EPS-HEP Conference 2021

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Measurement of the muon precession frequency in magnetic field at Fermilab

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Introduction

muon magnetic anomaly a_μ

- $\vec{\mu}_\mu = g_\mu \frac{q}{2m}\vec{L}, \quad a_\mu = \frac{g_\mu - 2}{2}$ $\begin{cases} \vec{\mu}_\mu & \text{muon magnetic momentum} \\ g_\mu & \text{muon gyromagnetic ratio} \\ a_\mu & \text{muon anomaly or anomalous magnetic moment} \end{cases}$
- a_μ measured to 0.46 ppm at FNAL in April 2021
  EPS-HEP presentation A. Keshavarzi, 28 Jul 14:45
- $a_\mu = \frac{\omega_a}{\omega_p} \cdot C$ $\begin{cases} \omega_a & \text{muon anomalous precession frequency in magnetic field} \\ \omega_p & \text{proton spin precession frequency in same magnetic field as } \omega_a \\ C & \text{precision metrology measurements} \end{cases}$

muon anomalous precession frequency ω_a

- this presentation
- April 2021 paper: Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ Experiment, doi:10.1103/PhysRevD.103.072002]

Anomalous muon precession frequency in uniform magnetic field, ω_a

ω_a = angular velocity of muon spin vs. momentum

$$\omega_s \quad - \quad \omega_c \quad = \quad \omega_a$$

$$\boxed{-g_\mu \frac{eB}{2m_\mu} - (1-\gamma)\frac{eB}{m_\mu\gamma}} - \boxed{-\frac{eB}{m_\mu\gamma}} = \boxed{-a_\mu \frac{eB}{m_\mu}}$$

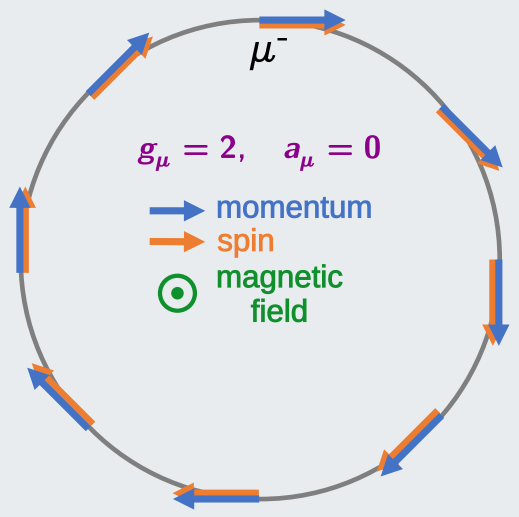
Larmor + Thomas spin precessions

cyclotron frequency

no γ!

- transverse motion in uniform magnetic field

polarized muons in magnetic storage ring



Anomalous muon precession frequency in uniform magnetic field, ω_a

ω_a = angular velocity of muon spin vs. momentum

$$\begin{array}{ccccc}
\omega_s & - & \omega_c & = & \omega_a \\
\boxed{-g_\mu \frac{eB}{2m_\mu} - (1-\gamma)\frac{eB}{m_\mu\gamma}} & - & \boxed{-\frac{eB}{m_\mu\gamma}} & = & \boxed{-a_\mu \frac{eB}{m_\mu}}
\end{array}$$

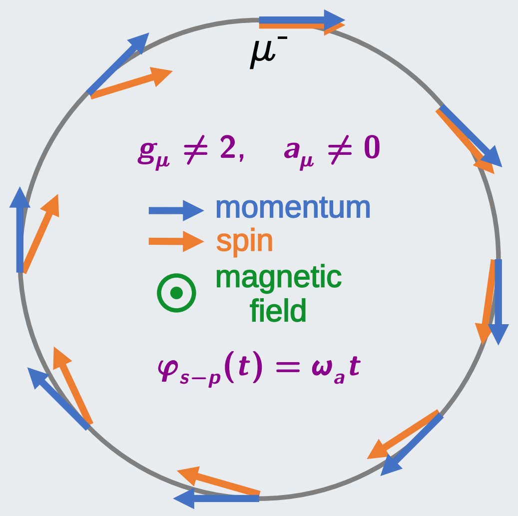
Larmor + Thomas spin precessions

cyclotron frequency

no γ!

- transverse motion in uniform magnetic field

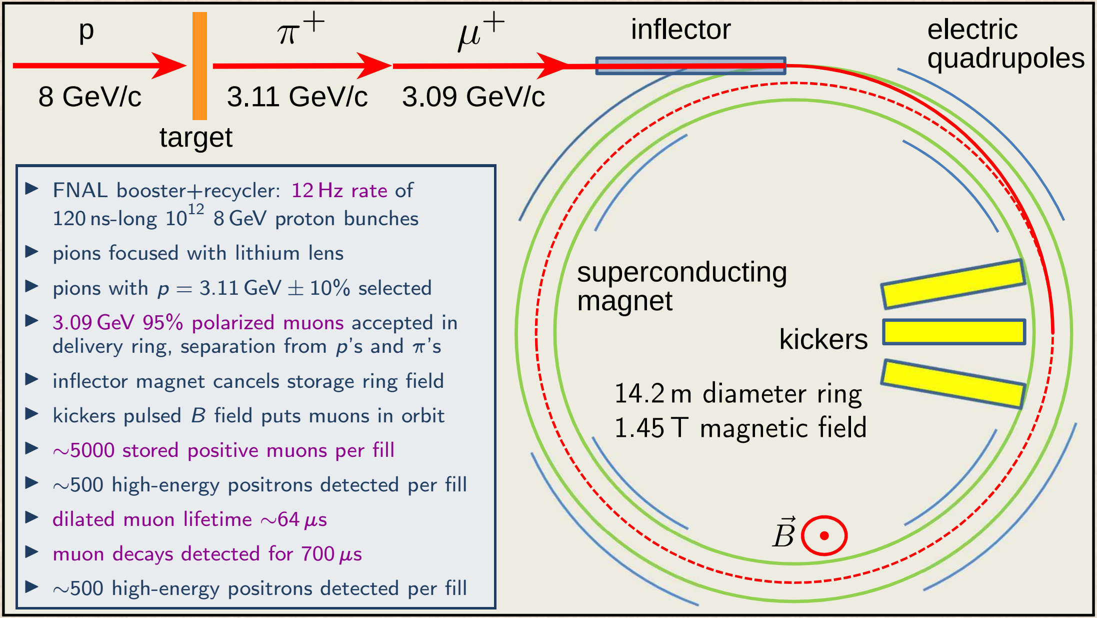
polarized muons in magnetic storage ring



