu\nu} \end{array}$$



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Can trivially include D8 ops. of Cl. 3 & 4 by updating h and g:

$$\begin{split} h_{IJ} &= \left[ 1 + \frac{\phi^4}{4} (c^{(8)}_{HD} + c^{(8)}_{HD2}) \right] \delta_{IJ} - 2c_{H\Box} \phi_I \phi_J \\ &+ \frac{1}{4} \left( c_{HD} + \phi^2 c^{(8)}_{HD2} \right) \Gamma^A_{IJ} \Gamma^A_{KL} \phi_K \phi_L \\ g_{AB} &= \delta_{AB} - 4 \left[ c_{HW} (1 - \delta_{A4}) + c_{HB} \delta_{A4} \right] \frac{\phi^2}{2} \delta_{AB} \\ &- 4 \left[ c^{(8)}_{HW} (1 - \delta_{A4}) + c^{(8)}_{HB} \delta_{A4} \right] \frac{\phi^4}{4} \delta_{AB} \\ &+ \left( c_{HWB} + c^{(8)}_{HWB} \frac{\phi^2}{2} \right) \left[ (\phi_I \Gamma^A_{IJ} \phi_J) (1 - \delta_{A4}) \delta_{B4} + A \leftrightarrow B \right] \\ &- c^{(8)}_{HW2} (\phi_I \Gamma^A_{IJ} \phi_J) (\phi_L \Gamma^B_{LK} \phi_K) (1 - \delta_{A4}) (1 - \delta_{B4}) \end{split}$$

