Form factors for semileptonic $B_{(s)}$ decays

Oliver Witzel

July 27, 2021

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in collaboration with

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RBC-UKQCD collaborations

EPS HEP2021 VIRTUAL EDITION

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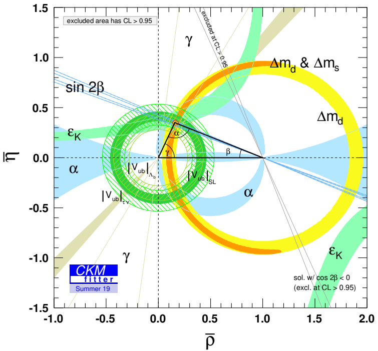
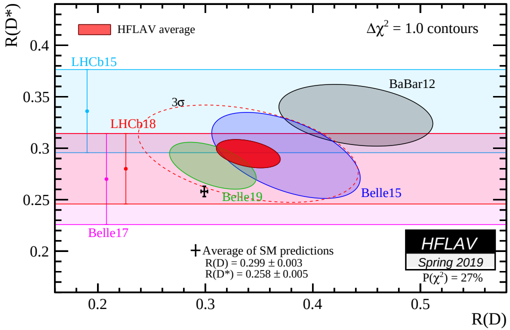
Motivation

[CKMfitter]

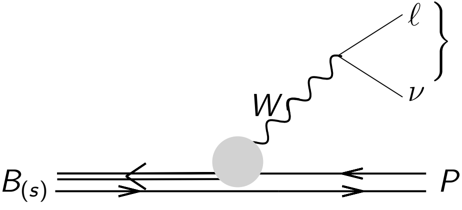


[HFLAV spring 2019]



- Determine fundamental parameters of the Standard Model e.g. $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$, $|V_{ts}|$
- May address interesting observations or challenge the Standard Model
  - $\rightarrow$ e.g. test lepton flavor universality via $R(D^{(*)})$

Exclusive semileptonic $B_{(s)}$ decays

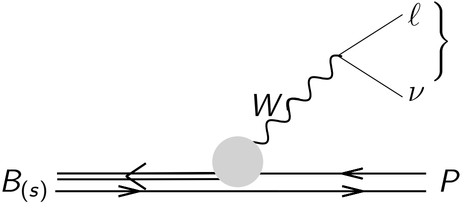


$$q^2 = M_{B_{(s)}}2 + M_P^2 - 2M_{B_{(s)}}E_P$$

pseudoscalar initial state   charged vector current   pseudoscalar final state

$$B \to \pi\ell\nu$$
$$B_s \to K\ell\nu$$
$$B_s \to D_s\ell\nu$$

Exclusive semileptonic $B_{(s)}$ decays



$$q^2 = M_{B_{(s)}}2 + M_P^2 - 2M_{B_{(s)}}E_P$$

- Conventionally parametrized ($B_{(s)}$ meson at rest)

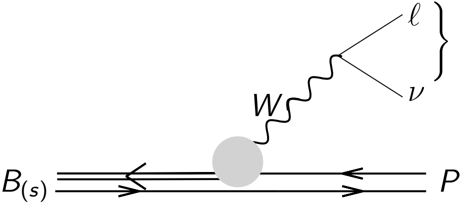
$$\frac{d\Gamma(B_{(s)} \to P\ell\nu)}{dq^2} = \frac{\eta_{EW} G_F^2 |V_{xb}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - M_P^2}}{q^4 M_{B_{(s)}}^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_{(s)}}^2 (E_P^2 - M_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_{B_{(s)}}^2 - M_P^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

Exclusive semileptonic $B_{(s)}$ decays



$$q^2 = M_{B_{(s)}} 2 + M_P^2 - 2 M_{B_{(s)}} E_P$$

- Nonperturbative form factors $f_+(q^2)$ and $f_0(q^2)$
  - → Parametrizes interactions due to (nonperturbative) strong force
  - → Use operator product expansion (OPE) to identify short distance contributions
  - → Calculate matrix element of flavor changing currents as point-like operators using lattice QCD