$$g_\mu \neq 2, \quad a_\mu \neq 0$$

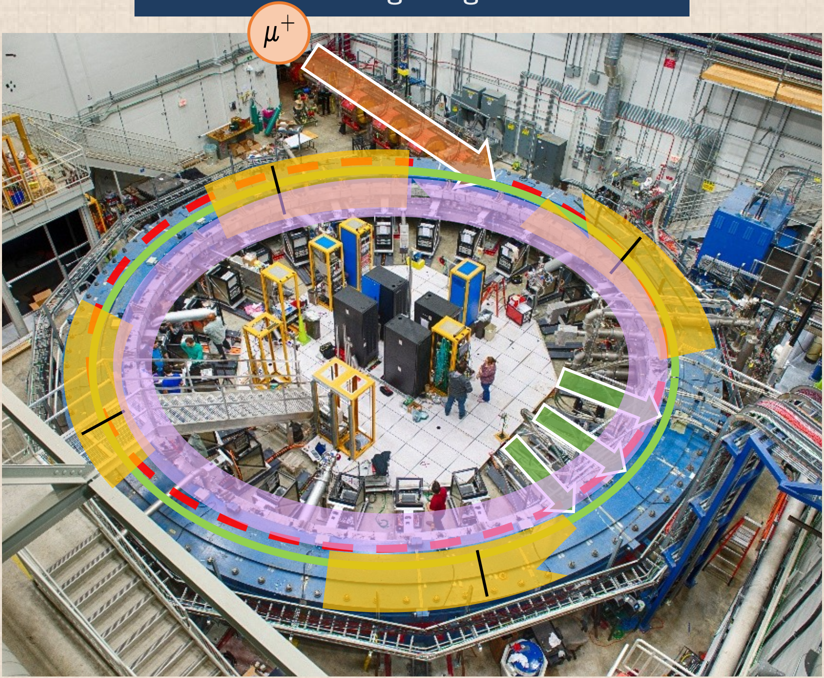
$$\varphi_{s-p}(t) = \omega_a t$$

Positive muon production, storage and decay at FNAL

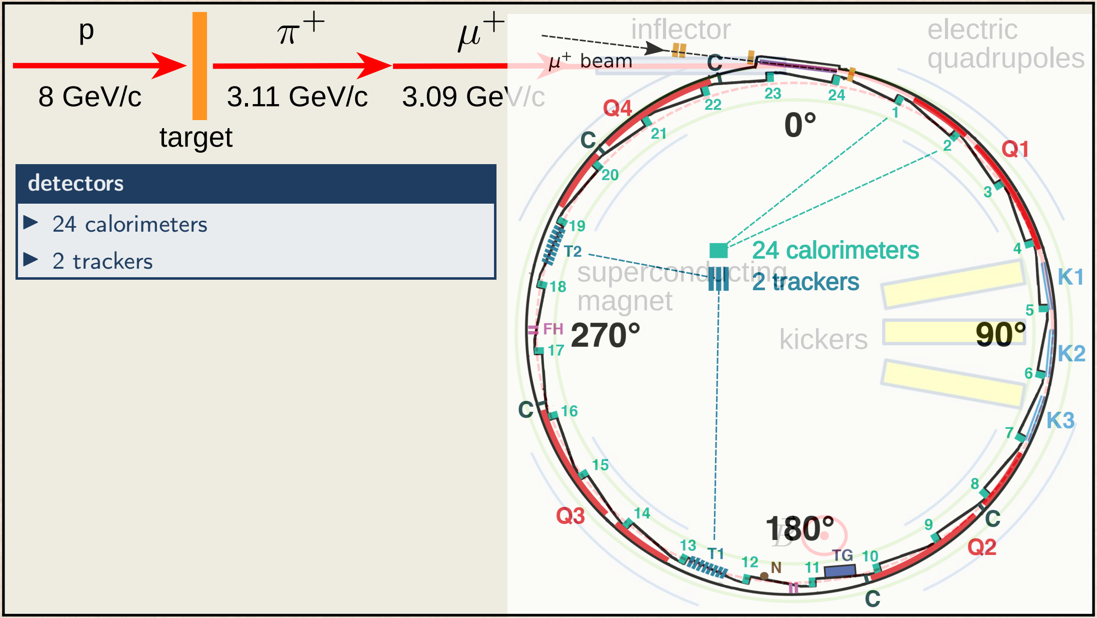


- FNAL booster+recycler: 12 Hz rate of 120 ns-long 10^{12} 8 GeV proton bunches
- pions focused with lithium lens
- pions with $p = 3.11\,\text{GeV} \pm 10\%$ selected
- 3.09 GeV 95% polarized muons accepted in delivery ring, separation from p's and π's
- inflector magnet cancels storage ring field
- kickers pulsed B field puts muons in orbit
- ∼5000 stored positive muons per fill
- ∼500 high-energy positrons detected per fill
- dilated muon lifetime ∼64 μs
- muon decays detected for 700 μs
- ∼500 high-energy positrons detected per fill

FNAL-E989 storage ring and detectors



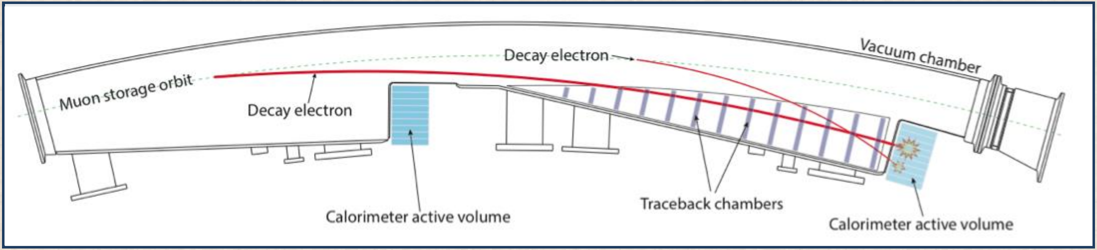
Positrons from positive muon decays are detected inside the storage ring



detectors

- 24 calorimeters
- 2 trackers

Calorimeter and tracker modules



calorimeter

- 6×9 PbF_2 crystals per module
- Cherenkov light collected ⇒ fast
- SiPM readout
- EM shower energy resolution 3% at 3 GeV
- laser-based gain monitoring
- measures positrons energy
- detects lost muons, identified with coincidences

tracker

- 8 modules × 4 layers × 32 straw chambers
- $\pm 7.5^\circ$ stereo angle w.r.t. vertical
- track positrons to measure beam profile
- also track lost muons

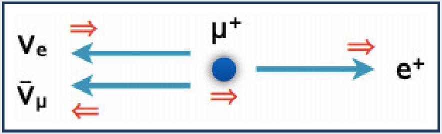
Run 1 data datasets, collected in 2018

Dataset	# Days (Apr-Jun 2018)	Tune (n)	Kicker (kV)	# fills $[10^4]$	# positrons $[10^9]$
1a	3	0.108	130	151	0.92
1b	7	0.120	137	196	1.28
1c	9	0.120	132	333	1.98
1d	24	0.107	125	733	4.00

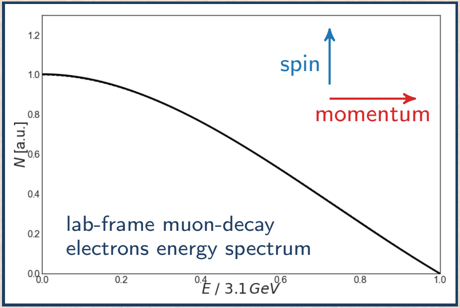
- four datasets, each with approximately stable beam conditions
- total of 8.2 billion positrons collected since 30 μs after injection and with $E > 1.7$ GeV corresponds to $\sim$1.2× the BNL statistics, 6% of the FNAL goal of 21×BNL
- $\omega_a \propto \omega_p$, but magnetic field actively stabilized to $\sim$1 ppm in whole Run 1

