NNLO QCD corrections to the B-meson mixing

Vladyslav Shtabovenko
Karlsruhe Institute of Technology
Institute for Theoretical Particle Physics

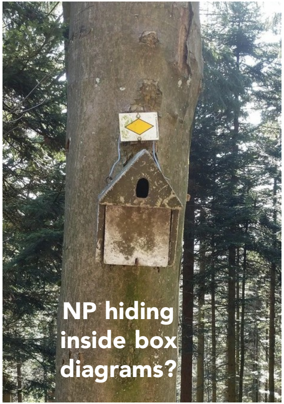
based on 2106.05979, 21XX.XXXXX
in collaboration with M. Gerlach, U. Nierste and M. Steinhauser

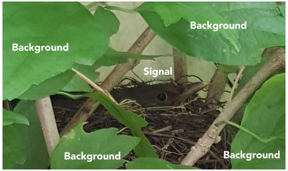
EPS-HEP Conference 2021 (virtual)
26th of July 2021

Karlsruhe Institute of Technology

Outline

- No new physics in sight at the high-energy frontier
- Growing importance of the precision frontier
- Flavor physics: increasing number of anomalies (LFU violation, muon $g-2$, ...) that challenge the validity of the SM
- New physics hiding around the corner?
- However: Working at the precision frontier is impossible without precision physics
- Precision physics implies precise *measurements* and precise *theoretical predictions*
- Theory and experiment must go hand in hand trying to decrease existing uncertainties in SM predictions





- Physical observables in $B_s^0 - \bar{B}_s^0$ oscillations: $|M_{12}|$, $|\Gamma_{12}|$, $\phi = \arg(-M_{12}/\Gamma_{12})$
- ΔM_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency $\Rightarrow \Delta M_s \approx 2|M_{12}|$
  t quark is dominant in SM, sensitivity to NP in the loops
- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference $\Rightarrow \Delta\Gamma_s \approx 2|\Gamma_{12}|$
  only u and c contribute, precision probe of SM, NP with light particles
- ϕ_s: CP-asymmetry in the mixing $a_{\text{fs}} = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s$
- Experimental value (HFLAV 2020 average)

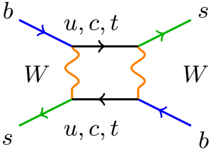
$$\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$$

- Theory prediction (NLO + n_f-piece of NNLO QCD corrections)
  [Beneke et al., 1999; Ciuchini et al., 2002, 2003; Lenz & Nierste, 2007; Asatrian et al., 2020, 2017]

$$\Delta\Gamma_{\overline{\text{MS}}} = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}$$

- Substantial uncertainty from uncalculated NNLO corrections (pert.), much larger than experimental errors!
- Need matching coefficients between $|\Delta|B = 1$ and $|\Delta|B = 2$ EFTs at 2- and 3-loops!

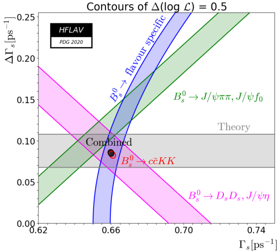
$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



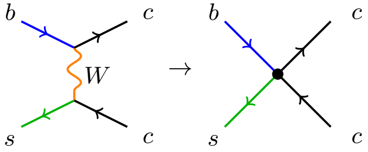
$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi_s$$

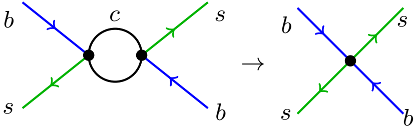
$$\phi_s \equiv \arg(-M_{12}/\Gamma_{12}) \approx 0$$



$|\Delta B| = 1$ effective theory ($m_b \ll m_W, m_t$)

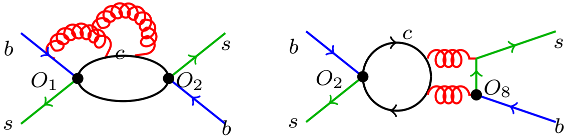


$|\Delta B| = 2$ effective theory (via HQE)



$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \ldots$$

- Calculation done using $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ in the CMM operator basis for $b \to s c \bar{c}$ [Chetyrkin et al., 1998]
- Representative diagrams in the $|\Delta B| = 1$ EFT needed for the NNLO accuracy