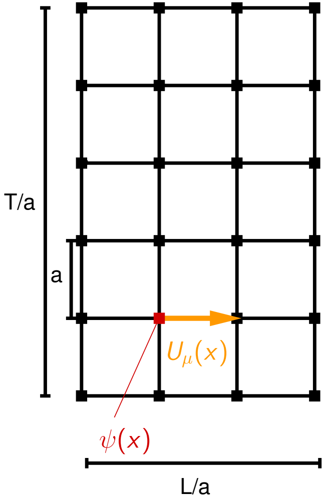
Lattice calculation

- Wick-rotate to Euclidean time $t \to i\tau$
- Discretize space-time and set up a hypercube of finite extent $(L/a)^3 \times T/a$ and spacing a
- Use path integral formalism

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

$\Rightarrow$ Large but finite dimensional path integral

- Finite lattice spacing $a \to$ UV regulator
  $\to$ Quark masses need to obey $am < 1$
- Finite volume of length $L/a \to$ IR regulator
  $\to$ Study physics in a finite box of volume $(aL)^3$

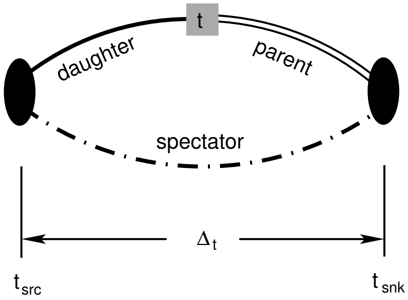


Lattice determination of form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle P|V^\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu$$

- Calculate the corresponding 3-point function
  - → Inserting source for spectator quark at t_{src}
  - → Allow it to propagate to t_{sink}
  - → Turn it into a sequential source for b quark
  - → Propagate light daughter quark from t_{src}
  - → Contract with b quark at t with $t_{\text{src}} \le t \le t_{sink}$



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- Prefer to compute

$$f_\parallel(E_P) = \langle P|V^0|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_\perp(E_P)p^i_P = \langle P|V^i|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}}$$

which are directly proportional to 3-point functions

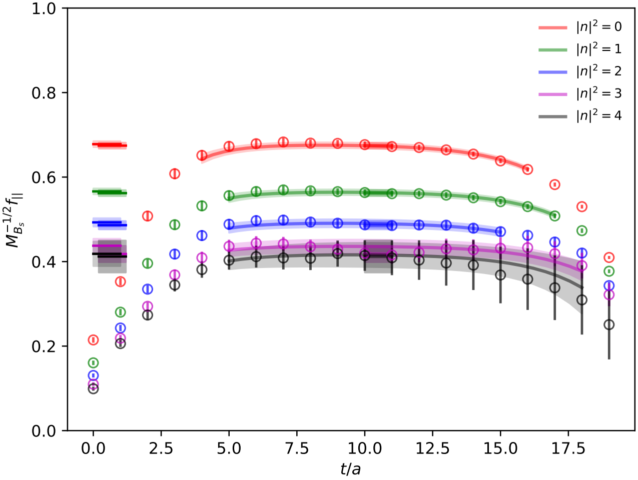
- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M^2_{B_{(s)}} - M^2_P}\left[(M_{B_{(s)}} - E_P)f_\parallel(E_P) + (E^2_P - M^2_P)f_\perp(E_P)\right]$$

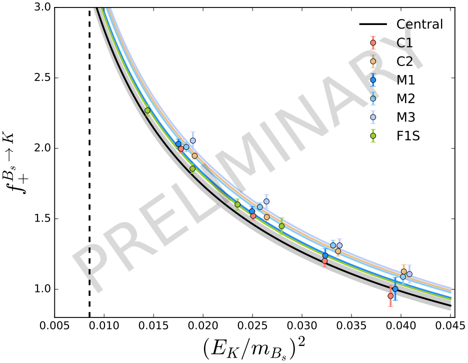
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}}\left[f_\parallel(E_P) + (M_{B_{(s)}} - E_P)f_\perp(E_P)\right]$$