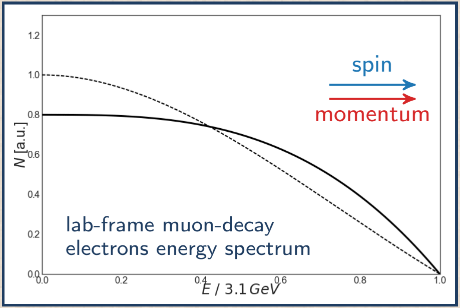
Measurement of angular velocity of muon spin w.r.t momentum

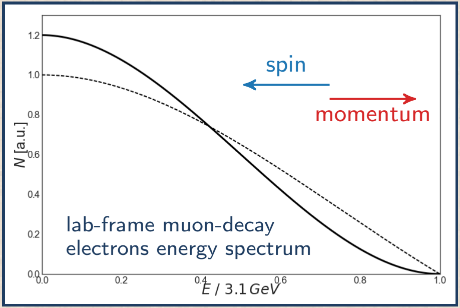
- parity violation in positive muon+ decay ⇒
  ⇒ decay positrons peak along muon spin



- positrons decaying along muon momentum have highest energy in laboratory frame
- $\boxed{N(E>E_{\text{thr}}, t) = N_0 e^{-t/\tau_\mu}(1 + A\cos\omega_a t)}$







Reconstruction of positron energy deposits in calorimeters

readout

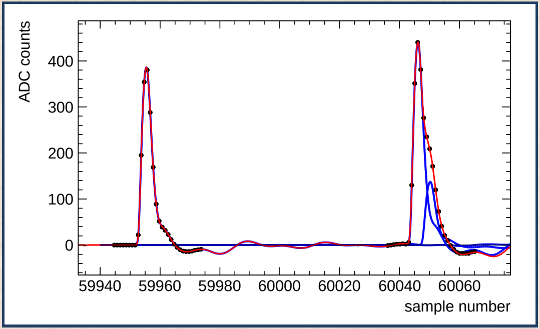
- record SiPM ADC counts for deposits $> 50\,\text{MeV}$

fit samples using crystal pulses templates

- distinct template for each crystal from data
- fit one or more superposed templates

two reconstruction algorithms

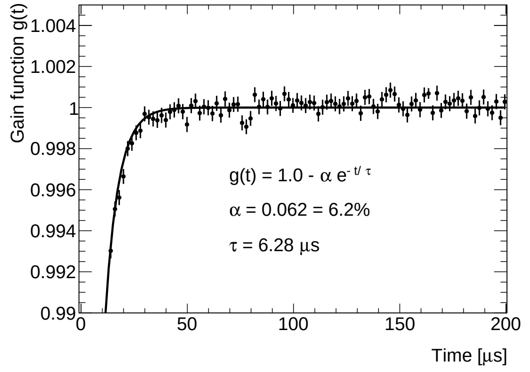
- local: fit individual crystals
- global: global fit over multiple crystals



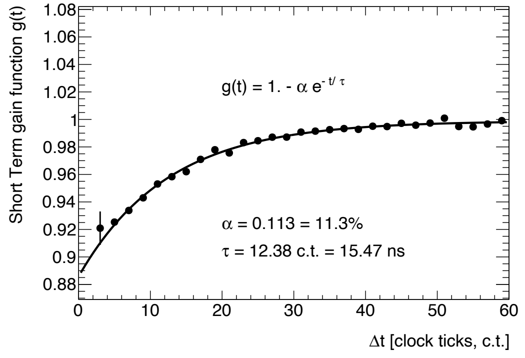
Calorimeter gain variation, measured with laser pulses and corrected

- SiPM gain is reduced by occurrence of preceding hits
- gain monitored by reading back reference laser light pulses injected in PbF_2 crystals
- positron energy measurement from SiPM readout corrected for average measured gain loss

μs time scale SiPM power supply recovery time



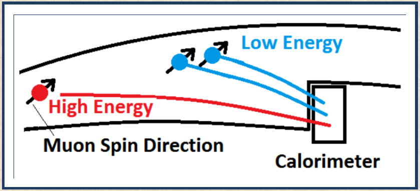
ns time scale SiPM pixel recovery time

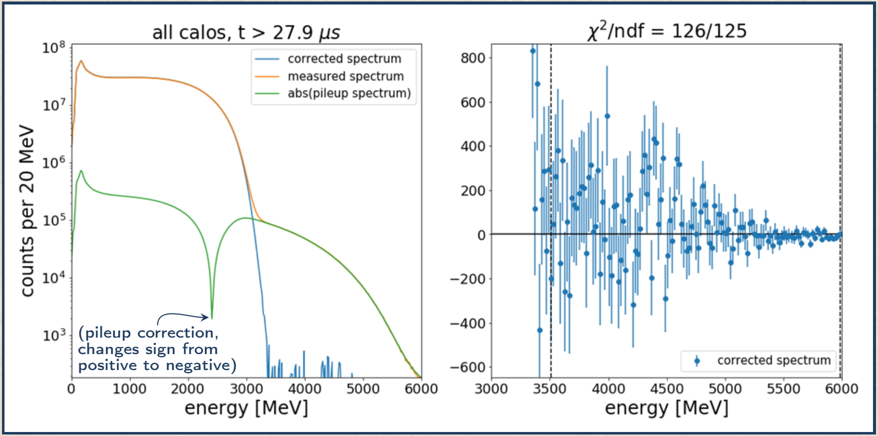


Pileup, statistically subtracted

3 methods of pileup estimation

- combine hit with 2nd hit in shadow window
  - (1) sum 2 hits using model to get E, t
  - (2) reconstruct pileup hit using all crystals ADC counts
- (3) convolve hit density $\rho(E, t)$, then use estimated pileup and iteratively solve for the pileup-subtracted $\rho'(E, t)$

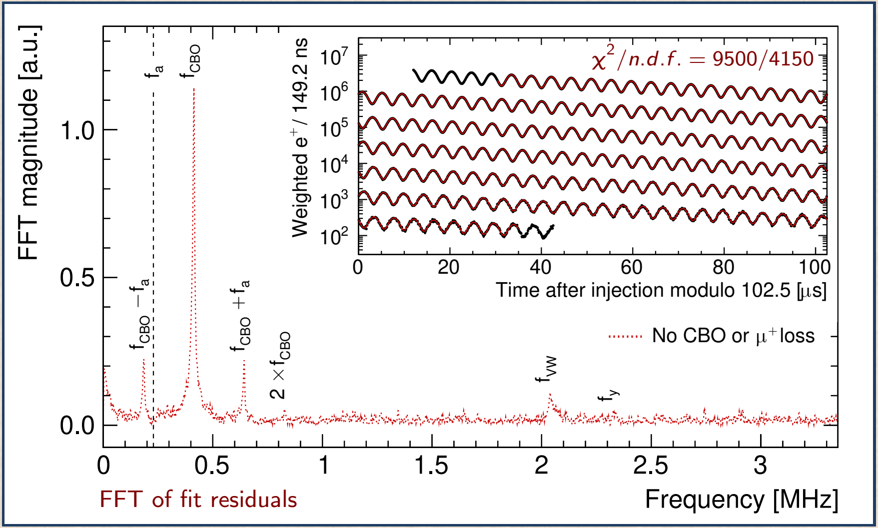




(pileup correction, changes sign from positive to negative)