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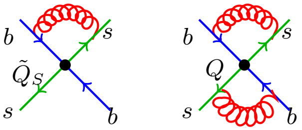

- 2-loop $O_{1-2} \times O_{3-6}$ available: $\Delta\Gamma_s^{p,12\times 36,\alpha_s}/\Delta\Gamma_s = 1.4\% (\overline{\text{MS}})$ [Gerlach, Nierste, VS, Steinhauser, 2021]
- All relevant 2-loop and 3-loop correlators already computed, two more publications in preparation

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to s c \bar{c}$ decays [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- Current operators

$$\begin{aligned}
Q_1 &= \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L, \\
Q_2 &= \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L, \\
Q_1^u &= \bar{s}_L \gamma_\mu T^a u_L \; \bar{u}_L \gamma^\mu T^a b_L, \\
Q_2^u &= \bar{s}_L \gamma_\mu u_L \; \bar{u}_L \gamma^\mu b_L, \\
Q_1^{cu} &= \bar{s}_L \gamma_\mu T^a u_L \; \bar{c}_L \gamma^\mu T^a b_L, \\
Q_2^{cu} &= \bar{s}_L \gamma_\mu u_L \; \bar{c}_L \gamma^\mu b_L, \\
Q_1^{uc} &= \bar{s}_L \gamma_\mu T^a c_L \; \bar{u}_L \gamma^\mu T^a b_L, \\
Q_2^{uc} &= \bar{s}_L \gamma_\mu c_L \; \bar{u}_L \gamma^\mu b_L,
\end{aligned}$$

- Penguin operators

$$\begin{aligned}
Q_3 &= \bar{s}_L \gamma_\mu b_L \textstyle\sum_q \bar{q} \gamma^\mu q\,, \\
Q_4 &= \bar{s}_L \gamma_\mu T^a b_L \textstyle\sum_q \bar{q} \gamma^\mu T^a q\,, \\
Q_5 &= \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q\,, \\
Q_6 &= \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q\,, \\
Q_8 &= \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G_{\mu\nu}^a\,,
\end{aligned}$$

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to sc\bar{c}$ decays [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$

$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

$\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]

$$\Gamma_{12} = -(\lambda_c^q)^2 \Gamma_{12}^{cc} - 2\lambda_c^q \lambda_u^q \Gamma_{12}^{uc} - (\lambda_u^q)^2 \Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

- Physical $|\Delta B| = 2$ operators

$$Q = \bar{s}_i \gamma^\mu (1-\gamma^5) b_i \; \bar{s}_j \gamma_\mu (1-\gamma^5) b_j \qquad \widetilde{Q}_S = \bar{s}_i (1-\gamma^5) b_j \; \bar{s}_j (1-\gamma^5) b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of evanescent operators)

$$\widetilde{Q} = \bar{s}_i \gamma^\mu (1-\gamma^5) b_j \; \bar{s}_j \gamma_\mu (1-\gamma^5) b_i\,, \qquad Q_S = \bar{s}_i (1-\gamma^5) b_i \; \bar{s}_j (1-\gamma^5) b_j\,,$$

$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- LO contributions to $\Delta\Gamma_s$
  - 1-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Hagelin, 1981; Franco et al., 1982; Chau, 1983; Buras et al., 1984; Khoze et al., 1987; Datta et al., 1987, 1988]
- NLO contributions to $\Delta\Gamma_s$ (z-exact)
  - 2-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_8$ correlators (z-exact) [Beneke et al., 1999]

$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- NNLO contributions to $\Delta\Gamma_s$
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, $\mathcal{O}(z^3)$)
  - 2-loop $O_{1-2} \times O_{3-6}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_{3-6} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1996]
  - 1-loop $O_{3-6} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop correlators at $\mathcal{O}(z)$ (including $O_8 \times O_8 \Rightarrow \mathrm{N^3LO}$)
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z^0)$
  - WIP: Final checks for the 3-loop result, higher order expansions in z, possibly z-exact results for selected correlators