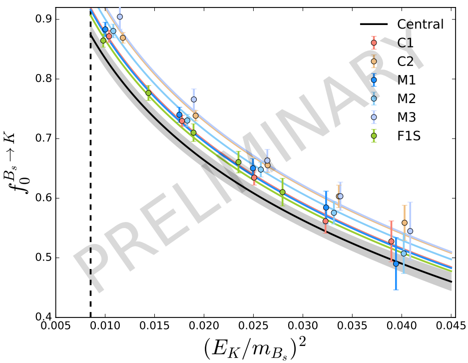
$B_s \to K\ell\nu$: form factor

- Example: form factor $f_{||}$ on coarse ensemble C1



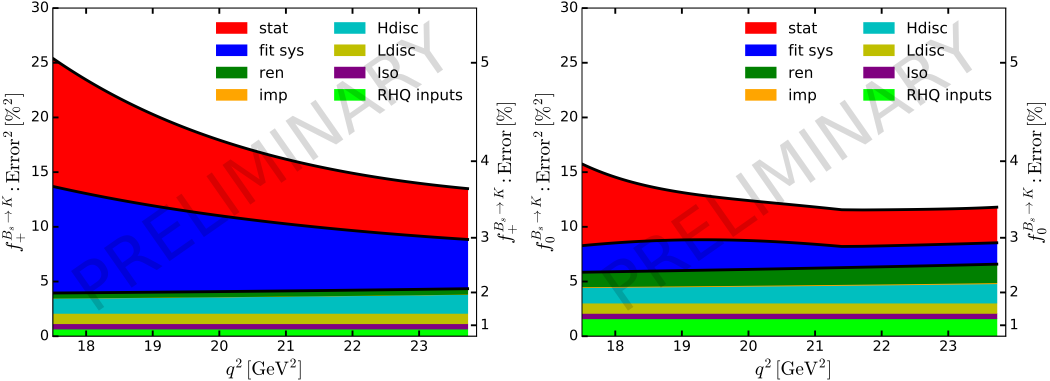
$B_s \to K\ell\nu$: chiral-continuum extrapolation



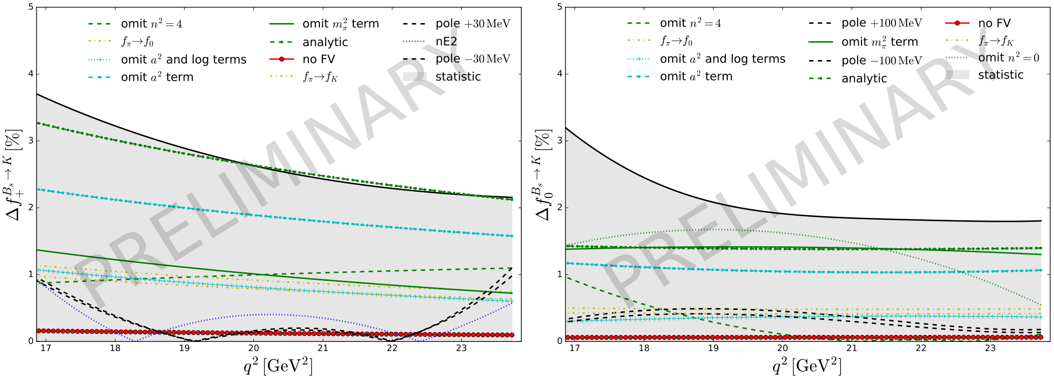


- Chiral-continuum extrapolation using SU(2) hard-kaon χPT
  - $\to f_{pole}(M_K, E_K, a^2) = \frac{c_0\Lambda}{E_K+\Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1\frac{M_\pi^2}{\Lambda^2} + c_2\frac{E_K}{\Lambda} + c_3\frac{E_K^2}{\Lambda^2} + c_4(a\Lambda)^2\right]$
  - $\to \delta f$ non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$

$B_s \to K\ell\nu$: error budget



$B_s \to K\ell\nu$: error budget



Kinematical extrapolation (z-expansion)

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \qquad \text{(two-particle production threshold)}$$
$$t_\pm = (M_{B_s} \pm M_K)^2 \qquad (\text{with } t_- = q^2_{max})$$
$$t_0 \equiv t_{\text{opt}} = t_* - \sqrt{t_*(t_* - t_-)} \qquad (\text{symmetrize range of } z)$$