Muon precession, 5 parameters ω_a fit model

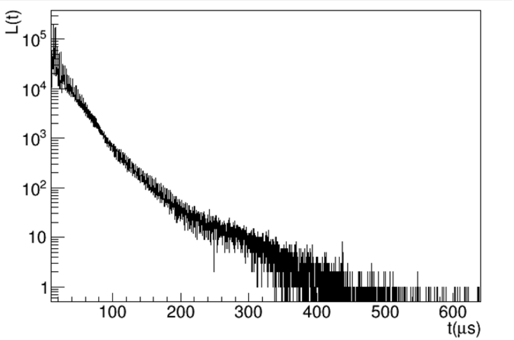
- number of positron decays with $E > \sim 1.7\,\text{GeV}$, binned over time, from 30 to 650 μs
- fit with $\boxed{N(t) = N_0 e^{-t/\tau_\mu}\,[1 + A\cos(\omega_a t + \varphi)]}$



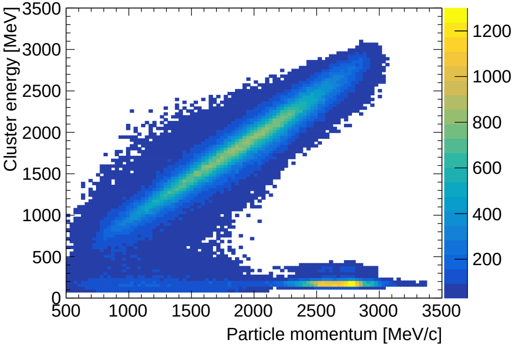
Extend ω_a fit model to account for lost muons on collimators

- some muons hit collimators and are lost
- muon loss rate during a fill measured with 3-4-5 coincidences of m.i.p. on calorimeters
- overall normalization of muon loss included as fit parameter

muon loss vs. time



energy in calorimeter vs. momentum in tracker



Extend ω_a fit model to account for beam dynamics

- transverse beam center and spread (waist) oscillate (betatron oscillations)
- beam oscillations modulate recorded positrons $N(t)$ because detector acceptance varies
- $N(t)$ effectively modulated by beatings of beam oscillations and cyclotron and anomaly frequencies
- operational issues in Run 1 make beame oscillations frequencies varying during fills

beam synamics with field index $n = 0.12$			f [MHz]	T [μs]
Anomalous precession	f_a		0.2291	4.3649
Cyclotron	f_c		6.7024	0.1492
Horizontal betatron	f_x	$= f_c\sqrt{1-n}$	6.2874	0.1590
Vertical betatron	f_y	$= f_c \cdot \sqrt{n}$	2.3218	0.4307
Coherent betatron oscillation	f_{CBO}	$= f_c - 1 \cdot fx$	0.4150	2.4097
Vertical oscillation	f_{VO}	$= f_c - 1 \cdot fy$	4.3806	0.2283
Vertical waist	f_{VW}	$= f_c - 2 \cdot fy$	2.0589	0.4857

Muon precession, 22-parameters ω_a fit

$$N(t) = N_0\, e^{-\frac{t}{\gamma\tau}} \left(1 + A \cdot A_{BO}(t) \cos(\omega_a t + \varphi + \varphi_{BO}(t))\right) \cdot N_{\text{CBO}}(t) \cdot N_{\text{VW}}(t) \cdot N_y(t) \cdot N_{\text{2CBO}}(t) \cdot \Lambda(t)$$

$$A_{\text{BO}}(t) = 1 + A_A \cos(\omega_{\text{CBO}}(t) \cdot t + \varphi_A) e^{-\frac{t}{\tau_{\text{CBO}}}}$$

$$\varphi_{\text{BO}}(t) = A_\varphi \cos(\omega_{\text{CBO}}(t) \cdot t + \varphi_\varphi) e^{-\frac{t}{\tau_{\text{CBO}}}}$$

$$N_{\text{CBO}}(t) = 1 + A_{\text{CBO}} \cos(\omega_{\text{CBO}}(t) \cdot t + \varphi_{\text{CBO}}) e^{-\frac{t}{\tau_{\text{CBO}}}}$$

$$N_{\text{2CBO}}(t) = 1 + A_{\text{2CBO}} \cos(2\omega_{\text{CBO}}(t) \cdot t + \varphi_{\text{2CBO}}) e^{-\frac{t}{2\tau_{\text{CBO}}}}$$

$$N_{\text{VW}}(t) = 1 + A_{\text{VW}} \cos(\omega_{\text{VW}}(t) \cdot t + \varphi_{\text{VW}}) e^{-\frac{t}{\tau_{\text{VW}}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) \cdot t + \varphi_y) e^{-\frac{t}{\tau_y}}$$

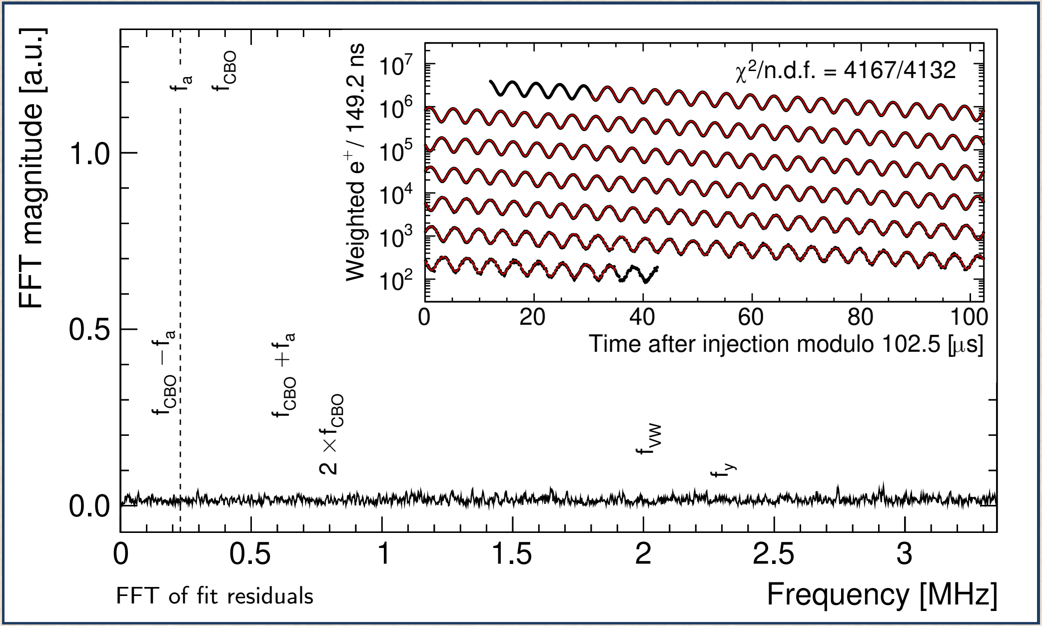
$$\Lambda(t) = 1 - k_{LM} \int_{t_0}^{t} L(t') e^{t'/\tau} dt'$$

$$\omega_{\text{CBO}}(t) = \omega_0^{CBO} + \frac{A}{t} e^{-\frac{t}{\tau_A}} + \frac{B}{t} e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F\omega_{\text{CBO}}(t) \sqrt{2\omega_c / F\omega_{\text{CBO}}(t) - 1}$$

$$\omega_{\text{VW}}(t) = \omega_c - 2\omega_y(t)$$

22 parameters ω_a fit has χ^2/n.d.o.f. consistent with 1



6 analysis groups, 4 analysis methods, 11 ω_a fits × 4 datasets

4 analysis methods

T (threshold): $\sum N(E)$	$E > 1.7\,\text{GeV}$
R (ratio): (see below)	$E > 1.7\,\text{GeV}$
A (asymmetry): $\sum A(E) \cdot N(E)$	$E > 1.0\,\text{GeV}$
Q (charge): $\sum$ energy deposits	no threshold

