NLO production of HH, ZH and ZZ by gluon fusion, in the high-energy limit

EPS-HEP Conference 2021 – Parallel T09

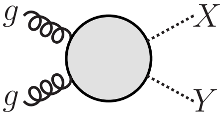
J. Davies

UNIVERSITY OF SUSSEX

July 27, 2021

Introduction

Gluon fusion amplitudes are interesting at the LHC despite loop suppression: enhanced by large gluon PDF.



HH
- gives access to Higgs self-coupling λ
- $-5.0 < \lambda/\lambda_{SM} < 12.0$ [CERN-EP-2019-099]
- *gg* is the dominant production channel (10x)

ZZ
- significant background to $H \to 4l$
- constrains Higgs width via $H \to ZZ$ diagrams
- sub-leading cf. quark-induced, but ∼60% of total NNLO

ZH
- $H \to b\bar{b}$ discovery
- sub-leading cf. quark-induced, but ∼10% of total
- large scale uncertainties

Introduction

Amplitude structure:

$$\mathcal{M}^{\mu\nu\rho\sigma} = \sum_i \mathcal{A}_i^{\mu\nu\rho\sigma} F_i$$

$HH : i = \{1, 2\}, \quad ZZ : i = \{1, \ldots, 18\}, \quad ZH : i = \{1, \ldots, 6\}.$

Two-loop computations of such form factors F_i are difficult:

- Depend on many scales, s, t, m_t, m_H, m_Z
- Feynman integrals are complicated

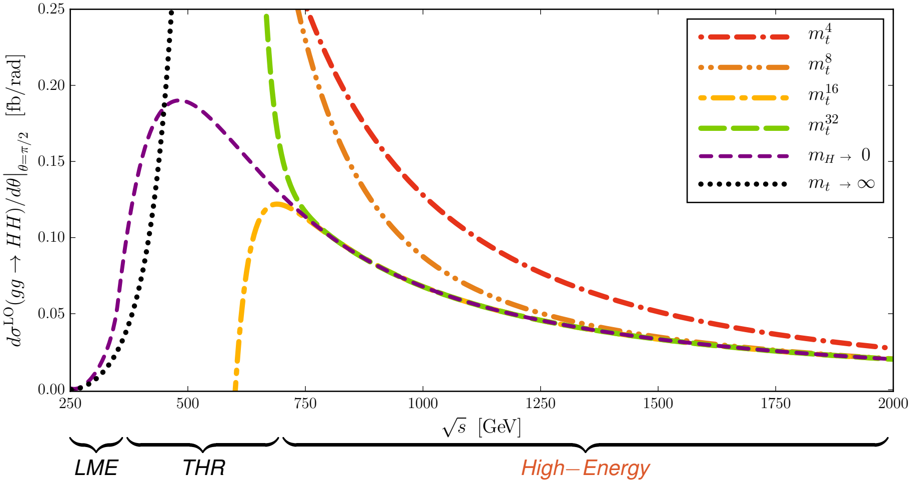
... not known analytically!

Numerical results exist, and expansions in various limits:

- large-m_t
- threshold
- small-p_T
- high-energy

Expansions

Seek an expansion in the region $s, t \gg m_t^2 > \{m_H^2, m_Z^2\}$:



(Incomplete) NLO Status

HH	• LO	[Glover,van der Bij '88]
	• NLO HEFT	[Dawson,Dittmaier,Spira '98]
	• NLO LME+THR Padé	[Gröber,Maier,Rauh '17]
	• NLO small-p_T	[Bonciani,Degrassi,Giardino,Gröber '18]
	• NLO numerical	[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke '16]
		[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '18]
ZZ	• LO	
	• NLO (massless)	[Caola,Melnikov,Röntsch,Tancredi '15]
	• NLO LME	[Dowling,Melnikov '15]
	• NLO numerical	[Agarwal,Jones,von Manteuffel '20]
		Brønnum-Hansen,Wang '21]
ZH	• LO	[Dicus,Kao '88, Kniehl '90]
	• NLO LME	[Hasselhuhn,Luthe,Steinhauser '17]
	• NLO small-p_T	[Alasfar,Degrassi,Giardino,Gröber,Vitti '21]
	• NLO numerical	[Chen,Heinrich,Jones,Kerner,Klappert,Schlenk '20]
	• NLO high-energy + numerical	[Wang, Xu, Xu, Yang '21]