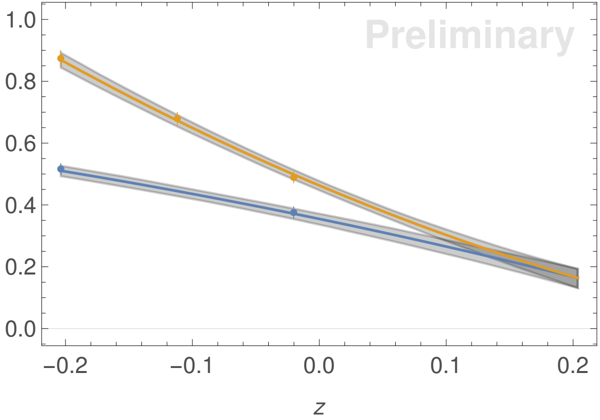
- BCL express form factor f_+ for $B \to \pi\ell\nu$

$$f_+(q^2) = \frac{1}{1 - q^2/M^2_{pole}} \sum_{k=0}^{K-1} b_k^+(t_0) z^k$$

- For other decays use product of factors for subthreshold poles for both f_+ and f_0 paralleling the Blaschke factors for a BGL fit to the same decay

Kinematical extrapolation (z-expansion)

- Perform fit in z-space with K parameters
- Then convert back to physical q^2
- BCL with pole $M_+ = B^* = 5.33$ GeV for f_+
- Exploit kinematic constraint $f_+ = f_0\Big|_{q^2=0}$
- Include HQ power counting to constrain size of f_+ coefficients (work in progress)
- Compare form factors to other determinations
  - → FNAL/MILC [Bazavov et al. arXiv:1901.02561]
  - → ALPHA [Bahr et al. PLB757(2016)473]
  - → RBC/UKQCD [Flynn et al. PRD 91 (2015) 074510]
  - → HPQCD [Bouchard et al. PRD 90 (2014) 054506][Monahan et al. PRD 98 (2014) 114509]



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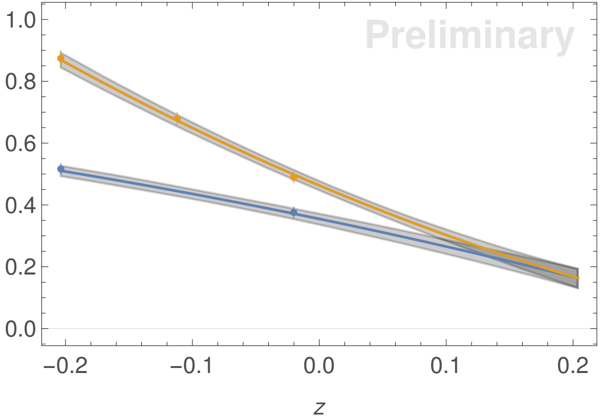
- Include HQ power counting to constrain size of f_+ coefficients (work in progress)
- Compare form factors to other determinations
  → Analytic predictions at $q^2 = 0$

  [Duplancic et al. PRD78 (2008) 054015]

  [Faustov et al. PRD87 (2013) 094028]

  [Wang et al. PRD86 (2012) 114025]

  [Khodjamirian et al. JHEP08 (2017) 112]



Ratios testing lepton flavor universality

Traditional R-ratio

$$R_P = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar\nu_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\ell\bar\nu_\ell)}{dq^2}}$$

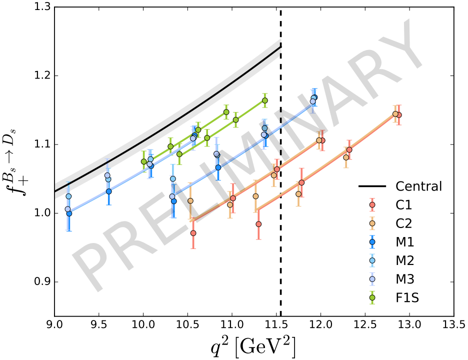
Alternative R-ratio

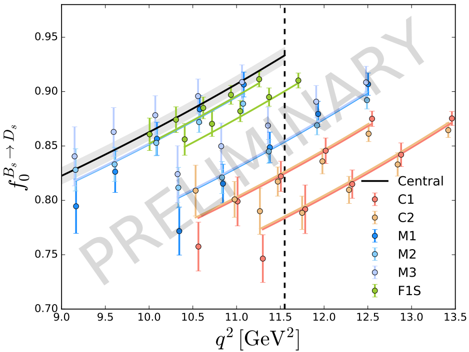
$$R_P^{\text{imp}} = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar\nu_\tau)}{dq^2}}{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)}\to P\ell\bar\nu_\ell)}{dq^2}}$$