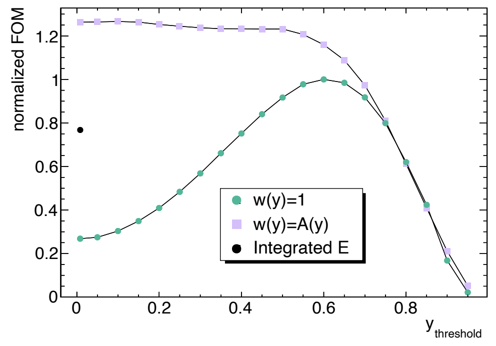
ratio method

- randomly split positron dataset in 4 subsets
- $$R(t) = \frac{n_1(t + \frac{T_a}{2}) + n_2(t - \frac{T_a}{2}) - n_3(t) - n_4(t)}{n_1(t + \frac{T_a}{2}) + n_2(t - \frac{T_a}{2}) + n_3(t) + n_4(t)} = A\cos(\omega_a t + \varphi)$$
- removed exponential decay envelope

two levels of blind analysis

- relative blindings between any ω_a analysis groups
- global blinding of the a_μ result

FOM vs. $y = E/E_{\max}$ threshold for T/R, A, Q



Run 1 $R(\omega_a)$ values and uncertainties (11 systematics categories)

Run 1a

R Run1a [ppb]	CA	EA	SA	WA	BT	CT	ET	ST	WT	BR	KQ
val	-28228.822	-28481.292	-28422.823	-28637.307	-28802.286	-28211.122	-28884.793	-28739.823	-28619.947	-28966.786	-29206.209
unc	1208.499	1201.164	1211.680	1221.856	1360.841	1338.227	1334.718	1335.002	1332.590	1361.011	2068.321
stat	1207.920	1193.750	1206.100	1218.350	1358.170	1337.677	1332.700	1331.350	1330.790	1359.810	2058.500
syst	37.401	133.251	116.155	92.493	85.226	38.373	73.370	98.683	69.237	57.160	201.322
Time randomization seed	6.536	26.020	27.000	16.000	20.200	6.711	31.370	22.500	16.000	22.500	0.000
Time correction	6.726	0.000	0.000	0.000	0.000	5.310	0.000	0.000	0.000	0.000	0.000
Cluster time assignment	0.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000	0.000
In-fill gain amplitude	4.411	21.431	1.900	14.300	7.790	3.905	23.620	2.500	15.300	2.718	5.000
In-fill gain time constant	5.612	0.000	1.000	0.000	20.222	5.532	0.000	2.300	0.000	11.689	8.000
STDP gain amplitude	0.177	0.099	0.000	2.600	0.091	0.005	0.103	0.000	0.800	0.053	0.000
STDP gain time constant	0.734	0.000	0.000	0.000	0.000	0.145	0.000	0.000	0.000	0.000	0.000
Pileup covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.000
Pileup amplitude	0.744	16.211	14.000	13.800	21.700	0.905	17.245	19.800	16.200	19.900	0.000
Pileup cluster time model	0.000	58.921	47.000	0.000	5.100	0.000	12.502	8.500	0.000	6.400	0.000
Pileup cluster energy model	0.000	7.005	11.000	0.000	11.000	0.000	11.658	12.000	0.000	10.900	0.000
Pileup phase	0.000	0.000	0.000	39.600	0.000	0.000	0.000	0.000	5.500	0.000	0.000
Pileup time/energy bias	0.150	0.000	0.000	0.000	0.000	0.027	0.000	0.000	0.000	0.000	0.000
Pileup rate error	0.033	0.000	0.000	0.000	0.000	0.193	0.000	0.000	0.000	0.000	0.000
Unseen pileup	0.946	1.100	0.300	10.000	0.800	3.413	5.300	5.400	10.000	0.600	0.000
Triple pileup correction	0.000	4.600	4.400	1.000	1.900	0.000	3.300	4.200	1.000	1.600	0.000
Pileup simulation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.000
Pileup artificial dead time	0.000	67.700	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss time cuts	0.808	0.000	1.000	0.000	0.300	0.876	0.000	1.000	0.000	0.300	0.000
Loss energy cuts	0.000	0.000	0.500	0.000	0.500	0.000	0.000	0.500	0.000	0.300	0.000
Loss statistics	1.596	0.000	0.000	0.000	0.000	1.522	0.000	0.000	0.000	0.000	0.000
Loss detection efficiency	1.075	0.000	0.000	0.000	0.000	1.952	0.000	0.000	0.000	0.000	0.000
Fixed loss scale	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.900	0.000
Higher-order coincidences	0.000	0.700	0.500	0.000	0.000	0.000	0.700	0.500	0.000	0.000	17.000
CBO frequency change	5.638	11.600	7.100	19.000	10.900	4.299	9.800	5.600	17.000	5.300	5.500
CBO decoherence envelope	24.046	39.700	19.700	26.000	38.300	23.226	22.000	17.500	26.500	5.500	4.100
CBO time constants	4.145	14.200	2.000	11.000	10.800	8.079	2.700	2.000	13.000	10.800	6.000
Fixed CBO time constant	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Vertical drift	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	198.000
Muon precession period	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.300	0.000
Muon lifetime	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ad hoc correction	5.593	5.000	77.700	27.000	28.700	0.445	12.000	78.000	4.000	22.900	0.000