- All computations done using our well-tested automatic setup
  - Diagram generation with QGRAF [Nogueira, 1993]
  - Insertion of Feynman rules and topology identification using Q2E/EXP [Seidensticker, 1999; Harlander et al., 1998] or TAPIR [Gerlach, Herren, 2021]
  - Feynman amplitude evaluation: in-house CALC setup written in FORM [Ruijl et al., 2017]
  - IBP-reduction: FIRE 6 [Smirnov & Chuharev, 2020]
  - Analytic computation of master integrals: HyperInt [Panzer, 2015], HyperlogProceedings [Schnetz], PolyLogTools [Duhr & Dulat, 2019]
  - All master integrals checked numerically using FIESTA [Smirnov, 2016] and pySecDec [Borowka et al., 2018]
- Cross-checks of selected intermediate results using FeynArts [Hahn, 2001], FeynRules [Christensen & Duhr, 2009; Alloul et al., 2014] and FeynCalc [VS et al., 2020]

- 23 on-shell 3-loop integrals with massive (solid) lines
- Only imaginary parts are relevant and turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

- Real parts (obtained as a byproduct) more complicated but irrelevant for $\Delta\Gamma_s$

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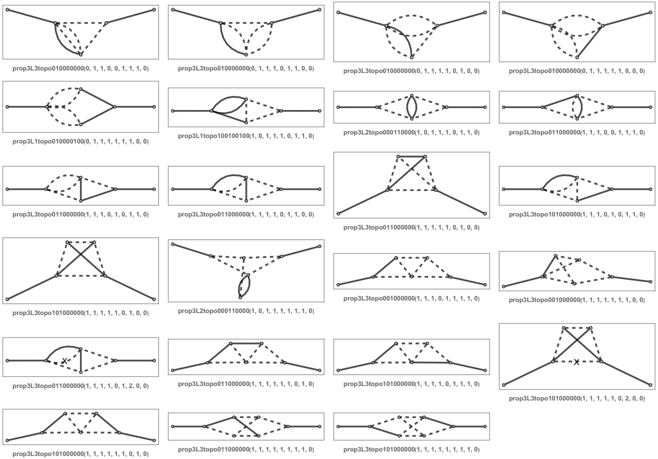

- New contributions to Γ_{12}^s computed in the course of this collaboration ($z = m_c^2/m_b^2$)

Correlator	Perturbative order	z-dependence
$O_{1,2} \times O_{3-6}$	2 loops	$\mathcal{O}(z)$
$O_{1,2} \times O_8$	2 loops	$\mathcal{O}(z)$
$O_{3-6} \times O_{3-6}$	2 loops	$\mathcal{O}(z)$
$O_{3-6} \times O_8$	2 loops	$\mathcal{O}(z)$
$O_8 \times O_8$	1 loop	exact
$O_8 \times O_8$	2 loops	$\mathcal{O}(z)$
$O_{1,2} \times O_{1,2}$	3 loops	$\mathcal{O}(z^0)$

- All 2-loop contributions to the NNLO correction already computed and cross-checked
- New theory predictions for the width difference $\Delta\Gamma_s$ and the CP asymmetry $a_{\rm fs}^s$ under way

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\mathrm{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right), \quad a_{\rm fs}^s = \mathrm{Im}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right)$$

- Ingredients

$$\Gamma_{12}^s = -(\lambda_t^s)^2 \left[\Gamma_{12}^{s,cc} + 2\frac{\lambda_u^s}{\lambda_t^s}\left(\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}\right) + \left(\frac{\lambda_u^s}{\lambda_t^s}\right)^2 \left(\Gamma_{12}^{s,uu} + \Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc}\right)\right]$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\underbrace{\langle B_s|Q|\bar{B}_s\rangle}_{\frac{8}{3}M_{B_s}^2 f_{B_s}^2 B_{B_s}} + \widetilde{H}_S^{ab}(z)\underbrace{\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle}_{\frac{1}{3}M_{B_s}^2 f_{B_s}^2 \widetilde{B}'_{S,B_s}}\right] + \mathcal{O}\left(\Lambda_{\rm QCD}/m_b\right)$$

$$M_{12} = (\lambda_t^s)^2 \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \hat{\eta}_B S_0\left(\frac{m_t^2}{M_W^2}\right) f_{B_s}^2 B_{B_s}$$

- Cancellation of $(\lambda_t^s)^2$, decay constants and to large extent bag parameters in the ratio Γ_{12}^s/M_{12}^s
- Following [Asatrian et al., 2020] we can calculate

$$\Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)\Delta M_s^{\rm exp}$$

- $|V_{cb}|$ controversy irrelevant!