- Follow idea proposed for $B \to V$ [Isidori and Sumensari EPJC80 (2020)1078]
  - → Common integration range: $q^2_{min} \geq m_\tau^2$ [Freytsis et al. PRD92(2015)054018] [Bernlochner and Ligeti PRD95(2017)014022] [Flynn et al. PoS ICHEP2020 436]
  - → Same weights in integrands for τ and ℓ

$$\omega_l(q^2) = \left(1 - \frac{m_l^2}{q^2}\right)^2 \left(1 + \frac{m_\ell^2}{2q^2}\right) \quad \text{for } l = e, \mu, \tau$$

$B_s \to D_s\ell\nu$: charm inter-/extrapolation + chiral-continuum extrapolation



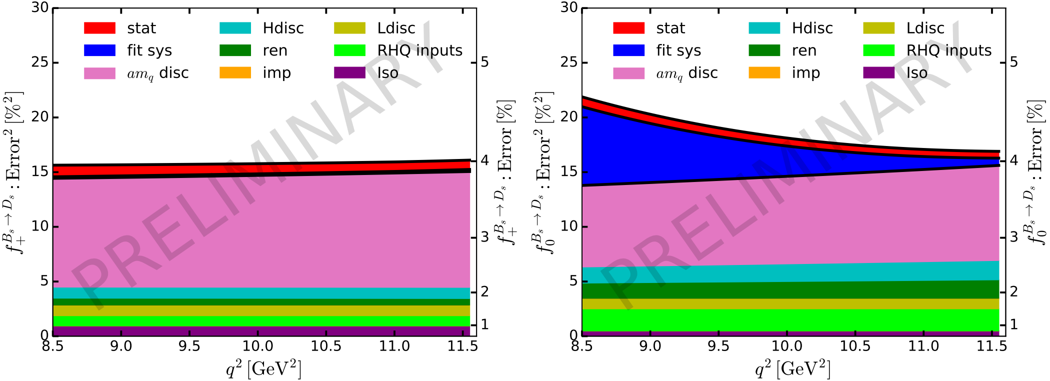


$$\Delta M_{D_s}^{-1} \equiv \left(\frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\text{phys}}} \right)$$

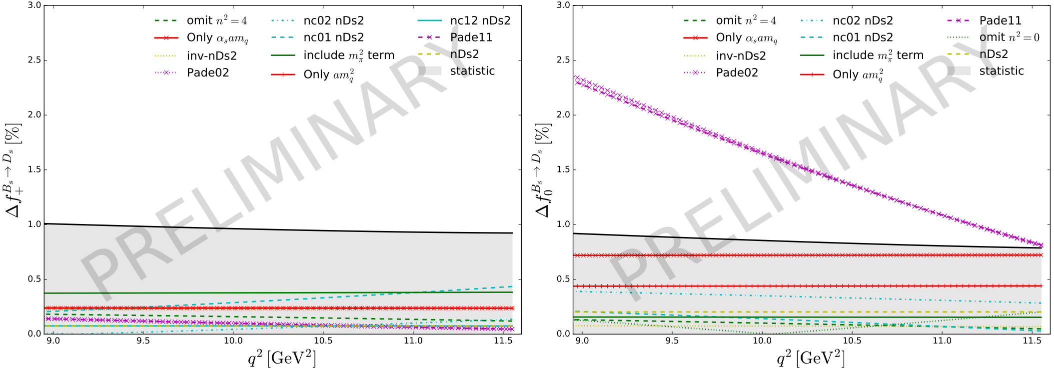
$$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + \sum_{j=1}^{n_{D_s}} c_{1,j} \left[\Lambda \Delta M_{D_s}^{-1}\right]^j + c_2 (a\Lambda)^2 \right] P_{a,b}\left(\frac{q^2}{M_{B_s}^2} \right)$$