Run 1b

R Run1b [ppb]	CA	EA	SA	WA	BT	CT	ET	ST	WT	BR	KQ
val	-26946.590	-27021.291	-27091.047	-26965.693	-27044.193	-27209.341	-27080.591	-27001.927	-27004.863	-27209.293	-24946.396
unc	1025.180	1018.880	1025.971	1035.843	1157.710	1135.268	1121.992	1129.788	1129.631	1158.484	1759.574
stat	1023.270	1012.020	1022.280	1030.180	1156.130	1133.584	1120.300	1128.100	1127.740	1157.420	1747.800
syst	62.548	118.034	86.946	108.166	60.455	61.811	61.587	61.745	65.338	49.651	203.212
Time randomization seed	5.484	23.600	17.100	13.000	19.100	5.663	26.300	19.500	14.000	20.100	0.000
Time correction	1.473	0.000	0.000	0.000	0.000	1.024	0.000	0.000	0.000	0.000	0.000
Cluster time assignment	0.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000	0.000
In-fill gain amplitude	2.373	6.475	6.400	20.900	3.282	3.857	6.662	3.500	22.600	0.529	4.000
In-fill gain time constant	3.668	0.000	2.300	0.000	9.776	10.372	0.000	7.700	0.000	3.231	2.000
STDP gain amplitude	0.075	0.089	0.000	0.400	0.042	0.009	0.068	0.000	1.400	0.042	0.000
STDP gain time constant	0.443	0.000	0.000	0.000	0.000	0.343	0.000	0.000	0.000	0.000	0.000
Pileup covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.000
Pileup amplitude	0.489	14.033	12.000	12.500	11.400	0.818	6.644	10.600	7.000	11.400	0.000
Pileup cluster time model	0.000	64.159	53.000	0.000	4.600	0.000	14.223	7.200	0.000	4.800	0.000
Pileup cluster energy model	0.000	8.041	8.000	0.000	4.800	0.000	11.189	11.000	0.000	7.200	0.000
Pileup phase	0.000	0.000	0.000	42.500	0.000	0.000	0.000	0.000	6.000	0.000	0.000
Pileup time/energy bias	0.162	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000
Pileup rate error	0.017	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000
Unseen pileup	2.912	2.900	1.600	10.000	1.300	1.764	0.500	8.100	10.000	0.100	0.000
Triple pileup correction	0.000	4.959	4.800	1.000	1.300	0.000	1.476	2.900	1.000	1.200	0.000
Pileup simulation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.000
Pileup artificial dead time	0.000	60.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss time cuts	1.490	0.000	1.000	0.000	0.100	1.451	0.000	1.000	0.000	0.100	0.000
Loss energy cuts	0.000	0.000	0.500	0.000	0.100	0.000	0.000	0.500	0.000	0.100	0.000
Loss statistics	0.770	0.000	0.000	0.000	0.000	0.727	0.000	0.000	0.000	0.000	0.000
Loss detection efficiency	0.618	0.000	0.000	0.000	0.000	0.577	0.000	0.000	0.000	0.000	0.000
Fixed loss scale	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.400	0.000
Higher-order coincidences	0.000	0.600	0.500	0.000	0.000	0.000	1.100	0.500	0.000	0.000	3.000
CBO frequency change	14.216	12.200	12.500	18.000	22.500	12.135	11.200	11.800	15.000	0.700	16.000
CBO decoherence envelope	2.490	7.100	2.800	10.000	3.700	8.044	13.700	2.900	7.000	9.100	1.000
CBO time constants	41.888	2.600	11.000	45.000	23.100	36.225	3.500	9.000	30.000	21.800	8.000
Fixed CBO time constant	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.800	0.000
Vertical drift	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.000
Muon precession period	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.300	0.000
Muon lifetime	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ad hoc correction	19.560	17.000	13.800	37.300	11.800	19.470	33.000	34.000	14.700	15.300	0.000

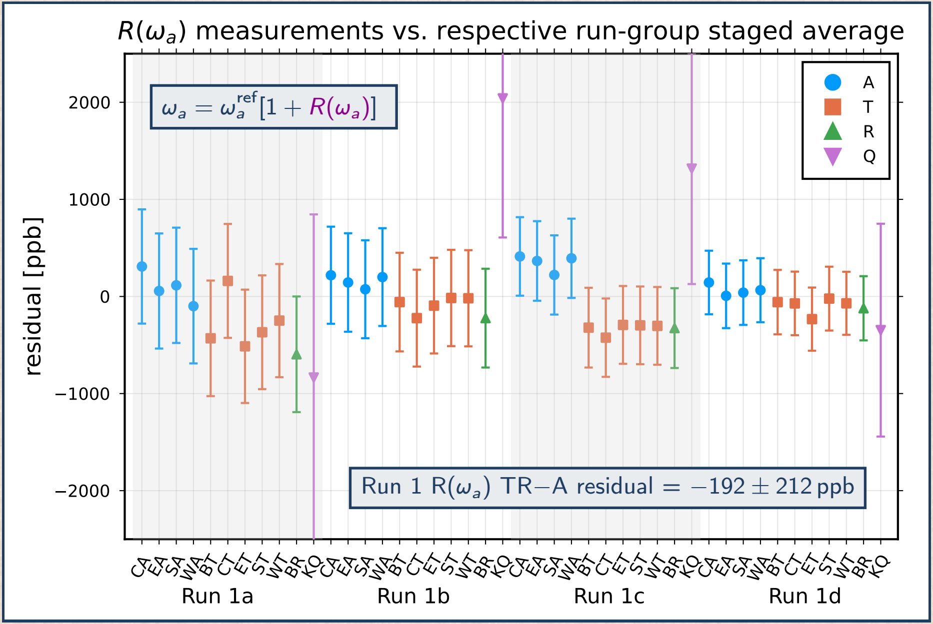
Run 1c

R Run1c [ppb]	CA	EA	SA	WA	BT	CT	ET	ST	WT	BR	KQ
val	-27553.151	-27599.792	-27744.007	-27572.324	-27917.094	-28020.191	-27888.992	-27893.477	-27898.924	-27921.794	-26279.397
unc	824.775	820.274	825.993	834.914	932.719	912.906	909.385	909.687	909.909	934.158	1447.337
stat	823.995	814.609	822.380	830.450	930.105	912.601	906.720	908.410	907.900	932.710	1403.200
syst	35.871	96.237	77.167	86.224	69.778	23.610	69.568	48.189	60.439	51.987	354.703
Time randomization seed	4.512	18.300	13.300	10.000	16.000	4.647	18.900	14.700	11.000	18.100	0.000
Time correction	1.178	0.000	0.000	0.000	0.000	1.142	0.000	0.000	0.000	0.000	0.000
Cluster time assignment	0.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000	0.000
In-fill gain amplitude	2.876	9.086	3.700	13.100	4.529	3.484	9.947	6.000	12.000	1.484	4.000
In-fill gain time constant	4.754	0.000	1.200	0.000	5.394	2.993	0.000	2.500	0.000	0.943	3.000
STDP gain amplitude	0.157	0.217	0.000	0.400	0.213	0.110	0.274	0.000	1.400	0.266	0.000
STDP gain time constant	0.727	0.000	0.000	0.000	0.000	0.570	0.000	0.000	0.000	0.000	0.000
Pileup covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.000
Pileup amplitude	0.370	9.751	8.000	10.000	8.100	0.557	5.263	8.000	6.000	10.100	0.000
Pileup cluster time model	0.000	57.190	44.000	0.000	5.500	0.000	12.330	6.700	0.000	5.600	0.000
Pileup cluster energy model	0.000	6.535	12.300	0.000	6.100	0.000	1.716	11.000	0.000	10.200	0.000
Pileup phase	0.000	0.000	0.000	38.300	0.000	0.000	0.000	0.000	5.300	0.000	0.000
Pileup time/energy bias	0.147	0.000	0.000	0.000	0.000	0.027	0.000	0.000	0.000	0.000	0.000
Pileup rate error	0.119	0.000	0.000	0.000	0.000	0.119	0.000	0.000	0.000	0.000	0.000
Unseen pileup	0.335	1.100	2.800	10.000	2.500	1.446	0.600	5.000	10.000	2.500	0.000
Triple pileup correction	0.000	3.900	3.600	1.000	1.000	0.000	1.360	2.300	1.000	1.100	0.000
Pileup simulation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.000
Pileup artificial dead time	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss time cuts	1.094	0.000	1.000	0.000	0.100	1.173	0.000	1.000	0.000	0.100	0.000
Loss energy cuts	0.000	0.000	0.500	0.000	0.100	0.000	0.000	0.500	0.000	0.100	0.000
Loss statistics	0.728	0.000	0.000	0.000	0.000	0.697	0.000	0.000	0.000	0.000	0.000
Loss detection efficiency	7.602	0.000	0.000	0.000	0.000	7.928	0.000	0.000	0.000	0.000	0.000
Fixed loss scale	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.100	0.000
Higher-order coincidences	0.000	0.700	0.500	0.000	0.000	0.000	0.800	0.500	0.000	0.000	2.000
CBO frequency change	12.971	17.300	13.100	21.000	21.300	11.399	15.000	11.600	18.000	1.600	48.400
CBO decoherence envelope	4.883	15.700	8.000	7.000	13.400	2.044	9.500	6.100	5.500	0.200	14.000
CBO time constants	7.536	15.900	6.000	9.000	23.400	4.965	35.500	12.000	25.000	30.900	28.000
Fixed CBO time constant	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.700	0.000
Vertical drift	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	342.000
Muon precession period	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.300	0.000
Muon lifetime	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ad hoc correction	21.403	17.000	4.000	47.600	24.400	6.620	18.000	7.600	22.400	19.100	0.000