+ various higher-order efforts, mostly HEFT and LME

High-Energy Expansion

Seek an expansion in the region $s, t \gg m_t^2 > \{m_H^2, m_Z^2\}$:

1. FFs in terms of Feynman integrals: $I(\{m_H^2, m_Z^2\}, m_t^2, s, t, \epsilon)$,
2. Expand for $\{m_H^2, m_Z^2\} \to 0$:
$$I(m_H^2, \ldots) = I(0, \ldots) + m_H^2 I'(0, \ldots) + \cdots ,$$
3. IBP reduce integrals to Master Integrals: $J(\{0, 0\}, m_t^2, s, t, \epsilon)$,
4. Determine MIs as an expansion around $m_t^2 = 0$:
$$J(\{0,0\}, m_t^2, s, t, \epsilon) = \sum_{i,j,k} C_{ijk}(s,t) \ \epsilon^i \ (m_t^2)^j \ \log(m_t^2)^k \, .$$

Tools: `qgraf`, `q2e/exp`, `FORM`, `FIRE`, `LiteRed`.

High-Energy Expansion

$$J(\{0,0\}, m_t^2, s, t, \epsilon) = \sum_{i,j,k} C_{ijk}(s,t)\ \epsilon^i\ (m_t^2)^j\ \log(m_t^2)^k$$

To determine $C_{ijk}(s,t)$:

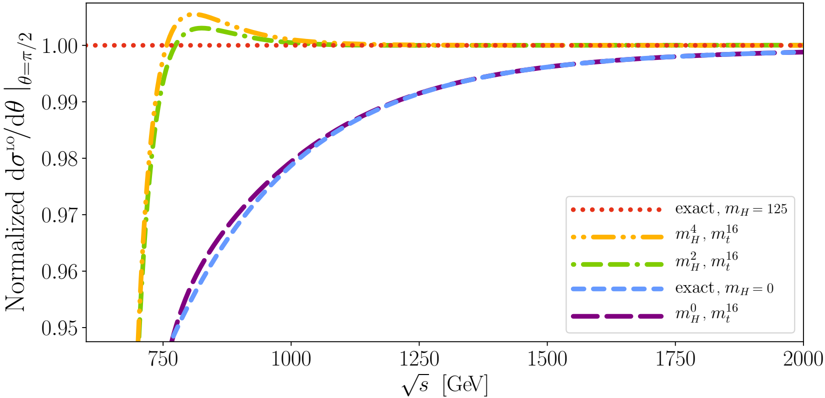
1. Differentiate MIs w.r.t. m_t^2, IBP reduce result:

$$\frac{d}{d\,m_t^2}\vec{J} = M(m_t^2, s, t, \epsilon)\ \cdot\ \vec{J},$$

2. Substitute ansatz $\rightarrow$ coupled system of *linear* equations for C_{ijk},
3. Boundary conditions: determine leading expansion terms using Expansion-by-Regions, solve linear system for higher C_{ijk}.
   [$C_{ijk}(s,t)$: Harmonic PolyLogarithms with polynomial coefficients].

High-Energy Expansion

Convergence of m_H expansion in LO $gg \to HH$:



[Davies, Mishima, Steinhauser, Wellmann '18]

m_H^0 : %-level agreement (reproduces exact $m_H = 0$ curve well)

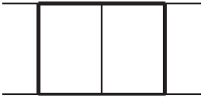
m_H^2, m_H^4: ‰-level agreement

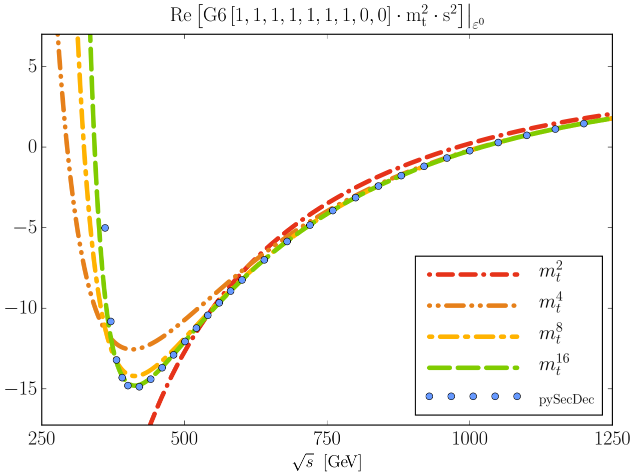
Master Integral Comparison

MI-level comparison: m_t^2 expansion vs. `pySecDec` numerical values:

[Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '18]

double box:
6 massive (m_t),
1 massless prop.
Real part, ϵ^0

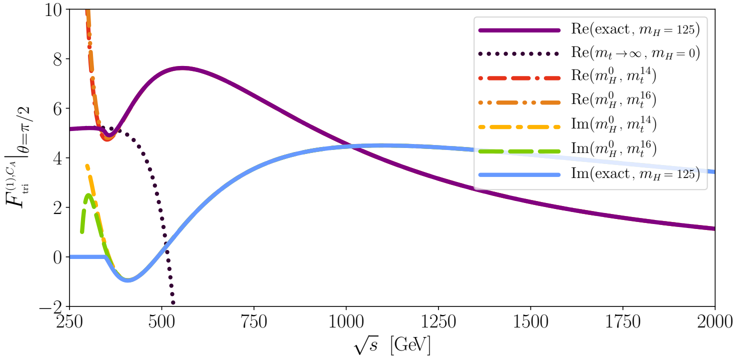




Form Factor Expansions

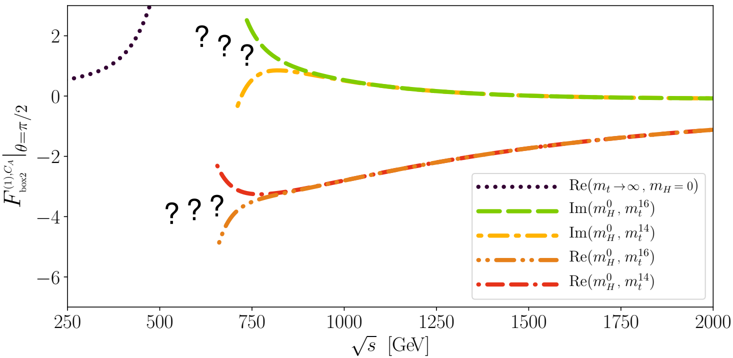
Renorm. and IR subtraction: $F_i^{(1),fin} = F_i^{(1),IR-div} - K_g^{(1)} F_i^{(0)}$.

In $gg \to HH$, NLO F_{tri} known analytically (from $gg \to H$):



Form Factor Expansions

$gg \to HH$, F_{box} form factors are not known analytically for comparison:



Form factors contain logs which diverge at $4m_t \approx 700$GeV.

Padé Approximants

Approximate a function $f(x)$ using a *rational polynomial*:

$$f(x) \approx [n/m](x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m}.$$

Use series coefficients of $f(x)$ to fix $a_0, \ldots, a_n, b_1, \ldots, b_m$.

- Agrees with series to order $n + m$ (but not beyond)
- Can be a better approximation of $f(x)$ than the truncated series.
- Even beyond the radius of convergence!

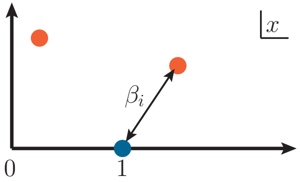
Padé Approximants: applied to m_t expansion

In small-m_t expansions:

- Replace $m_t^2 \to m_t^2 x$,
- Set variables (m_t, s, t, etc) to numerical values,
- Construct Padé approximants at $x = 0$, then evaluate at $x = 1$: m_t^{32} exp. $\implies$ $n + m \leq 16$: $[7/8], [8/7], [7/9], [8/8], [9/7]$.