$$P_{a,b}(x) = \frac{1 + \sum_{i=1}^{a} a_i x^i}{1 + \sum_{i=1}^{b} b_i x^i}$$

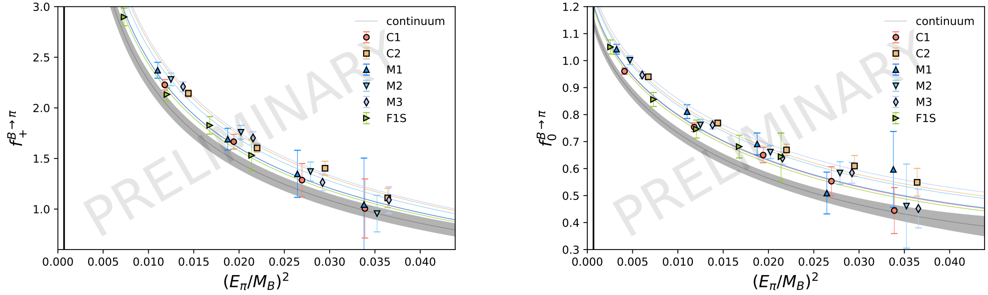
$B_s \to D_s\ell\nu$: error budget



$B_s \to D_s\ell\nu$: error budget

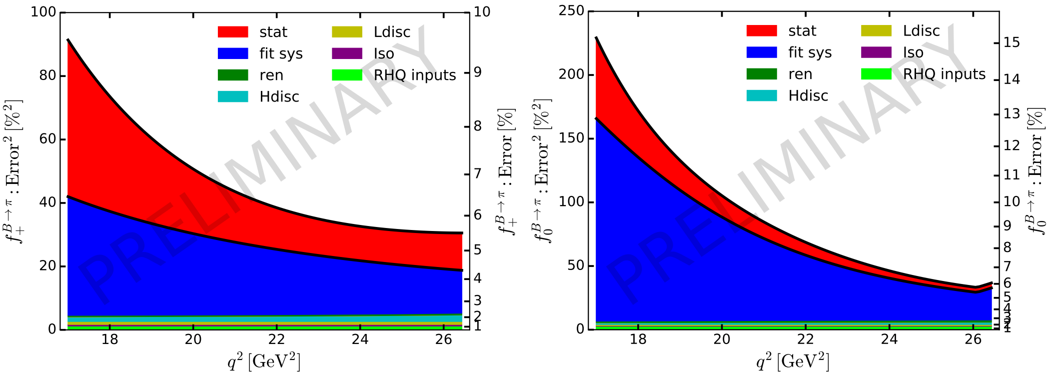


$B \to \pi\ell\nu$: chiral-continuum extrapolation



- Chiral-continuum extrapolation using SU(2) hard-pion χPT
  $\to$ $f_{pole}(M_\pi, E_\pi, a^2) = \frac{c_0\Lambda}{E_\pi+\Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1\frac{M_\pi^2}{\Lambda^2} + c_2\frac{E_\pi}{\Lambda} + c_3\frac{E_\pi^2}{\Lambda^2} + c_4(a\Lambda)^2\right]$
  $\to$ δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_\pi/E_\pi \to 0$
- Substantially reduced statistical errors compared to [Flynn et al. PRD 91 (2015) 074510]

$B \to \pi\ell\nu$: error budget



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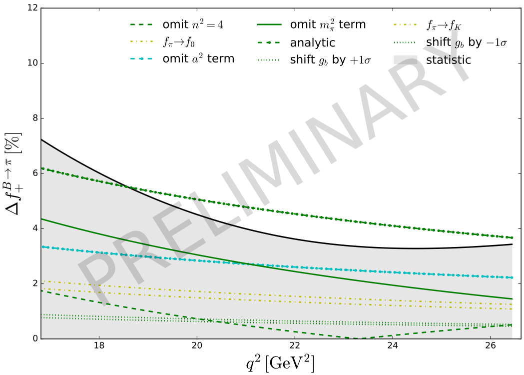

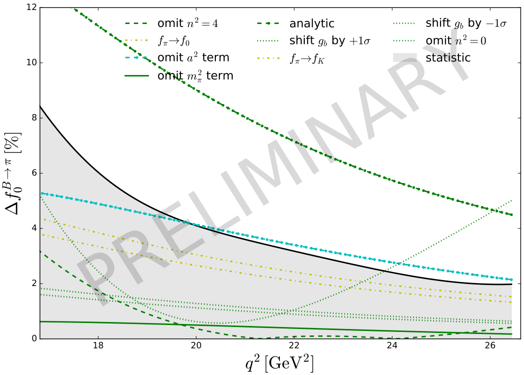

Summary

- Publication on $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ under preparation
- Update on $B \to \pi\ell\nu$ next
- Subsequently $B \to D\ell\nu$, vector final states, rare decays, B_c decays, ...