Run 1d

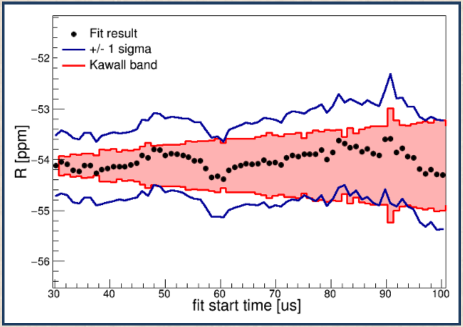
R Run1d [ppb]	CA	EA	SA	WA	BT	CT	ET	ST	WT	BR	KQ
val	-27590.211	-27727.592	-27694.497	-27669.444	-27701.994	-27715.241	-27877.192	-27665.847	-27714.364	-27765.394	-27990.498
unc	677.405	672.330	677.201	682.522	761.733	748.292	744.367	747.218	744.932	759.068	1288.346
stat	675.823	667.947	672.860	679.850	758.400	747.400	743.493	744.140	743.690	757.600	1269.000
syst	46.264	76.641	76.558	60.337	71.176	36.532	36.069	67.757	42.992	47.187	222.428
Time randomization seed	3.632	12.600	11.900	6.000	12.300	3.737	10.900	11.000	7.000	13.700	0.000
Time correction	1.024	0.000	0.000	0.000	0.000	3.218	0.000	0.000	0.000	0.000	0.000
Cluster time assignment	0.000	1.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000	0.000
In-fill gain amplitude	0.042	3.850	5.100	9.800	2.368	0.048	0.643	9.000	14.100	1.032	8.000
In-fill gain time constant	0.694	0.000	3.300	0.000	13.293	0.808	0.000	7.300	0.000	6.585	2.000
STDP gain amplitude	0.122	0.086	0.000	2.600	0.074	0.095	1.200	0.000	0.800	0.095	0.000
STDP gain time constant	0.594	0.000	0.000	0.000	0.000	0.504	0.000	0.000	0.000	0.000	0.000
Pileup covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.000
Pileup amplitude	0.355	8.809	7.000	6.300	10.100	0.348	10.378	5.600	9.500	9.400	0.000
Pileup cluster time model	0.000	47.500	41.000	0.000	5.000	0.000	7.527	5.600	0.000	4.800	0.000
Pileup cluster energy model	0.000	7.000	7.000	0.000	10.000	0.000	8.439	10.000	0.000	6.800	0.000
Pileup phase	0.000	0.000	0.000	34.900	0.000	0.000	0.000	0.000	4.400	0.000	0.000
Pileup time/energy bias	0.101	0.000	0.000	0.000	0.000	0.019	0.000	0.000	0.000	0.000	0.000
Pileup rate error	0.089	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000
Unseen pileup	0.142	0.800	3.500	10.000	4.100	0.232	0.700	0.200	10.000	2.400	0.000
Triple pileup correction	0.000	4.637	3.900	1.000	1.600	0.000	2.330	1.400	1.000	1.300	0.000
Pileup simulation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.000
Pileup artificial dead time	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss covariance matrix	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Loss time cuts	2.035	0.000	1.000	0.000	0.100	2.024	0.000	1.000	0.000	0.100	0.000
Loss energy cuts	0.000	0.000	0.500	0.000	0.100	0.000	0.000	1.000	0.000	0.500	0.000
Loss statistics	0.802	0.000	0.000	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.000
Loss detection efficiency	1.171	0.000	0.000	0.000	0.000	0.861	0.000	0.000	0.000	0.000	0.000
Fixed loss scale	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.000
Higher-order coincidences	0.000	0.400	0.500	0.000	0.000	0.000	1.200	0.500	0.000	0.000	3.000
CBO frequency change	0.634	25.000	13.300	23.000	22.200	0.606	1.400	0.500	21.000	8.500	33.000
CBO decoherence envelope	38.083	5.000	3.200	1.000	25.300	32.052	9.600	9.200	1.500	18.000	38.000
CBO time constants	7.053	0.600	1.000	2.000	10.500	3.316	0.800	8.000	6.000	9.800	3.000
Fixed CBO time constant	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Vertical drift	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	208.000
Muon precession period	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.300	0.000
Muon lifetime	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ad hoc correction	3.544	6.000	38.000	7.000	18.800	0.518	13.200	58.000	12.000	6.700	0.000

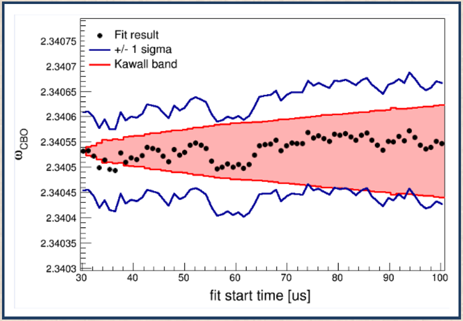
In each of 4 datasets, 11 ω_a measurements are consistent between each-other