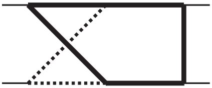
Central value and error:

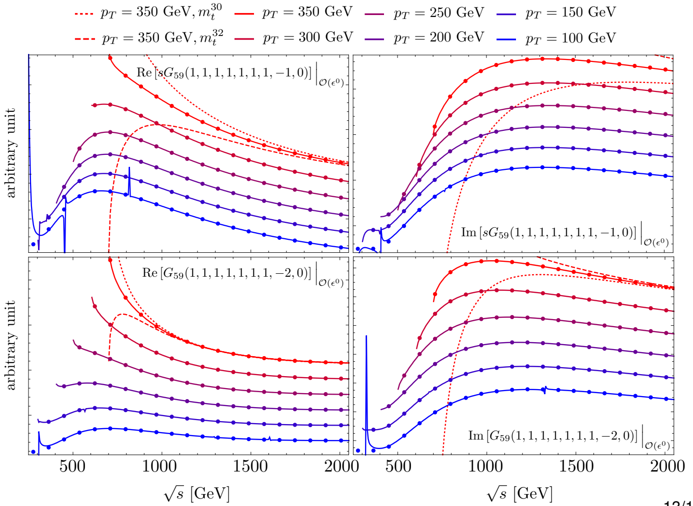
- Padé approximants have poles in the complex x plane, which can lead to poor behaviour if close to $x = 1$.
- Compute a weighted average and deviation, with $\omega_i = \beta_i^2 / \sum_j \beta_j^2$. β_i is the distance from $x = 1$ to the nearest pole.



Padé Approximants: Master Integral Level

Dashed lines: m_t^{30}, m_t^{32}. Smaller p_T values: series doesn't converge.
Solid lines: Padé approximant. Points: `pySecDec`.

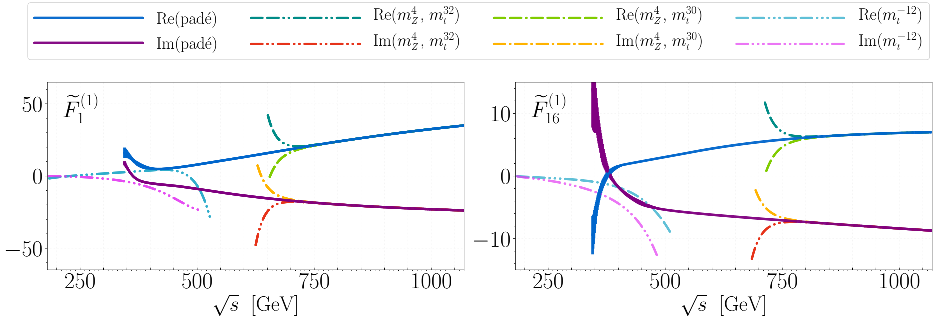




Form Factor Approximations

Two example Form Factors for $gg \to ZZ$:

- the high-energy exp. diverges around $\sqrt{s} \approx 750$GeV as usual
- the Padé-based approximation continues to lower $\sqrt{s}$ values

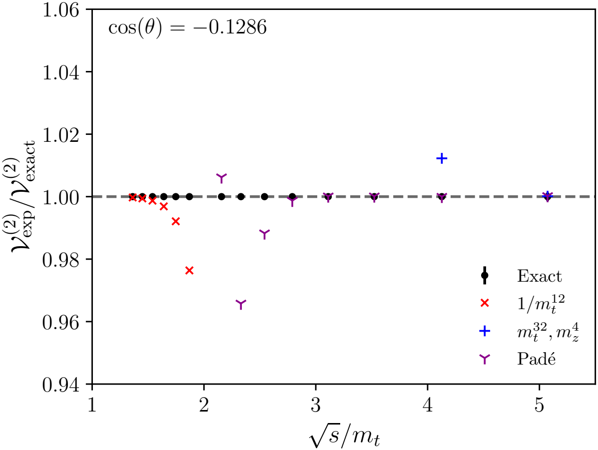


[Davies,Mishima,Steinhauser,Wellmann '20]

Comparison with $gg \to ZZ$ numerical V_{fin}

V_{fin}: "virtual finite cross-section".