- Theoretical predictions for the $\overline{\text{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\text{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\text{MS}}(m_b)\right)^2$ in the $\overline{\text{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- NLO result for M_{12} from [Buras et al., 1990]
- $1/m_b$ LO corrections to Γ_{12} [Beneke et al., 1996; Lenz & Nierste, 2007] are included
-
- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$
- Numerical input [Tanabashi et al., 2018; Dowdall et al., 2019; Bazavov et al., 2018; Amhis et al., 2021]

$$
\begin{aligned}
M_{B_s} &= 5366.88\ \text{MeV} \quad f_{B_s} = (0.2307 \pm 0.0013)\ \text{GeV}\,, \\
B_{B_s} &= 0.813 \pm 0.034\,, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09\,, \\
\frac{\lambda_u^s}{\lambda_t^s} &= -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i \\
\Delta M_s^{\text{exp}} &= (17.749 \pm 0.020)\ \text{ps}^{-1}
\end{aligned}
$$

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- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$
- **Preliminary** results (no scale variation, 3-loop corrections not included)

Included correlators	$\text{Re}\left(\Gamma_{12}^s/M_{12}^s\right)$	$\Delta\Gamma_s$
$O_{1,2} \times O_{1,2}$ (2 loops), $O_{1,2} \times O_{3-6}$ (1 loop), $O_{1,2} \times O_8$ (2 loops), $O_{3-6} \times O_{3-6}$ (1 loop)	$(5.31 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.73 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{1,2} \times O_{3-6}$ (2 loops)	$(5.21 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.71 \pm 0.08) \times 10^{-3}$ (pole)	$(0.093 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{3-6} \times O_{3-6}$ (2 loops)	$(5.24 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.74 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)
as above + $O_{3-6} \times O_8$ (2 loops) and $O_8 \times O_8$ (1 loop and 2 loops)	$(5.24 \pm 0.67) \times 10^{-3}$ ($\overline{\text{MS}}$) $(4.73 \pm 0.08) \times 10^{-3}$ (pole)	$(0.094 \pm 0.012)\text{ps}^{-1}$ ($\overline{\text{MS}}$) $(0.084 \pm 0.014)\text{ps}^{-1}$ (pole)

- The error is dominated by the uncertainty in the hadronic ME entering $1/m_b$ LO corrections
- The main contribution comes from the current-penguin $O_{12} \times O_{3-6}$ 2-loop corrections

- Theoretical predictions for the $\overline{\mathrm{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\mathrm{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\mathrm{MS}}(m_b)\right)^2$ in the $\overline{\mathrm{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\mathrm{MS}}(m_b)/m_b^{\mathrm{MS}}(m_b))^2$
- NLO result for M_{12} from [Buras et al., 1990]
- $1/m_b$ LO corrections to Γ_{12} [Beneke et al., 1996; Lenz & Nierste, 2007] are included
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- Experimental value (HFLAV 2020 average): $\Delta\Gamma_s^{\mathrm{exp}} = (0.085 \pm 0.004)\ \mathrm{ps}^{-1}$
- **Preliminary** results (no scale variation, 3-loop corrections not included)

$$
\begin{aligned}
a_{\mathrm{fs}}^{s,2-\mathrm{loop}} &= (2.02 \pm 0.08) \times 10^{-5} (\overline{\mathrm{MS}}),\\
a_{\mathrm{fs}}^{s,2-\mathrm{loop}} &= (2.05 \pm 0.08) \times 10^{-5} (\overline{\mathrm{pole}})
\end{aligned}
$$

Summary

- Experimental precision of $\Delta\Gamma_s$ calls for the NNLO calculation!
- We calculated all building blocks needed to obtain the NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- All the occurring 3-loop MI from the current-current contribution calculated analytically (for $m_c = 0$)
- The result for the 2-loop current-penguin contribution already published [Gerlach, Nierste, VS, Steinhauser, 2021]

Outlook

- Results for all the remaining 2-loop contributions and the 3-loop current-current piece to appear soon
- New theory predictions for $\Delta\Gamma_s$ and the CP asymmetry a_{fs}^s
- Higher order expansions in $z \equiv m_c^2/m_b^2$, ideally z-exact results at least for the 2-loop contributions