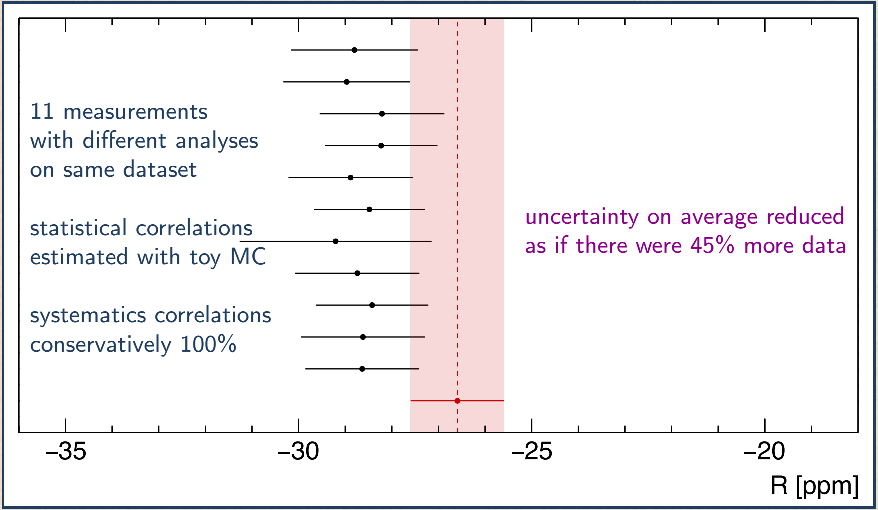
A large number of consistency checks have been completed

- fit results ought to be stable vs. chosen start time
- similar checks check performed vs.
  - calorimeter station
  - bunch number
  - Run number
  - time of day
  - positron energy bin
  - position within calorimeter
  - . . .



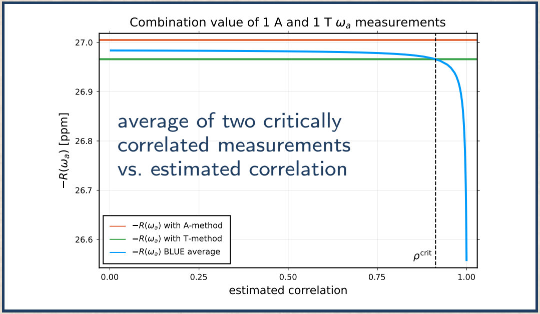


Average of 11 ~critically correlated measurements with imprecise correlation

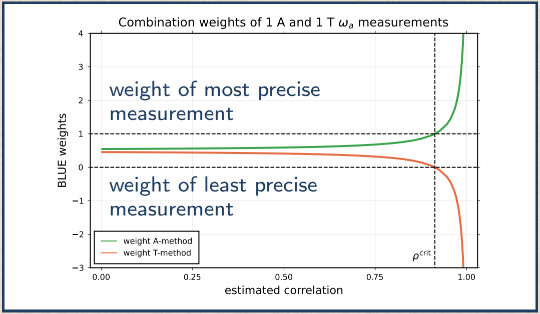


- statistical correlations estimations have limited precision
- systematic correlations estimated conservatively to avoid underestimation
- ⇒ minimum χ^2 combination (=BLUE) not reliable

Critical correlation: $C_{ij}^{\text{crit}} = \rho^{\text{crit}} = \min(\sigma_i, \sigma_j)/\max(\sigma_i, \sigma_j) \quad (i \neq j)$

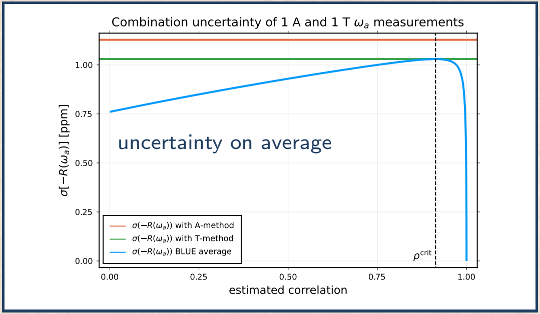

Combination value of 1 A and 1 T ω_a measurements

average of two critically correlated measurements vs. estimated correlation


Combination weights of 1 A and 1 T ω_a measurements

weight of most precise measurement

weight of least precise measurement


Combination uncertainty of 1 A and 1 T ω_a measurements

uncertainty on average

Least χ^2 average of 2 meas. around $\rho = \rho^{\text{crit}}$

- unstable vs. value of estimated correlation ρ
- Glen Cowan, Stat. Data Analysis, sec. 7.6.1
- Valassi & Chierici 2014, EPJC 74 (2014) 2017
- but no literature really appropriate for our case

Average the four most statistically precise A-method analyses

(on each of four datasets)

even weights, disregarding imprecise correlations

- 50% of weight to 1 A-method analysis using one of the two reconstructions
- 50% of weight to 3 A-method analyses using the other reconstruction
  - weight = 1/3 of 50% each
- robust procedure, close to optimal combination

measurements with methods T, R, Q used for checks

- T and R methods are ∼11% less precise than A (Q is even less precise)
- optimal combination T, R, Q weights non negligible only if T and/or R and/or Q measurements had much smaller systematics than A, which is not observed in present estimations

Synthetic ω_a^m FNAL Run 1 measurement

$R(\omega_a) = (\omega_a - \omega_a^{\text{ref}}) \cdot 10^9$	FNAL Run 1 [ppb]	FNAL goal [ppb]	BNL [ppb]
value	-82357		
total uncertainty	437	122	487
statistical uncertainty	434	100	460
systematic uncertainty	56	70	160
- Gain changes	8	20	120
- Pileup	35	40	80
- Coherent Betatron Oscillation	38	30	70
- Time randomization (to remove “fast rotation”)	9		
- Residual slow term	17		
- Other	3		

notes

- the FNAL total systematics goal includes other systematics that are not discussed here

Conclusions

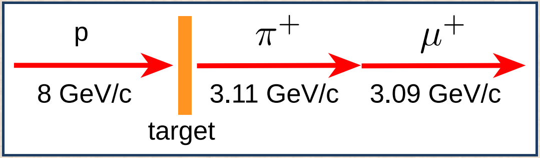
- positive muon anomalous precession frequency ω_a^m measured at FNAL to 0.44 ppm
- with 6% of design goal statistics, ω_a^m systematics quite close to design goal

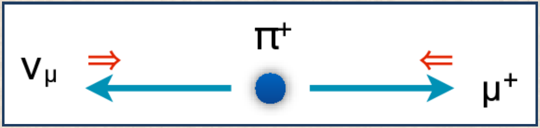
Thanks for your attention!

Backup Slides

Production of polarized muons

- dump 8 GeV protons on target to produce pions
- select pions with momemtum $p \simeq 3.11$ GeV
- let them decay into muons
- in pion rest frame, because of parity violation in pion decay, μ^- spin is aligned with momentum (μ^+ spin is anti-aligned with momentum)
- in laboratory frame, highest energy muons are >90% polarized





- with 8 GeV protons on target, μ^+ are produced ∼4× more frequently than μ^-

Subsamples ω_a^m measurements

$R(\omega_a^m)$	1a	1b	1c	1d
value	-83123	-81738	-82347	-82395
uncentainty	1209	1025	825	676
statistical	1207	1022	823	675
systematic	64	70	54	49

statistical correlation

	1a	1b	1c	1d
1a	1.0000	0.0000	0.0000	0.0000
1b	0.0000	1.0000	0.0000	0.0000
1c	0.0000	0.0000	1.0000	0.0000
1d	0.0000	0.0000	0.0000	1.0000

systematic correlation

	1a	1b	1c	1d
1a	1.0000	0.9935	0.9884	0.9812
1b	0.9935	1.0000	0.9820	0.9936
1c	0.9884	0.9820	1.0000	0.9669
1d	0.9812	0.9936	0.9669	1.0000