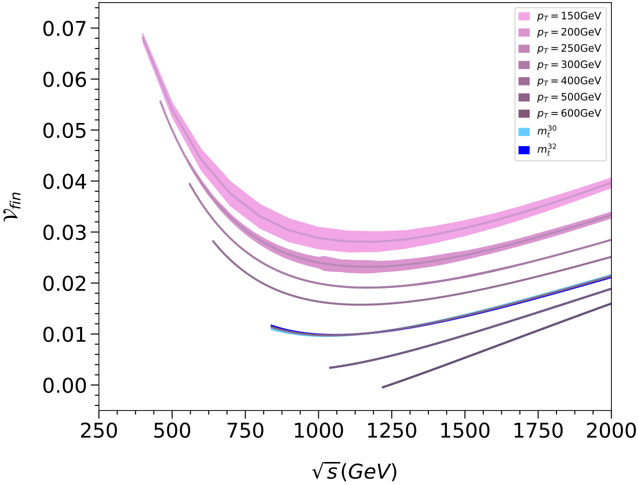
Padé-improved V_{fin} shows excellent agreement with `pySecDec`-based evaluation above $\sqrt{s} \approx 3m_t$. [Agarwal,Jones,von Manteuffel '20]



$gg \to ZH$ V_{fin}

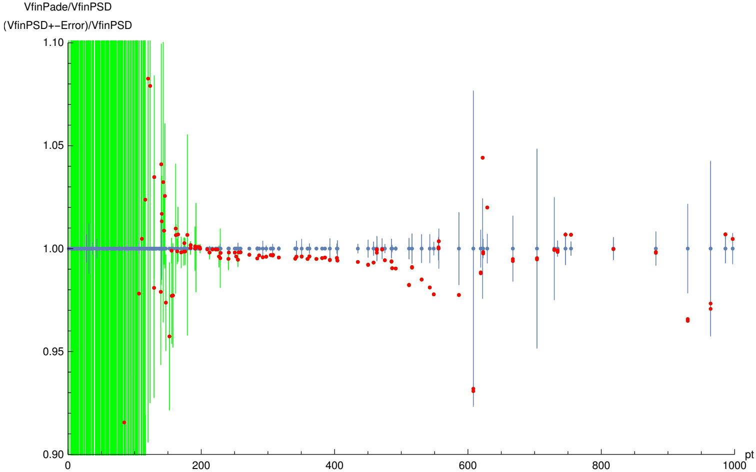
Similarly, we construct a V_{fin} for ZH. [Davies,Mishima,Steinhauser '20]

High-energy exp does not converge for $p_T \leq 300$.



Comparison with $gg \to ZH$ numerical V_{fin}

In progress: $gg \to ZH$ comparison. Blue: `pySecDec`, Red+Green: Padé

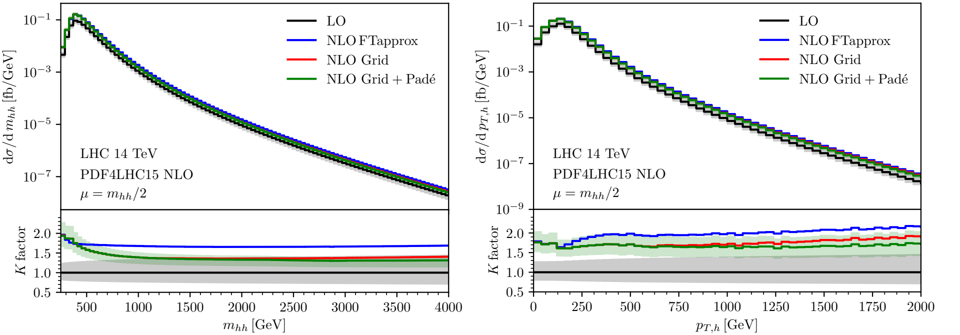


`pySecDec` data points: [Chen,Heinrich,Jones,Kerner,Klappert,Schlenk]

$gg \to HH$ Distributions

Add real radiation, convolute with PDF. Impact of including high-energy points in `hhgrid` V_{fin}:

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann '19]



Conclusions

Two-loop $2 \to 2$ amplitudes which depend on extra mass scales are not known analytically.

Nonetheless we can learn about their behaviour through expansions in various limits, and direct numerical evaluations.

High-energy expansions give the behaviour of the amplitudes in a region which is difficult to describe precisely by numerical evaluation.

- Padé approximants significantly improve the high-energy description